SOME ASPECTS OF ADS/CFT CORRESPONDENCE

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THESIS

by

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Some aspects of ADS/CFT correspondence

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By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is presented in this thesis entitled **Some aspects of ADS/CFT correspondence** in the partial fulfillment of the requirements for the award of the degree of **Master of Science** and submitted in the **Department of Physics, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July 2021 to June 2022 under the supervision of Dr. Debajyoti Sarkar, Assistant Professor, Department of Physics, Indian Institute of Technology Indore.

The matter presented in this thesis has not been submitted by me or by anyone else for the award of any other degree of this or any institute.

7/6/2022

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(Satyabrata Sahoo)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

Signature of the Supervisor

(Dr. Debajyoti Sarkar)

Satyabrata Sahoo has successfully given his M.Sc. Oral Examination held on **03 June 2022**.

Signature of supervisor of MSc thesis

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Date: 07.06.2022

Signature of PSPC member no. 2

Date: 08/06/2022

Science is an alliance of free spirits in all cultures rebelling against the local tyranny that each culture imposes on its children. Insofar as I am a scientist, my vision of the universe is not reductionist or anti-reductionist. I have no use for Western isms of any kind. I feel myself a traveler on the "Immense Journey" of the paleontologist Loren Eisely, a journey that is far longer than the history of nations and philosophies, longer even than the history of our species.

-Freeman Dyson
(The Scientist as Rebel)

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IIT Indore - 453 552 **Satyabrata Sahoo**

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Dedicated to my parents for giving me this opportunity and every single person working to fight the COVID-19 pandemic.

ABSTRACT

In the context of holography in string theory we study AdS/CFT which is the best candidate for quantum gravity and it gives a concrete theoretic realization to it. In AdS/CFT the extrapolate dictionary can be understood as the bulk fields being reconstructed from modular flow of boundary operators which reduces to OPE block reconstruction of bulk fields. Then we compute various OPE blocks with different spins in 2 and d dimensions and show that OPE blocks in 2 dimensions are Fourier modes of the modular Hamiltonian. Thus OPE blocks can be used as CFT input for bulk reconstructions.

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Acronyms and Nomenclature

- AdS:- Anti-de-sitter
- CFT:- Conformal field theory
- RT:- Ryu-Takayanagi
- OPE:- Operator product expansion
- Bulk:- The AdS space
- Boundary:- The CFT space
- (L,R):- corresponds to subregion of left and right endpoints of the causal diamond.
- (T,B):- corresponds to subregion of top and bottom endpoints of the causal diamond.
- H_{mod}:- modular Hamiltonian

Chapter 1

Introduction

All of physics is about studying interactions among different entities and the forces responsible for them. At present, the universe's functioning can be described by four forces: Gravitational force, Weak force, Electromagnetic force, and Strong Nuclear force, in the order of weakest to strongest. Even though we have achieved some breakthroughs in understanding the other three forces, gravity still remains a puzzle. Gravity, even if weak, must apply in the microworld. The theory of general relativity by Einstein was a turning point in the field, because of which we understand gravity at a classical level. However, still, it is not successful in explaining gravity at a quantum level. It is possible to embed the curved geometry of general relativity into how quantum field theory deals with space and time. Nevertheless, that approach completely fails when you have strong gravitational effects on the more minor scales of space and time, like the central singularity of the black hole or at the instant of the Big Bang. For that, you need a true quantum theory of gravity. General relativity and quantum mechanics seem to describe all of observable reality. However, it is not easy to merge them. We were able to formulate a quantum field theoretical description for the other three forces in which the fundamental object we consider is a 0 - dimensional point particle. Nevertheless, this approach was not successful in formulating a theory of quantum gravity.

However, if we consider a one-dimensional string as the fundamental object, we achieve a consistent formulation of quantum gravity. This theoretical model is called string theory. Quantum gravity in string theory incorporates the principles of both GR and QFT and can describe spacetime's microstructure at Planck's scale. In this, the gravitational waves themselves are quantized.

We live in a universe with three dimensions of space and one temporal dimension. Up, down, left, right, forward, back, past, future. 3+1 dimensions, or so our primitive Pleistocene-evolved brains find it helpful to believe. Moreover, we cling to this intuition, even as physics shows us that this view of reality may be only a very narrow perception. One of the most startling possibilities is that our 3+1 dimensional universe may be better described as resulting from a spacetime one dimension lower – like a hologram projected from a surface infinitely far away. In recent years, string theory and quantum field theory have been studied together in the form of holography, which connects quantum gravity in a specific (d+1)-dimensional spacetimes with corresponding (conformal) field theories on a d-dimensional flat spacetime. These developments and connections have deepened our understanding of quantum gravity, cosmology, particle physics, and intermediate-scale physics, such as condensed matter systems, the quark-gluon plasma, and disordered systems.
Leonard Susskind laid out the first steps towards how this could be achieved using string theory, which is traditionally interpreted as a realization of a vague "holographic principle" according to which quantum gravity in bulk spacetimes is controlled, in one way or other, by "boundary field theories" on effective spacetime boundaries, such as event horizons. but ultimately, it was Juan Maldacena who figured out a concrete string theoretic realization of the holographic principle with AdS/CFT correspondence.

In the next chapter, we will be seeing what the AdS/CFT duality is, the extrapolate dictionary, and various metrics and coordinate systems used in this thesis. The third chapter deals with a literature review of some of the past works like the HKLL prescription [1, 2, 3, 4], OPE blocks and the intersecting modular Hamiltonian. The final chapter interprets the extrapolate dictionary as bulk reconstruction from boundary modular flow and shows how the bulk fields can be constructed from the boundary OPE blocks of different spins. Thus OPE blocks can be used as CFT input for bulk reconstruction.

Chapter 2

Duality in field theory

We know a great deal about conformal field theory, and ever since the revolutionary work by Juan Maldacena [5], recovering AdS (or bulk) physics from the boundary CFT to understand quantum gravity has been our primary goal. So, if you take a cylinder the volume inside the cylinder is a 3 dimensional space whereas the boundary is like a 2 dimensionalspace, if the AdS sits in the volume or bulk of the cylinder the CFT is on the boundary of it. In itself, the boundary CFT isn't a string theory instead, it is a quantum field theory like the ones that give us our standard model of particle physics. It is invariant to the scaling of grid sizes. Due to the duality say the boundary 2 dimensional space becomes a 3 dimensional space. While the original space is flat, the new space has negative curvature – it is a hyperbolic, anti-de Sitter, or AdS space. The conformal field theory in the original space does not include gravity, but in the higher-dimensional space, it becomes a complete quantum theory of gravity. This is AdS/CFT duality.
In [5] the authors claim that any conformal field theory on $\mathbb{R} \times \mathbb{S}^{d-1}$ is equivalent to a theory of quantum gravity in an asymptotically AdS_{d+1} spacetime. This brings us to the question of how the observable on both the sides (boundary and bulk) are mapped. The answer is the AdS/CFT dictionary, which is still a work in progress but many things are known about it. It can be viewed as an isomorphism between the Hilbert spaces:

$$\phi : \mathscr{H}_{AdS} \longrightarrow \mathscr{H}_{CFT}$$

where ϕ is the map between the bulk and the boundary. This gives us the extrapolate dictionary

$$\lim_{r\to\infty} r^{\Delta}\ \phi_i(r,t,\Omega) = \mathscr{O}_i(t,\Omega)$$

where r is the radial bulk direction, Δ_i is the scaling dimension of conformal primary $\mathscr{O}_i$. This can be interpreted as we are approaching the boundary from the bulk limit, which is the extrapolate dictionary formalism. One such extrapolate dictionary formalism is the HKLL prescription in which a bulk field is equal to a boundary operator being smeared over a causal diamond using a kernel. We will discussing more about HKLL in the later sections.

We will now be taking a digression and talking about the different metrics in which the HKLL prescription is usually written down. The general CFT_d metric is

$$ds^2 = -dt^2 + dr^2 + r^2\ d\Omega_{d-2}^2 = g_{\mu\nu}\ dx^\mu\ dx^\nu \tag{2.1}$$

and then via the coordinate transformations

$$r = R\frac{\sinh\rho}{(\cosh\rho + \cosh\chi)},\quad t = R\frac{\sinh\chi}{(\cosh\rho+\cosh\chi)},\quad \Omega_{d-2} = \Omega_{d-2} \quad \text{with} \quad \Omega = \frac{R}{(\cosh\rho+\cosh\chi)} \tag{2.2}$$

where R is the radius of the spherical sub-region that we are considering in the boundary. The causal diamond of the sub-region R is mapped to

$$ds^2 = \frac{R^2}{(\cosh\rho+\cosh\chi)^2}\left(-d\chi^2 + d\rho^2 + \sinh^2\rho\ d\Omega_{d-2}^2\right) = \Omega^2\ g^R_{\mu\nu}\ dx^\mu_R\ dx^\nu_R\ . \tag{2.3}$$

However, later on we will be using lightcone coordinates consisting of r and t . So we coordinate transform to Poincaré and Rindler lightcone coordinates as $(z,\bar{z}) = (r-t, r+t)$ and $(w,\bar{w}) = (\rho-\chi, \rho+\chi)$ respectively. Then z and w relate to each other as

$$\begin{aligned} w &= \rho - \chi = f(z) = \log\tfrac{r-t+R}{R-r+t} = \log\tfrac{z+R}{R-z} \\ \bar{w} &= \rho + \chi = f(\bar{z}) = \log\tfrac{r+t+R}{R-r-t} = \log\tfrac{\bar{z}+R}{R-\bar{z}}\ . \end{aligned} \tag{2.4}$$

In these coordinates, the CFT metrics in (2.1) and (2.3) take the form

$$ds^2 = dz\ d\bar{z} + \left(\frac{z+\bar{z}}{2}\right)^2 d\Omega_{d-2}^2 \tag{2.5}$$

and

$$ds^2 = \Omega^2 \left(dw\ d\bar{w} + \sinh^2\left(\frac{w+\bar{w}}{2}\right) d\Omega_{d-2}^2\right) \quad . \tag{2.6}$$

Now, talking about the bulk metric, The metric for the AdS space is

$$ds^2 = -dU^2 - dv^2 + dX^2 + dY^2 \tag{2.7}$$

along with the constraint $-U^2 - V^2 + X^2 + Y^2 = -R_{AdS}^2$.

