Motor imagery EEG based brain-computer interfacing using Fourier-Bessel series expansion

M.Tech. Thesis

By

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DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY INDORE

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DEPARTMENT OF ELECTRICAL ENGINEERING

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Motor imagery EEG based brain-computer interfacing using Fourier-Bessel series expansion"** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DEPARTMENT OF ELECTRICAL ENGINEERING, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July 2021 to May 2022 under the supervision of Prof. Ram Bilas Pachori.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

Ainwad

09/06/2022

Signature of the student with date
SUNILKUMAR AINWAD

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

09.06.2022
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Sunilkumar Ainwad has successfully given his M.Tech. Oral Examination was held on 06-06-2022.

09.06.2022
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Date: 09/06/2022

10/06/2022
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Signature of PSPC Member #2
(Dr. Hem Chandra Jha) 9/6/2022
Date:

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Abstract

The electroencephalography (EEG)-based brain-computer interface (BCI) is a most emerging technology incorporated into treating patients suffering from cognitive or physical impairments. EEG signals are the recording of brain electrical activity. One of the most well-known ways to deal with BCI is motor imagery (MI). The MI-based BCI is an intuitive interface that directly controls computer applications from brain activity. We present a Fourier-Bessel series expansion (FBSE) based classification framework for MI from recorded EEG signals for enhancing the BCI application. The FBSE spectrum has a better spectral resolution for the non-stationary signals than the Fourier spectrum. It provides a representation of real signals in terms of positive frequencies, and it does not require the use of a window function in order to obtain the spectrum of the signal. Due to its unique and compact representation, The FBSE decomposition method is used for MI-specific rhythm separation from EEG signals.The proposed work aims to enhance rhythm separation and feature extraction and provide improved classification of MI-EEG motor imagery task. Then multi-domain features were extracted from EEG rhythm, namely Hjorth and band power. The Hjorth and band power features are estimated from enhanced EEG signals. Further, the obtained features were tested based on different classifier networks, To classifyof the right hand, left hand, both feet, and tongue movement of MI task. The classifier, namely linear discrimination analysis (LDA), k-nearest neighbours (k-NN), and ensemble k-NN, are employed. In comparison, classification network k-NN provides the best classification accuracy on obtained features from FBSE based rhythm.

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Chapter 1

Introduction

1.1 Introduction

The brain-computer interface (BCI) bridges the gap between human-computer interaction. BCI relies on analysing signals transmitted by the brain and converting them into commands. BCI is extensively used these days due to its wide range of biomedical applications. Such as bionic vision devices, memory augmentation techniques, and rehabilitation of several neurological diseases like Alzheimer's disease and Parkinson's disease. Also, BCI training has shown improved results in the cases of spinal cord injury and cortical dysfunction cases. BCI has proved to be an efficient technology in preventing, diagnosing, and rehabilitation of several diseases. The development of new methods in BCI opens a wide area for the neurorehabilitation of several patients suffering from brain injuries and physical disabilities. The primary application of motor BCI involves the development of robotic prosthetics, which can help paralysed patients and amputees to gain control over the developed prosthetic. EEG is an effective invasive tool which is commonly used for BCI applications. However, there are several limitations involving BCI using EEG. Accurate responsiveness of BCI towards the motory signals acquisition and controlling of assistive devices for the disabled person has a major problem. The MI-BCI is notably difficult to analyse, requiring significant training time and having limited BCI channels, which introduce difficulties in practical modelling. To overcome these limitations, a decomposition method is required which will improve the extraction of meaningful discriminative features for different motor imagery classifications. This chapter presents some basic details about MI-EEG signals and BCI, the problem background, and the motivation behind our research. Finally, an organisation of the thesis has been presented.

1.2 Electroencephalogram (EEG) and BCI

EEG is an extensively used electrophysiological assessment technique for clinically recognising and managing various neurological disorders [1]. In order to measure the neurophysiological activity of the brain, electrodes are placed on the scalp or intracranially on the cerebral cortex or in special cases, on deeper brain tissues. Measured activity in the form of traces is known as EEG or brain waves [2], [3]. Based on the position of the electrode used for measuring the activity, EEG is of two types: Non-invasive EEG, in which electrodes are placed on the scalp and another type is intracranial EEG, in which electrodes are implanted in the deep brain. A brain-computer interface (BCI) is a communications system between hardware and software that activate human to interact with its surrounding only by using an EEG signals as a control signals instead of peripheral nerves and muscles [1], [4], [5].

In the past 20 years many paradigms for constructing EEG-based BCI systems were tried out [6], [7]. The paradigms differ from well-known EEG phenomena that occur when stimuli interact with the brain, to the use of biofeedback, to paradigms using sophisticated machine learning algorithms to classify the EEG [8]. It shows effectiveness in different MI tasks such as each strategy has its own set of benefits and drawbacks. The important question for the future of BCI technology is the degree to which performance can be increased, and its minute-to-minute and day-to-day variation can be decreased. Such advancements rely on the ability to systematically evaluate and contrast different BCI approaches, allowing the most promising approaches to be discovered.

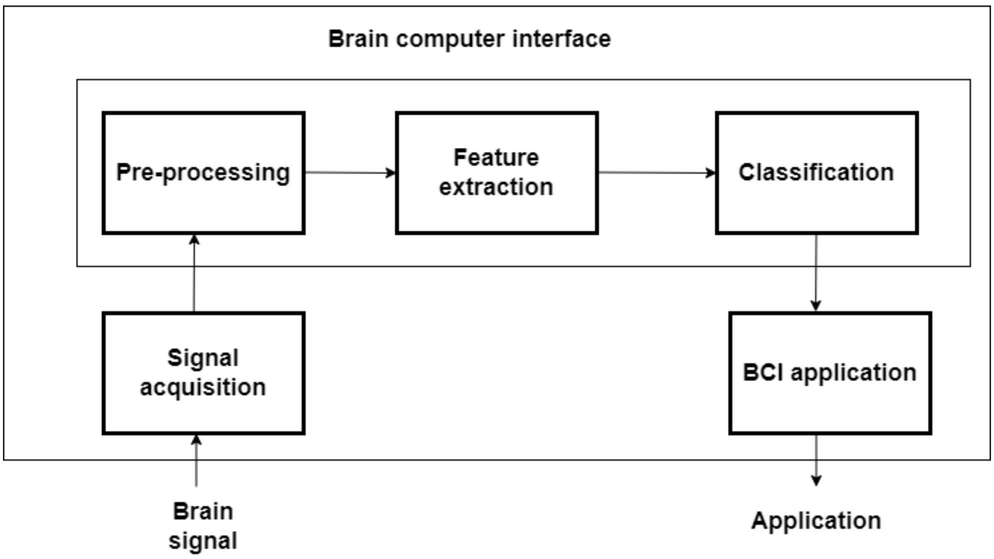


Figure 1.1: BCI block diagram.

