Some studies on the recent flavor anomalies

M.Sc. Thesis

By

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Department of Physics
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Some studies on the recent flavor anomalies

A Thesis

Submitted in partial fulfillment of the requirements for the award of the degree

of

Master of Science

By

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Candidate's Declaration

I hereby certify that the work which is being presented in thesis entitled **Some studies on the recent flavor anomalies** in the partial fulfillment of the requirements for the award of the degree of **Master of Science** and submitted in the **Department of Physics, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July 2021 to June 2022 under the supervision of **Dr. Dipankar Das, Indian Institute of Technology Indore.**

The matter presented in thesis has not been submitted by me for the award of any other degree of this or any institute.

अमरेन्द्र
08/06/2022
Signature of student with date
(Amarendra Kumar Verma)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

08/06/22
Signature of the Supervisor of
M.Sc. thesis (with date)
(**Dr. Dipankar Das**)

Amarendra Kumar Verma has successfully given his M.Sc. Oral Examination held on **03 June 2022**.

Signature of supervisor of MSc thesis
Date: 08/06/22

Convener, DPGC
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I am very thankful to my family and friends for their constant love, support, and kindness.

Jibana ta dukha , Sansara ta moh maya

Dedication

I dedicate my work to Covid and my depression which helps me understand things better.
I also dedicate this work to people who remind me that there's hope in this world.

"What one fool can understand, an other can." -Richard Feynman

Abstract

This thesis discuss the rare decays $b \to s\ell^+\ell^-$ which shows significant deviations from Standard Model predictions. Here, we discussed the Standard Model, Flavour Structure, Neutral currents, Charged currents, and possibilities of FCNC at tree level and loop levels. We try to reproduce the Standard Model prediction of R_k values. We try to understand statistical methods to understand data and graphs of experiments of R_k.

The LHCb detector at CERN collected in beauty quarks decays, $R_K - R_{K*}$ measurements show the violation of lepton universality and 3.1σ deviation from standard model prediction. Further experimental runs give us more confidence in the search for the Beyond Standard Model.

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Chapter 1

Introduction

We always desire to find a simple and elegant theory to explain every process in the universe; Einstein's general theory of relativity describes the idea of Gravitation and quantum mechanics, which leads to Quantum Field Theory, then to Standard Model explains strong, weak, and electromagnetic interaction of elementary particles. The Standard Model(SM) develop in the 1960-70s through many experimental observations which explain elementary particles and their dynamics. Higgs boson predicted by SM which was found in 2012, results in near completion of SM theory. Some recent experiments, the Anomalous magnetic dipole moment of the muon, Rare decays experimental results point toward failure of SM, which motivate us to look at physics Beyond the Standard Model. Hierarchy problem and other theoretical concepts also indicate there is physics beyond the standard model.

As for now, we don't have observe a direct indication of BSM, so we can speculate that BSM occurs in a high energy scale that cannot be reached right now with present experimental technology or that BSM has very weak coupling strength. Studies of rare decays open up a window to look for BSM; small effects of BSM could be hidden away in SM in abundant decays, that's why looking for rare decays. Also, in rare decay experiments, we are testing the compatibility of experimental results with standard model predictions, and these rare decay experiments can be performed right now with the present level of technology. If rare decay experiment hints for BSM, we can then invest money and efforts to find a direct search.

In rare decays, in this thesis, we look for $B \to K\ell^+\ell^-$ ($b \to s\ell^+\ell^-$) decays, the LHCb detector at CERN collected in beauty quarks decays , $R_K - R_{K*}$ measurements show the violation of lepton universality and 3.1σ deviation from standard model prediction. Further experimental runs give us more confidence in the search of BSM.

In SM, different leptons have the same interaction strength except for the lepton-Higgs interaction, which gives leptons different masses. In the standard model, strong force doesn't couple with lepton; therefore, $B^+ \to K^+e^+e^-$ and $B^+ \to K^+\mu^+\mu^-$ decays identically, giving R_k nearly equal to 1. Beyond SM predicts new virtual particles which can interact non universally to leptons which can explain experimentally found branching fraction of $B^+ \to K^+\ell^+\ell^-$. Based on data collected in LHCb, CERN there is lepton universality violation in beauty-quark decays.

$$R_H = \frac{Br(B \to H\mu^+\mu^-)}{Br(B \to He^+e^-)} \tag{1.1}$$

For $H = K^+$,ratio denotes R_k and for $H = K^{*0}$ ratio called $R_{k^{*0}}$

Measured R_k values

$$R_k(1.1 < q^2 < 6.0\ GeV^2/c^4) = 0.846^{+0.042+0.013}_{-0.039-0.012}\ [3]$$

SM expectation is $R_k = 1.00 \pm 0.01$. [3]

The discrepancy of 3.1σ with SM [3], gives evidence of a violation of lepton universality.

Chapter 2

Standard Model

The standard model(SM) of Particle Physics is a theory of elementary particles and their behavior. SM consist of 12 elementary particles (Fermions) :- 6 Quarks.

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \quad \text{and} \quad u_{\mathrm{R}}, d_{\mathrm{R}}, c_{\mathrm{R}}, s_{\mathrm{R}}, t_{\mathrm{R}}, b_{\mathrm{R}}$$

and 6 leptons as

$$\begin{pmatrix} e^- \\ v_e \end{pmatrix}_{\mathrm{L}}, \begin{pmatrix} \mu^- \\ v_\mu \end{pmatrix}_{\mathrm{L}}, \begin{pmatrix} \tau^- \\ v_\tau \end{pmatrix}_{\mathrm{L}}, \quad \text{and} \quad \left(e^-\right)_{\mathrm{R}}, \left(\mu^-\right)_{\mathrm{R}}, \left(\tau^-\right)_{\mathrm{R}}$$

Chirality of lepton arises due to weak interaction in SM. SM's interactions/Forces carriers (Boson) are Gluon, Z Boson, W Boson, Photon, and Higgs Boson. Massless photons and gluons ignore the Higgs field, whereas quarks, leptons, and particles interact with the Higgs field; because of this interaction, these particles have mass. SM can accurately anticipate the results of many collider experiments and has predicted the existence of several elementary particles before they were discovered.

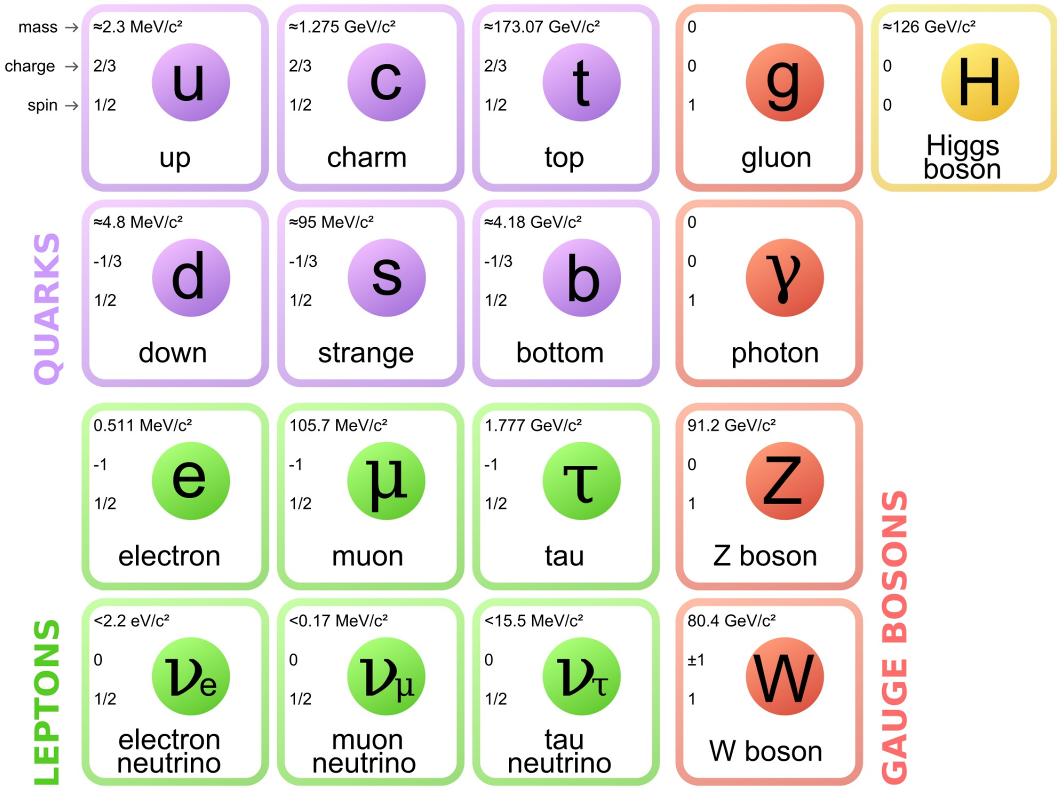


Figure 2.1: Standard Model of Elementary Particles

source:`https://en.wikipedia.org/wiki/Standard_Model`

In the Standard Model, we have to following gauge group [1] :

$$SU(3)_c \times SU(2)_L \times U(1)_Y \,, \tag{2.1}$$

where $SU(3)_c$ is the Quantum chromodynamics or QCD gauge group and $SU(2)_L \times U(1)_Y$ is the electroweak part. After spontaneously symmetry breaking, this symmetry ends up into,

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_Q. \tag{2.2}$$

Here Y and Q denote the weak hypercharge and the electric charge generators, respectively. The QCD part describes the strong interactions of particle physics. Strong interactions are mediated by eight gluons G_a. The $SU(2)_L \times U(1)_Y$ is called the electroweak part of the Standard Model and describes both electromagnetic and weak interactions. These interactions

are mediated by Photon(γ),W Boson($W^{\pm}$), Z Boson(Z_0) and the neutral Higgs boson H.
Quantum number of $SU(3)_c$ called color, $SU(2)_L$ called weak isospin, and $U(1)_Y$ is called hypercharge. The Gell-Mann-Nishima relation gives electric charge Q.

$$Q = T_3 + \frac{Y}{2} \tag{2.3}$$

where T_3 is the third component of weak spin and Y is hypercharge.
Let's look Standard Model Lagrangian, the first kinetic term of fermions

$$\begin{aligned} \mathcal{L}^{SM}_{fermion} &= \sum_{j=1,2,3} \overline{L}_j i \not{D} L_j + \overline{e}_{R_j} i \not{D} e_{R_j} + \overline{Q}_j i \not{D} Q_j \\ &\quad + \overline{u}_{R_j} i \not{D} u_{R_j} + \overline{d}_{R_j} i \not{D} d_{R_j} \end{aligned} \tag{2.4}$$

where $\not{D} = D_\mu \gamma^\mu$.
Covariant derivatives are given as
For LH lepton doublets L_j

$$D_\mu = \partial_\mu - i g_1 Y B_\mu - i g_2 \frac{\sigma^a}{2} W^a_\mu \tag{2.5}$$

For RH lepton singlets e_{R_j}

$$D_\mu = \partial_\mu - i g_1 Y B_\mu \tag{2.6}$$

For LH quarks doublets Q_j

$$D_\mu = \partial_\mu - i g_1 Y B_\mu - i g_2 \frac{\sigma^a}{2} W^a_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu \tag{2.7}$$

For RH quarks singlets u_{R_j}, d_{R_j}

$$D_\mu = \partial_\mu - i g_1 Y B_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu \tag{2.8}$$

here, Y denotes hypercharge, Pauli matrices denoted by σ^a and Gell-Mann matrices by λ^a.

Kinetic term of gauge boson is as follows :

$$\mathcal{L}^{SM}_{gauge} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.9}$$

where,

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu, \tag{2.10}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc}W^b_\mu W^c_\nu, \tag{2.11}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.12}$$

Mass terms of both fermions and gauge bosons are not allowed in SM. To resolve this issue, we turn to the Higgs mechanism. Therefore, Higgs doublets Φ is also part of Lagrangian :

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \tag{2.13}$$

Yukawa sector of SM describes the coupling of Higgs doublets to fermions. It determines the flavor structure of SM. Lagrangian of Yukawa part is given by:

$$\mathcal{L}_Y = -\overline{Q}\Phi Y^D d_R - \overline{Q}\Phi^c Y^U u_R - \overline{L}\Phi Y^E e_R + h.c. \tag{2.14}$$

where $\Phi^c = i\sigma_2\Phi^*$.

Therefore, Total SM Lagrangian is given by adding eqn. 2.4,2.9,2.13,2.14.

$$
\begin{aligned}
\mathcal{L}^{SM} &= \mathcal{L}^{SM}_{fermion} + \mathcal{L}^{SM}_{gauge} + \mathcal{L}_{\Phi} + \mathcal{L}_{Y} \qquad (2.15)\\
\mathcal{L}^{SM} &= \sum_{j=1,2,3} \overline{L}_j i\not{D} L_j + \overline{e}_{R_j} i\not{D} e_{R_j} + \overline{Q}_j i\not{D} Q_j + \overline{u}_{R_j} i\not{D} u_{R_j} + \overline{d}_{R_j} i\not{D} d_{R_j}\\
&\quad -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\\
&\quad +(D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)\\
&\quad -\overline{Q}\Phi Y^D d_R - \overline{Q}\Phi^c Y^U u_R - \overline{L}\Phi Y^E e_R + h.c. \qquad (2.16)
\end{aligned}
$$