Then going to the Rindler patch which is a space of all the space-like connected points from a bulk field the metric becomes

$$ds^2 = -\frac{r^2 - r_+^2}{R_{AdS}^2} dt_R^2 + \frac{R_{AdS}^2}{r^2 - r_+^2} dr^2 + r^2\ d\tilde{\phi}^2 \tag{2.8}$$

If we perform the following coordinate transformations

$$\tilde{r} = \frac{r}{r_+}, \quad \rho = \frac{r_+\ \tilde{\phi}}{R_{AdS}}, \quad \chi = \frac{r_+\ t_R}{R_{AdS}^2}$$

the metric for the Rindler patch becomes

$$ds^2 = R_{AdS}^2 \left(-(\tilde{r}^2 - 1)\ d\chi^2 + \tilde{r}^2\ d\rho^2 + \frac{d\tilde{r}^2}{\tilde{r}^2 - 1}\right) \tag{2.9}$$

In what follows unless otherwise specified we will be writing the equations in the above mentioned metrics.

Chapter 3

AdS/CFT dictionary

In this chapter we will be discussing bulk reconstruction via 'HKLL' prescription, OPE blocks and intersecting modular Hamiltonians.

3.1 HKLL prescription

We start with a particular boundary to bulk map known as the HKLL prescription [1, 2, 3, 4]. This is a Lorentzian AdS/CFT correspondence in which we construct local operators in the bulk from non local operators in the boundary using a kernel, which is also known as the smearing function. The conformal primary is smeared over a region on the boundary which consists of all the points that are spacelike separated from the bulk field. This region is the causal diamond.

Now, the bulk field ϕ in (χ, ρ) coordinates can be written using the HKLL prescription as

$$\phi_{Rindler}(\tilde{r}, \chi, \rho) = C_{\Delta} \int_{\sigma>0} d\rho' \; d\chi' \left(\lim_{\tilde{r}' \to \infty} K(\tilde{r}, \chi, \rho | \tilde{r}', \chi + \chi', \rho + \rho') \right) \mathcal{O}_{\Delta}(\chi + \chi', \rho + i\rho') \tag{3.1}$$

where $\tilde{r}$ is a bulk parameter (not to be confused with r which is a boundary parameter) and K is the smearing function and which is given by

$$\lim_{\tilde{r}'\to\infty} K(\tilde{r},\chi,\rho|\tilde{r}',\chi+\chi',\rho+\rho') = \lim_{\tilde{r}'\to\infty}(\sigma/\tilde{r}')^{\Delta-2} = \left[\tilde{r}\left(\cos\rho' - (1-\tilde{r}^{-2})^{1/2}\cosh\chi'\right)\right]^{\Delta-2} . \tag{3.2}$$

The dictionary consists of natural boundary operators whose duals are simple, diffeomorphism-invariant bulk operators.

3.2 OPE blocks

We will look at a certain boundary quantity known as the OPE block. It was shown in [6] that the OPE blocks are dual to integrals of bulk fields along a minimal surface area (a geodesic in AdS_3) in the AdS_{d+1}. We are interested in one of the application of this in particuar in the studies of the modular Hamiltonian.

Now,we will define the modular Hamiltonian briefly. Consider a time slice and two points x_1 and x_2 on the boundary

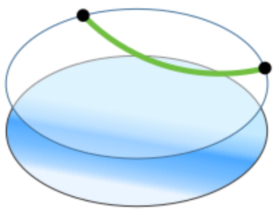

Figure 3.1: Two boundary points x_1 and x_2 on a time slice.

these two points form a causal diamond on the boundary as shown in figure 3.2.

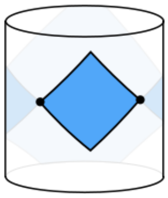

Figure 3.2: A causal diamond of two points (x_1, x_2) on the boundary.

Now, consider a subregion on the causal diamond whose density matrix $\rho_{x_1 x_2}$ is defined as $-\log\rho = 2\pi H_{mod}$ where H_{mod} is the boundary modular Hamiltonian. The line in the bulk connecting the points x_1 and x_2 denotes the minimal surface area in the bulk of the corresponding causal diamond in that time slice. The boundary modular Hamiltonian is an OPE block, and in case of the stress tensor OPE block commutes with the bulk fields on the minimal surface area in the bulk.

Now we come back to OPE blocks. In CFT, primaries $\mathscr{O}_i(0)$ and their descendants $\partial_\mu\partial_\nu\ldots\mathscr{O}_i(0)$ form a complete basis of states [6]. Consider a product of two scalar boundary operators which can be expanded (around x=0) in this basis as

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k C_{ijk}\ |x|^{\Delta_k-\Delta_i-\Delta_j}\left(1 + b_1 x^\mu\partial_\mu + b_2 x^\mu x^\nu\partial_\mu\partial_\nu + \ldots\right)\mathcal{O}_k(0)\ . \tag{3.3}$$

In more general terms, we get

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = |x_1 - x_2|^{-\Delta_i-\Delta_j}\sum_k C_{ijk}\ \mathcal{B}_k^{ij}(x_1, x_2)\ . \tag{3.4}$$

The $\mathcal{B}_k^{ij}(x_1, x_2)$ is known as the OPE block. Expressing it in lightcone coordinates (2.5) and applying the initial conditions (in CFT$_2$), we write it in terms of kinematic boundary-to-bulk propagators and get [6]

$$\mathcal{B}_k^{ij}(x_1, x_2) = \frac{\Gamma(2h_k)\Gamma(2\bar{h}_k)}{\Gamma(h_k)^2\Gamma(\bar{h}_k)^2}\int_{\Diamond 12} du\ d\bar{u}\left(\frac{(u-z_1)(z_2-u)}{z_2-z_1}\right)^{h_k-1}\left(\frac{(\bar{u}-\bar{z}_1)(\bar{z}_2-\bar{u})}{\bar{z}_2-\bar{z}_1}\right)^{\bar{h}_k-1} \tag{3.5}$$

where $(u, \bar{u})$ are also Rindler lightcone coordinates. In this way the product of scalar operators $\mathcal{O}_i(x_1)\mathcal{O}_j(x_2)$ can be expanded in terms of quasiprimary operators (in CFT$_d$) that are smeared over the causal diamond. This can also be shown in a different way relating to shadow operators, in which the OPE block (in CFT_d) becomes

$$\mathcal{B}_k^{ij}(x_1,x_2) \propto \int d^d z \; |x_1-x_2|^{\Delta_i+\Delta_j} \; \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\tilde{\mathcal{O}}_{k\mu\nu\ldots}(z)\rangle \mathcal{O}_k^{\mu\nu\cdots}(z) \ . \tag{3.6}$$

This way of writing the OPE block in terms of correlation functions is known as shadow operator formalism [7] in which $\tilde{\mathcal{O}}_{k\mu\nu\ldots}(z)$ is the shadow operator of $\mathcal{O}_k^{\mu\nu\cdots}(z)$. The dimensions of the shadow operator is $(d-\Delta)$ where Δ is the dimension of the boundary operator $\mathcal{O}_k^{\mu\nu\cdots}(z)$. Here the OPE block is of $\mathcal{O}_i(x_1)$ and $\mathcal{O}_j(x_2)$ in the $\mathcal{O}_k^{\mu\nu\cdots}(z)$ channel. This is better suited for higher dimensional cases. In case of two dimensions the OPE block can be written in terms of the boundary stress tensor, in which case (3.5) becomes

$$\mathcal{B}(x_1,x_2) = 6\int_{z_1}^{z_2} dw \; \frac{(z_2-w)(w-z_1)}{z_2-z_1} T(w) \ . \tag{3.7}$$

Here the OPE block is in stress tensor channel. The stress tensor OPE block is identical to the modular Hamiltonian, and therefore gives the commutation relation between the OPE block and the modular Hamiltonian. We will use this observation later in the thesis.

3.3 Bulk physics from intersecting modular Hamiltonians

In this section we discuss the method of reconstructing the bulk from intersecting modular Hamiltonians as shown in [8] and also from the OPE blocks.

We start with an ansatz (3.8), where $\Phi(X)$ commutes with the bulk modular Hamiltonian. Which is particular to $\mathrm{AdS}_3/\mathrm{CFT}_2$.

$$\Phi(X) = \int dt' \; dy' \; g(p,q) \; \mathcal{O}(q,p) \tag{3.8}$$

Then, we demand two such modular Hamiltonians for the same bulk field where the Ryu-Takayanagi surfaces (minimal area surfaces in AdS_3) of both the modular Hamiltonians are intersecting at the $\Phi(X)$ field point A as show in figure 3.3 . We thereby get the commutation of both the H_{mod} with the $\Phi(X)$ as

$$\left[\tilde{H}_{mod}^{x1,x2}, \Phi(\xi,\bar{\xi})\right] = 0, \left[\tilde{H}_{mod}^{x1',x2'}, \Phi(\xi,\bar{\xi})\right] = 0 \tag{3.9}$$

where $\tilde{H}_{mod,B} = H_{mod,B} - H_{mod,\bar{B}}$.