In Fig. 1.1 basic block diagram of BCI we will collect signals from human brain and signals acquisition will the done. Here in pre-processing we have used band pass filter with pass band frequency 0.1 Hz to 100 Hz to remove artifacts. This artifact generate while we recording EEG signals. It may be due to power fluctuation, human movements. these feature are extracted from EEG signals [9]–[13]. This features used as input to the classifier. Using different classifier we are able to classifies our task. BCI allow paralyzed people to control the prosthetic limbs with their mind, transmit the visual image to the mind of a blind person which allows them to see. Transmit auditory data to mind of a deaf person, which allows them to bear [14]–[18], allow mute person to have their thought to displayed and spoken by the computer. Along with some advantages, there are some disadvantages. An electrode outside of the skull can detect very few electrical signals from the brain. An electrode placed inside the head creates scar tissue in the brain. The present BCI technology is clumsy and ethical issues may prevent its development.

1.3 Amplitude and spectral characteristics of EEG signals

Signals from both cerebral and non-cerebral origins are present in EEG. Depending on the vicinity of electrodes to neurons, as in the case of subdural electrodes, the amplitude of depth electrodes is usually larger than that of scalp electrodes [19]–[21]. It has been observed that the amplitude of scalp electrode signals varies from 20 µV to 100 µV, while depth electrode amplitude varies from 100 µV to 2 mV. The frequency range of EEG is from 0.5 Hz to about 500 Hz. Five frequency bands out of this spectral bandwidth are of clinical importance: (a) delta, (b) theta, (c) alpha, (d) beta, and (e) gamma, which are explained below. These bands are obtained by considering mean frequency corresponding to the frequency range [delta: 1-4 Hz, theta: 4-8 Hz, alpha: 8-13 Hz, beta: 13-30 Hz and gamma: 30-84 Hz from the decomposed intrinsic mode function (IMF) of the empirical decomposition method [22].

- **Delta waves:** These waves have a spectral content of about 3 Hz. It is a wave with least frequency and highest amplitude [22].

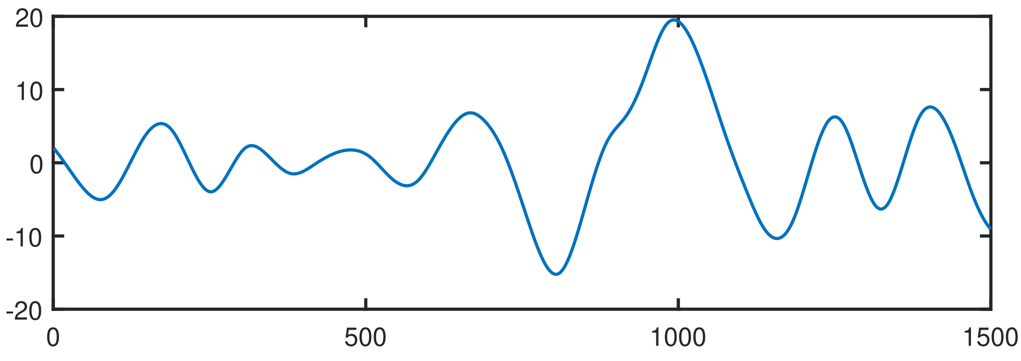


Figure 1.2: Delta band of EEG signals waveform

- **Theta waves:** These slow waves have a frequency content ranging from 4 Hz to 7 Hz. Closing of eyes with relaxation emerges these waves. This is normally found in EEG signals of young children and adults [22].

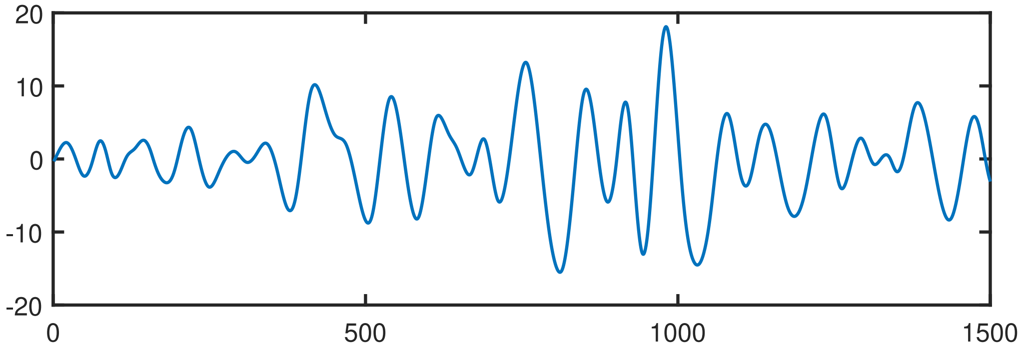


Figure 1.3: Theta band of EEG signals waveform

- **Alpha waves:** Commonly found in adults with a spectral range from 7 Hz to 12 Hz. These waves arise in rhythms on both sides of the head. On the non-dominant side, it has a slightly higher amplitude. They disappear with stress/opening of eyes and are regarded as normal waveforms [22].

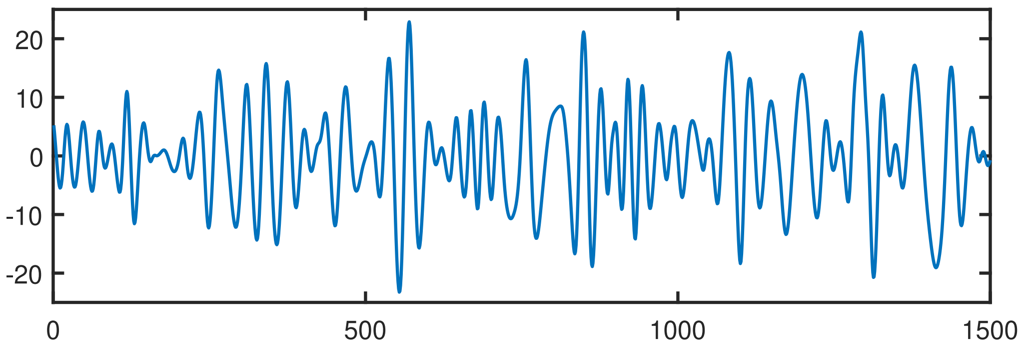


Figure 1.4: Alpha band of EEG signals waveform

- **Beta waves:** Characterized by small amplitude and fast activity, these waves have a frequency range of 14 Hz to 30 Hz. These dominant rhythms are present in patients, who are alert or anxious with their eyes open. These are usually seen on both sides with symmetrical distribution, dominant in frontal areas. In areas of cortical damage, it may be absent or diminished. These waves are most dominant in the frontal and central portions of the brain. The amplitude of beta waves is less than 30 µV [22].