Chapter 3

Flavor structure of SM

3.1 Quark Sector

Yukawa Lagrangian for quarks can be written as [1]

$$\begin{aligned}\mathcal{L}_Y &= -(Y^D_{11}\overline{Q}_{1L}\Phi d_{1R} + Y^D_{12}\overline{Q}_{1L}\Phi d_{2R} + Y^D_{13}\overline{Q}_{1L}\Phi d_{3R} + Y^D_{21}\overline{Q}_{2L}\Phi d_{1R} + Y^D_{22}\overline{Q}_{2L}\Phi d_{2R} \\ &\quad + Y^D_{23}\overline{Q}_{2L}\Phi d_{3R} + Y^D_{31}\overline{Q}_{3L}\Phi d_{1R} + Y^D_{32}\overline{Q}_{3L}\Phi d_{2R} + Y^D_{33}\overline{Q}_{3L}\Phi d_{3R} \\ &\quad + Y^U_{11}\overline{Q}_{1L}\Phi^c u_{1R} + Y^U_{12}\overline{Q}_{1L}\Phi^c u_{2R} + Y^U_{13}\overline{Q}_{1L}\Phi^c u_{3R} + Y^U_{21}\overline{Q}_{2L}\Phi^c u_{1R} + Y^U_{22}\overline{Q}_{2L}\Phi^c u_{2R} \\ &\quad + Y^U_{23}\overline{Q}_{2L}\Phi^c u_{3R} + Y^U_{31}\overline{Q}_{3L}\Phi^c u_{1R} + Y^U_{32}\overline{Q}_{3L}\Phi^c u_{2R} + Y^U_{33}\overline{Q}_{3L}\Phi^c u_{3R} + h.c.) \qquad (3.1)\end{aligned}$$

where Q is $SU(2)_L$ doublet ,

$$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \qquad (3.2)$$

Using spontaneous symmetry breaking and taking Φ to be ;

$$\Phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad (3.3)$$

Quarks mass terms be ;

$$\begin{aligned}\mathcal{L}_{mass} &= -\frac{v}{\sqrt{2}}(Y^D_{11}\overline{d}_{1L}\Phi d_{1R} + Y^D_{12}\overline{d}_{1L}\Phi d_{2R} + Y^D_{13}\overline{d}_{1L}\Phi d_{3R} + Y^D_{21}\overline{d}_{2L}\Phi d_{1R} + Y^D_{22}\overline{d}_{2L}\Phi d_{2R} \\ &+Y^D_{23}\overline{d}_{2L}\Phi d_{3R} + Y^D_{31}\overline{d}_{3L}\Phi d_{1R} + Y^D_{32}\overline{d}_{3L}\Phi d_{2R} + Y^D_{33}\overline{d}_{3L}\Phi d_{3R} \\ &+Y^U_{11}\overline{u}_{1L}\Phi^c u_{1R} + Y^U_{12}\overline{u}_{1L}\Phi^c u_{2R} + Y^U_{13}\overline{u}_{1L}\Phi^c u_{3R} + Y^U_{21}\overline{u}_{2L}\Phi^c u_{1R} + Y^U_{22}\overline{u}_{2L}\Phi^c u_{2R} \\ &+Y^U_{23}\overline{u}_{2L}\Phi^c u_{3R} + Y^U_{31}\overline{u}_{3L}\Phi^c u_{1R} + Y^U_{32}\overline{u}_{3L}\Phi^c u_{2R} + Y^U_{33}\overline{u}_{3L}\Phi^c u_{3R} + h.c.)\end{aligned} \tag{3.4}$$

Above equations can be written in matrice form

$$\begin{aligned}\mathscr{L}_{\text{mass}} = \quad & -\frac{v}{\sqrt{2}}\begin{pmatrix}\bar{d}_{1L} & \bar{d}_{2L} & \bar{d}_{3L}\end{pmatrix}\begin{pmatrix}Y^D_{11} & Y^D_{12} & Y^D_{13}\\ Y^D_{21} & Y^D_{22} & Y^D_{23}\\ Y^D_{31} & Y^D_{32} & Y^D_{33}\end{pmatrix}\begin{pmatrix}d_{1L}\\ d_{2L}\\ d_{3L}\end{pmatrix} \\ & -\frac{v}{\sqrt{2}}\begin{pmatrix}\bar{u}_{1L} & \bar{u}_{2L} & \bar{u}_{3L}\end{pmatrix}\begin{pmatrix}Y^U_{11} & Y^U_{12} & Y^U_{13}\\ Y^U_{21} & Y^U_{22} & Y^U_{23}\\ Y^U_{31} & Y^U_{32} & Y^U_{33}\end{pmatrix}\begin{pmatrix}u_{1L}\\ u_{2L}\\ u_{3L}\end{pmatrix} + h.c.\end{aligned} \tag{3.5}$$

Here,

$$M_d = \frac{v}{\sqrt{2}}\begin{pmatrix}Y^D_{11} & Y^D_{12} & Y^D_{13}\\ Y^D_{21} & Y^D_{22} & Y^D_{23}\\ Y^D_{31} & Y^D_{32} & Y^D_{33}\end{pmatrix}, \quad M_u = \frac{v}{\sqrt{2}}\begin{pmatrix}Y^U_{11} & Y^U_{12} & Y^U_{13}\\ Y^U_{21} & Y^U_{22} & Y^U_{23}\\ Y^U_{31} & Y^U_{32} & Y^U_{33}\end{pmatrix} \tag{3.6}$$

M_d and M_u are mass matrices for down-type quarks and up-types quarks. These matrices are not diagonal, so fields u_i and d_i do not represent physical particles. We have to diagonalize these matrices to get physical fields.

Bi-unitary Transformation : For any matrix A, we can find two unitary matrices U_L and U_R such that $U_L A U_R^{\dagger}$ is diagonal, with real positive entries.
Bi-unitary transformation signifies that the left-chiral and right-chiral fields change differently when unitary transformation. Therefore,

$$m_A = U_L^{\dagger}.m_A^{diag}.U_R \tag{3.7}$$

The left-handed down-type fields transform as

$$\begin{pmatrix} d_{1L} \\ d_{2L} \\ d_{3L} \end{pmatrix} = U_L^{\dagger} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \implies \begin{pmatrix} \bar{d}_{1L} & \bar{d}_{2L} & \bar{d}_{3L} \end{pmatrix} = \begin{pmatrix} \bar{d}_L & \bar{s}_L & \bar{b}_L \end{pmatrix} U_L \tag{3.8}$$

Similarly, for the right-handed down-type field

$$\begin{pmatrix} d_{1R} \\ d_{2R} \\ d_{3R} \end{pmatrix} = U_R^{\dagger} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} . \tag{3.9}$$

And for the left-chiral up-type fields,

$$\begin{pmatrix} u_{1L} \\ u_{2L} \\ u_{3L} \end{pmatrix} = V_L^{\dagger} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \implies \begin{pmatrix} \bar{u}_{1L} & \bar{u}_{2L} & \bar{u}_{3L} \end{pmatrix} = \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} V_L \tag{3.10}$$

Right-chiral field transforms as

$$\begin{pmatrix} u_{1R} \\ u_{2R} \\ u_{3R} \end{pmatrix} = V_R^{\dagger} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} . \tag{3.11}$$

The mass terms for the down-type and up-type quarks can then be rewritten as

$$\begin{aligned}\mathscr{L}_{\text{mass}} = \quad & \begin{pmatrix}\bar{d}_L & \bar{s}_L & \bar{b}_L\end{pmatrix} U_L \cdot M_d \cdot U_R^\dagger \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \\ & \begin{pmatrix}\bar{u}_L & \bar{c}_L & \bar{t}_L\end{pmatrix} V_L \cdot M_u \cdot V_R^\dagger \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + h.c. \end{aligned} \tag{3.12}$$

Here, $D_d = U_L \cdot M_d \cdot U_R^\dagger$ and $D_u = V_L \cdot M_u \cdot V_R^\dagger$ are diagonal matrices.

3.2 Lepton Sector

Yukawa interaction Lagrangian for the lepton sector [1]:

$$\begin{aligned}\mathscr{L}_Y &= -(Y^l_{11}\bar{L}_{1L}\Phi e_{1R} + Y^l_{12}\bar{L}_{1L}\Phi e_{2R} + Y^l_{13}\bar{L}_{1L}\Phi e_{3R} + Y^l_{21}\bar{L}_{2L}\Phi e_{1R} + Y^l_{22}\bar{L}_{2L}\Phi e_{2R} \\ &\quad + Y^l_{23}\bar{L}_{2L}\Phi e_{3R} + Y^l_{31}\bar{L}_{3L}\Phi e_{1R} + Y^l_{32}\bar{L}_{3L}\Phi e_{2R} + Y^l_{33}\bar{L}_{3L}\Phi e_{3R} + h.c), \quad (3.13)\end{aligned}$$

where L is the $SU(2)_L$ doublet of the form

$$L_{iL} = \begin{pmatrix} \nu_{iL} \\ e_L \end{pmatrix}. \quad (3.14)$$

After spontaneous symmetry breaking, using 3.3, the mass terms are

$$\begin{aligned}\mathscr{L}_{\text{mass}} &= -\frac{v}{\sqrt{2}}(Y^l_{11}\bar{e}_{1L}e_{1R} + Y^l_{12}\bar{e}_{1L}e_{2R} + Y^l_{13}\bar{e}_{1L}e_{3R} + Y^l_{21}\bar{e}_{2L}e_{1R} + Y^l_{22}\bar{e}_{2L}e_{2R} \\ &\quad + Y^l_{23}\bar{e}_{2L}e_{3R} + Y^l_{31}\bar{e}_{3L}e_{1R} + Y^l_{32}\bar{e}_{3L}e_{2R} + Y^l_{33}\bar{e}_{3L}e_{3R} + h.c.)\,, \quad (3.15)\end{aligned}$$

$$\mathscr{L} = -\frac{v}{\sqrt{2}}\sum_{i,j=1}^{3}(Y^l_{ij}\bar{e}_{iL}e_{jR} + h.c.)\,. \quad (3.16)$$

In the generation indices Y^l is a matrix. We can redefine our fields in a way such that the matrix Y^l becomes diagonal.

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_L = e_L^\dagger \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}, \quad \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_R = E_R^\dagger \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}. \quad (3.17)$$

Here, e, μ and τ represent physical fields. After changing the basis, the mass terms are written as

$$\mathscr{L}_{mass} = -\frac{v}{\sqrt{2}}\begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} e_L.M_d.e_R^\dagger \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + h.c.\,. \quad (3.18)$$

As we can see, there are no mass terms for neutrinos because right-handed neutrinos do not exist in the Standard Model. There is only one kind of mass term, and those are for the charged leptons as given in 3.18. We could work on the basis of generations where these terms are diagonal.

There was no need to write the lepton fields; we could have started with the physical fields.

3.3 Spontaneous Symmetry Breaking in SM

In SM,Higg field breakdown electroweak to electromagnetic gauge symmetry [1];

$$SU(2)_L \times U(1)_Y \to U(1)_{em} \tag{3.19}$$

To derive mass of M_W and M_Z , we see,

$$\begin{aligned} D_\mu &= \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a \\ &= \partial_\mu - i \begin{pmatrix} g_1 Y B_\mu + \frac{g_2}{2} W_\mu^3 & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & g_1 Y B_\mu - \frac{g_2}{2} W_\mu^3 \end{pmatrix} \end{aligned} . \tag{3.20}$$

where $W_\mu^{1,2}$ replaced by $W_\mu^\pm$

$$W_\mu^- = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2), W_\mu^+ = [W_\mu^-]^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) \tag{3.21}$$

which are eigenvectors of T_3 and Q due to $Y = 0$;

$$T_3 \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \end{pmatrix} = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} -iW_\mu^2 \\ iW_\mu^1 \\ 0 \end{pmatrix} \tag{3.22}$$

Therefore,

$$T_3 W_\mu^\pm = Q W_\mu^\pm = \pm W_\mu^\pm \tag{3.23}$$

which means $W_\mu^\pm$ have electric charge ± 1 , whereas W_μ^3 and B_μ are electrically neutral.

Due to mixing between W_μ^3 and B_μ, they can not be physical mass eigen-

states. Therefore we do an orthogonal transformation with one mass-less state A_μ and a second one massive Z_μ.
Orthogonal transformation characterized by Weinberg mixing angle θ_w given as:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \tag{3.24}$$

or its inverse be;

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \tag{3.25}$$

After inserting back the value of B_μ and W_μ^3; we can get the following relations;

$$\tan\theta_w = \frac{g_1}{g_2}, e = g_2 \sin\theta_w = g_1 \cos\theta_w \tag{3.26}$$

The boson mass can be calculated from;

$$(D_\mu \Phi)^\dagger D^\mu \Phi \stackrel{!}{=} M_W^2 W_\mu^- W^{\mu+} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} m_\gamma A_\mu A^\mu \tag{3.27}$$

$$M_W = \frac{g_2 v}{2}, M_Z = \frac{g_2 v}{2\cos\theta_w}, m_\gamma = 0 \tag{3.28}$$

$$M_W = \frac{g_2 v}{2}, M_Z = \frac{g_z v}{2}, g_z = \frac{g_2}{\cos\theta_w} = \sqrt{g_1^2 + g_2^2}, \frac{M_W}{M_Z} = \cos\theta_w \tag{3.29}$$

3.4 Neutral Currents

To derive couplings of quarks and leptons to photon and z-boson [1] , we need covariant derivatives
For LH lepton doublets L_j

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W^a_\mu \tag{3.30}$$

For RH lepton singlets e_{R_j}

$$D_\mu = \partial_\mu - ig_1 Y B_\mu \tag{3.31}$$

For LH quarks doublets Q_j

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W^a_\mu - ig_s \frac{\lambda^a}{2} G^a_\mu \tag{3.32}$$

For RH quarks singlets u_{R_j}, d_{R_j}

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_s \frac{\lambda^a}{2} G^a_\mu \tag{3.33}$$

we put A_μ and Z_μ in above equations; or we can directly use following equation for doublets with proper value of Y;

$$\begin{aligned} D_\mu &= \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W^a_\mu \\ &= \partial_\mu - i \begin{pmatrix} g_1 Y B_\mu + \frac{g_2}{2} W^3_\mu & \frac{g_2}{\sqrt{2}} W^+_\mu \\ \frac{g_2}{\sqrt{2}} W^-_\mu & g_1 Y B_\mu - \frac{g_2}{2} W^3_\mu \end{pmatrix} \end{aligned} \tag{3.34}$$

For lepton doublets L_j , we get

$$D_\mu = \partial_\mu - ie \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}, \tag{3.35}$$

where,

$$\begin{aligned}
K_{11} &= (Y + \frac{1}{2})A_\mu - (\frac{2Y\tan^2\theta_w - 1}{2\tan\theta_w} Z_\mu) \\
K_{11} &= \frac{1}{2\sin\theta_w\cos\theta_w} Z_\mu, && (3.36) \\
K_{12} &= \frac{1}{\sqrt{2}\sin\theta_w} W_\mu^+, && (3.37) \\
K_{21} &= \frac{1}{\sqrt{2}\sin\theta_w} W_\mu^-, && (3.38) \\
K_{22} &= (Y - \frac{1}{2})A_\mu - (\frac{2Y\tan^2\theta_w + 1}{2\tan\theta_w} Z_\mu) \\
K_{22} &= -A_\mu - \frac{-\sin^2\theta_w + \frac{1}{2}}{\sin\theta_w\cos\theta_w} Z_\mu. && (3.39)
\end{aligned}$$

above , we put the value of $Y = -\frac{1}{2}$ for LH leptons.