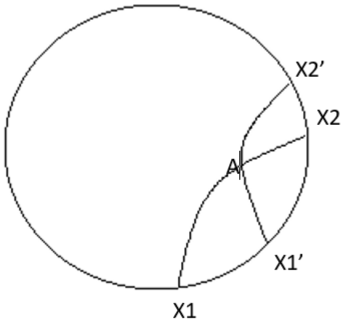

Figure 3.3: The intersection point is where the bulk field ϕ is

From [9] we know that the boundary modular flow is dual to the bulk modular flow and thereby reconstruct the bulk scalar field $\Phi(X)$ from boundary operators and get

$$\Phi\left(Z, X_0\right) = c_\Delta \int_{t'^2+y'^2<Z^2} dt'\ dy' \left(Z^2 - t'^2 - y'^2\right)^{\Delta-2}\ \mathcal{O}\left(t', X_0 + iy'\right)\ . \tag{3.10}$$

Which is indeed what we obtain from the HKLL prescription [1, 2, 3, 4] .

In the next chapter we will see how the OPE blocks in two dimensions are related to the bulk fields in AdS_3 which will then generalize to higher dimensions. We will then discuss some of the ongoing works in which we consider the higher dimensional case of the OPE blocks where the CFT operators may have spin. We will also be discussing some of the future prospects of this project which include finding the commutation between two different types of OPE blocks, reconstructing the bulk field from OPE blocks for higher dimensional cases with spin.

Chapter 4

Bulk reconstruction from OPE blocks

4.1 Rindler HKLL prescription

In order to reconstruct the AdS_3 bulk from the boundary CFT_2 OPE blocks, we start with the HKLL prescription (3.1). And for a given subregion (say $[L,R]$) the modular Hamiltonian is given by

$$H_{mod} = 2\pi \left\{ \int_L^R dz\, T_{zz}(z) \frac{(L-z)(R-z)}{L-R} + \int_L^R d\bar{z}\, T_{\bar{z}\bar{z}}(\bar{z}) \frac{(L-\bar{z})(R-\bar{z})}{L-R} \right\} \tag{4.1}$$

which generates a modular flow in the χ direction.

Writing the HKLL prescription with the boundary operator $\mathcal{O}$ in real space flowing under the action of the H_{mod}

$$\begin{aligned} &\phi_{\text{Rindler}}(\tilde{r}=1,\chi,\rho) \\ &= -iC_\Delta \int_{-i\pi/2}^{i\pi/2} ds_\rho \int_{-\infty}^{\infty} ds_\chi \, (\cosh s_\rho)^{\Delta-2} \; e^{-\frac{i\left(H_{\text{ mod },\rho} s_\rho + H_{\text{ mod }} s_\chi\right)}{2\pi}} \; \mathcal{O}_\Delta(\chi,\rho) \\ &e^{\frac{i\left(H_{\text{ mod }} s_\chi + H_{\text{ mod },\rho} s_\rho\right)}{2\pi}} \\ &= -iC_\Delta \int_{-i\pi/2}^{i\pi/2} ds_\rho \int_{-\infty}^{\infty} ds_\chi \, (\cosh s_\rho)^{\Delta-2} \; e^{-\frac{iH_{\text{ mod },\rho} s_\rho}{2\pi}} \; \mathcal{O}_\Delta\left(\chi + s_\chi, \rho\right) \; e^{\frac{iH_{\text{ mod },\rho} s_\rho}{2\pi}} \end{aligned} \tag{4.2}$$

where we have taken the $\tilde{r}=1$ limit, with $s_\chi = \chi'$ and $s_\rho = i\rho'$

And then by taking 'ω_{s_χ}' to be the Fourier mode of 's_χ' and a separate Fourier mode 'k_χ'

of ‘χ. So, we have

$$\mathcal{O}_\Delta\left(\rho,\chi+s_\chi\right)=\int_{-\infty}^{\infty}d\omega_{s_\chi}\ e^{-i\omega_{s_\chi}(\chi+s_\chi)}\ \mathcal{O}_\Delta\left(\rho,\omega_{s_\chi},\chi\right) \tag{4.3}$$

Then by doing the s_χ integral ω_{s_χ} goes to 0 and (72) becomes

$$\phi_{\text{Rindler}}\left(\tilde{r}=1,\chi,\rho\right)=-iC_\Delta\int_{-i\pi/2}^{i\pi/2}ds_\rho\left(\cosh s_\rho\right)^{\Delta-2}\ e^{\frac{-iH_{\text{ mod },\rho}\rho'}{2\pi}}\ \mathcal{O}_\Delta\left(\omega_{s\chi}=0,\rho,\chi\right)\ e^{\frac{+iH_{\text{ mod },\rho}s_\rho}{2\pi}}. \tag{4.4}$$

Then we take the Fourier mode of ρ as k_ρ

$$\begin{aligned}
&\phi_{\text{Rindler}}\left(\tilde{r}=1,\chi,\rho\right)\\
&=\int_{-\infty}^{\infty}dk_\rho\ e^{ik_\rho\rho}\ \phi_{\text{Rindler}}\ \left(\tilde{r}=1,\omega_{s_\chi}=0,k_\rho,\chi\right)\\
&=-iC_\Delta\int_{-\infty}^{\infty}dk_\rho\ e^{ik_\rho\rho}\left(\int_{-i\pi/2}^{i\pi/2}ds_\rho\left(\cosh s_\rho\right)^{\Delta-2}\ e^{-\frac{iH_{\text{ mod },\rho}s_\rho}{2\pi}}\ \mathcal{O}_\Delta\left(\omega_{s_\chi}=0,k_\rho,\chi\right)\ e^{\frac{iH_{\text{ mod },\rho}s_\rho}{2\pi}}\right)\\
&=2\pi^{\frac{3}{2}}\ C_\Delta\ \frac{\Gamma\left(\frac{\Delta-1}{2}\right)}{\Gamma\left(\frac{\Delta}{2}\right)}\int_{-\infty}^{\infty}dk_\rho\ e^{ik_\rho\rho}\ \mathcal{O}_\Delta\left(\omega_{s_\chi}=0,k_\rho,\chi\right).
\end{aligned} \tag{4.5}$$

The corresponding Fourier mode of χ is k_χ so, we have

$$\begin{aligned}
\mathcal{O}(\omega_{s_\chi}=0,k_\rho,\chi)&=\int_{-\infty}^{+\infty}dk_\chi\ e^{-ik_\chi\chi}\ \mathcal{O}(\omega_{s_\chi}=0,k_\chi,k_\rho)\\
\mathcal{O}(\omega_{s_\chi}=0,k_\rho,k_\chi)&=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\chi\ e^{+ik_\chi\chi}\ \mathcal{O}(\omega_{s_\chi}=0,\chi,k_\rho)
\end{aligned} \tag{4.6}$$

Taking an inverse Fourier transformation of (4.5) we get

$$\mathcal{O}(\omega_{s_\chi}=0,k_\rho,\chi)=\left(4\pi^{5/2}\ \mathcal{C}_\Delta\ \frac{\Gamma(\frac{\Delta-1}{2})}{\Gamma(\frac{\Delta}{2})}\right)^{-1}\int_{-\infty}^{+\infty}d\rho\ e^{-ik_\rho\rho}\ \phi_{Rindler}(\tilde{r}=1,\chi,\rho) \tag{4.7}$$

substituting (4.7) in to (4.6) we get

$$\begin{aligned}
\mathcal{O}(\omega_{s_\chi}=0,k_\rho,k_\chi)=\left(8\pi^{7/2}\ \mathcal{C}_\Delta\ \frac{\Gamma(\frac{\Delta-1}{2})}{\Gamma(\frac{\Delta}{2})}\right)^{-1}\ \int_{-\infty}^{+\infty}d\chi\ e^{+ik_\chi\chi}\ \int_{-\infty}^{+\infty}d\rho\ e^{-ik_\rho\rho}\\
\phi_{Rindler}(\tilde{r}=1,\chi,\rho)
\end{aligned} \tag{4.8}$$

Now going back to OPE blocks, a more generalised form of (3.5) which for scalar exchange becomes

$$\mathcal{B}^k_{\Delta,l}(L,R) = C_{\mathcal{O}} \int_{D_{z\bar{z}}} dz\, d\bar{z} \left(\frac{(z-L)(\bar{z}-L)}{(R-z)(R-\bar{z})}\right)^{h_R-h_L} \left(\frac{(z-L)(R-z)}{R-L}\right)^{h-1} \left(\frac{(\bar{z}-R)(L-\bar{z})}{R-L}\right)^{\bar{h}-1} \tag{4.9}$$

where [L,R] is the boundary sub-region with L and R as the horizontal endpoints of the boundary causal diamond, $D_{z\bar{z}}$ is the boundary domain of dependence of the sub-region [L,R] and $C_{\mathscr{O}}$ is the normalisation constant.