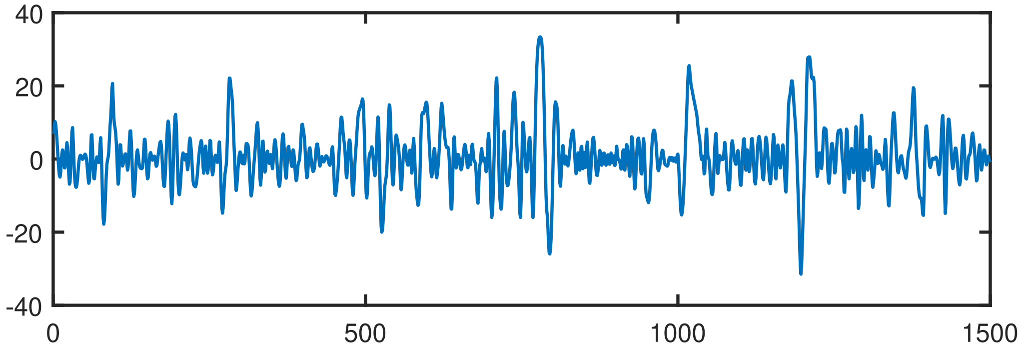


Figure 1.5: Beta band of EEG signals waveform

1.4 Clinical approach for MI-EEG classification

Motor imagery (MI)-EEG based brain-compute interface (BCI) has clinical applications in both rehabilitation and communication. The patient who lost their motor function due to injury or neurogenerative disease such as amyotrophic lateral sclerosis [23]. Epilepsy affects over 1% of the world's population, and BCI for seizure prediction can improve the quality of life for epileptic patients. A BCI system for seizure forecasting, for example, can assist epileptic patients have a better quality of life by using EEG, which is a measure of brain waves. In order for such a BCI system to work effectively, computational algorithms must be able to accurately identify periods of heightened seizure risk. If the cause of seizures could be pinpointed, it might be possible to create gadgets that would alert patients, allowing them to avoid potentially harmful activities such as swimming or driving. Medications could also be used only when absolutely necessary to prevent seizures [24]. In another clinical study, BCI find effective use in the case of a paralyzed patient, diagnosed with severe cerebral palsy, who was trained over a period of several months to use an EEG-based brain-computer interface (BCI) for verbal communication. In clinical rehabilitation practice, EEG feedback training is performed in the patient's home (clinic), with the help of a tele-monitoring system, supervised from a distant laboratory. Also, online feedback computation is done with single-trial analysis and classification of MI- EEG task. Here experts analyze MI-EEG task-related changes in brain oscillations over the course of training steps were investigated by quantifying time-frequency maps of event-related de-synchronization for BCI-controlled spelling. The conclusion of the above discussion is proposed BCI-MI based on EEG signals can enhance feature extraction and classification, The MI task may improve the actual level of communication ability in locked-in patients [25].

Additionally, It's critical to develop an EEG task which use automatic computer-aided diagnosis (CAD) categorization framework in which EEG signals shows inherently time varying and nonlinear nature that are additive with non stationary qualities at different frequency scales.

1.5 Literature review

In the literature, several computer-aided diagnosis-based BCI (BCI-CAD) related works have been employed to achieve different motor imagery tasks. It highlights various experiments that work with certain limitations. Park et al. [26] introduced a multivariate extension of empirical mode decomposition for motor imagery classification, which consists of filtering, feature extraction, aggregative convolutional block, and classification block. The method allows the use of a noise-assisted mode of operation for extracting the highly localized time-frequency representation with common spatial pattern features, which progressively highly localized time-frequency representation. Experimental results show that this model obtains 94% classification accuracy. Kevric et al. [27] have improved the classification of controlling commands for wheelchairs with the detection of motor imagery gestures such as foot class and right-hand class. It uses empirical mode decomposition, discrete wavelet transform, and wavelet packet decomposition method to extract improved features namely adaptive higher-order statistics (HOS), and delivered classification accuracy of 92.8%. Gutierrez et al. [28] presented a new classification approach based on common spatial filtering from EEG signals through time-varying autoregressions (TVAR) as a preface step of the common spatial pattern (CSP) method for BCI application. It employed Mahalanobis distance-based classifier on extracted most significant eigen component-based features to overcome excessive use of filter banks and achieved average performance above 80%. It proved the suitability of the proposed method for practical BCI applications by optimizing training and validation parameters such as the number of trials and channels. In Dutta et al. [29], for multivariate time series data decomposition into a large number of IMF groups, a new feature extraction approach based on the multivariate autoregressive (VARM) model of the sensitive intrinsic mode function (IMF) groups in the multivariate empirical mode decomposition (MEMD) domain was proposed. The recited feature vectors is classified by least squares support vector machine (LS-SVM) classifier with three different kernel functions namely, linear, polynomial, and radial basis functions on three non-motor cognitive tasks in EEG based BCI system tested approaches on EEG data sets of three subjects on the mental task. With this, for binary classification an average classification accuracy of 94.43% using polynomial kernel and 91.65% were achieved. For the mental arithmetic and mental letter composing task using radial basis function (RBF), an average classification accuracy of 77.7% was obtained.

Siuly et al. [30] introduced a new modified cross-correlation based logistic regression algorithm and used three diverse feature sets for motor imagery tasks of EEG signals to overcome the problem of cross-correlation based logistic regression. In this, a six-feature set yields the highest accuracy in each of the three folds for the proposed LR classifier. Cross-correlation technique is capable of feature extraction. Here, the feature set is two-feature sets (mean, standard deviation) four-feature set (mean, standard deviation, skewness, and kurtosis), and a six-feature set (mean, standard deviation, skewness, kurtosis, maximum values, and minimum values) as the input to the logistic regression (LR) classifier and it has the ability to solve a pattern recognition task in BCI applications. These three sets of features are extracted from each cross-correlation sequence technique for different channels of the EEG data of each and every MI class of a subject. The algorithm that has been proposed was compared with eight of the most recently reported well-known methods in addition to the BCI III Winner algorithm. And gives the average classification accuracy of the proposed algorithm is 93.91%. Cross-correlation method has been successfully used in many applications like gait signals processing, ECG beat detection, emotional speech recognition, heart rate variability classification, signal-to-noise enhancement, and seizure prediction. Kamble et al. [31]. The development of optimal models for feature extraction and signals classification is one of the most critical steps in BCI, Adaptive signals decomposition is a relatively new approach such as Empirical Mode decomposition (EMD), empirical wavelet transform (EWT), variational mode decomposition (VMD), and variational nonlinear chirp mode decomposition (VN-CMD) Decompose EEG signals into various modes using a time-frequency approach. And then linear and nonlinear time-domain features are extracted from the modes, and VMD is discovered to be a superior adaptive signals decomposition method to the others thant hree tested decomposition methods (EWT, EMD, and VNCMD). Classification is performed using four recent ML-based algorithms.The performance of the machine learning algorithm is evaluated using the parameters of accuracy, recall, specificity, precision, F1-score, and area under the curve. Four popular ML algorithms, decision tree With medium kernels, a KNN with different kernels, ensemble bagged tree, and SVM with radial basis function and quadratic kernels, are employed for the classification task. The highest level of categorization accuracy was attained that is 89.6 ± 4.6% in binary and 61.1 ± 5.1% in multiclass (seven classes) signals. Jayalaxmi et al. [32], the proposed framework is assessed using three different RNN architectures namely long short-term memory, bi-directional long short-term memory, and gated recurrent units (GRU). Results show that wavelet scattering coefficients extracted from the dominant mode of EWT decomposition recorded better performance of 90.23% for EEG and 84.25% for MEG signals using GRU as the classifier. A greater degree of computational and time complexities will be reduced by the proposed method.