For RH lepton singlets e_{R_j},

$$\begin{aligned}
D_\mu &= \partial_\mu - i\frac{e}{\cos\theta_w} Y(\cos\theta_w A_\mu - \sin\theta_w Z_\mu)|_{Y=-1} \\
&= \partial_\mu + ieA_\mu - ie\tan\theta_w Z_\mu && (3.40)
\end{aligned}$$

The couplings to neutral bosons be;

$$\begin{aligned}
\mathcal{L}_I^{SM} &\supset \sum_{j=1,2,3} \overline{L}_j i\not{D} L_j + \overline{e}_{R_j} i\not{D} e_{R_j} \\
&\supset \sum_{j=1,2,3} \Big[\frac{e}{2\sin\theta_w\cos\theta_w} \overline{\nu}_{Lj}\gamma^\mu Z_\mu \nu_{Lj} - e\overline{e}_{Lj}\gamma^\mu A_\mu e_{Lj} \\
&\quad -e\overline{e}_{Rj}\gamma^\mu A_\mu e_{Rj} + \frac{e}{\sin\theta_w\cos\theta_w}(-\frac{1}{2} + \sin^2\theta_w)\overline{e}_{Lj}\gamma^\mu Z_\mu e_{Lj} \\
&\quad + \frac{e}{\sin\theta_w\cos\theta_w}\sin^2\theta_w \overline{e}_{Rj}\gamma^\mu Z_\mu e_{Rj} \Big] \qquad (3.41)
\end{aligned}$$

We can conclude that photons couple similarly with LH and RH particles. We can do the same calculation for quarks too, and we can come to the following conclusion;

$$\mathcal{L}_I^{SM} \supset \frac{e}{\sin\theta_w\cos\theta_w}(T_3 - \sin^2\theta_w Q_f)\overline{f}\gamma^\mu Z_\mu f + eQ_f\overline{f}\gamma^\mu A_\mu f \qquad (3.42)$$

where f denotes fermions (leptons or quarks) with weak isospin T_3 ($T_3 = \frac{1}{2}$ for LH fermions and $T_3 = 0$ for RH fermions) and Q_f is electric charge in unit of electron charge (e). Eq.(3.42) is valid for all three generations; the CKM matrix (we will discuss later) does not occur in photon and Z boson interactions. Due to the unitarity of the CKM matrix , for $V_\mu = Z_\mu$, G_μ or A_μ neutral currents does not change flavor on rotation from flavor state to mass eigenstates:

$$\sum_{j=1,2,3} \overline{d}_{Lj}\gamma_\mu d_{Lj} V^\mu \to \sum_{j,k=1,2,3} \overline{d}_{Lj} \underbrace{(V^\dagger_{CKM} V_{CKM})_{jk}}_{=\delta_{jk}} \gamma_\mu d_{Lk} V^\mu \qquad (3.43)$$

The above result is called Tree-level GIM (Glashow, Iliopoulos, and Maiani) mechanism, and it's a significant result.
Due to the unitarity of the CKM matrix, there are no flavor-changing neutral currents (FCNCs) at the tree level in SM. GIM mechanism gives the

possibilities of FCNCs in loop level but not in tree-level within SM.

3.5 Charged Currents

As discussed in Sec.3.1 , the matrix M_u and M_d are not necessarily be diagonalized by same matrix. Cabibbo gives the mismatch between the LH up-type and down-type quark sector–Kobayashi–Maskawa(CKM) matrix. For charged current interactions. [1];

$$\mathcal{L}^{CC} = \frac{g_2}{\sqrt{2}} \overline{u}_{Lj} \gamma_\mu W^{\mu +} d_{Lj}. \tag{3.44}$$

After field rotation;

$$\begin{aligned} \mathcal{L}^{CC} &= \frac{g_2}{\sqrt{2}} (\overline{u}_L V_L \gamma_\mu W^{\mu +} U_L^\dagger d_L), & (3.45) \\ &= \frac{g_2}{\sqrt{2}} (\overline{u}_L V_L U_L^\dagger \gamma_\mu W^{\mu +} d_L), & (3.46) \\ &= \frac{g_2}{\sqrt{2}} (\overline{u}_L V_{CKM} \gamma_\mu W^{\mu +} d_L) & (3.47) \end{aligned}$$

CKM matrix may also refer to as the quark mixing matrix because the charged current interactions couple any up-type quark to a down-type quark of any generation. The above section shows that the mixing does not appear in neutral currents involving the Z boson. The neutral current interactions do not change quark flavor. There are no flavor-changing interactions in the SM in the lepton sector because we don't have any rotation matrices for leptons.

The CKM matrix is given as

$$V_{\mathrm{CKM}} = V_L \cdot U_L^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{3.48}$$

Chapter 4

Lepton Flavour Universality

SM lagrangian states that leptons don't interact with gluons, which implies leptons don't take part in strong interactions. Electromagnetic and weak interaction also doesn't have different interactions for different generations (flavor) of leptons. In SM, different leptons have the same interaction strength except for the lepton-Higgs interaction, which gives leptons different masses. Lepton Flavour Universality (LFU) is an essential feature of SM.

Couplings of the Z boson, W boson, and photons have been probed directly Large Electron-Positron Collider (LEP) experiments, via precision measurements and the ratios of these partial widths, are in good agreement with unity, which implies these interactions are flavor universal.([21]- [25])

$$\frac{\Gamma_{Z\to\mu^+ + \mu^-}}{\Gamma_{Z\to e^+ + e^-}} = 1.0009 \pm 0.0028; \tag{4.1}$$

$$\frac{\Gamma_{Z\to\mu^+ + \mu^-}}{\Gamma_{Z\to\tau^+ + \tau^-}} = 1.0029 \pm 0.0032; \tag{4.2}$$

Measurements also exist (at LEP and LHC) comparing the W $\pm$ decays:

$$\frac{B(W^- \to e^-\overline{\nu_e})}{B(W^- \to \mu^-\overline{\nu_\mu})} = 1.004 \pm 0.008 \tag{4.3}$$

LFU holds in the limit of mass-less leptons as mass enters the phase-space factor. In reality, corrections due to non-zero lepton masses have to be applied. Usually, these corrections are minor for electrons and muons, which are light but can be large for the heavy tau lepton. LFU can be tested not only directly by studying couplings of leptons to the gauge bosons but also in the decays of hadrons. For example, a charged pion decay, mediated by a weak charged current, can be used to measure the following ratio:

$$\frac{\Gamma_{\pi^- \to e^+ \overline{\nu_e}}}{\Gamma_{\pi^- \to \mu^+ \overline{\nu_\mu}}} = (1.230 \pm 0.004) \times 10^{-4} \tag{4.4}$$

which is in a good agreement with the SM prediction of $(1.2352 \pm 0.0001) \times 10^{-4}$.It should be noted that LFU does not imply the ratio to be equal to unity in this particular case.

LFV may insist there is some part of SM we still didn't discover, or the standard model is not a complete theory; we look for BSM to resolve issues with SM. Recent rare decay experimental results show deviation from SM predictions maybe suggest physics beyond SM.

Chapter 5

$R_K - R_{K*}$ $(b \to s\ell^+\ell^-)$

In SM, different leptons have the same interaction strength except for the lepton-Higgs interaction, which gives leptons different masses. In the standard model, strong force doesn't couple with lepton; therefore, $B^+ \to K^+e^+e^-$ and $B^+ \to K^+\mu^+\mu^-$ decays identically, giving R_k nearly equal to 1. Beyond SM predicts new virtual particles which can interact non universally with leptons which can explain experimentally found branching fraction of $B^+ \to K^+\ell^+\ell^-$. Based on data collected in LHCb, CERN, there is lepton universality violation in beauty-quark decays.

$$R_H = \frac{Br(B \to H\mu^+\mu^-)}{Br(B \to He^+e^-)} \tag{5.1}$$

For $H = K^+$,ratio denotes R_k and for $H = K^{*0}$ ratio called $R_{k^{*0}}$

Measured R_k values

$$R_k(1.1 < q^2 < 6.0\ GeV^2/c^4) = 0.846^{+0.042+0.013}_{-0.039-0.012}\ [3]$$

SM expectation is $R_k = 1.00{\pm}0.01$. [3]

The discrepancy of 3.1σ with SM, gives evidence of a violation of lepton universality.

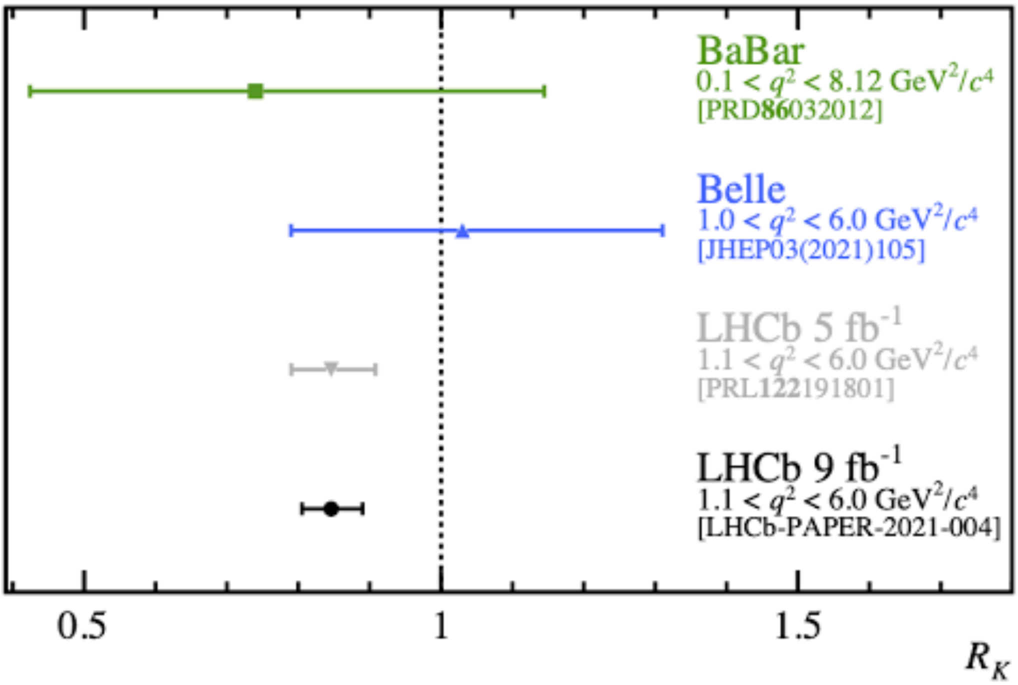


Figure 5.1: R_k measurements. [3]

Chapter 6

Standard Model Calculation

6.1 Effective Hamiltonian

The basic starting point to do phenomenology of weak decays of hadrons is the effective Hamiltonian which has the following generic structure [11],

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} C_i(\mu) O_i(\mu) \tag{6.1}$$

Here G_F is the Fermi coupling constant, V_{CKM} are the Cabibo-Kobayashi and Maskawa(CKM) matrix elements, $O_i(\mu)$ are the four-quark operators, and $C_i(\mu)$ are the corresponding Wilson coefficients at the energy scale μ. Now the amplitude for the decay of meson M to a final state meson F can be written as

$$\begin{aligned} A(M \to F) &= \langle F \left| H_{eff} \right| M \rangle & (6.2) \\ &= \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) \langle F \left| O_i(\mu) \right| M \rangle & (6.3) \end{aligned}$$

Wilson coefficients give the short distance effects, whereas the long-distance effects involve the matrix elements of the operators in Eq.(6.2) between initial and final state mesons. The explicit form of the operators, which are sandwiched between the initial and final state meson, can be written as

Current Current Operators

$$O_1 = (\bar{c}_\alpha b_\beta)_{V-A} (\bar{s}_\beta c_\alpha)_{V-A} \quad (6.4)$$

$$O_2 = (\bar{c}b)_{V-A}(\bar{s}c)_{V-A} \quad (6.5)$$

QCD-Penguins

$$O_3 = (\bar{s}b)_{V-A} \sum_{q=i,d,s,c,b} (\bar{q}q)_{V-A} \quad (6.6)$$

$$O_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=i,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A} \quad (6.7)$$

$$O_5 = (\bar{s}_b)_{V-A} \sum_{q=i,d,s,c,b} (\bar{q}q)_{V+A} \quad (6.8)$$

$$O_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=i,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A} \quad (6.9)$$

Electroweak penguins

$$O_7 = \tfrac{3}{2}(\bar{s})_{V-A} \textstyle\sum_{q=u,d,s,c,b} e_q(\bar{q}q)_{V+A} \quad (6.10)$$

$$O_g = \tfrac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \textstyle\sum_{q-u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A} \quad (6.11)$$

$$O_9 = \tfrac{3}{2}(\bar{s}b)_{V-A} \textstyle\sum_{q=u,d,s,cb} e_q(\bar{q}q)_{V-A} \quad (6.12)$$

$$O_{10} = \tfrac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \textstyle\sum_{q-i,d,_{s,c,b}} (\bar{q}\beta q_\alpha)_{V-A} \quad (6.13)$$

Magnetic Penguins

$$O_{7\gamma} = \tfrac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma^5)\, b_\alpha F_{\mu\nu} \quad (6.14)$$

$$O_{3G} = \tfrac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma^5)\, T^a_{\alpha\beta} b_\beta G^a_{\mu\nu} \quad (6.15)$$

Semileptonic Operators

$$O_9 = (\bar{s}b)_{V-A}(\bar{\ell}\ell)_V \quad (6.16)$$

$$O_{10} = (\bar{s}b)_{V-A}(\bar{\ell}\ell)_A \quad (6.17)$$

$$O_{LQ} = (\bar{s}b)_{V-A}(\bar{\nu}\nu)_{V-A} \quad (6.18)$$

$$O_{\ell\ell} = (\bar{s}b)_{V-A}(\bar{\ell})_{V-A} \quad (6.19)$$

The above set of operators characterizes the interplay of QCD and electroweak effects. As already mentioned earlier, this thesis deals with rare decays of B mesons into a final state hadron with lepton-antilepton pair, so the operators responsible for these decays are electromagnetic penguin operator $O_{7\gamma}$ given in Eq.(6.14) and the semileptonic operators given in Eq.(6.16).