As discussed earlier L and R are the two endpoints of the causal diamond at which the corresponding CFT operators are of which the OPE block we have considered. The condition of $\tilde{r}=1$ where $\tilde{r}$ is a bulk parameter, ensures that the bulk field is on the Ryu-Takyanagi surface. Therefore we can write the OPE block in terms of the correlation functions after which (4.9) takes the form

$$\mathcal{B}_{(\Delta,l)}(L,R)^k = \frac{C_{\mathcal{O}}}{C_{h_L h_R \tilde{\mathcal{O}}}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\chi\, d\rho\, \langle \mathcal{O}_{h_L}(0,-\infty)\mathcal{O}_{h_R}(0,\infty)\tilde{\mathcal{O}}_{\tilde{\Delta}}(\chi,\rho)\rangle\, \mathcal{O}_{\Delta}(\chi,\rho) \tag{4.10}$$

where $C_{h_L h_R \tilde{\mathcal{O}}}$ is the OPE coefficient that appears in the conformal 3 point function. This is also known as 'Shadow operator formalism' of writing the OPE block [7]. Therefore, (4.10) becomes

$$\mathcal{B}_{(\Delta,l)}(L,R)^k = C_{\mathcal{O}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\chi\, d\rho\, e^{-ik_\rho \rho}\, \mathcal{O}_{\Delta}(\chi,\rho) = 4\pi^2\, C_{\mathcal{O}}\, \mathcal{O}(k_\chi=0,k_\rho) \tag{4.11}$$

Now, for $\mathcal{B}^k_{\mathcal{O}\mathcal{O}\mathcal{O}}$ in (L,R) case (4.8) is the zero χ mode i.e. $k_\chi=0$ so, we have

$$\mathcal{O}(\omega_{s_\chi}=0,k_\rho,k_\chi=0) = \left(4\pi^{5/2}\,\mathcal{C}_\Delta\,\frac{\Gamma(\frac{\Delta-1}{2})}{\Gamma(\frac{\Delta}{2})}\right)^{-1} \int_{-\infty}^{+\infty} d\rho\, e^{-ik_\rho \rho}\, \phi_{Rindler}(\tilde{r}=1,\chi,\rho). \tag{4.12}$$

Now, comparing (4.11) and (4.12) with a $zero^{th}$ χ mode i.e. $k_\chi=0$ we have

$$\frac{1}{4\pi^2 \mathcal{C}_{\mathcal{O}}}\mathcal{B}^k_{(\Delta,l)}(L,R) = \left(4\pi^{5/2}\,\mathcal{C}_\Delta\,\frac{\Gamma(\frac{\Delta-1}{2})}{\Gamma(\frac{\Delta}{2})}\right)^{-1} \int_{-\infty}^{+\infty} d\rho\, e^{-ik_\rho \rho}\, \phi_{Rindler}(\tilde{r}=1,\chi,\rho) \tag{4.13}$$

which is OPE blocks without spin in scalar channel as bulk fields integrated over the 'RT' surface. This can be thought of as the Rindler HKLL prescription at a given time slice but we have derived this from the boundary OPE blocks. In the next sub-section we will be discussing different OPE blocks with spin.

4.2 OPE blocks with different spins and dimensions

In this section we will be discussing the OPE blocks of the conformal spinning primaries. Since we will be considering the case of higher dimensions [10], instead of taking the boundary operators at the sides of the causal diamond, we will be considering the boundary operators at top and bottom positions of the causal diamond. The generalised expression then is

$$\begin{aligned}\mathcal{B}^k_{(\Delta,\ell)} &= C_d \int_D \sqrt{-g}\ dt\ dr\ d\Omega_{d-2} \left\langle J^{\mu_1\ldots\mu_{\ell_B}}_{\Delta_B,\ell_B}(B)\mathcal{O}_{\Delta_T,\ell_T=0}(T)\tilde{\mathcal{O}}_{\tilde{\Delta},\ell=0}(x)\right\rangle \mathcal{O}_{\Delta,\ell=0}(x)\\ &= \int_D \sqrt{-g}\ dt\ dr\ d\Omega_{d-2}\frac{C_d C_{J\mathcal{O}\tilde{\mathcal{O}}}\left(Z^{\mu_1}\ldots Z^{\mu_\ell B} - \text{ traces }\right)\mathcal{O}_{\Delta,\ell=0}(X)}{|(x-B)|^{\Delta_B+\tilde{\Delta}-\Delta_T-\ell_B}|(x-T)|^{\Delta_T+\tilde{\Delta}-\Delta_B+\ell_B}|(B-T)|^{\Delta_B+\Delta_T-\tilde{\Delta}-\ell_B}}\ .\end{aligned} \tag{4.14}$$

Here we have taken the bottom CFT operator to have spin and the top CFT operator to be a scalar but both are in higher dimensions. The second line of (4.14) has the expansion of the 3 point function.

We can write the correlator in (4.14) between a higher spin conserved current and two scalars in a different way where the indices of the higher spin conserved current are contracted by a null vector n^μ. (4.14) then becomes

$$\begin{aligned}\mathcal{B}^k_{(\Delta,\ell)} &= C_d \int_D \sqrt{-g}\ dt\ dr\ d\Omega_{d-2} \left\langle J_{\Delta_B,\ell_B}(B)\mathcal{O}_{\Delta_T,\ell_T=0}(T)\tilde{\mathcal{O}}_{\tilde{\Delta},\ell=0}(x)\right\rangle \mathcal{O}_{\Delta,\ell=0}(x)\\ &= \int_D \sqrt{-g}\ dt\ dr\ d\Omega_{d-2}\frac{C_d C_{J\mathcal{O}\tilde{\mathcal{O}}}\left(n_\mu Z^\mu\right)^{\ell_B}\mathcal{O}_{\Delta,\ell=0}(X)}{|(x-B)|^{\Delta_B+\tilde{\Delta}-\Delta_T-\ell_B}|(x-T)|^{\Delta_T+\tilde{\Delta}-\Delta_B+\ell_B}|(B-T)|^{\Delta_B+\Delta_T-\tilde{\Delta}-\ell_B}}\end{aligned} \tag{4.15}$$

with

$$n_\mu Z^\mu = \frac{n\cdot x_{B_x}}{x^2_{B_x}} - \frac{n\cdot x_{BT}}{x^2_{BT}}\ . \tag{4.16}$$

Then finally doing a coordinate transformation of (4.14) from (r,t) coordinates to (χ,ρ) Rindler coordinates we get

$$\begin{aligned}\mathcal{B}^k_{(\Delta,\ell)} &= C_d\, C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g(w)}\; dw\; d\bar{w}\; e^{i\chi(\ell_B+\Delta_{TB})} \\ &\left[\int d\Omega_{d-2}\;\left(Z^{\mu_1}\ldots Z^{\mu_\ell} - \text{traces}\right)\right] \mathcal{O}_{\Delta,\ell=0}(w,\bar{w},\phi) \\ &= C_d\, C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\; d\rho\; d\chi\; e^{i\chi(\ell_B+\Delta_{TB})} \\ &\left[\int d\Omega_{d-2}\;\left(Z^{\mu_1}\ldots Z^{\mu_\ell} - \;\text{traces}\;\right)\right] \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi)\;.\end{aligned} \tag{4.17}$$

similarly (4.15) becomes

$$\mathcal{B}^k_{(\Delta,\ell)} = C_d\, C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\; d\rho\; d\chi\; e^{i\chi(\ell_B+\Delta_{TB})} \left[\int d\Omega_{d-2}\,(n_\mu Z^\mu)^{\ell_B}\right] \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi)\;. \tag{4.18}$$

We see that the $n_\mu Z^\mu$ part takes care of the spin.

4.2.1 OPE block of (T,B) in d dimensions in case of scalars at 'T' and 'B' in scalar channel: $B_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}}$

We had shown the scalar OPE block in scalar channel for the (L,R) case in (4.11). Now we will compute it for the (T,B) case. We start with the general expression of the OPE block

$$\mathcal{B}^k_{(\Delta,\ell)} = C_d \int_D \sqrt{-g}\; dt\; dr\; d\Omega_{d-2} \left\langle \mathcal{O}_{\Delta_T}(T)\mathcal{O}_{\Delta_B}(B)\tilde{\mathcal{O}}_{\tilde{\Delta},\ell_x=0}(x)\right\rangle \mathcal{O}_{\Delta,\ell_x=0}(x)$$

Now, coordinate transforming to lightcone coordinates (2.5) $(z,\bar{z})$ we get

$$\begin{aligned} &= C_d \int_D \sqrt{-g(z)}\; dz\; d\bar{z}\; d\Omega_{d-2} \left\langle \mathcal{O}_{\Delta_T}(T,\bar{T})\mathcal{O}_{\Delta_B}(B,\bar{B})\tilde{\mathcal{O}}_{\tilde{\Delta},\ell_x=0}(x,\bar{x})\right\rangle \mathcal{O}_{\Delta,\ell_x=0}(x,\bar{x}) \\ &= C_d \int_D \sqrt{-g(z)}\; dz\; d\bar{z}\; d\Omega_{d-2} \frac{C_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}}\mathcal{O}_{\Delta,\ell_x=0}(x,\bar{x})}{|(B-T)|^{\frac{\Delta_B+\Delta_T-\tilde{\Delta_x}}{2}}|(x-T)|^{\frac{\Delta_T+\tilde{\Delta}-\Delta_b}{2}}|(x-B)|^{\frac{\Delta_B+\tilde{\Delta}-\Delta_T}{2}}} \\ &\frac{1}{|(\bar{B}-\bar{T})|^{\frac{\Delta_T+\Delta_B-\tilde{\Delta_x}}{2}}|(\bar{x}-\bar{T})|^{\frac{\Delta_T+\tilde{\Delta}-\Delta_B}{2}}|(\bar{x}-\bar{B})|^{\frac{\Delta_B+\tilde{\Delta}-\Delta_T}{2}}} \\ &= C_d\, C_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g(z)}\; dz\; d\bar{z}\; d\Omega_{d-2}\; I_1\; I_2\; \mathcal{O}_{\Delta,\ell_x=0}(x,\bar{x}) \end{aligned} \tag{4.19}$$

where

$$
\begin{aligned}
I_1 &= \left(\frac{|x-T||x-B||\bar{x}-\bar{T}||\bar{x}-\bar{B}|}{|B-T||\bar{B}-\bar{T}|}\right)^{\frac{l_x-\tilde{\Delta}}{2}} = \left(\frac{\partial \bar{z}}{\partial \bar{w}} \cdot \frac{\partial z}{\partial w}\right) \\
I_2 &= \left(\frac{|x-B||\bar{x}-\bar{B}|}{|x-T||\bar{x}-\bar{T}|}\right)^{\frac{\Delta_{TB}}{2}} = e^{\chi \Delta_{TB}}
\end{aligned}
\tag{4.20}
$$

Here a factor of $(|B-T||\bar{B}-\bar{T}|)^{\frac{-\Delta_T-\Delta_B}{2}}$ is absorbed in C_d.