Li et al. [33] researcher introduced an improved common spatial subspace decomposition algorithm (CSSD) It's a technique for extracting features. EEG signals that are influenced by physical conditions, emotion, body position, and other factors are simple to examine. Improved CSSD provides a solid theoretical and experimental framework for developing online BCI systems based on mental EEG imagery movement. The improved CSSD algorithm is effective in resolving eigenvalue disability and low recognition rate. Zhu et al. [34] developed a novel heuristic technique for selecting the best CSP channels CSP's performance has been shown to deteriorate in the past, particularly when the number of channels is considerable relative to the number of training datasets. Furthermore, the CSP method's accuracy in identifying two classes of motor imagery tasks is not determined by the number of channels. This indicates that having more channels does not ensure better accuracy, but it does add to the computational effort. However, in order to decrease computing time while maintaining high classification accuracy, it is necessary to pick the best subset of the entire channel. This can be accomplished by employing a common spatial pattern (CSP) technique, which produces a smaller number of channels. Based on L1 NORM, a channel score is assigned, and the channel with the highest score is selected. On the basis of motor imagery signals, Xue et al. [35] suggest several convolutional neural network typologies. For classification of MI-EEG signals, one solution offered is an end-to-end innovative multi-branch hybrid neural network. A bidirectional gated recurrent unit was also included to recognise EEG. Four-class motor imagery is one of the features here, and collecting high-quality EEG data can be difficult at times, causing classification accuracy to deteriorate. To improve this, The researchers developed a new method for frequency domain segmentation swapping to improve the classification of MI-EEG signals accuracy. On both datasets, the BCI IV 2a data set was employed, and the High gamma suggested multi-branch hybrid neural network achieved 95.04% and 86.15%. Tang et al. [36] introduce a new method of upper triangle filter bank autoencoder to improve the classification performance of MI EEG signals. UTFB mostly decomposes EEG signals into sub-band applying the common spatial pattern to each sub-band feature are extracted, merging of all this CSP feature into a vector, Using an extreme learning machine autoencoder, this vector dimensionality is reduced afterword we use LDA classifier basic MI operation are left hand, right hand, foot, tongue. The classification performance of the UTFB-AE method is higher than that of tensor-based schemes (TBS), common spatial pattern with support vector machine (SVM), non-negative multi-way factorization, and power spectral density 2.68%, 3.38%, 10.18%, and 17.98% respectively. With this, it improves the classification performance of MI-EEG signals.

To overcome the limitations of traditional signals decomposition approaches, Fourier Bessel Seriend Expansion based classification framework [36] is suggested in this area for the first time to our knowledge. The fundamental merits of the proposed technique are :

When compared to the Fourier transform, the FBSE has a higher resolution. Real signals are represented by real Bessel basis functions. The frequency points in the spectrum based on the FBSE are equal to the signal length. It expresses real-world signals in terms of positive frequencies. It does not require use of window function in order to obtain spectrum of the signals. It represents real signals in terms of real Bessel basis functions. The basis functions include amplitude modulation(AM) in the representation.

The following are the study's significant contributions

- For the creation of a reliable BCI system, an unique FBSE based classification framework is developed and verified for automatic classification of MI-EEG data.
- For MI-EEG signals, an adaptive boundary estimation criterion is implemented in the FBSE spectrum for accurate reconstruction of original signals component as compared to FT spectrum.
- Four different MI-EEG classes with three channels were used for experiment. We have extracted features i.e Hjorth (mobility, complexity) and band power. It can extract useful information both in time-domain and frequency-domain through simple computation. Hjorth parameter can also be used as good feature in real time application [37].
- The four classifier frameworks namely RF, KNN, ensemble kNN, and LDA are used and achieved highest classification accuracy in the case of MI-EEG tasks.

1.6 Objectives

The main objective of our work is to analyze the acquired raw data using a signals processing tool and extract the features from them and then classify them into different classes. Our work is focused on the decomposition of EEG signals using Fourier-Bessel series expansion (FBSE). Then extracting some non-linear highly discriminative features from decomposed signals and using different classifiers maximum accuracy can be obtained. This results in easy to classify motor imagery operation. To achieve this:

- Segmented EEG signals decomposed into a spectrum using FBSE.
- Filter is used to separate out the boundary of the spectrum by finding the mid of two peaks and their location [38].
- Band separation based on mean frequency.
- Mojor feature are experimented and obtained feature fed into a classifier which are LDA, kNN, ensemble kNN, random forest improve the classification result. Here it experimented two features Hjorth and band power which improved classification performance. We can determine motor imagery based on the classifier output.

1.7 Motivations and contributions

The EEG signals can be used for the following purpose in BCI studies:

- Physically disabled: Person with a physical disability unable to move body part because the damaged muscle block brain signals. here BCI plays an important role. The objective of BCI is to translate human intentions into motion commands for robots, wheelchairs, and other devices.
- Abnormalities associated with brain tumors and epilepsy seizures: Now day epilepsy seizures are one of the most common neurological disorders these abnormalities can be recognized using EEG signals.
- Sleeping disorders can be detected with BCI assistance.
- Brain stroke: When the brain cell suddenly dies due to lack of oxygen it creates a problem for blood flow, because of this unable to speak, also there is a memory problem or one portion of the body can be paralyzed. The solution to this involves brain signals. Using BCI brain injury could be reorganized and also damage motor function restore [39]. A brain stroke, disability,

psychological disorder, tumor brain disorder, sleep disorder, smoking, alcoholism, or motion sickness motivated us to study brain signals which will resolve the above problem.