6.2 Calculation

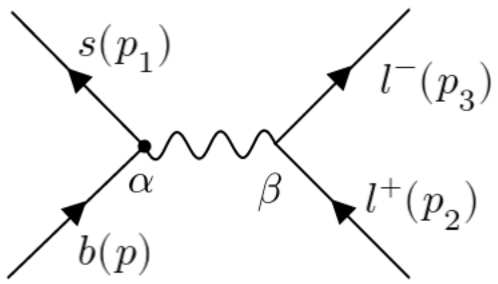


Figure 6.1: Feynman Diagram

Effective Lagrangian is

$$
\begin{aligned}
L &= C_{7R}\bar{s}\sigma^{\mu\nu}P_R b F_{\mu\nu} + C_{7L}\bar{s}\sigma^{\mu\nu}P_L b F_{\mu\nu} && (6.20)\\
&\quad + C^*_{7R}\bar{b}\sigma^{\mu\nu}P_L s F_{\mu\nu} + C^*_{7R}\bar{b}\sigma^{\mu\nu}P_R s F_{\mu\nu} + ie\overline{\psi}\gamma^\mu\psi A_\mu\,, \\
L_1 &= C_{7R}\bar{s}\sigma^{\mu\nu}P_R b\,(\partial_\mu A_\nu - \partial_\nu A_\mu)\,, && (6.21)\\
L_1 &= C_{7R}\bar{s}\sigma^{\mu\nu}P_R b\,(\partial_\mu g_{\nu\alpha} - \partial_\nu g_{\mu\alpha})A^\alpha && (6.22)
\end{aligned}
$$

Using Feynman Convention , we can derive the vertex term for vertex α in **??** and other vertex is QED vertex therefore, we can write the Feynman Amplitude as follow,

$$
M = \overline{u}_{s_1}(p_1)(-i\Gamma_\alpha)u_s(p)\frac{-ig^{\alpha\beta}}{(p-p_1)^2}\overline{u}_{s_3}(p_3)(-ie\gamma_\beta)v_{s_2}(p_2)\,, \quad (6.23)
$$

where vertex term for vertex α is ,

$$
(-i\Gamma_\alpha) = -i\Big\{\{C_{7R}\sigma^{\mu\nu}P_R + C_{7L}\sigma^{\mu\nu}P_L\}\{ik_\mu g_{\nu\alpha} - ik_\nu g_{\mu\alpha}\}\Big\}\,. (6.24)
$$

then Feynman Amplitude be,

$$M = ie\overline{u}_{s_1}(p_1)\Gamma_\alpha u_s(p)\frac{g^{\alpha\beta}}{(p-p_1)^2}\overline{u}_{s_3}(p_3)\gamma_\beta v_{s_2}(p_2)\,, \tag{6.25}$$

Reduction of β index by $g^{\alpha\beta}$ gives

$$M = ie[\overline{u}_{s_1}(p_1)\Gamma_\alpha u_s(p)]\frac{1}{(p-p_1)^2}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)]\,. \tag{6.26}$$

Now we find the value of M^* (Introducing β, ρ, δ as indices so we can not confuse it with indices of M)

$$M^* = -ie[\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)]\frac{1}{(p-p_1)^2}[\overline{u}_{s_1}(p_1)\Gamma_\beta u_s(p)]^*\,. \tag{6.27}$$

It is easier to calculate complex conjugate of QED vertex; now we find $\bar{[}u_{s_1}(p_1)\Gamma_\beta u_s(p)]^*$

$$\begin{aligned}
[\overline{u}_{s_1}(p_1)\Gamma_\beta u_s(p)]^* &= (u_s(p))^\dagger(\Gamma_\beta)^\dagger(\gamma^0)^\dagger((u^\dagger_{s_1}(p_1))^\dagger\,,\\
&= (u_s(p))^\dagger(\{C_{7R}\sigma^{\rho\delta}P_R + C_{7L}\sigma^{\rho\delta}P_L\}\{ik_\rho g_{\delta\beta} - ik_\delta g_{\rho\beta}\})^\dagger(\gamma^0)^\dagger((u^\dagger_{s_1}(p_1))^\dagger\,,\\
&= u^\dagger_s(p)(\{C_{7R}\sigma^{\rho\delta}P_R + C_{7L}\sigma^{\rho\delta}P_L\}\{ik_\rho g_{\delta\beta} - ik_\delta g_{\rho\beta}\})^\dagger(\gamma^0)^\dagger u_{s_1}(p_1)\,,\\
&= u^\dagger_s(p)(\{-ik_\rho g_{\delta\beta} + ik_\delta g_{\rho\beta}\}\{C^*_{7R}P^\dagger_R\sigma^{\rho\delta\dagger} + C^*_{7L}P^\dagger_L\sigma^{\rho\delta\dagger}\})(\gamma^0)^\dagger u_{s_1}(p_1)
\end{aligned}$$

Projection operator and Sigma matrices

$$P_R = (1+\gamma^5)/2 \tag{6.28a}$$
$$P_L = (1-\gamma^5)/2 \tag{6.28b}$$
$$\gamma^{5\dagger} = \gamma^5 \tag{6.28c}$$
$$P^\dagger_R = P_R \tag{6.28d}$$
$$P^\dagger_L = P_L \tag{6.28e}$$
$$(\sigma^{\mu\nu})^\dagger = \gamma^0\sigma^{\mu\nu}\gamma^0 \tag{6.28f}$$

Using definition of Projection operator and Sigma matrices, we get

$$\begin{aligned}
[\overline{u}_{s_1}(p_1)\Gamma_\beta u_s(p)]^* &= u^\dagger_s(p)(\{-ik_\rho g_{\delta\beta} + ik_\delta g_{\rho\beta}\}\{C^*_{7R}P_R\gamma^0\sigma^{\rho\delta}\gamma^0 + C^*_{7L}P_L\gamma^0\sigma^{\rho\delta}\gamma^0\})(\gamma^0)^\dagger u_{s_1}(p_1)\\
&= \overline{u}_s(p)(\{-ik_\rho g_{\delta\beta} + ik_\delta g_{\rho\beta}\}\{C^*_{7R}P_L\sigma^{\rho\delta} + C^*_{7L}P_R\sigma^{\rho\delta}\})u_{s_1}(p_1)
\end{aligned}$$

We used following gamma matrices relation to solve above equation ;

$$\gamma^{0\dagger} = \gamma^0 \,, \tag{6.29a}$$

$$(\gamma^0)^2 = 1 \,, \tag{6.29b}$$

$$\{\gamma^5, \gamma^\mu\} = 0 \,. \tag{6.29c}$$

Now,$\overline{|M|^2}$ is absolute square of the Feynman amplitude suitably summed and averaged;

$$\overline{|M|^2} = \frac{1}{2}\sum_{spin} \|M\|^2 \quad = \quad \frac{1}{2}\sum_{spin} M^* M \,, \tag{6.30}$$

$$\frac{1}{2}\sum_{spin}\|M\|^2 = \frac{1}{2}\sum_{spin}\{-ie[\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)]\frac{1}{(p-p_1)^2}\left[\overline{u}_s(p)(\{-ik_\rho g_{\delta\beta} + ik_\delta g_{\rho\beta}\}\{C^*_{7R}P_L\sigma^{\rho\delta} + C^*_{7L}P_R\sigma^{\rho\delta}\})u_{s_1}(p_1)\right.$$
$$\{ie[\overline{u}_{s_1}(p_1)(\{C_{7R}\sigma^{\mu\nu}P_R + C_{7L}\sigma^{\mu\nu}P_L\}\{ik_\mu g_{\nu\alpha} - ik_\nu g_{\mu\alpha}\})u_s(p)]\frac{1}{(p-p_1)^2}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)]$$

Using $\Gamma_\alpha, \Gamma^\dagger_\beta$ we can simplify,

$$\begin{aligned}\frac{1}{2}\sum_{spin}\|M\|^2 &= \frac{e^2}{2(p-p_1)^4} \times \sum_{s_2,s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)][\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)] \\ &\quad \times \sum_{s_1,s}\left[\overline{u}_s(p)\Gamma^\dagger_\beta u_{s_1}(p_1)\right][\overline{u}_{s_1}(p_1)\Gamma_\alpha u_s(p)] \,,\end{aligned} \tag{6.31}$$

where,

$$\Gamma_\alpha = \{C_{7R}\sigma^{\mu\nu}P_R + C_{7L}\sigma^{\mu\nu}P_L\}\{ik_\mu g_{\nu\alpha} - ik_\nu g_{\mu\alpha}\} \tag{6.32a}$$

$$\Gamma^\dagger_\beta = \{-ik_\rho g_{\delta\beta} + ik_\delta g_{\rho\beta}\}\{C^*_{7R}P_L\sigma^{\rho\delta} + C^*_{7L}P_R\sigma^{\rho\delta}\} \,. \tag{6.32b}$$

Doing spin sum ,

$$\sum_{s_2,s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)][\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)] \quad = \quad \sum_{s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha]\sum_{s_2}[v_{s_2}(p_2)\overline{v}_{s_2}(p_2)][\gamma^\beta u_{s_3}(p_3)] \,,$$

Using Spin sum of Dirac Spinors ,

$$\sum_{s_2}[v_{s_2}(p_2)\overline{v}_{s_2}(p_2)] = (\not{p}_2 - m_{l^+})\,. \tag{6.33}$$

$$\begin{aligned}
\sum_{s_2,s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)][\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)] &= \sum_{s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha](\not{p}_2 - m_{l^+})[\gamma^\beta u_{s_3}(p_3)] \\
\gamma^\alpha(\not{p}_2 - m_{l^+})\gamma^\beta &= Q\,. \qquad (6.34)\\
\sum_{s_2,s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)][\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)] &= \sum_{s_3}[\overline{u}_{s_3}(p_3)]_i Q_{ij}[u_{s_3}(p_3)]_j\,, \\
&= Q_{ij}\sum_{s_3}[u_{s_3}(p_3)\overline{u}_{s_3}(p_3)]_{ji}\,, \\
\sum_{s_3}[u_{s_3}(p_3)\overline{u}_{s_3}(p_3)] &= (\not{p}_3 + m_{l^-})\,. \qquad (6.35)\\
\sum_{s_2,s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)][\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)] &= Q_{ij}(\not{p}_3 + m_{l^-})_{ji}\,, \\
&= Tr[Q(\not{p}_3 + m_{l^-})]\,, \\
&= Tr[\gamma^\alpha(\not{p}_2 - m_{l^+})\gamma^\beta(\not{p}_3 + m_{l^-})] \qquad (6.36)
\end{aligned}$$

After doing the above calculation now problem finding spin sum change to calculation of trace,

$$\sum_{s_2,s_3}[\overline{u}_{s_3}(p_3)\gamma^\alpha v_{s_2}(p_2)][\overline{v}_{s_2}(p_2)\gamma^\beta u_{s_3}(p_3)] = Tr[\gamma^\alpha(\not{p}_2 - m_{l^+})\gamma^\beta(\not{p}_3 + m_{l^-})] \tag{6.37}$$

Similarly as above we can solve and find the following equation,

$$\sum_{s_1,s}\left[\overline{u}_s(p)\Gamma^\dagger_\beta u_{s_1}(p_1)\right][\overline{u}_{s_1}(p_1)\Gamma_\alpha u_s(p)] = Tr[\Gamma^\dagger_\beta(\not{p}_1 + m_s)\Gamma_\alpha(\not{p} + m_b)] \tag{6.38}$$

Now solving eq.6.37,

$$\begin{aligned}
Tr[\gamma^\alpha(\not{p}_2 - m_{l^+})\gamma^\beta(\not{p}_3 + m_{l^-})] &= Tr[\gamma^\alpha(\gamma^\mu p_{2\mu} - m_{l^+})\gamma^\beta(\gamma^\nu p_{3\nu} + m_{l^-})]\,, \qquad (6.39a)\\
&= Tr[\gamma^\alpha\gamma^\mu p_{2\mu}\gamma^\beta\gamma^\nu p_{3\nu} - \gamma^\alpha\gamma^\beta(m_{l^+})(m_{l^-})]\,, \qquad (6.39b)\\
&= p_{2\mu}p_{3\nu}Tr[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu] - Tr[\gamma^\alpha\gamma^\beta](m_l)^2\,. \qquad (6.39c)
\end{aligned}$$

Used above trace of odd gamma matrices is zero.

Using Trace of gamma matrices as following,

$$\begin{aligned} Tr[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu] &= 4(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\nu}g^{\mu\beta})\,, && (6.40a)\\ Tr[\gamma^\alpha\gamma^\beta] &= 4g^{\alpha\beta}\,. && (6.40b)\end{aligned}$$

eq.6.39c becomes,

$$\begin{aligned} Tr[\gamma^\alpha(\not{p}_2 - m_{l^+})\gamma^\beta(\not{p}_3 + m_{l^-})] &= 4p_{2\mu}p_{3\nu}(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\nu}g^{\mu\beta}) - 4g^{\alpha\beta}(m_l)^2\\ &= 4(p_2^\alpha p_3^\beta + p_3^\alpha p_2^\beta - g^{\alpha\beta}[p_2.p_3 + m_l^2])\end{aligned}$$

We can find trace of gamma matrices using FeynCalc in Mathematica (A.1).

To solve Tr of eq.6.38 , first we simplify the Γ_α and $\Gamma_\beta^\dagger$

$$\Gamma_\alpha = \Big\{\{C_{7R}\sigma^{\mu\nu}P_R + C_{7L}\sigma^{\mu\nu}P_L\}\{ik_\mu g_{\nu\alpha} - ik_\nu g_{\mu\alpha}\}\Big\}\,, \qquad (6.41)$$

Using definition of $\sigma^{\mu\nu}$,

$$\sigma^{\mu\nu} = \frac{i}{2}\{\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\}\,. \qquad (6.42)$$

Γ_α becomes,

$$\begin{aligned} \Gamma_\alpha &= \frac{-1}{2}\Big\{\{C_{7R}\{\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\}P_R + C_{7L}\{\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\}P_L\}\{k_\mu g_{\nu\alpha} - k_\nu g_{\mu\alpha}\}\Big\}\,, \qquad ,\\ &= \frac{-1}{2}\Big\{[C_{7R}(\gamma^\mu\gamma^\nu k_\mu g_{\nu\alpha} - \gamma^\nu\gamma^\mu k_\mu g_{\nu\alpha})P_R - C_{7R}(\gamma^\mu\gamma^\nu k_\nu g_{\mu\alpha} - \gamma^\nu\gamma^\mu k_\nu g_{\mu\alpha})P_R]\\ &\quad +[C_{7L}(\gamma^\mu\gamma^\nu k_\mu g_{\nu\alpha} - \gamma^\nu\gamma^\mu k_\mu g_{\nu\alpha})P_L - C_{7L}(\gamma^\mu\gamma^\nu k_\nu g_{\mu\alpha} - \gamma^\nu\gamma^\mu k_\nu g_{\mu\alpha})P_L]\Big\}\,,\\ &= \frac{-1}{2}\Big\{C_{7R}(\not{k}\gamma_\alpha - \gamma_\alpha\not{k})P_R - C_{7R}(\gamma_\alpha\not{k} - \not{k}\gamma_\alpha)P_R + C_{7L}(\not{k}\gamma_\alpha - \gamma_\alpha\not{k})P_L\\ &\quad -C_{7R}(\gamma_\alpha\not{k} - \not{k}\gamma_\alpha)P_L\Big\}\,,\\ &= \frac{-1}{2}\Big\{2\,C_{7R}(\not{k}\gamma_\alpha - \gamma_\alpha\not{k})P_R + 2\,C_{7L}(\not{k}\gamma_\alpha - \gamma_\alpha\not{k})P_L\Big\}\,,\\ \Gamma_\alpha &= (C_{7R}P_R + C_{7L}P_L)[\gamma_\alpha\not{k} - \not{k}\gamma_\alpha]\,,\end{aligned}$$