Now, using the following relations we will transform it to Poincaré coordinates $(w, \bar{w})$ and then to Rindler coordinates (ρ, χ).

$$
\sqrt{-g(z)} = \frac{1}{2}\left(\frac{z+\bar{z}}{2}\right)^{d-2} \tag{4.21}
$$

$$
\sqrt{-g(w)} = \frac{1}{2}\left[\sinh\left(\frac{w+\bar{w}}{2}\right)\right]^{d-2} \tag{4.22}
$$

$$
\frac{\sqrt{-g(z)}}{\sqrt{-g(w)}} = \left(\frac{R}{2\ \cosh\frac{w}{2}\ \cosh\frac{\bar{w}}{2}}\right)^{d-2} \tag{4.23}
$$

$$
\partial w\ \partial \bar{w} = \partial z\ \partial \bar{z} \left(\frac{\partial \bar{z}}{\partial \bar{w}} \cdot \frac{\partial z}{\partial w}\right)^{-1} \tag{4.24}
$$

$$
\mathcal{O}_{\Delta,l=0}(w,\bar{w},\phi) = \left(\frac{\partial \bar{z}}{\partial \bar{w}} \cdot \frac{\partial z}{\partial w}\right)^{\frac{\Delta}{2}} \mathcal{O}_{\Delta,l=0}(z,\bar{z},\phi) \tag{4.25}
$$

$$
\frac{1}{\cosh^2\frac{w}{2}} = \frac{4}{B-T}\frac{\partial z}{\partial w} \quad \text{and} \quad \frac{1}{\cosh^2\frac{\bar{w}}{2}} = \frac{4}{\bar{T}-\bar{B}}\frac{\partial \bar{z}}{\partial \bar{w}} \tag{4.26}
$$

we get,

$$
\begin{aligned}
B^k_{(\Delta,l)} &= C_d\ C_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g(w)}\ dw\ d\bar{w}\ d\Omega_{d-2} \left(\frac{R}{2cosh\frac{w}{2}cosh\frac{\bar{w}}{2}}\right)^{d-2} \left(\frac{\partial \bar{z}}{\partial \bar{w}} \cdot \frac{\partial z}{\partial w}\right)^{1-\frac{d}{2}} \\
&\quad e^{i\chi(\Delta_{TB})} \mathcal{O}_{\Delta,l=0}(w,\bar{w},\phi) \\
B^k_{(\Delta,l)} &= C_d\ C_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g(w)}\ dw\ d\bar{w}\ d\Omega_{d-2}\ e^{i\chi(\Delta_{TB})}\ \mathcal{O}_{\Delta,l=0}(w,\bar{w},\phi)
\end{aligned}
\tag{4.27}
$$

Here the factor $\left(\frac{4R^2}{(B-T)(\bar{T}-\bar{B})}\right)^{\frac{d-2}{2}}$ is equal to 1.

The OPE block of (T,B) in d dimensions in case of scalars at 'T' and 'B' in scalar channel in Rindler coordinates is

$$
B^k_{(\Delta,l)} = C_d\ C_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ d\Omega_{d-2}\ e^{i\chi(\Delta_{TB})}\ \mathcal{O}_{\Delta,l=0}(\rho,\chi,\phi). \tag{4.28}
$$

4.2.2 OPE block of (T,B) and (L,R) in 2 dimensions with a stress tensor as one of the operators in scalar channel: $\mathcal{B}_{T\mathcal{O}\tilde{\mathcal{O}}}$

If we specify the case of two dimensions but with operators inserted on top and bottom points of the causal diamond then the conformal spinning primary is the stress tensor.

Therefore, we can write the OPE block in terms of the stress tensor

$$\begin{aligned}
\mathcal{B}^{k\ (T,B)}_{T_{zz}\mathcal{O}\mathcal{O}} &= C_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{\chi(2+\Delta_{TB})} \left[(Z^{\mu_1} \dots Z^{\mu_{\ell_B}} -\ \text{traces}\ )\right] \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \\
&= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{\frac{\bar{w}-w}{2}(2+\Delta_{TB})} \left(e^{2w}\right) \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \\
&= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{\frac{\bar{w}+w}{2}(2)}\ e^{\frac{\bar{w}-w}{2}(\Delta_{TB})} \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \\
&= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{2\rho}\ e^{\chi\Delta_{TB}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi)\ .
\end{aligned} \tag{4.29}$$

in (4.29) we have used T_{zz}, similarly if we use $T_{\bar{z}\bar{z}}$ we get

$$\begin{aligned}
\mathcal{B}^{k\ (T,B)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} &= C_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi`e^{\chi(2+\Delta_{TB})} \left[(Z^{\mu_1} \dots Z^{\mu_{\ell_B}} -\ \text{traces}\ )\right] \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \\
&= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{\frac{\bar{w}-w}{2}(2+\Delta_{TB})} \left(e^{-2\bar{w}}\right) \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \\
&= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{-2\rho}\ e^{\chi\Delta_{TB}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi)\ .
\end{aligned} \tag{4.30}$$

Similarly for the (L,R) case we have

$$\mathcal{B}^{k\ (L,R)}_{T_{zz}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{2\chi}\ e^{-\rho\Delta_{LR}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.31}$$

$$\mathcal{B}^{k\ (L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{-2\chi}\ e^{-\rho\Delta_{LR}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.32}$$

We have now computed the expression of the OPE blocks for two dimensional case with one operator having spin.
We see that the expression of the OPE block in two dimensions in case of higher spins is trivial as the relevant components are either $zz...z$ or $\bar{z}\bar{z}...\bar{z}$ components. Then the trace part becomes zero as the metric is again g_{zz} or $g_{\bar{z}\bar{z}}$ and finally we will get l_B powers of Z_z

or $Z_{\bar{z}}$. The final expression for (T,B) case is then

$$\mathcal{B}^{k\ (T,B)}_{J_{zz\ldots z}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{i\rho\ell_{BT}}\ e^{i\chi\Delta_{TB}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.33}$$

$$\mathcal{B}^{k\ (T,B)}_{J_{\Sigma\bar{z}\ldots\bar{z}}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{-i\rho\ell_{BT}}\ e^{i\chi\Delta_{TB}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi). \tag{4.34}$$

And for (L,R) case is

$$\mathcal{B}^{k\ (L,R)}_{J_{zz\ldots z}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{i\chi\ell_{LR}}\ e^{-i\rho\Delta_{LR}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.35}$$

$$\mathcal{B}^{k\ (L,R)}_{J_{\Sigma\bar{z}\ldots\bar{z}}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{J\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{-i\chi\ell_{LR}}\ e^{-i\rho\Delta_{LR}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi). \tag{4.36}$$

4.2.3 OPE block of (T,B) in d dimensions in case of scalar at 'T' and a stress tensor at 'B' in scalar channel: $\mathcal{B}_{T\mathcal{O}\tilde{\mathcal{O}}}$

Now, we are going to compute the OPE block in higher dimensions but with the bottom (or can take top) operator to be the stress tensor (2 spin operator) in a scalar CFT operator channel as for more than 2 dimensions the (L,R) case becomes invalid because say in 3 dimensions the causal diamond is like two cones stuck together along their bottom circular plane and we have only two endpoints which are the top and bottom endpoints. The metric for higher dimensional CFT is

$$ds^2 = dz\ d\bar{z} + \left(\frac{z+\bar{z}}{2}\right)^2 d\Omega^2_{d-2}. \tag{4.37}$$

The expression for the OPE block is then

$$\mathcal{B}^{k}_{T_{zz}}\mathcal{O}\mathcal{O} = \tilde{C}_d\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{2i\rho}\ e^{i\chi\Delta_{TB}} \left(\int d\Omega_{d-2}\right)\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.38}$$

and

$$\mathcal{B}^{k}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} = \tilde{C}_d\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{-2i\rho}\ e^{i\chi\Delta_{TB}} \left(\int d\Omega_{d-2}\right)\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi). \tag{4.39}$$

But in case of more than two dimensions we have the ϕ component along with z and $\bar{z}$ which gives us more spin parts such as that of $\langle T_{\phi\phi}\mathcal{O}\mathcal{O}\rangle$, $\langle T_{\bar{z}\phi}\mathcal{O}\mathcal{O}\rangle$ and $\langle T_{z\phi}\mathcal{O}\mathcal{O}\rangle$. To calculate the OPE block of each of these components we need the respective spin parts which are

$$
\begin{aligned}
\langle T_{z\bar{z}}\mathcal{O}\mathcal{O}\rangle|_{spin\ part} &= Z_z Z_{\bar{z}} - \frac{1}{d}\ g_{z\bar{z}}\ Z^2 \\
\langle T_{\phi\phi}\mathcal{O}\mathcal{O}\rangle|_{spin\ part} &= Z_\phi Z_\phi - \frac{1}{d}\ g_{\phi\phi}\ Z^2 \\
\langle T_{\bar{z}\phi}\mathcal{O}\mathcal{O}\rangle|_{spin\ part} &= Z_z Z_\phi \\
\langle T_{z\phi}\mathcal{O}\mathcal{O}\rangle|_{spin\ part} &= Z_{\bar{z}} Z_\phi
\end{aligned}
\tag{4.40}
$$

where

$$
Z^2 = g_{z\bar{z}}\ Z^z Z^{\bar{z}} + g_{z\bar{z}}\ Z^{\bar{z}} Z^z + g_{\phi\phi}\ Z^\phi Z^\phi \tag{4.41}
$$