Testing and training of automated learning systems, which perform EEG signals analysis and classification are very important.

1.8 Organization of the thesis

The rest of the thesis is organized as follows: Chapter 2 deals with a detailed description of FBSE, where signals decomposition using this method is discussed. Chapter 3 contains the description of the database used and the proposed approach. Chapter 4 contains results and a discussion of the proposed approach using different classifiers and a comparison of obtained results with other existing methodologies. Chapter 5 discusses the conclusions drawn and the scope of future work.

1.9 Summary

People living with a locked-in syndromes or disability such as amyotrophic lateral sclerosis (ALS), traumatic brain disorders, spinal cord injury, brain stem stroke, and other severe motor disabilities. Physically locked-in syndromes or disability patients may find it difficult to interact or communicate with the help of computer either through speech, gesture, or touch pseudocoma is used to describe a condition. Because practically all voluntary muscles have been paralysed except for eye movement and blinking, the patient is unable to move or speak verbally. Therefore solving the disability problem is clinically important. to overcome drawback improve performance in MI-task Here we have proposed the FBSE method for MI - EEG-based brain-computer Interfacing, here using different classifiers we are able to classify motor imagery. Here we have used raw EEG signals data using the FBSE method and different classifiers we are able to solve the above problem.

Chapter 2

Database used and Fourier-Bessel series expansion

2.1 Introduction

This chapter describes the databases used in this work and the Fourier Bessel series expansion (FBSE) techniques used on the MI- EEG signals. We have used only one dataset, i.e. BCI competition IV dataset 2A. This data set is used for motor imagery task problems. FBSE is used to decompose the EEG signals as Fourier spectrum analysis provides the ability to perform spectral analysis of nonstationary signals based on short-time stationarity of the signals. Unlike the sinusoids, however, Bessel functions of the first kind are quasiperiodic with successive zero-crossing intervals slowly increasing toward π. The representation of a first-order Bessel function is thought to be more efficient than the Fourier domain description. Similar to the Fourier series coefficients, The FB coefficients are unique for a given signals Unlike the sinusoidal basis functions in the Fourier series, Within the range a, the Bessel functions decay, similar to the rise and fall of speech within a pitch interval.

2.2 Organisation of chapter

The rest of the chapter is organized as follows: The datasets used in this work are discussed in Section 2.3. Section 2.4 gives the basics of the FBSE method used here. FBSE spectrum of MI-EEG signals is shown in section 2.5. Section 2.6 conclude the chapter.

2.3 Database used and pre-processing

2.3.1 BCI competetion dataset 2A

In the present work, we have used a publicly available database consisting of nine subjects. The BCI model here of different motor imagery tasks, namely imagination of movement of the left hand for class 1, right hand for class 2, both feet for class 3, and tongue for class 4. For the subject, two sessions were taped on different days. Six runs are separated by short rests in each session. A single run contains 48 trials, 12 for each of the four classes, for a total of 288 trials per session. To estimate the EOG influence, a recording of approximately 5 minutes was made at the start of each session. The recording was broken down into three sections: (1) eyes open with two minutes (looking at a screen with a fixation cross), (2) eyes closed with one minute and (3) eye movements with one minute. The Scheduling for a single session and the Scheduling of the paradigm are shown below in Fig. 2.1.

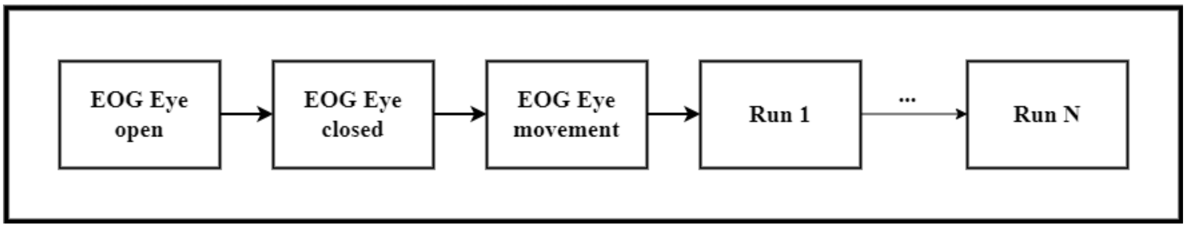


Figure 2.1: Scheduling for a single session

On the comfortable armchair in front of the computer screen Subjects were sitting. The timing scheme of the paradigm is shown in Fig. 2.2 at the beginning of trial t = 0 second. On black screen fixation of the cross appeared t = 2 second arrow pointing to right, left, down or up(one of four types of classes) appeared and stay on the screen for 1.25 second. This indicates that the subject performs MI tasks until the cross disappears from the screen at t = 6 second. After a brief pause, the screen went blank once more.

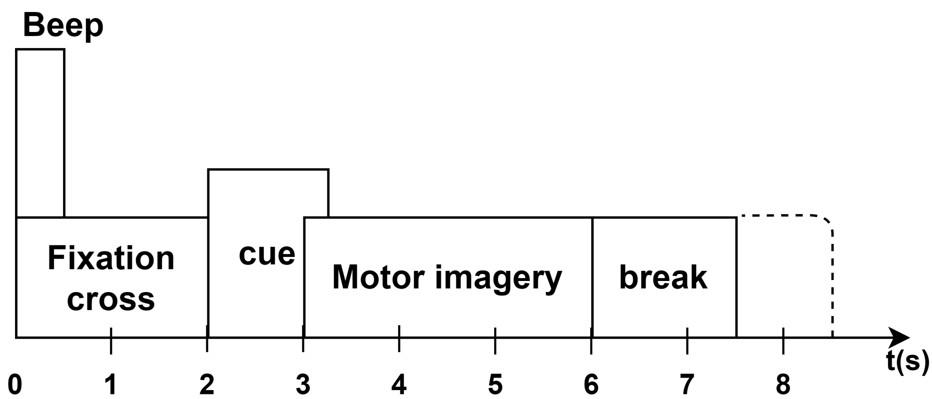


Figure 2.2: Scheduling of the paradigm.

Table 2.1: List of all file contain in data set.