Using eq.6.28a,6.28b;

$$\begin{aligned}
\Gamma_\alpha &= \left\{\left[\frac{C_{7R}+C_{7L}}{2}\right]+\left[\frac{C_{7R}-C_{7L}}{2}\right]\gamma^5\right\}(\gamma_\alpha \not{k} - \not{k}\gamma_\alpha)\,, \\
\Gamma_\alpha &= (A+B\gamma^5)(\gamma_\alpha \not{k} - \not{k}\gamma_\alpha)\,. \\
\text{where}, A &= \left[\frac{C_{7R}+C_{7L}}{2}\right], \\
B &= \left[\frac{C_{7R}-C_{7L}}{2}\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Gamma^\dagger_\beta &= (\{-ik_\rho g_{\delta\beta}+ik_\delta g_{\rho\beta}\}\{C^*_{7R}P_L\sigma^{\rho\delta}+C^*_{7L}P_R\sigma^{\rho\delta}\})\,, \\
&= \frac{-1}{2}(\{-k_\rho g_{\delta\beta}+k_\delta g_{\rho\beta}\})\left[C^*_{7R}P_L\{\gamma^\rho\gamma^\delta-\gamma^\delta\gamma^\rho\}+C^*_{7L}P_R\{\gamma^\rho\gamma^\delta-\gamma^\delta\gamma^\rho\}\right]\,, \\
&= \frac{-1}{2}\left[k_\delta g_{\rho\beta}\,C^*_{7R}\,P_L\{\gamma^\rho\gamma^\delta-\gamma^\delta\gamma^\rho\}-k_\rho g_{\delta\beta}\,C^*_{7R}\,P_L\{\gamma^\rho\gamma^\delta-\gamma^\delta\gamma^\rho\}\right. \\
&\quad \left.+k_\delta g_{\rho\beta}\,C^*_{7L}\,P_R\{\gamma^\rho\gamma^\delta-\gamma^\delta\gamma^\rho\}-k_\rho g_{\delta\beta}\,C^*_{7L}\,P_R\{\gamma^\rho\gamma^\delta-\gamma^\delta\gamma^\rho\}\right]\,, \\
&= \frac{-1}{2}\left[C^*_{7R}P_L(\gamma_\beta\not{k}-\not{k}\gamma_\beta)-C^*_{7R}P_L(/k\gamma_\beta-\gamma_\beta\not{k})+C^*_{7L}P_R(\gamma_\beta\not{k}-\not{k}\gamma_\beta)\right. \\
&\quad \left.-C^*_{7L}P_R(\not{k}\gamma_\beta-\gamma_\beta\not{k})\right]\,, \\
&= \frac{-1}{2}\left\{2\,C^*_{7R}P_L(\gamma_\beta\not{k}-\not{k}\gamma_\beta)+2\,C^*_{7L}P_R(\gamma_\beta\not{k}-\not{k}\gamma_\beta)\right\}, \\
&= (C^*_{7R}P_L+C^*_{7L}P_R)(\not{k}\gamma_\beta-\gamma_\beta\not{k})\,, \\
\Gamma^\dagger_\beta &= \left\{\left[\frac{C^*_{7R}+C^*_{7L}}{2}\right]+\left[\frac{C^*_{7L}-C^*_{7R}}{2}\right]\gamma^5\right\}(\not{k}\gamma_\beta-\gamma_\beta\not{k})
\end{aligned}$$

$$\Gamma^\dagger_\beta = (M+N\gamma^5)(\not{k}\gamma_\beta-\gamma_\beta\not{k}) \tag{6.43}$$

where,

$$M = \left[\frac{C^*_{7R}+C^*_{7L}}{2}\right], \tag{6.44}$$

$$N = \left[\frac{C^*_{7L}-C^*_{7R}}{2}\right]. \tag{6.45}$$

Tr of eq.6.38 becomes,

$$\begin{aligned} Tr[\Gamma^{\dagger}_{\beta}(\not{p}_1 + m_s)\Gamma_{\alpha}(\not{p} + m_b)] &= Tr[(M + N\gamma^5)(\not{k}\gamma_{\beta} - \gamma_{\beta}\not{k})(\not{p}_1 + m_s) \\ &\quad \times (A + B\gamma^5)(\gamma_{\alpha}\not{k} - \not{k}\gamma_{\alpha})(\not{p} + m_b)] \end{aligned} \tag{6.46}$$

Tr of eq.6.46 from FeynCalc (A.2),

In[13]:=
```
Tr[C2.bk.(p1s + ms).C1.ak.(ps + mb)]
```

Out[13]=

$$\begin{aligned} 16\Big(& A M \mathrm{mb}\,\mathrm{ms}\,\overline{k}^2\,\bar{g}^{\alpha\beta} - 2 A M \bar{g}^{\alpha\beta}(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p1}}) + A M \overline{k}^2\,\bar{g}^{\alpha\beta}(\overline{p}\cdot\overline{\mathrm{p1}}) - A M \mathrm{mb}\,\mathrm{ms}\,\overline{k}^{\alpha}\,\overline{k}^{\beta} - \\ & A M \overline{k}^2\,\overline{p}^{\beta}\,\overline{\mathrm{p1}}^{\alpha} - A M \overline{k}^2\,\overline{p}^{\alpha}\,\overline{\mathrm{p1}}^{\beta} + A M \overline{k}^{\alpha}\,\overline{\mathrm{p1}}^{\beta}(\overline{k}\cdot\overline{p}) + A M \overline{k}^{\alpha}\,\overline{p}^{\beta}(\overline{k}\cdot\overline{\mathrm{p1}}) - A M \overline{k}^{\alpha}\,\overline{k}^{\beta}(\overline{p}\cdot\overline{\mathrm{p1}}) + \\ & A M \overline{k}^{\beta}\,\overline{\mathrm{p1}}^{\alpha}(\overline{k}\cdot\overline{p}) + A M \overline{k}^{\beta}\,\overline{p}^{\alpha}(\overline{k}\cdot\overline{\mathrm{p1}}) - i A N \overline{k}^{\alpha}\,\bar{\epsilon}^{\beta\overline{k}\overline{p}\overline{\mathrm{p1}}} + i A N \overline{k}^{\beta}\,\bar{\epsilon}^{\alpha\overline{k}\overline{p}\overline{\mathrm{p1}}} - \\ & i A N \overline{k}^2\,\bar{\epsilon}^{\alpha\beta\overline{p}\overline{\mathrm{p1}}} - 2 i A N(\overline{k}\cdot\overline{\mathrm{p1}})\bar{\epsilon}^{\alpha\beta\overline{k}\overline{p}} + B \mathrm{mb}\,\mathrm{ms}\,N \overline{k}^2\,\bar{g}^{\alpha\beta} + 2 B N \bar{g}^{\alpha\beta}(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p1}}) - \\ & B N \overline{k}^2\,\bar{g}^{\alpha\beta}(\overline{p}\cdot\overline{\mathrm{p1}}) - i B M \overline{k}^{\alpha}\,\bar{\epsilon}^{\beta\overline{k}\overline{p}\overline{\mathrm{p1}}} + i B M \overline{k}^{\beta}\,\bar{\epsilon}^{\alpha\overline{k}\overline{p}\overline{\mathrm{p1}}} - i B M \overline{k}^2\,\bar{\epsilon}^{\alpha\beta\overline{p}\overline{\mathrm{p1}}} + \\ & 2 i B M(\overline{k}\cdot\overline{p})\bar{\epsilon}^{\alpha\beta\overline{k}\overline{\mathrm{p1}}} - B \mathrm{mb}\,\mathrm{ms}\,N \overline{k}^{\alpha}\,\overline{k}^{\beta} + B N \overline{k}^2\,\overline{p}^{\beta}\,\overline{\mathrm{p1}}^{\alpha} + B N \overline{k}^2\,\overline{p}^{\alpha}\,\overline{\mathrm{p1}}^{\beta} - B N \overline{k}^{\alpha}\,\overline{\mathrm{p1}}^{\beta}(\overline{k}\cdot\overline{p}) - \\ & B N \overline{k}^{\alpha}\,\overline{p}^{\beta}(\overline{k}\cdot\overline{\mathrm{p1}}) + B N \overline{k}^{\alpha}\,\overline{k}^{\beta}(\overline{p}\cdot\overline{\mathrm{p1}}) - B N \overline{k}^{\beta}\,\overline{\mathrm{p1}}^{\alpha}(\overline{k}\cdot\overline{p}) - B N \overline{k}^{\beta}\,\overline{p}^{\alpha}(\overline{k}\cdot\overline{\mathrm{p1}})\Big) \end{aligned}$$

Detour:- Muon Decay

Using Decay calculation of Muon Decay , we need to solve

$$\begin{aligned} \overline{|M|^2} &= \frac{G_F^2}{4}\Big\{Tr[(\not{k}_2 + m_e)\gamma^{\alpha}(1-\gamma_5)\not{q}_1\gamma^{\beta}(1-\gamma_5)] \\ &\quad \times Tr[\not{q}_2\gamma_{\alpha}(1-\gamma_5)(\not{k}_1 + m_{\mu})\gamma_{\beta}(1-\gamma_5)]\Big\} \end{aligned} \tag{6.47}$$

Comparing from Sec.(7.2.2) of A First Book of Quantum Field Theory by Amitabha Lahiri, Palash B. Pal [2] where ,

$$k_1 = p, k_2 = p', q_1 = k', q_2 = k. \tag{6.48}$$

Using FeynCalc we find the solution of eq.6.47,
We get

```
In[10]:= Simplify[Contract[Tr[(k2s + me).a.L.q1s.b.L] × Tr[(q2s).a.L.(k1s + mu).b.L]]]
Out[10]= 256 (k1 · q1) (k2 · q2)
```

$$\overline{|M|^2} \;=\; 64\, G_F^2 (k_1.q_1)(k_2.q_2) \tag{6.49}$$

Above result is same as eq.(7.49) of A First Book of Quantum Field Theory 2nd Edition by Amitabha Lahiri, Palash B. Pal [2] .

Now coming back to our problem,

$$\begin{aligned}\frac{1}{2}\sum_{spin}\|M\|^2 \;=\;& \frac{e^2}{2(p-p_1)^4}\times Tr[(M+N\gamma^5)(\not{k}\gamma_\beta-\gamma_\beta\not{k})(\not{p}_1+m_s)\\ &\times(A+B\gamma^5)(\gamma_\alpha\not{k}-\not{k}\gamma_\alpha)(\not{p}+m_b)]\\ &\times Tr[\gamma^\alpha(\not{p}_2-m_{l^+})\gamma^\beta(\not{p}_3+m_{l^-})]\end{aligned} \tag{6.50}$$

Mass of particle and antiparticle is same ($m_{l^+}=m_{l^-}=m_l$).

We can directly solve the following eq.(6.51) in FeynCalc (A.4),

$$\begin{aligned}Tr[(M+N\gamma^5)(\not{k}\gamma_\beta-\gamma_\beta\not{k})(\not{p}_1+m_s)(A+B\gamma^5)(\gamma_\alpha\not{k}-\not{k}\gamma_\alpha)(\not{p}+m_b)]\\ \times Tr[\gamma^\alpha(\not{p}_2-m_{\ell^+})\gamma^\beta(\not{p}_3+m_{\ell^-})]\end{aligned} \tag{6.51}$$

We get,

Out[23]=

$$\begin{aligned}&32\Big(-(\overline{p}-\overline{p1})^2\big((\text{C7L}+\text{C7R})\big((\overline{p}\cdot\overline{p1})(\text{ml}^2-\overline{p2}\cdot\overline{p3})+2\big((\overline{p}\cdot\overline{p3})(\overline{p1}\cdot\overline{p2})+(\overline{p}\cdot\overline{p2})(\overline{p1}\cdot\overline{p3})\big)\big)+\\ &\quad \text{mb ms}(\text{C7LR}+\text{C7RL})(\overline{p2}\cdot\overline{p3}+3\,\text{ml}^2)\big)-\\ &\quad 2((\overline{p}-\overline{p1})\cdot\overline{p2})\big(((\overline{p}-\overline{p1})\cdot\overline{p3})\big((\text{C7L}+\text{C7R})(\overline{p}\cdot\overline{p1})+\text{mb ms}(\text{C7LR}+\text{C7RL})\big)-\\ &\quad (\text{C7L}+\text{C7R})(\overline{p}\cdot\overline{p3})((\overline{p}-\overline{p1})\cdot\overline{p1})\big)+2(\text{C7L}+\text{C7R})(\overline{p}\cdot(\overline{p}-\overline{p1}))\\ &\quad \big(2\,\text{ml}^2((\overline{p}-\overline{p1})\cdot\overline{p1})+(\overline{p1}\cdot\overline{p2})((\overline{p}-\overline{p1})\cdot\overline{p3})+(\overline{p1}\cdot\overline{p3})((\overline{p}-\overline{p1})\cdot\overline{p2})\big)+\\ &\quad 2(\text{C7L}+\text{C7R})(\overline{p}\cdot\overline{p2})((\overline{p}-\overline{p1})\cdot\overline{p1})((\overline{p}-\overline{p1})\cdot\overline{p3})\Big)\end{aligned}$$

Now to calculate decay width , we put value of $\overline{|M|^2}$ in decay width formula

$$\begin{aligned} \Gamma &= \frac{1}{2E_i}\int\prod_f \frac{d^3p_f}{(2\pi)^3 2E_f}(2\pi)^4\delta^4\left(p_i-\sum_f p_f\right)\overline{|M|^2} \\ \Gamma &= \frac{1}{2E_b}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^4\left(p-(p_1+p_2+p_3)\right)\overline{|M|^2} \end{aligned} \quad .$$

After doing the phase space integral, we should get

$$\Gamma(b \to s\ell^+\ell^-) \propto |C_{7R}|^2 + |C_{7L}|^2 \tag{6.52}$$

which is same as seen in literature ,

$$\Gamma(b \to s\ell^+\ell^-) \propto |C_7|^2 \tag{6.53}$$

Phase space [16] integration could not be done so far. I am still working on phase integration and further calculations. So right now, I assume the ratio R_k is one as stated in the literature; on completing the math, I can clearly say that this is true.

Chapter 7

Understanding the data

The statistical analysis of data is important part of research. In here we discuss the data from [4] to get the value of R_k and R_{k^*}. We extract the data using `https://automeris.io/WebPlotDigitizer` of fractions of candidate to the value of q^2. Below are the plots from which data is extracted.

7.1 $\Delta\chi^2$ analysis in Mathematica

$$R_K = \frac{Br(B \to K\mu^+\mu^-)}{Br(B \to Ke^+e^-)} \tag{7.1}$$

Taking R_K as z and $B \to K\mu^+\mu^-$, $B \to Ke^+e^-$ as x,y respectively.

$$z = f(x,y) = \frac{x}{y} \tag{7.2}$$

Propagation of Error be

$$\sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 \tag{7.3}$$

$$f_x = \frac{\partial f}{\partial x} = \frac{1}{y} \tag{7.4}$$

$$f_y = \frac{\partial f}{\partial y} = -\frac{x}{y^2} \tag{7.5}$$

We here discuss $\Delta\chi^2$ graphs of data and then compare our results with the research paper [4].