The Z_ϕ goes to zero as the bottom point is a pole for the ϕ component, thereby the OPE blocks for $\langle T_{\bar{z}\phi}\mathcal{O}\mathcal{O}\rangle$ and $\langle T_{z\phi}\mathcal{O}\mathcal{O}\rangle$ becomes zero

Now, the first line of (4.42) becomes

$$
\langle T_{z\bar{z}}\mathcal{O}\mathcal{O}\rangle|_{spin\ part} = \frac{e^{-2\chi}}{(z_{BT})^2}\left(\frac{1}{4} - \frac{1}{2d}\right) \tag{4.42}
$$

and the corresponding OPE block is

$$
\mathcal{B}^k_{T_{z\bar{z}}\mathcal{O}\mathcal{O}} = C_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{\chi(2+\Delta_{TB})}\left[\frac{e^{-2\chi}}{(z_{BT})^2}\left(\frac{1}{4} - \frac{1}{2d}\right)\right]\left(\int d\Omega_{d-2}\right)\mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.43}
$$

The second line of (4.40) also simplifies to

$$
\begin{aligned}
\langle T_{\phi\phi}\mathcal{O}\mathcal{O}\rangle|_{spinpart} &= -\frac{1}{d}\ g_{\phi\phi}\ Z^z Z^{\bar{z}} \\
&= -\frac{1}{d}\left(\frac{z_x + \bar{z}_x}{2}\right)^2 \frac{e^{-2\chi}}{(z_{BT})^2}
\end{aligned}
\tag{4.44}
$$

and the corresponding OPE block is

$$
\begin{aligned}
\mathcal{B}^k_{T_{\phi\phi}\mathcal{O}\mathcal{O}} =& C_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{\chi(2+\Delta_{TB})}\left[-\frac{1}{d}\left(\frac{R\cdot 4\ \sinh\rho}{2(2\ \cosh\rho + \cosh\chi)}\right)^2 \frac{e^{-2\chi}}{(z_{BT})^2}\right]\left(\int d\Omega_{d-2}\right) \\
&\mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi)
\end{aligned}
\tag{4.45}
$$

4.2.4 OPE blocks with spin as bulk fields integrated over the RT surface weighted with appropriate momentum in two dimensions

We have calculated the OPE block of (T,B) subregion with a stress tensor at 'B'. Now we will see the corresponding bulk field which will be nothing but a bulk field integrated over the 'RT surface' of the Rindler causal wedge with appropriate momentum.

Now, let's first take $\mathcal{B}^k_{T_{zz}\mathcal{O}\mathcal{O}}$ (4.29) in (T,B) case in two dimensions where we have a non zero χ mode for the bulk field integrated over the RT surface (4.8). We begin with the OPE block

$$\mathcal{B}^k_{T_{zz}\mathcal{O}\mathcal{O}} = \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{2\rho}\ e^{\chi\Delta_{TB}}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \tag{4.46}$$

Defining $-l_{TB} = -ik_\rho$, $\Delta_{TB} = ik_\chi$ we get,

$$\begin{aligned}
\mathcal{B}^k_{T_{zz}\mathcal{O}\mathcal{O}} &= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}} \int_D \sqrt{-g^R}\ d\rho\ d\chi\ e^{-ik_\rho\rho}\ e^{ik_\chi\chi}\ \mathcal{O}_{\Delta,\ell=0}(\rho,\chi,\phi) \\
\mathcal{B}^k_{T_{zz}\mathcal{O}\mathcal{O}} &= \tilde{C}_2\ C_{T\mathcal{O}\tilde{\mathcal{O}}}\ \mathcal{O}(k_\rho,k_\chi)
\end{aligned} \tag{4.47}$$

The constants were absorbed in $\tilde{C}_2$. Equating (4.47) and (4.8) we get

$$\mathcal{B}^k_{T_{zz}\mathcal{O}\mathcal{O}} = [constants...] \int_{-\infty}^{+\infty} d\chi\ e^{-ik_\chi\chi} \int_{-\infty}^{+\infty} d\rho\ e^{-ik_\rho\rho}\ \phi_{Rindler}(\tilde{r}=1,\chi,\rho) \tag{4.48}$$

upto a factor of some constants. Similarly for $\mathcal{B}^k_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}$ we will define $-l_{TB} = ik_\rho$ and $\Delta_{TB} = ik_\chi$ we get,

$$\mathcal{B}^k_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} = [constants...] \int_{-\infty}^{+\infty} d\chi\ e^{-ik_\chi\chi} \int_{-\infty}^{+\infty} d\rho\ e^{-ik_\rho\rho}\ \phi_{Rindler}(\tilde{r}=1,\chi,\rho) \tag{4.49}$$

Which is the OPE blocks with spin in scalar channel as bulk fields integrated over the RT surface with appropriate momentum as k_χ in two dimensions.

In higher dimensions the OPE blocks are not equal to the bulk field integrated over the RT surface.

4.3 Modular Hamiltonian and OPE blocks

In this section we establish the relation between H_{mod} and scalar OPE blocks in stress tensor channel and then see the commutation of H_{mod} and $H_{mod,\rho}$ with spinning OPE blocks in scalar channel.
Where

$$\begin{aligned} H_{mod} &= H^R_{mod} + H^L_{mod} \\ &= 2\pi \Big\{ \int_L^R dz\, T_{zz}(z) \frac{(L-z)(R-z)}{L-R} + \int_L^R d\bar{z}\, T_{\bar{z}\bar{z}}(\bar{z}) \frac{(L-\bar{z})(R-\bar{z})}{L-R} \Big\} \end{aligned} \tag{4.50}$$

which creates a modular flow in 'χ' direction and

$$\begin{aligned} H_{mod,\rho} &= H^R_{mod} - H^L_{mod} \\ &= 2\pi \Big\{ \int_L^R dz\, T_{zz}(z) \frac{(L-z)(R-z)}{L-R} - \int_L^R d\bar{z}\, T_{\bar{z}\bar{z}}(\bar{z}) \frac{(L-\bar{z})(R-\bar{z})}{L-R} \Big\} \end{aligned} \tag{4.51}$$

which creates a modular flow in 'ρ' direction.

4.3.1 OPE block of (L,R) in 2 dimension in stress tensor channel and it's similarities with modular Hamiltonian: $\mathcal{B}_{\mathcal{O}\mathcal{O}\tilde{T}}$

Here we are trying to prove that scalar OPE block in stress tensor channel is equal to modular Hamiltonian. It was already done in [6] but we are doing it in the 'Shadow operator formalism' and for both (L,R) and (T,B) cases. We start with the general expression of the OPE block

$$\begin{aligned} \mathcal{B}^k_{(\Delta,\ell)} &= C_2 \int_D \sqrt{-g}\, dt\, dr\, \Big\langle \mathcal{O}_{\Delta_R}(R) \mathcal{O}_{\Delta_L}(L) \tilde{T}_{\tilde{\Delta},\ell_x=2}(x) \Big\rangle T_{\Delta,\ell_x=2}(x) \\ &= C_2 \int_D \sqrt{-g(z)}\ dz\ d\bar{z} \Big\langle \mathcal{O}_{\Delta_R}(R,\bar{R}) \mathcal{O}_{\Delta_L}(L,\bar{L}) \tilde{T}_{\tilde{\Delta},\ell_x=2}(x,\bar{x}) \Big\rangle T_{\Delta,\ell_x=2}(x,\bar{x}) \\ &= C_2 \int_D \sqrt{-g(z)}\ dz\ d\bar{z} \frac{C_{\mathcal{O}\mathcal{O}\tilde{T}} T_{\Delta,\ell_x=2}(x,\bar{x})}{|(L-R)|^{\frac{\Delta_R+\Delta_L-\tilde{\Delta}+\ell_x}{2}} |(R-x)|^{\frac{\Delta_R+\tilde{\Delta}-\Delta_L-\ell_x}{2}} |(x-L)|^{\frac{\Delta_L+\tilde{\Delta}-\ell_x-\Delta_R}{2}}} \\ &\quad \frac{1}{|(\bar{L}-\bar{R})|^{\frac{\Delta_R+\Delta_L-\tilde{\Delta}_x+\ell_x}{2}} |(\bar{R}-\bar{x})|^{\frac{\Delta_R+\tilde{\Delta}-\Delta_L-\ell_x}{2}} |(\bar{x}-\bar{L})|^{\frac{\Delta_L+\tilde{\Delta}-\ell_x-\Delta_R}{2}}} \\ &= C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)}\ dz\ d\bar{z}\ I_1\ I_2\ T_{\Delta,\ell_x=2}(x,\bar{x}) \end{aligned} \tag{4.52}$$

where

$$I_1 = \left(\frac{|R-x||x-L||\bar{R}-\bar{x}||\bar{x}-\bar{L}|}{|L-R||\bar{L}-\bar{R}|} \right)^{\frac{l_x - \tilde{\Delta}}{2}}$$
$$I_2 = \left(\frac{|R-x||\bar{R}-\bar{x}|}{|x-L||\bar{x}-\bar{L}|} \right)^{\frac{\Delta_{LR}}{2}} . \tag{4.53}$$

Here a factor of $(|L-R||\bar{L}-\bar{R}|)^{\frac{\Delta_R + \Delta_L}{2}}$ is absorbed.