Patient ID	Training	Evaluation
1	A-01-T.gdf	A-01-E.gdf
2	A-02-T.gdf	A-02-E.gdf
3	A-03-T.gdf	A-03-E.gdf
4	A-04-T.gdf	A-04-E.gdf
5	A-05-T.gdf	A-05-E.gdf
6	A-06-T.gdf	A-06-E.gdf
7	A-07-T.gdf	A-07E-.gdf
8	A-08-T.gdf	A-08E-.gdf
9	A-09-T.gdf	A-09-E.gdf

Fig. 2.3 22 Ag/AgCl electrodes with inter-electrode distances of 3.5cm were used to record the EEG signals montage of electrodes based on the international 10-20 system. Aside from the 22 EEG channels, three monopolar EOG channels in our data set, which is the sample with 250 Hz. In our work, we used BCI competition IV dataset 2A. It has nine subjects of training and nine subject evaluations. here in this study we are using three channels C3 (8), C4 (12), and Cz (10), out of 25 channels. We consider the smallest epoch size = 262 out of 18 subject epochs (9 training and 9 evaluation).

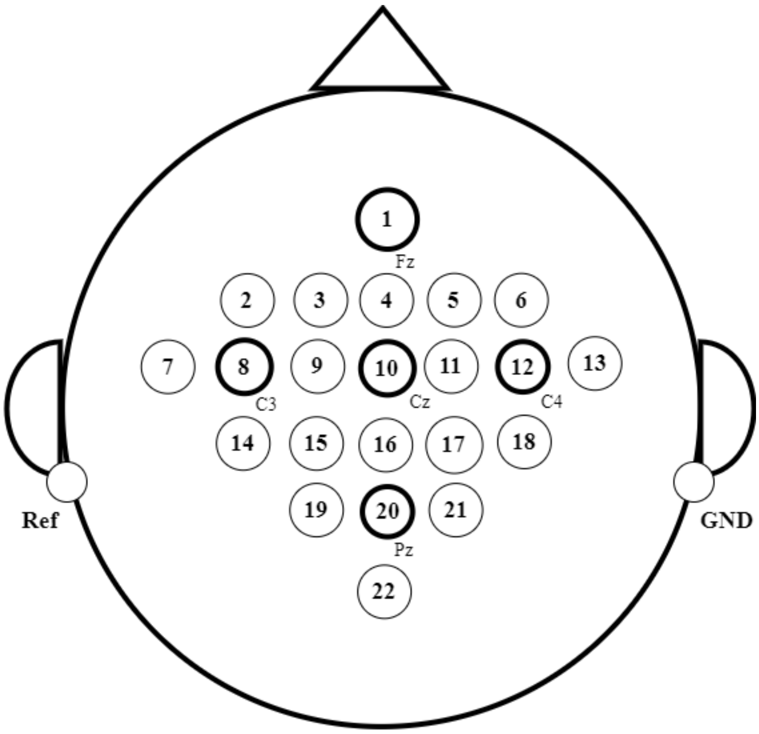


Figure 2.3: Arrangement of electrodes in accordance with the International 10-20 system.

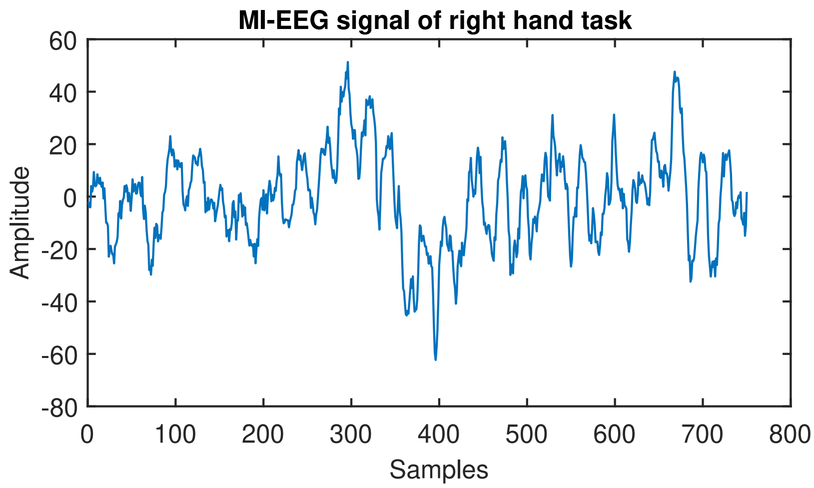


Figure 2.4: MI-EEG signals of left hand task.

The EOG block for subject is shorter due to technical issues for subject A-04-T. It contains eye movement only and RUN are separated by 100 missing value, which is encoded as not-a-number (NaN).

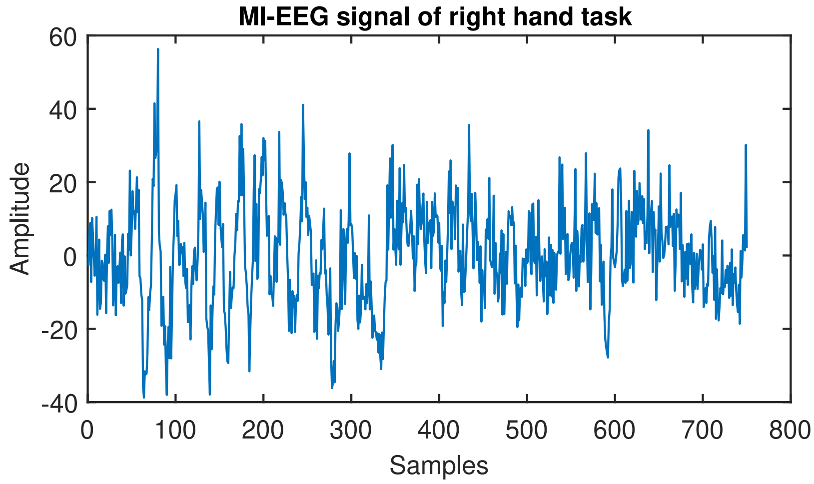


Figure 2.5: MI-EEG signals of right hand task.

In Fig. 2.4 represent the EEG signals waveform of subject, when subject perform the MI task which is movement of left hand for short duration of time. Fig. 2.4 x-axis represent number of sample and y-axis represent amplitude of EEG signals. In Fig. 2.5 represent of EEG signals waveform of subject, when subject perform the MI task which is movment of righ hand. Fig. 2.5 x-axis represent the no of sample and y-axis represent amplitude of EEG signals.

2.3.2 Pre-processing

Electroencephalography (EEG) is a basicaly an non-invasive step that records and captures brain signals. Raw EEG signals are contain with a lot of noise, such as movement of muscle is a electromyography artifacts,power line interference and eye blinking is a electrooculography artifacts. Removing these types of noise can produce a clean signals. The noise-contaminated with EEG signals will affect the actual result during the analyzing stage. Therefore, in pre-processing, to remove this, we used 5^{th} order Butterworth band-pass filter. The band-pass filter of range 0.5 Hz to 100 Hz and the 50 Hz notch filter enabled to suppression of the line noise, and the amplifier's sensitivity was set to 100μvolt. Here the band-pass filter used the sampling frequency of 64 Hz. The three EOG channels provided for the subsequent application of the artifact processing method. Due to artifact, 8 out of a total of 9 data sets were analyzed. The file is available in .gdf form, and it can be loaded using the toolbox BioSig. EOG block is shorter Due to technical problems for subject A-04-T, and it contains only eye movement. Each subject and session has its own file.; one session contains the class label of all trials.