First we define χ^2 function;

$$\begin{aligned}\chi^2(R) \;=\; & \frac{|R-R1|^2}{\sigma_1^2}+\frac{|R-R2|^2}{\sigma_2^2}+\frac{|R-R3|^2}{\sigma_3^2}+\frac{|R-R4|^2}{\sigma_4^2} \\ & +\frac{|R-R5|^2}{\sigma_5^2}+\frac{|R-R6|^2}{\sigma_6^2}+\frac{|R-R7|^2}{\sigma_7^2}+\frac{|R-R8|^2}{\sigma_8^2} \\ & +\frac{|R-R9|^2}{\sigma_9^2}+\frac{|R-R10|^2}{\sigma_{10}^2} \qquad (7.6)\end{aligned}$$

here, R1,R2.... represents the values R_K we get from data, σ represents the uncertainty in values of R_K.

$$\Delta\chi^2(R) \;=\; \chi^2(R)-\chi^2_{min}(R) \qquad (7.7)$$

.

We perform all the calculations and plot the data with the help of mathematica.

7.1.1 For low-q^2 value

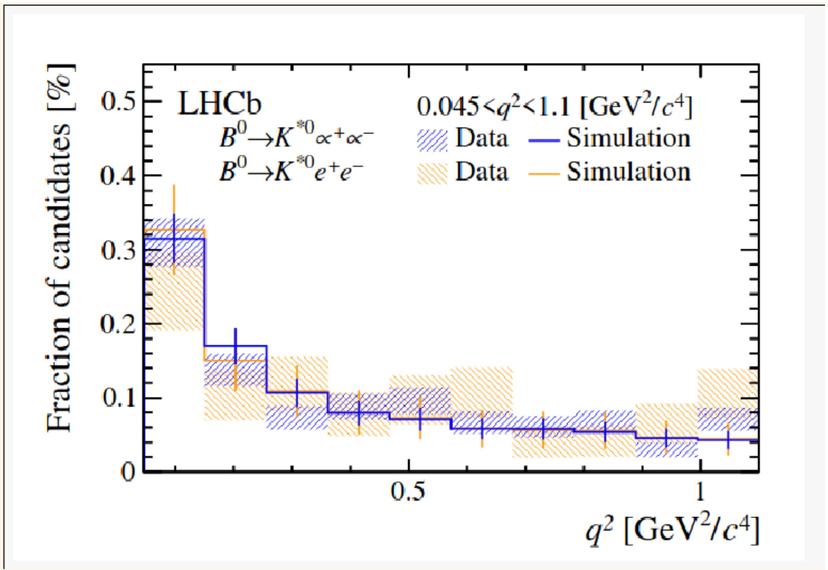


Table of $0.045 < q^2 < 1.1$ GeV^2

S.No.	q^2 (GeV^2)	$B \to K e^- e^+$	Mean (y)	bin (σ_y)	$B \to K \mu^- \mu^+$	Mean (x)	bin (σ_x)
1	0.045 - 0.15	0.191 - 0.295	0.243	±0.052	0.276 - 0.341	0.3085	±0.0325
2	0.15 - 0.25	0.069 - 0.138	0.1035	±0.0345	0.116 - 0.158	0.137	±0.021
3	0.25 - 0.36	0.0817 - 0.155	0.11835	±0.03665	0.055 - 0.086	0.0705	±0.0155
4	0.36 - 0.47	0.047 - 0.106	0.0765	±0.0295	0.070 - 0.106	0.088	±0.018
5	0.47 - 0.57	0.064 - 0.130	0.097	±0.033	0.076 - 0.114	0.095	±0.019
6	0.57 - 0.68	0.072 - 0.141	0.1065	±0.0345	0.051 - 0.082	0.0665	±0.0155
7	0.68 - 0.79	0.019 - 0.059	0.039	±0.02	0.046 - 0.075	0.0605	±0.0145
8	0.79 - 0.89	0.020 - 0.063	0.0415	±0.0215	0.053 - 0.082	0.0675	±0.0145
9	0.89 - 1	0.039 - 0.091	0.065	±0.026	0.019 - 0.040	0.0295	±0.0105
10	1 - 1.1	0.070 - 0.139	0.1045	±0.0345	0.054 - 0.087	0.0705	±0.0165

.

Table for R_K and σ_R

$R = \frac{\Gamma(B \to K \mu^- \mu^+)}{\Gamma(B \to K e^- e^+)}$

$\frac{B \to K \mu^- \mu^+}{B \to K e^- e^+}$

$z = f(x, y) = \frac{x}{y}$

$\sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$

$fx = \frac{\partial f}{\partial x} = \frac{1}{y}$, $fy = \frac{\partial f}{\partial y} = \frac{-x}{y^2}$

Table of $0.045 < q^2 < 1.1\ \mathrm{GeV}^2$

S.No.	Mean $(B \to K \mu^- \mu^+)$ x	Mean $(B \to K e^- e^+)$ y	$R = \frac{x}{y}$	fx	fy	fx^2	fy^2	σ_x	σ_y	σ_x^2	σ_y^2	σ_R^2	σ_R
1	0.3085	0.243	1.269547325	4.115226337	-5.224474589	16.93508781	27.29513473	±0.0325	±0.052	0.00105625	0.002704	0.09169373081	0.3028097271
2	0.137	0.1035	1.323671498	9.661835749	-12.7890966	93.35107004	163.5609917	±0.021	±0.0345	0.000441	0.00119025	0.2358462923	0.4856400851
3	0.0705	0.11835	0.5956907478	8.449514153	-5.033297404	71.39428942	25.33408276	±0.0155	±0.03665	0.00024025	0.0013432225	0.05118178801	0.2262339232
4	0.088	0.0765	1.150326797	13.07189542	-15.0369516	170.87445	226.1099134	±0.018	±0.0295	0.000324	0.00087025	0.2521354739	0.5021309331
5	0.095	0.097	0.9793814433	10.30927835	-10.09671591	106.2812201	101.9436722	±0.019	±0.033	0.000361	0.001089	0.1493841795	0.3865024961
6	0.0665	0.1065	0.6244131455	9.389671362	-5.86303423	88.16592828	34.37517039	±0.0155	±0.0345	0.00024025	0.00119025	0.06209691082	0.2491925176
7	0.0605	0.039	1.551282051	25.64102564	-39.77646285	657.4621959	1582.166997	±0.0145	±0.02	0.00021025	0.0004	0.7710982255	0.8781219878
8	0.0675	0.0415	1.626506024	24.09638554	-39.19291624	580.6357962	1536.084684	±0.0145	±0.0215	0.00021025	0.00046225	0.8321338212	0.9122136927
9	0.0295	0.065	0.4538461538	15.38461538	-6.982248521	236.6863905	48.7517944	±0.0105	±0.026	0.00011025	0.000676	0.05905088757	0.2430038839
10	0.0705	0.1045	0.6746411483	9.56937799	-6.455896156	91.57299512	41.67859518	±0.0165	±0.0345	0.00027225	0.00119025	0.07453869584	0.2730177574

Values of R_K

In[1]:=
```
Clear["Global`*"]
```

In[2]:=
```
R1 = 1.269547325
R2 = 1.323671498
R3 = 0.5956907478
R4 = 1.150326797
R5 = 0.9793814433
R6 = 0.6244131455
R7 = 1.551282051
R8 = 1.626506024
R9 = 0.4538461538
R10 = 0.6746411483
```

Values of σ_R

In[12]:=
```
σ21 = 0.3028097271
σ22 = 0.4856400851
σ23  = 0.2262339232
σ24 = 0.5021309331
σ25 = 0.3865024961
σ26 = 0.2491925176
σ27 = 0.8781219878
σ28 = 0.9122136927
σ29 = 0.2430038839
σ20 = 0.2730177574
```

χ^2 function

In[22]:=
```
χ2[R_] := (Abs[R - R1])^2/σ21^2 + (Abs[R - R2])^2/σ22^2 + (Abs[R - R3])^2/σ23^2 + (Abs[R - R4])^2/σ24^2 + (Abs[R - R5])^2/σ25^2 + (Abs[R - R6])^2/σ26^2 + (Abs[R - R7])^2/σ27^2 + (Abs[R - R8])^2/σ28^2 + (Abs[R - R9])^2/σ29^2 + (Abs[R - R10])^2/σ20^2
```

In[23]:=
```
χ2min = NMinValue[{χ2[R], 0 ≤ R ≤ 2 }, {R}]
```

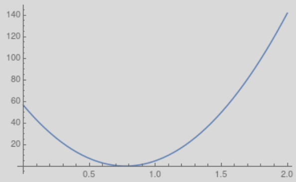
Out[23]=
```
9.30991
```

In[24]:=
```
NMinimize[{χ2[R], 0 ≤ R ≤ 2 }, {R}]
```

Out[24]=
```
{9.30991, {R → 0.773868}}
```

In[25]:=
```
Δχ2[R_] := χ2[R] - χ2min;
plot1 = Plot[Δχ2[R], {R, 0, 2}]
```

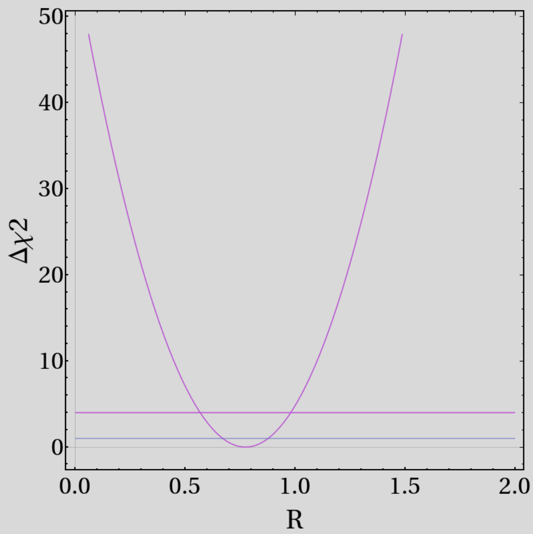
Out[26]=



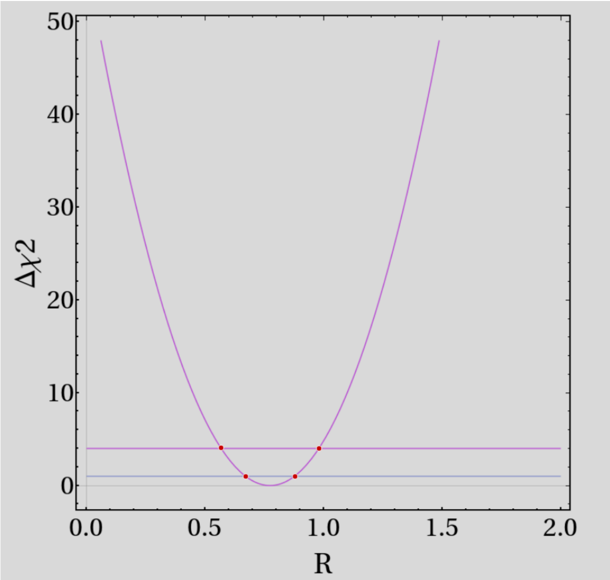
$\Delta\chi^2$ graph

```
In[34]:= plot1 = Plot[{Δχ2[R], 1, 4}, {R, 0, 2}, Axes → False, FrameLabel → {"R", "Δχ2"}, BaseStyle → {(*Bold,*) FontFamily → "Times", (*Italic,*) FontSize → 24}, PlotTheme → "Scientific",
  PlotStyle → {ColorData["HTML"]["MediumOrchid"], Opacity[0.6]}, PlotPoints → 100, BaseStyle → {(*Bold,*) FontFamily → "Times", (*Italic,*) FontSize → 24}, PlotTheme → "Scientific",
  LabelStyle → Directive[Black, 40],
  LabelStyle → Directive[Black, Large, Background → Transparent],
  FrameTicksStyle → {{Directive[Black, 35], Directive[FontOpacity → 0, FontSize → 0]}, {Directive[Black, 35], Directive[FontOpacity → 0, FontSize → 0]}}, ImageSize → {800, 800}, AspectRatio → 1,
  Background → Transparent, FrameStyle → Directive[Black, Thick]]
```

Out[•]=



Extracting values of 1σ and 2σ region using `https://automeris.io/WebPlotDigitizer`



1σ points $(R = 0.6713483146067415, \Delta\chi^2 = 0.98743267504487 87)$ and $(R = 0.8792134831460675, \Delta\chi^2 = 0.9874326750448787)$

2σ points $(R = 0.5674157303370786, \Delta\chi^2 = 4.075403949730699)$ and $(R = 0.9803370786516853, \Delta\chi^2 = 4.003590664272885)$

7.1.2 For central-q^2 value

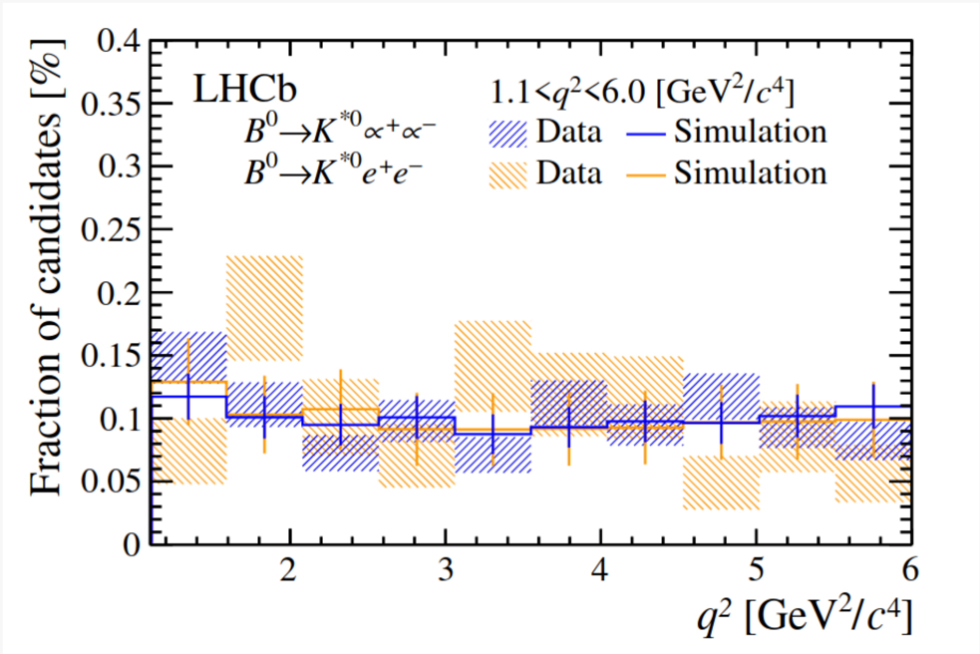


Table of 1.1 < q^2 < 6.0 GeV^2

S.No	GeV^2	e^- e^+	Mean	bin	μ^- μ^+	Mean	bin
1	1.1 - 1.6	0.047 - 0.099	0.073	±0.026	0.127 - 0.168	0.1475	±0.0205
2	1.6 - 2.1	0.145 - 0.228	0.1865	±0.0415	0.092 - 0.128	0.11	±0.018
3	2.1 - 2.6	0.07 - 0.13	0.1	±0.03	0.057 - 0.086	0.0715	±0.0145
4	2.6 - 3.1	0.045 - 0.095	0.07	±0.025	0.081 - 0.114	0.0975	±0.0165
5	3.1 - 3.6	0.104 - 0.176	0.14	±0.036	0.056 - 0.085	0.0705	±0.0145
6	3.6 - 4.05	0.085 - 0.151	0.118	±0.033	0.092 - 0.129	0.1105	±0.0185
7	4.05 - 4.55	0.083 - 0.148	0.1155	±0.0325	0.078 - 0.111	0.0945	±0.0165
8	4.55 - 5	0.027 - 0.069	0.048	±0.021	0.097 - 0.135	0.116	±0.019
9	5 - 5.55	0.057 - 0.113	0.085	±0.028	0.076 - 0.108	0.092	±0.016
10	5.55 - 6	0.033 - 0.078	0.0555	±0.0225	0.066 - 0.098	0.082	±0.016