Say in time slice $t = 0$ the coordinates are as follows:

$[L(z), \bar{L}(\bar{z}) = (R, -R)$; $[R(z), \bar{R}(\bar{z})] = (-R, R)$ and $e^w = \frac{z-L}{R-z}$; $e^{\bar{w}} = \frac{\bar{z}-\bar{R}}{\bar{L}-\bar{z}}$.

Here the spin parts do not appear as in other OPE blocks because of the stress tensor being the channel operator and not the OPE operator. Therefore the OPE block becomes the sum of two components where one component is the T_{zz} part,

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}} = C_2 \, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)} \; dz \; d\bar{z} \left[\frac{|R-z||z-L||\bar{R}-\bar{z}||\bar{z}-\bar{L}|}{|L-R||\bar{L}-\bar{R}|} \right]^{\frac{l_x - \tilde{\Delta}}{2}} \left[\frac{|R-z||\bar{R}-\bar{z}|}{|z-L||\bar{z}-\bar{L}|} \right]^{\frac{\Delta_{LR}}{2}} T_{\bar{z}\bar{z}} \tag{4.54}$$

and the other component is the $T_{\bar{z}\bar{z}}$ part

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}} = C_2 \, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)} \; dz \; d\bar{z} \left[\frac{|R-z||z-L||\bar{R}-\bar{z}||\bar{z}-\bar{L}|}{|L-R||\bar{L}-\bar{R}|} \right]^{\frac{l_x - \tilde{\Delta}}{2}} \left[\frac{|R-z||\bar{R}-\bar{z}|}{|z-L||\bar{z}-\bar{L}|} \right]^{\frac{\Delta_{LR}}{2}} T_{\bar{z}\bar{z}}. \tag{4.55}$$

Therefore the OPE blocks is:

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}} + B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}} = C_2 \, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)} \; dz \; d\bar{z} \left[\frac{|R-z||z-L||\bar{R}-\bar{z}||\bar{z}-\bar{L}|}{|L-R||\bar{L}-\bar{R}|} \right]^{\frac{l_x - \tilde{\Delta}}{2}} \left[\frac{|R-z||\bar{R}-\bar{z}|}{|z-L||\bar{z}-\bar{L}|} \right]^{\frac{\Delta_{LR}}{2}} \big\{ T_{zz} + T_{\bar{z}\bar{z}} \big\}. \tag{4.56}$$

Now, if we absorb $\sqrt{-g(z)} = \frac{1}{2}$ in constants, put $\tilde{\Delta} = 0$, $\Delta_{LR} = 0$, $l_x = 2$ and integrate out the $d\bar{z}$ in $B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}}$ part and dz in $B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}}$ we get

$$
\begin{aligned}
B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}} + B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}} = C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \Bigg\{ & \left[\int_D dz \left(\frac{|R-z||z-L|}{|L-R|} \right) T_{zz} \right] \\
& + \left[\int_D d\bar{z} \left(\frac{|\bar{R}-\bar{z}||\bar{z}-\bar{L}|}{|\bar{L}-\bar{R}|} \right) T_{\bar{z}\bar{z}} \right] \Bigg\}
\end{aligned} \tag{4.57}
$$

This is similar to the expression of the modular Hamiltonian in (4.50).

4.3.2 OPE block of (T,B) in 2 dimension in stress tensor channel and it's similarities with modular Hamiltonian : $\mathcal{B}_{\mathcal{O}\mathcal{O}\tilde{T}}$

We start with the general expression of the OPE block in (T,B) case

$$
\begin{aligned}
\mathcal{B}^k_{(\Delta,\ell)} &= C_2 \int_D \sqrt{-g}\, dt\, dr\, \left\langle \mathcal{O}_{\Delta_T}(T)\mathcal{O}_{\Delta_B}(B)\tilde{T}_{\tilde{\Delta},\ell_x=2}(x) \right\rangle T_{\Delta,\ell_x=2}(x) \\
&= C_2 \int_D \sqrt{-g(z)}\; dz\; d\bar{z} \left\langle \mathcal{O}_{\Delta_T}(T,\bar{T})\mathcal{O}_{\Delta_B}(B,\bar{B})\tilde{T}_{\tilde{\Delta},\ell_x=2}(z,\bar{z}) \right\rangle T_{\Delta,\ell_x=2}(x,\bar{x}) \\
&= C_2 \int_D \sqrt{-g(z)}\; dz\; d\bar{z} \frac{C_{\mathcal{O}\mathcal{O}\tilde{T}} T_{\Delta,\ell_x=2}(z,\bar{z})}{|(T-B)|^{\frac{\Delta_T+\Delta_B-\tilde{\Delta}+\ell_x}{2}} |(T-z)|^{\frac{\Delta_T+\tilde{\Delta}-\Delta_B-\ell_x}{2}} |(z-B)|^{\frac{\Delta_B+\tilde{\Delta}-\ell_x-\Delta_T}{2}}} \\
&\quad \frac{1}{|(\bar{B}-\bar{T})|^{\frac{\Delta_T+\Delta_B-\tilde{\Delta}+\ell_x}{2}} |(\bar{T}-\bar{z})|^{\frac{\Delta_T+\tilde{\Delta}-\Delta_B-\ell_x}{2}} |(\bar{z}-\bar{B})|^{\frac{\Delta_B+\tilde{\Delta}-\ell_x-\Delta_T}{2}}} \\
&= C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)}\; dz\; d\bar{z}\; I_1\; I_2\; T_{\Delta,\ell_x=2}(z,\bar{z})
\end{aligned} \tag{4.58}
$$

where

$$
\begin{aligned}
I_1 &= \left(\frac{|T-z||z-B||\bar{T}-\bar{z}||\bar{z}-\bar{B}|}{|T-B||\bar{B}-\bar{T}|} \right)^{\frac{l_z-\tilde{\Delta}}{2}} \\
I_2 &= \left(\frac{|T-z||\bar{T}-\bar{z}|}{z-B||\bar{z}-\bar{B}|} \right)^{\frac{\Delta_{TB}}{2}} .
\end{aligned} \tag{4.59}
$$

Here a factor of $(|T-B||\bar{T}-\bar{B}|)^{\frac{\Delta_T+\Delta_B}{2}}$ is absorbed.

Say in time slice $t=0$ the coordinates are as follows:

$[T(z),\bar{T}(\bar{z}) = (-R,R)$; $[B(z),\bar{B}(\bar{z})] = (R,-R)$ and $e^w = \frac{z-T}{B-z}$; $e^{\bar{w}} = \frac{\bar{z}-\bar{B}}{\bar{T}-\bar{z}}$.

Here also as in the (L,R) case the spin parts do not appear. Therefore the OPE block becomes the sum of two components where one component is the T_{zz} part,

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}} = C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)}\; dz\; d\bar{z} \left[\frac{|T-z||z-B||\bar{T}-\bar{z}||\bar{z}-\bar{B}|}{|T-B||\bar{T}-\bar{B}|}\right]^{\frac{l_x-\tilde{\Delta}}{2}} \left[\frac{|T-z||\bar{T}-\bar{z}|}{|z-B||\bar{z}-\bar{B}|}\right]^{\frac{\Delta_{TB}}{2}} T_{zz} \tag{4.60}$$

and the other component is the $T_{\bar{z}\bar{z}}$ part

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}} = C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)}\; dz\; d\bar{z} \left[\frac{|T-z||z-B||\bar{T}-\bar{z}||\bar{z}-\bar{B}|}{|T-B||\bar{T}-\bar{B}|}\right]^{\frac{l_x-\tilde{\Delta}}{2}} \left[\frac{|T-z||\bar{T}-\bar{z}|}{|z-B||\bar{z}-\bar{B}|}\right]^{\frac{\Delta_{TB}}{2}} T_{\bar{z}\bar{z}}. \tag{4.61}$$

Therefore the OPE blocks is:

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}} + B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}} = C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \int_D \sqrt{-g(z)}\; dz\; d\bar{z} \left[\frac{|T-z||z-B||\bar{T}-\bar{z}||\bar{z}-\bar{B}|}{|T-B||\bar{T}-\bar{B}|}\right]^{\frac{l_x-\tilde{\Delta}}{2}} \left[\frac{|T-z||\bar{T}-\bar{z}|}{|z-B||\bar{z}-\bar{B}|}\right]^{\frac{\Delta_{TB}}{2}} \big\{T_{\bar{z}\bar{z}} + T_{zz}\big\}. \tag{4.62}$$

Now, if we absorb $\sqrt{-g(z)} = \frac{1}{2}$ in constants, put $\tilde{\Delta} = 0$, $\Delta_{TB} = 0$, $l_x = 2$ and integrate out the $d\bar{z}$ in $B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}}$ part and dz in $B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}}$ we get

$$B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{zz}} + B^k_{\mathcal{O}\mathcal{O}\tilde{T}_{\bar{z}\bar{z}}} = C_2\, C_{\mathcal{O}\mathcal{O}\tilde{T}} \Bigg\{ \left[\int_D dz \left(\frac{|T-z||z-B|}{|T-B|}\right) T_{zz})\right] + \left[\int_D d\bar{z} \left(\frac{|\bar{T}-\bar{z}||\bar{z}-\bar{B}|}{|\bar{T}-\bar{B}|}\right) T_{\bar{z}\bar{z}}\right] \Bigg\} \tag{4.63}$$

which is the modular Hamiltonian up to a constant factor.