Here, we have used three channels C3, C4, and Cz. Each channel has nine subject data (9 training and evaluation). If we consider one subject, i.e., subject A-04-T, for easy to understand, it has a sample length of 600915 and epoch 262 of 3 seconds. It performs four motor imagery operations, i.e., left hand, right hand, both feet, and tongue. We are considering only left-hand and right-hand MI tasks for our convenience. After extracting 3 second data of subject A-04-T which have 750 rows (3 seconds) into 262 (epoch) *18 (subject) = 4716 (column). In the 4716 column, MI tasks are performed sequentially in which (1 - 11), (12 - 23), (24 - 35), and (36 - 47) segments correspond to the classes 1, 2, 3, and 4, respectively.

In total, the 4716 evaluation samples were used from 1 to 2358, and the training samples were used from 2359 to 4716. In our work, we have taken an epoch of 240, and we combined two columns, and the total epoch is 120 with five-run and 48 trials. We have used exactly 120 trials, which are further used for classification. In the pre-processing, the band-pass filter was used to remove noise using a specific cutoff frequency. Further, the FBSE method was used for decomposing the signals into FB coefficients.

2.4 Fourier-Bessel series expansion method

2.4.1 Introduction

signals and data analysis are vital for practical research applications. Understanding acquired data is crucial and tough since data gathered nowadays from a variety of sources is chaotic in nature and enormous in volume. One of the most difficult tasks for any data analyst is estimating parameters or establishing a model for extracting hidden information from enormous amounts of real data. Short data spans, non-linearity, non-stationarity of data, and combining of key information with noise or other irrelevant information are among challenges experienced during massive data analysis [40], [41]. An earlier attempt in this area was spectral analysis utilising Fourier transforms (FT), which provided a broad way of signals and data analysis. The disadvantage of Fourier analysis is that it cannot handle non-stationary data or data from non-linear systems. Therefore, an better method is needed to address the issue in the existing FT based methods [42]–[45]. The spectrum derived using FBSE has been used in place of the traditional FT-based spectrum in this work. Because of the non-stationary characteristics of the Bessel functions, FBSE coefficients are suited to spectral analysis of non-stationary and non-periodic signals [46]–[51].

In the literature, the potential of FBSE is utilized in the different capacity. In [46], FBSE was used to improve non-stationary signals analysis. In [52], the authors extended FBSE method for multivariate signals application and improved classification accuracy related to human emotion recognition. It's important to remember that, The FBSE approach extracts narrow-band frequency using an adaptive decomposition mechanism. components by considering the realtion between order and correspoding extracted component. Further [43], [44], [53], [54], it decomposes voiced speech signals with FBSE based decomposition after detecting the boundary frequencies in the FT-based spectrum. It provides correlation between voiced speech and the Bessel functions generated and delivered suitability of FBSE spectrum for speech applications. In the real signals analysis [52], [55]–[59] FBSE method proved it's efficacy in the computer assistive technology for clinical application. In this, researchers explored different modalities such EEG, ECG, EMG and achieved satisfactory disease classification. Furthermore [40], [41], [49], [50], [60] various variants of FBSE were developed, in which researcher explored the potential of FBSE for various automated disease classification such as heart disease, diabetic retinopathy and diabetic macular edema, epilepsy, glaucoma, sleep related disease and many more.

2.4.2 Fourier-Bessel series expansion method

The FBSE is a powerful method to represent non-stationary, FB series is used to represent EEG signals which are non-stationary. It is demonstrated that the coefficients of the FB series can be modeled reliably by a sum of sinusoidal whose parameters can be estimated accurately. signals expansion using the FB series is used to analyze signals. FB series expansion of a signals becomes a way to analyze the signals, and its coefficient is used to reconstruct the signals. The non-stationary property is used a basis function in the case of Bessel functions in FBSE. This is why FBSE is more suited to analyze signals which are non-stationary as compared to Fourier transform. The frequency spectrum of a signal x(n) in the frequency range [0, π] is obtained using the FBSE method. The analytical expression for the FBSE approach, which is based on the zero-order Bessel function, is shown below. [53], [54], [61]–[63]:

$$x(n) = \sum_{k=1}^{k} R_k J_0\left(\frac{\gamma_k n}{K}\right), n = 0, 1, 2, 3, ...K-1 \tag{2.1}$$

Here R_k represents the FBSE coefficients of the signal x(n) which can be written as [64]

$$R_k = \frac{2}{K^2\left(J_1\left(\gamma_k\right)\right)^2} \sum_{n=0}^{K-1} n x(n) J_0\left(\frac{\gamma_k n}{K}\right) \tag{2.2}$$

The zero-order and first-order Bessel functions are represented as $J_0(\cdot)$ and $J_1(\cdot)$ respectively. γ_k where, $k = 1, 2, 3, ..., K$. denotes the positive roots of a zero-order Bessel function (J_0 (γ) = 0 arranged in increasing order. The FBSE coefficients of the k^{th} order are represented in terms of the corresponding continuous frequency $W_k(Hz)$ as shown below, [61] [64]:

$$\gamma_k \approx \frac{2\pi W_k K}{W_s}, \text{ where } \quad \gamma_k \approx \gamma_{k-1} + \pi \approx k\pi \tag{2.3}$$

Here W_s represents the sampling rate in Hz. The above equation is given as [65] [66]

$$k \approx \frac{2\ W_k K}{W_s} \tag{2.4}$$

It is important to note that in order to cover the whole bandwidth of the signals, the range of k should vary from 1 to K, where K is represent the length of the signals. The FBSE spectrum is displayed between the absolute value of the FBSE coefficients (R_k) and the frequency (W_k) [67].

As the output of the band-pass filter is given as input to the FBSE. It generate the coefficient of the FBSE, which further shows the peak value present in the FBSE spectrum we have to set the threshold value for the amplitude range. We have a maximum threshold value of 40. Threshold value was selected as normalised value 0.2 to select most of the frequency bands, such as band 1: 6-24 Hz, band

2: 8-12 Hz, and band 3: 16-24 Hz. The number of peak points and their locations are important for the selection of boundaries. Details discussion will be in chapter 3.