Table for R_K and σ_R

Table of $1.1 < q^2 < 6.0\ GeV^2$

S.No.	Mean $(B \to K\mu^- \mu^+)$ x	Mean $(B \to K e^- e^+)$ y	$R = \frac{x}{y}$	fx	fy	fx^2	fy^2	σ_x	σ_y	${\sigma_x}^2$	${\sigma_y}^2$	${\sigma_R}^2$	σ_R
1	0.1475	0.073	2.020547945	13.69863014	-27.67873898	187.6524676	766.1125913	±0.0205	±0.026	0.00042025	0.000676	0.5967530612	0.7724979361
2	0.11	0.1865	0.5898123324	5.361930295	-3.162532614	28.75029649	10.00161253	±0.018	±0.0415	0.000324	0.00172225	0.02654037325	0.1629121642
3	0.0715	0.1	0.715	10	-7.15	100	51.1225	±0.0145	±0.03	0.00021025	0.0009	0.06703525	0.2589116645
4	0.0975	0.07	1.392857143	14.28571429	-19.89795918	204.0816327	395.9287797	±0.0165	±0.025	0.00027225	0.000625	0.3030167118	0.5504695376
5	0.0705	0.14	0.5035714286	7.142857143	-3.596938776	51.02040816	12.93796855	±0.0145	±0.036	0.00021025	0.001296	0.02749464806	0.165815102
6	0.1105	0.118	0.936440678	8.474576271	-7.935937949	71.81844298	62.97911113	±0.0185	±0.033	0.00034225	0.001089	0.09316411413	0.3052279707
7	0.0945	0.1155	0.8181818182	8.658008658	-7.083825266	74.96111392	50.18058039	±0.0165	±0.0325	0.00027225	0.00105625	0.07341140131	0.2709453844
8	0.116	0.048	2.416666667	20.83333333	-50.34722222	434.0277778	2534.842785	±0.019	±0.021	0.000361	0.000441	1.274549696	1.128959564
9	0.092	0.085	1.082352941	11.76470588	-12.73356401	138.4083045	162.1436525	±0.016	±0.028	0.000256	0.000784	0.1625531495	0.4031788059
10	0.082	0.0555	1.477477477	18.01801802	-26.62121581	324.6489733	708.6891312	±0.016	±0.0225	0.000256	0.00050625	0.4418840098	0.664743567

Values of R_K

```
Clear["Global`*"]

R1 = 2.020547945
R2  = 0.5898123324
R3 = 0.715
R4  = 1.392857143
R5  = 0.5035714286
R6  = 0.936440678
R7  =0.8181818182
R8  = 2.416666667
R9  = 1.082352941
R10  = 1.477477477
```

.

.

Values of σ_R

```
σ21 = 0.7724979361
σ22 = 0.1629121642
σ23 = 0.2589116645
σ24 = 0.5504695376
σ25 = 0.165815102
σ26 = 0.3052279707
σ27 = 0.2709453844
σ28 = 1.128959564
σ29 = 0.4031788059
σ20 = 0.664743567
```

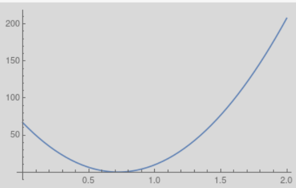
χ^2 function

$$\chi2[R\_] := \frac{(\mathrm{Abs}[R-R1])^2}{\sigma21^2}+\frac{(\mathrm{Abs}[R-R2])^2}{\sigma22^2}+\frac{(\mathrm{Abs}[R-R3])^2}{\sigma23^2}+\frac{(\mathrm{Abs}[R-R4])^2}{\sigma24^2}+\frac{(\mathrm{Abs}[R-R5])^2}{\sigma25^2}+\frac{(\mathrm{Abs}[R-R6])^2}{\sigma26^2}+\frac{(\mathrm{Abs}[R-R7])^2}{\sigma27^2}+\frac{(\mathrm{Abs}[R-R8])^2}{\sigma28^2}+\frac{(\mathrm{Abs}[R-R9])^2}{\sigma29^2}+\frac{(\mathrm{Abs}[R-R10])^2}{\sigma20^2}$$

```
χ2min = NMinValue[{χ2[R], 0 ≤ R ≤ 2}, {R}]
11.6682
```

```
NMinimize[{χ2[R], 0 ≤ R ≤ 2}, {R}]
{11.6682, {R → 0.723793}}
```

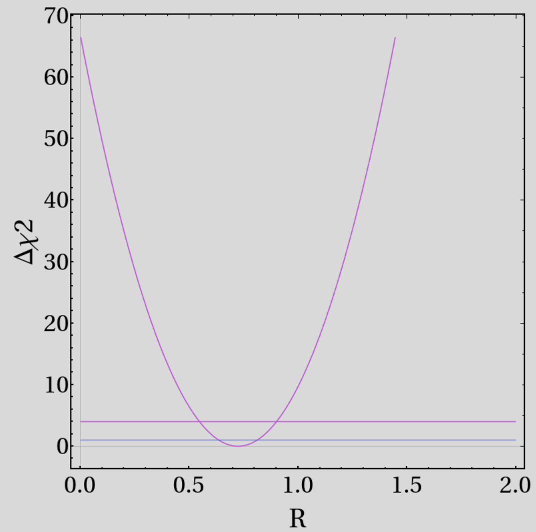
```
Δχ2[R_] := χ2[R] - χ2min;
plot1 = Plot[Δχ2[R], {R, 0, 2}]
```



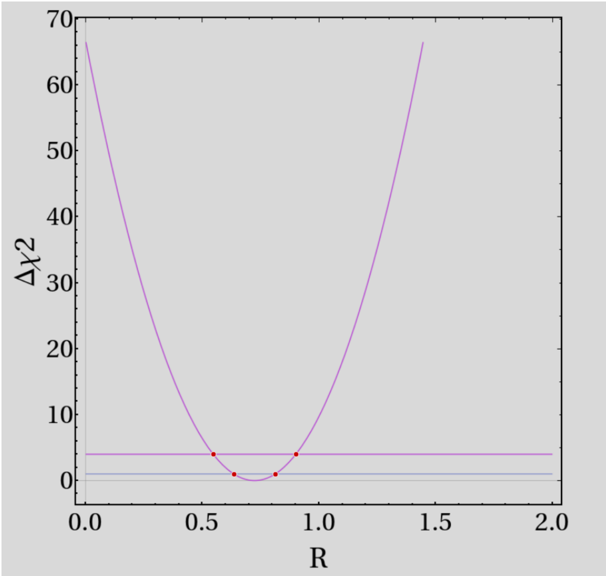
.

$\Delta\chi^2$ graph

```
Plot[{Δχ2[R], 1, 4}, {R, 0, 2}, Axes → False, FrameLabel → {"R", "Δχ2"}, BaseStyle → {FontFamily → "Times", FontSize → 24}, PlotTheme → "Scientific", PlotStyle → {ColorData["HTML"]["MediumOrchid"], Opacity[0.6`]},
 PlotPoints → 100, BaseStyle → {FontFamily → "Times", FontSize → 24}, PlotTheme → "Scientific", LabelStyle → Directive[Black, 40], LabelStyle → Directive[Black, Large, Background → Transparent],
 FrameTicksStyle → {{Directive[Black, 35], Directive[FontOpacity → 0, FontSize → 0]}, {Directive[Black, 35], Directive[FontOpacity → 0, FontSize → 0]}}, ImageSize → {800, 800}, AspectRatio → 1,
 Background → Transparent, FrameStyle → Directive[Black, Thick]]
```



Extracting values of 1σ and 2σ region using `https://automeris.io/WebPlotDigitizer`



1σ points $(R = 0.638176638176638, \Delta\chi^2 = 1.0579345088161176)$ and $(R = 0.8148148148148148, \Delta\chi^2 = 1.0579345088161176)$

2σ points $(R = 0.5498575498575498, \Delta\chi^2 = 4.080604534005033)$ and $(R = 0.9031339031339031, \Delta\chi^2 = 4.080604534005033)$.

R_{K^*} value for low-q^2 value found to be $0.77^{+0.10}_{-0.10}$ from $\Delta\chi^2(R)$ data analysis which is in region of 1σ of stated value $0.66^{+0.11}_{-0.07} \pm 0.03$ [4] in research paper.
Similarly for R_{K^*} value for central-q^2 value found to be $0.72^{+0.09}_{-0.08}$ which is nearly same to the stated value $0.69^{+0.11}_{-0.07} \pm 0.05$ [4].

The given data from [4]

	low- q^2	central- q^2
$R_{K^{*0}}$	$0.66^{+0.11}_{-0.07} \pm 0.03$	$0.69^{+0.11}_{-0.07} \pm 0.05$
95.4%CL	$[0.52, 0.89]$	$[0.53, 0.94]$
99.7%CL	$[0.45, 1.04]$	$[0.46, 1.10]$

.

.

Calculated data from $\Delta\chi^2(R)$ graphs (7.1.1,7.1.2)

	low- q^2	central- q^2
$R_{K^{*0}}$	$0.77^{+0.10}_{-0.10}$	$0.72^{+0.09}_{-0.08}$
95.4%CL	$[0.67, 0.87]$	$[0.63, 0.81]$
99.7%CL	$[0.56, 0.98]$	$[0.55, 0.90]$

Chapter 8

Conclusion and Future Plan

Recent experimental results encourage us to look more closely at rare decay. Significance deviation of 3.1σ with SM of current data leads us to look for BSM interactions to explain this discrepancy. In this thesis, we look closely at the Standard Model, its flavor structure, and the R_K experiment, which help us to get to the result that SM may not give us the complete picture of nature. We will try to complete the missing calculations and look for a new kind of interaction that can describe the Lepton Flavour Violation.

Appendix A

FeynCalc in Mathematica

A.1 Trace of gamma matrices

[13]

Calculation of traces in FeynCalc ;
Procedure to install FeynCalc in Mathematica ,
Simply type below text in Mathematica,

```
Import["https://raw.githubusercontent.com/FeynCalc/feyncalc/master/install.m"];

InstallFeynCalc[]
```

After installation of FeynCalc , you can find traces using FeynCalc ,
Whenever you start new Mathematica session you need to load FeynCalc first ,
You can find FeynCalc guide here:- http://www.feyncalc.org/documentation/FCGuide-pre4.2.0.pdf ,
Notations and how to use FeynCalc given in guide.

In[ ]:=
```
<< FeynCalc`
```

FeynCalc 9.3.1 (stable version). For help, use the documentation center, check out the wiki or visit the forum.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

In[ ]:=
```
Tr[DiracMatrix[α].DiracMatrix[μ].DiracMatrix[β].DiracMatrix[ν]]
```
Out[ ]=
$$4\left(\overline{g}^{\alpha\nu}\,\overline{g}^{\beta\mu}+\overline{g}^{\alpha\mu}\,\overline{g}^{\beta\nu}-\overline{g}^{\alpha\beta}\,\overline{g}^{\mu\nu}\right)$$

In[ ]:=
```
Tr[DiracMatrix[α].DiracMatrix[β]]
```
Out[ ]=
$$4\,\overline{g}^{\alpha\beta}$$

We can directly solve for the Tr of leptons using Dirac slash and Dirac matrices notations in FeynCalc,

In[ ]:=
```
Tr[DiracMatrix[α].(DiracSlash[p2] - ml).DiracMatrix[β].(DiracSlash[p3] + ml)]
```
Out[ ]=
$$-4\left(\mathrm{ml}^2\,\overline{g}^{\alpha\beta}+\overline{g}^{\alpha\beta}\left(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}\right)-\overline{\mathrm{p2}}^{\beta}\,\overline{\mathrm{p3}}^{\alpha}-\overline{\mathrm{p2}}^{\alpha}\,\overline{\mathrm{p3}}^{\beta}\right)$$

A.2 Trace of eq.6.46

In[2]:=
```
(*Here DiracSlash[p1] means γ^μ.p1_μ , DiracMatrix[α] means γ^α ,
DiracMatrix[5] means γ^5 , C1 = A+Bγ^5 , C2 = M+Nγ^5 ,
ak= (γ^α γ^μ.k_μ - γ^μ.k_μ γ^α) , similarly for bk *)
p1s = DiracSlash[p1];
ps = DiracSlash[p];
ks = DiracSlash[k];
a = DiracMatrix[α];
b = DiracMatrix[β];
C1 = A + B * DiracMatrix[5];
C2 = M + N * DiracMatrix[5];
ak = a.ks - ks.a;
bk = ks.b - b.ks;
p2s = DiracSlash[p2];
p3s = DiracSlash[p3];
```

A.3 Muon Decay

We can also directly find traces of matrices and contract them in FeynCalc , here shown the trace contraction of muon decay,

```
In[2]:= k1s = DiracSlash[k1];
        q2s = DiracSlash[q2];
        k2s = DiracSlash[k2];
        q1s = DiracSlash[q1];
        a = DiracMatrix[α];
        b = DiracMatrix[β];
        L = 1 - DiracMatrix[5];

In[9]:= Contract[Tr[(k2s + me).a.L.q1s.b.L] × Tr[(q2s).a.L.(k1s + mu).b.L]]
```

Out[9]= $128\,(\overline{k1}\cdot\overline{q1})\,(\overline{k2}\cdot\overline{q2}) + 128\,(\overline{k1}\cdot\overline{k2})\,(\overline{q1}\cdot\overline{q2}) + 64\,(2\,(\overline{k1}\cdot\overline{q1})\,(\overline{k2}\cdot\overline{q2}) - 2\,(\overline{k1}\cdot\overline{k2})\,(\overline{q1}\cdot\overline{q2}))$

```
In[10]:= Simplify[Contract[Tr[(k2s + me).a.L.q1s.b.L] × Tr[(q2s).a.L.(k1s + mu).b.L]]]
```