4.3.3 Commutation of modular Hamiltonian with spinning OPE blocks in 2 dimensional CFT

In this sub-section we see the commutation relations between H_{mod} and different OPE blocks. We begin with commutator of H_{mod} and a boundary primary field [8]

$$[H^{(R)}_{mod}, O(z,\bar{z})] = \Theta((z-T)(B-z))\frac{2\pi i}{(B-T)}\big[h(B+T-2z)+(z-T)(B-z)\partial_z\big]O(z,\bar{z}) \tag{4.64}$$

$$[H^{(L)}_{mod}, O(z,\bar{z})] = -\Theta((\bar{T}-\bar{z})(\bar{z}-\bar{B}))\frac{2\pi i}{(\bar{T}-\bar{B})}\big[h(\bar{T}+\bar{B}-2z)+(\bar{T}-\bar{z})(\bar{z}-\bar{B})\partial_{\bar{z}}\big]O(z,\bar{z}). \tag{4.65}$$

Then by using (4.64) we start with the commutation of H_{mod} and spinning OPE blocks in scalar channel in two dimensions for (T,B) case

$$\begin{aligned}
&[H^{(R)}_{mod}, B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}}] = \\
&\tilde{C}_{T\mathcal{O}\tilde{\mathcal{O}}}\int_D dz\,d\bar{z}\left(\frac{(z-T)(B-z)}{(B-T)}\right)^{h-1}\left(\frac{(\bar{T}-\bar{z})(\bar{z}-\bar{B})}{(\bar{T}-\bar{B})}\right)^{\bar{h}-1} \\
&\times\left(\frac{(B-z)(\bar{z}-\bar{B})}{(z-T)(\bar{T}-\bar{z})}\right)^{\frac{\Delta_{TB}+l_B}{2}}\left(\frac{(z-T)}{(B-z)(B-T)}\right)^{2}[H^{(L)}_{mod}, O(z,\bar{z})] \\
&= \frac{2\pi i}{(B-T)}\tilde{C}_{T\mathcal{O}\tilde{\mathcal{O}}}\int_D dz\,d\bar{z}\left(\frac{(z-T)(B-z)}{(B-T)}\right)^{h-1}\left(\frac{(\bar{T}-\bar{z})(\bar{z}-\bar{B})}{(\bar{T}-\bar{B})}\right)^{\bar{h}-1} \\
&\left(\frac{(B-z)(\bar{z}-\bar{B})}{(z-T)(\bar{T}-\bar{z})}\right)^{\frac{\Delta_{TB}+l_B}{2}}\times\left(\frac{(z-T)}{(B-z)(B-T)}\right)^{2}\big[h(B+T-2z)+(z-T)(B-z)\partial_z\big]O(z,\bar{z}) \\
&= \frac{2\pi i}{(B-T)^{h+2}}\tilde{C}_{T\mathcal{O}\tilde{\mathcal{O}}}\int_D dz\,d\bar{z}\frac{(z-T)^{h+1-\frac{\Delta_{TB}+l_B}{2}}(B-z)^{h-3+\frac{\Delta_{TB}+l_B}{2}}(\bar{T}-\bar{z})^{\bar{h}-1-\frac{\Delta_{TB}+l_B}{2}}}{(\bar{T}-\bar{B})^{\bar{h}-1}} \\
&\times(\bar{z}-\bar{B})^{\bar{h}-1+\frac{\Delta_{TB}+l_B}{2}}\big[h(B+T-2z)+(z-T)(B-z)\partial_z\big]O(z,\bar{z}) \\
&= \frac{2\pi i}{(B-T)^{h+2}}\tilde{C}_{T\mathcal{O}\tilde{\mathcal{O}}}\int_D dz\,d\bar{z}\frac{(z-T)^{h+1-\frac{\Delta_{TB}+l_B}{2}}(B-z)^{h-3+\frac{\Delta_{TB}+l_B}{2}}(\bar{T}-\bar{z})^{\bar{h}-1-\frac{\Delta_{TB}+l_B}{2}}}{(\bar{T}-\bar{B})^{\bar{h}-1}} \\
&\times(\bar{z}-\bar{B})^{\bar{h}-1+\frac{\Delta_{TB}+l_B}{2}}\big[h(B+T-2z)+(T.D.)-h(B+T-2z)-(2-\frac{\Delta_{TB}+l_B}{2})(B-T)\big] \\
&\qquad\qquad\times O(z,\bar{z}) \\
&= \frac{2\pi i}{(B-T)^{h+1}}\tilde{C}_{T\mathcal{O}\tilde{\mathcal{O}}}\int_D dz\,d\bar{z}\frac{(z-T)^{h+1-\frac{\Delta_{TB}+l_B}{2}}(B-z)^{h-3+\frac{\Delta_{TB}+l_B}{2}}(\bar{T}-\bar{z})^{\bar{h}-1-\frac{\Delta_{TB}+l_B}{2}}}{(\bar{T}-\bar{B})^{\bar{h}-1}} \\
&\qquad\qquad\times(\bar{z}-\bar{B})^{\bar{h}-1+\frac{\Delta_{TB}+l_B}{2}}(\frac{\Delta_{TB}+l_B}{2}-2)O(z,\bar{z}) \\
&= 2\pi i\,(\frac{\Delta_{TB}+l_B}{2}-2)\;B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}}
\end{aligned} \tag{4.66}$$

Similarly by using (4.65) we get

$$[H^{(L)}_{mod}, B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}}] = 2\pi i \; (\frac{\Delta_{TB} + l_B}{2}) \; B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.67}$$

and for $\mathcal{B}^{(T,B)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}$

$$[H^{(L)}_{mod}, B^{(T,B)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = 2\pi i \; (\frac{\Delta_{TB} + l_B}{2} - 2) \; B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.68}$$

$$[H^{(R)}_{mod}, B^{(T,B)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = 2\pi i \; (\frac{\Delta_{TB} + l_B}{2}) \; B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.69}$$

Therefore

$$[H_{mod}, B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}}] = 2\pi i \; \Delta_{TB} \; B^{(T,B)}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.70}$$

$$[H_{mod}, B^{(T,B)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = 2\pi i \; \Delta_{TB} \; B^{(T,B)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} \tag{4.71}$$

and

$$[H_{mod,\rho}, \mathcal{B}_{T_{zz}\mathcal{O}\mathcal{O}}] = -2\pi i \; l_{BT} \; \mathcal{B}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.72}$$

$$[H_{mod,\rho}, \mathcal{B}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = 2\pi i \; l_{BT} \; \mathcal{B}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} \tag{4.73}$$

The commutation of H_{mod} and spinning OPE blocks in scalar channel in two dimensions for (L,R) case

$$[H^{(R)}_{mod}, \mathcal{B}^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}}] = 2\pi i \; \mathcal{B}^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}}(2 + \frac{\Delta_{LR} - l_{LR}}{2}) \tag{4.74}$$

$$[H^{(L)}_{mod}, \mathcal{B}^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}}] = -2\pi i \; \mathcal{B}^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}}(\frac{\Delta_{LR} - l_{LR}}{2}) \tag{4.75}$$

and for $\mathcal{B}^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}$

$$[H^{(R)}_{mod}, \mathcal{B}^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = 2\pi i \; \mathcal{B}^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}(\frac{\Delta_{LR} - l_{LR}}{2}) \tag{4.76}$$

$$[H^{(L)}_{mod}, \mathcal{B}^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = -2\pi i \; \mathcal{B}^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}(2 + \frac{\Delta_{LR} - l_{LR}}{2}) \tag{4.77}$$

Therefore

$$[H_{mod}, B^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}}] = 2\pi i \; l_{LR} \; B^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.78}$$

$$[H_{mod}, B^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = -2\pi i \; l_{LR} \; B^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} \tag{4.79}$$

and

$$[H_{mod,\rho}, B^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}}] = 2\pi i \ \Delta_{LR} \ B^{(L,R)}_{T_{zz}\mathcal{O}\mathcal{O}} \tag{4.80}$$

$$[H_{mod,\rho}, B^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}}] = 2\pi i \ \Delta_{LR} \ B^{(L,R)}_{T_{\bar{z}\bar{z}}\mathcal{O}\mathcal{O}} \tag{4.81}$$

We see that in two dimensions the spinning OPE block is a eigenmode of H_{mod} . In higher dimensions the OPE blocks do not commute with the H_{mod} which also explains why the OPE blocks in higher dimensions were not equal to the bulk fields integrated over the RT surface as the H_{mod} and ϕ_{bulk} commute with each other [9].

CONCLUSION

In this thesis, we have discussed a particular boundary operator OPE block and its usage in constructing the bulk. In 4.1 we interpret the extrapolate dictionary as the bulk field equal to the modular flow of the boundary operator in the χ direction (4.2) and is equal to different boundary operators as different modes of the integrated bulk field where the zero mode is on the RT surface. Then in (4.13) we show that the scalar OPE block in the scalar channel is equal to the bulk field integrated over the RT surface as the zero χ mode. We compute OPE blocks in higher dimensions with a stress tensor as one of the operators in section 4.2.3 where we show different components of the OPE block. Though the OPE blocks in higher dimensions are not modes of the bulk field integrated over the RT surface, they will definitely be some bulk fields in the Rindler wedge from the extrapolate dictionary. This might be a future prospect of this project to find the corresponding bulk fields of higher dimensional OPE blocks. Then in 4.2.4 we find that in two dimensions the OPE block with spin is the bulk field integrated over the RT surface but with an appropriate momentum as k_χ. This might be the bulk field spread out over the light sheet covering the entire Rindler wedge in the bulk, which gives us a different perspective of the bulk reconstruction. Then finally from (4.66) to (4.81) we see that OPE blocks with spin are eigenmodes of modular Hamiltonian in two dimensions, which was indicative when OPE blocks were equal to the bulk field integrated over the RT surface or vice-versa as the bulk fields commute with modular Hamiltonian [9]. These results show that OPE blocks are a suitable candidate for CFT inputs in bulk reconstruction and thereby helps us in developing and better understanding the extrapolate dictionary.

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