Out[10]= $256\,(\overline{k1}\cdot\overline{q1})\,(\overline{k2}\cdot\overline{q2})$

A.4 M^2 of $b \to s\ell^+\ell^-$

In[16]:=

```
Contract[Tr[C2.bk.(p1s + ms).C1.ak.(ps + mb)] × Tr[a.(p2s − mlp).b.(p3s + mlm)]]
```

Out[16]=

$$
\begin{aligned}
&-192\, A\, M \,\mathrm{mb}\,\mathrm{mlm}\,\mathrm{mlp}\,\mathrm{ms}\,\overline{k}^2 - 64\, A\, M\,\mathrm{mb}\,\mathrm{ms}\,\overline{k}^2\,(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) - 128\, A\, M\,\mathrm{mb}\,\mathrm{ms}\,(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{k}\cdot\overline{\mathrm{p3}}) - \\
&64\, A\, M\,\mathrm{mlm}\,\mathrm{mlp}\,\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p1}}) + 256\, A\, M\,\mathrm{mlm}\,\mathrm{mlp}\,(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p1}}) - 128\, A\, M\,\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) - \\
&128\, A\, M\,\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) + 64\, A\, M\,\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p1}})(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) - 128\, A\, M(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{p}\cdot\overline{\mathrm{p1}}) + \\
&128\, A\, M(\overline{k}\cdot\overline{\mathrm{p1}})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{p}\cdot\overline{\mathrm{p2}}) + 128\, A\, M(\overline{k}\cdot\overline{\mathrm{p1}})(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{p}\cdot\overline{\mathrm{p3}}) + \\
&128\, A\, M(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) + 128\, A\, M(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) - 192\, B\,\mathrm{mb}\,\mathrm{mlm}\,\mathrm{mlp}\,\mathrm{ms}\, N\overline{k}^2 - \\
&64\, B\,\mathrm{mb}\,\mathrm{ms}\, N\overline{k}^2\,(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) - 128\, B\,\mathrm{mb}\,\mathrm{ms}\, N(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{k}\cdot\overline{\mathrm{p3}}) + 64\, B\,\mathrm{mlm}\,\mathrm{mlp}\, N\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p1}}) - \\
&256\, B\,\mathrm{mlm}\,\mathrm{mlp}\, N(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p1}}) + 128\, B\, N\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) + 128\, B\, N\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) - \\
&64\, B\, N\overline{k}^2\,(\overline{p}\cdot\overline{\mathrm{p1}})(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) + 128\, B\, N(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{p}\cdot\overline{\mathrm{p1}}) - 128\, B\, N(\overline{k}\cdot\overline{\mathrm{p1}})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{p}\cdot\overline{\mathrm{p2}}) - \\
&128\, B\, N(\overline{k}\cdot\overline{\mathrm{p1}})(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{p}\cdot\overline{\mathrm{p3}}) - 128\, B\, N(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) - 128\, B\, N(\overline{k}\cdot\overline{p})(\overline{k}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}})
\end{aligned}
$$

In[17]:=

```
Simplify[Contract[Tr[C2.bk.(p1s + ms).C1.ak.(ps + mb)] × Tr[a.(p2s − mlp).b.(p3s + mlm)]]]
```

Out[17]=

$$
\begin{aligned}
-64 \Big(&2\,((\overline{k}\cdot\overline{\mathrm{p2}})((\overline{k}\cdot\overline{\mathrm{p3}})((\overline{p}\cdot\overline{\mathrm{p1}})(A M - B N) + \mathrm{mb}\,\mathrm{ms}\,(A M + B N)) + (\overline{k}\cdot\overline{\mathrm{p1}})(\overline{p}\cdot\overline{\mathrm{p3}})(B N - A M)) - \\
&\qquad (\overline{k}\cdot\overline{p})(A M - B N)(2\,\mathrm{mlm}\,\mathrm{mlp}\,(\overline{k}\cdot\overline{\mathrm{p1}}) + (\overline{k}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) + (\overline{k}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}})) + \\
&\qquad (\overline{k}\cdot\overline{\mathrm{p1}})(\overline{k}\cdot\overline{\mathrm{p3}})(\overline{p}\cdot\overline{\mathrm{p2}})(B N - A M)) + \\
&\overline{k}^2\,((\overline{p}\cdot\overline{\mathrm{p1}})(A M - B N)(\mathrm{mlm}\,\mathrm{mlp} - \overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) + 2\,(\overline{p}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}})(A M - B N) + \\
&\qquad A M\,\mathrm{mb}\,\mathrm{ms}\,(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) + 2\, A M(\overline{p}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) + B\,\mathrm{mb}\,\mathrm{ms}\, N(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) - \\
&\qquad 2\, B N(\overline{p}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) + 3\, A M\,\mathrm{mb}\,\mathrm{mlm}\,\mathrm{mlp}\,\mathrm{ms} + 3\, B\,\mathrm{mb}\,\mathrm{mlm}\,\mathrm{mlp}\,\mathrm{ms}\, N)\Big)
\end{aligned}
$$

After above simplification we change back to previous convection,

$$\begin{aligned} A &= \frac{(C_{7R}+C_{7L})}{2} && \text{(A.1a)} \\ B &= \frac{(C_{7R}-C_{7L})}{2} && \text{(A.1b)} \\ M &= \frac{(C^*_{7R}+C^*_{7L})}{2} && \text{(A.1c)} \\ N &= \frac{(C^*_{7L}-C^*_{7R})}{2} && \text{(A.1d)} \end{aligned}$$

Now,

$$\begin{aligned} AM-BN &= \frac{(|C_{7R}|^2+|C_{7L}|^2)}{2}, && \text{(A.2a)} \\ BN-AM &= \frac{-(|C_{7R}|^2+|C_{7L}|^2)}{2}, && \text{(A.2b)} \\ AM+BN &= \frac{(C_{7R}C^*_{7L}+C_{7L}C^*_{7R})}{2} && \text{(A.2c)} \\ AM &= \frac{(|C_{7R}|^2+|C_{7L}|^2+C_{7R}C^*_{7L}+C_{7L}C^*_{7R})}{4} && \text{(A.2d)} \\ BN &= \frac{(-|C_{7R}|^2-|C_{7L}|^2+C_{7R}C^*_{7L}+C_{7L}C^*_{7R})}{4} && \text{(A.2e)} \end{aligned}$$

New notations used for,

$$\begin{aligned} |C_{7R}|^2 &= C7R && \text{(A.3a)} \\ |C_{7L}|^2 &= C7L && \text{(A.3b)} \\ C_{7R}C^*_{7L} &= C7RL && \text{(A.3c)} \\ C_{7L}C^*_{7R} &= C7LR && \text{(A.3d)} \end{aligned}$$

```
In[19]:= Simplify[
  -64 (2 ((ScalarProduct[k, p2]) (ScalarProduct[k, p3] ((ScalarProduct[p, p1]) (C7R + C7L) / 2 +
              mb ms (C7RL + C7LR) / 2) - (ScalarProduct[k, p1] × ScalarProduct[p, p3]
             (C7R + C7L) / 2)) - (ScalarProduct[k, p]) ((C7R + C7L) / 2 )
          (2 ml ^ 2 (ScalarProduct[k, p1]) + (ScalarProduct[k, p3]) (ScalarProduct[p1, p2]) +
            (ScalarProduct[k, p2]) (ScalarProduct[p1, p3])) -
         (C7R + C7L) / 2 (ScalarProduct[k, p1]) (ScalarProduct[k, p3])
          (ScalarProduct[p, p2])) + (ScalarProduct[k, k])
       ((ScalarProduct[p, p1]) (C7R + C7L) / 2 (ml ^ 2 - ScalarProduct[p2, p3]) +
         2 (ScalarProduct[p, p3]) (ScalarProduct[p1, p2]) (C7R + C7L) / 2 +
         (C7R + C7L + C7RL + C7LR) / 4 mb ms (ScalarProduct[p2, p3]) +
         2 (C7R + C7L + C7RL + C7LR) / 4 (ScalarProduct[p, p2]) (ScalarProduct[p1, p3]) +
         (- C7R - C7L + C7RL + C7LR) / 4 mb ms (ScalarProduct[p2, p3]) -
         2 (-C7R - C7L + C7RL + C7LR) / 4 (ScalarProduct[p, p2]) (ScalarProduct[p1, p3]) +
         3 (C7R + C7L + C7RL + C7LR) / 4 mb ms ml ^ 2 + 3 (-C7R - C7L + C7RL + C7LR) / 4 mb ms ml ^ 2))]
```

Out[19]=

$$
\begin{aligned}
32 \Big(-\overline{k}^2 \big(&(\mathrm{C7L}+\mathrm{C7R})\,(\overline{p}\cdot\overline{\mathrm{p1}})\,(\mathrm{ml}^2-\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) + 2\,(\mathrm{C7L}+\mathrm{C7R})\,(\overline{p}\cdot\overline{\mathrm{p3}})\,(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) +\\
&2\,\mathrm{C7L}\,(\overline{p}\cdot\overline{\mathrm{p2}})\,(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) + \mathrm{C7LR}\,\mathrm{mb}\,\mathrm{ms}\,(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) + 2\,\mathrm{C7R}\,(\overline{p}\cdot\overline{\mathrm{p2}})\,(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}}) +\\
&\mathrm{C7RL}\,\mathrm{mb}\,\mathrm{ms}\,(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}) + 3\,\mathrm{C7LR}\,\mathrm{mb}\,\mathrm{ml}^2\,\mathrm{ms} + 3\,\mathrm{C7RL}\,\mathrm{mb}\,\mathrm{ml}^2\,\mathrm{ms}\big) -\\
&2\,(\overline{k}\cdot\overline{\mathrm{p2}})\,\big((\overline{k}\cdot\overline{\mathrm{p3}})\,((\mathrm{C7L}+\mathrm{C7R})\,(\overline{p}\cdot\overline{\mathrm{p1}}) + \mathrm{mb}\,\mathrm{ms}\,(\mathrm{C7LR}+\mathrm{C7RL})) - (\mathrm{C7L}+\mathrm{C7R})\,(\overline{k}\cdot\overline{\mathrm{p1}})\,(\overline{p}\cdot\overline{\mathrm{p3}})\big) +\\
&2\,(\mathrm{C7L}+\mathrm{C7R})\,(\overline{k}\cdot\overline{p})\,\big(2\,\mathrm{ml}^2\,(\overline{k}\cdot\overline{\mathrm{p1}}) + (\overline{k}\cdot\overline{\mathrm{p3}})\,(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}}) + (\overline{k}\cdot\overline{\mathrm{p2}})\,(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}})\big) +\\
&2\,(\mathrm{C7L}+\mathrm{C7R})\,(\overline{k}\cdot\overline{\mathrm{p1}})\,(\overline{k}\cdot\overline{\mathrm{p3}})\,(\overline{p}\cdot\overline{\mathrm{p2}})\Big)
\end{aligned}
$$

In all the calculation we didn't put the value of k , from vertex α we can see $k = (p - p_1)$ Here we substituting value of k , In[23]:=

```
FullSimplify[
  -64 (2 ((ScalarProduct[k, p2]) (ScalarProduct[k, p3] ((ScalarProduct[p, p1]) (C7R + C7L) / 2 +
            mb ms (C7RL + C7LR) / 2) - (ScalarProduct[k, p1] × ScalarProduct[p, p3]
           (C7R + C7L) / 2)) - (ScalarProduct[k, p]) ((C7R + C7L) / 2 )
        (2 ml ^ 2 (ScalarProduct[k, p1]) + (ScalarProduct[k, p3]) (ScalarProduct[p1, p2]) +
          (ScalarProduct[k, p2]) (ScalarProduct[p1, p3])) -
       (C7R + C7L) / 2 (ScalarProduct[k, p1]) (ScalarProduct[k, p3])
        (ScalarProduct[p, p2])) + (ScalarProduct[k, k])
     ((ScalarProduct[p, p1]) (C7R + C7L) / 2 (ml ^ 2 - ScalarProduct[p2, p3]) +
       2 (ScalarProduct[p, p3]) (ScalarProduct[p1, p2]) (C7R + C7L) / 2 +
       (C7R + C7L + C7RL + C7LR) / 4 mb ms (ScalarProduct[p2, p3]) +
       2 (C7R + C7L + C7RL + C7LR) / 4 (ScalarProduct[p, p2]) (ScalarProduct[p1, p3]) +
       (-C7R - C7L + C7RL + C7LR) / 4 mb ms (ScalarProduct[p2, p3]) -
       2 (-C7R - C7L + C7RL + C7LR) / 4 (ScalarProduct[p, p2]) (ScalarProduct[p1, p3]) +
       3 (C7R + C7L + C7RL + C7LR) / 4 mb ms ml ^ 2 +
       3 (-C7R - C7L + C7RL + C7LR) / 4 mb ms ml ^ 2))] /. {k → (p - p1)}
```

Out[23]=

$$
\begin{aligned}
&32\,\Big(-(\overline{p}-\overline{\mathrm{p1}})^2\,\Big((\mathrm{C7L}+\mathrm{C7R})\big((\overline{p}\cdot\overline{\mathrm{p1}})(\mathrm{ml}^2-\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}})+2\big((\overline{p}\cdot\overline{\mathrm{p3}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}})+(\overline{p}\cdot\overline{\mathrm{p2}})(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}})\big)\big)+\\
&\qquad\qquad \mathrm{mb}\,\mathrm{ms}\,(\mathrm{C7LR}+\mathrm{C7RL})\,(\overline{\mathrm{p2}}\cdot\overline{\mathrm{p3}}+3\,\mathrm{ml}^2)\Big)-\\
&\quad 2\,\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p2}}\big)\Big(\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p3}}\big)\big((\mathrm{C7L}+\mathrm{C7R})(\overline{p}\cdot\overline{\mathrm{p1}})+\mathrm{mb}\,\mathrm{ms}\,(\mathrm{C7LR}+\mathrm{C7RL})\big)-\\
&\qquad (\mathrm{C7L}+\mathrm{C7R})(\overline{p}\cdot\overline{\mathrm{p3}})\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p1}}\big)\Big)+2\,(\mathrm{C7L}+\mathrm{C7R})\big(\overline{p}\cdot(\overline{p}-\overline{\mathrm{p1}})\big)\\
&\quad\ \big(2\,\mathrm{ml}^2\,\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p1}}\big)+(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p2}})\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p3}}\big)+(\overline{\mathrm{p1}}\cdot\overline{\mathrm{p3}})\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p2}}\big)\big)+\\
&\quad 2\,(\mathrm{C7L}+\mathrm{C7R})(\overline{p}\cdot\overline{\mathrm{p2}})\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p1}}\big)\big((\overline{p}-\overline{\mathrm{p1}})\cdot\overline{\mathrm{p3}}\big)\Big)
\end{aligned}
$$

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