2.5 Conclusion

Here we have discussed the data set, the format of data stored in our dataset and also we have discussed how subjects perform different tasks. We have also shown this with the help of graphs of i.e timing scheme of different paradigms . We have also discussed 10-20 electrode montages using international standards. 25-channels out of which 22-channels we have used discussion of all of this we have done in section 2.3.1. The plotted graph represents a specific task in our case we have used our left hand and right hand movement task. Furthermore, we have discussed preprocessing in section 2.3.2 here we have discussed a filter which is used for removing noise. Also type of artifact is present in our data set. Removing of this artifact using filter is discussed in this section. Filtered EEG data further used for decomposition, in our work we have used FBSE method which help us to decompose EEG signals. The FBSE decomposes the signals and we are able to separate out the EEG rhythm with the help of the given method.

Chapter 3

Proposed framework

3.1 Introduction

The databases used and the Fourier Bessel series expansion considered in this work are discussed in chapter 2. This method used in MI-EEG classification problem. Pre-processing done on decomposed signals and different feature are extracted and used as input to the classifier. Various machine learning and deep learning based classifier need to be consider to check and compare accuracy, sensitivity, specificity in this proposed framework we are comparing accuracy. In this chapter, frameworks which include the different classifier model proposed. The proposed frameworks are discussed in some details here. Using these frameworks automated MI-EEG classification is achieved and the accuracy obtained are analyzed and compare in upcoming chapter.

3.2 Organisation of chapter

The rest of the chapter is organized as follows.An automated MI-EEG classification framework with machine learning and deep learning classifier such as K-NN, random forest, ensemble K-NN, LDA are discussed in section 3.3. Feature extraction such as Hjorth (mobility, complexity), band power are also discussed here. Section 3.5 classification evaluation index has discussed. Different type of classifier discussed in section 3.5. Further K-fold cross-validation has been discussed in section 3.6. Conclusion and remarks are in section 3.7.

3.3 Automated MI-EEG classification model

The EEG signals which are extracted from dataset i.e the BCI competition IV dataset 2A are used in our work. Each of these EEG signals have 25 channels. It was observed that out these 25 channels, 3 channels were more important in the MI–EEG signals. Three channels suggested were C_3, C_4, C_z are 8, 12, 10 channel respectively. Selection of channel is important because it increases the redundancy due to noise and creates high-dimensional data. Main reasons to reduce the number of EEG signals channels is feasibility of BCI implementation, BCI cost reduction, and improvement of the BCI performance. Fig. 3.1 shows block diagram of proposed framework for automated MI-EEG classification model describe in this work.

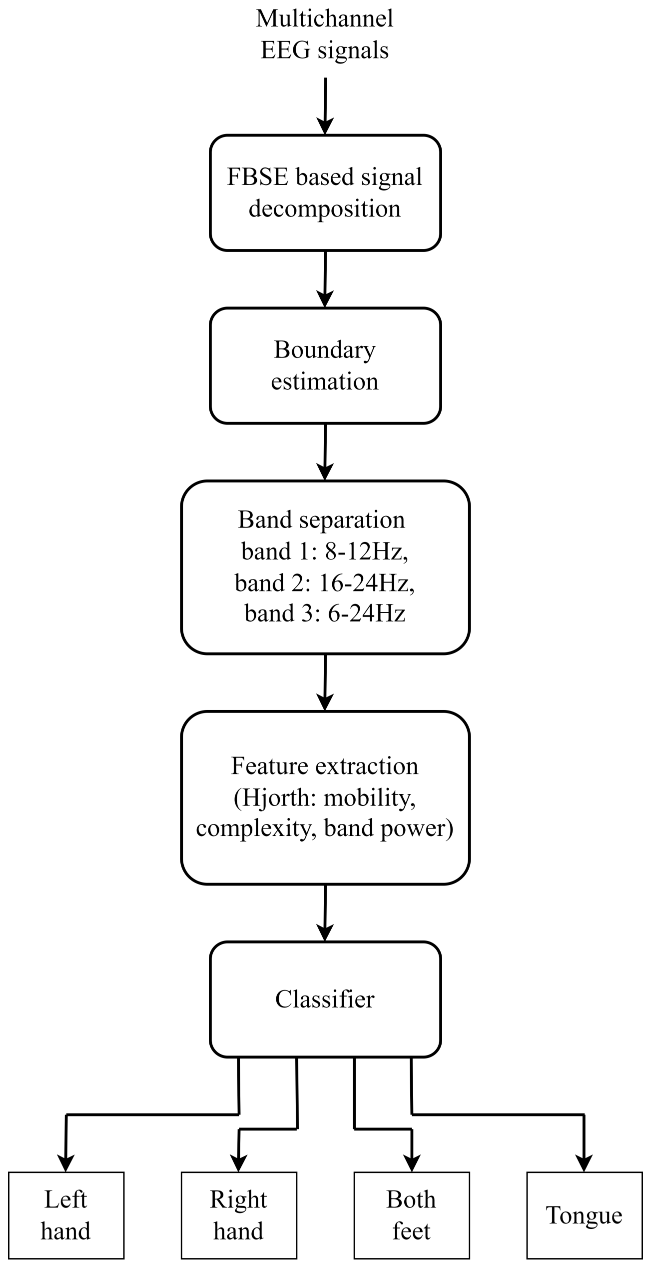


Figure 3.1: MI-EEG classification framework model.

In this work all channel consider individually. Three second epochs were extracted from the last 8 sec of EEG signals, where each subject is recorded of 44 minutes (672528 sample per subject) respectively. Depending upon the number of samples recorded by particular subject there are six runs for each one subject and each run has 48 trials (12 for class 1, 12 for class 2, 12 for class 3, 12 for class 4). In three seconds, we have 262 epochs for each subject. For each subject starting 5-minute data (7500 samples) removed. Remain sample divided with 2000 samples (8 sec) five epoch of subject one for 8 second. Three seconds (1500 samples) are taker from 8 seconds (2000 samples) by considering timing scheme of paradigm shown in Fig. 2.2.

3.3.1 Boundary detection and band separation

As discussed in chapter 2, peak value and its location detected. The boundary detection method [68] was utilized to segment the FBSE spectrum into N subband signals and to find the best $N+1$ boundary frequencies. By examining successive middle maxima, between 0 and π. Where $N-1$ intermediary boundary frequencies are extracted. After adaptively recognizing intermediate order ranges, the FBSE algorithm separates the spectrum into an approximated N number of fixed border segments, whch is shown in Fig. 3.2. The middle of two progressive local maxima, which is given by equation 3.1, can be used to calculate the border of each segment [38] [67].

$$W_i = \frac{F_{c_i} + F_{c_{i+1}}}{2} \tag{3.1}$$

Where F_{c_i} and $F_{c_{i+1}}$ two frequencies and the N-number of boundaries set is denoted by i= 1, 2,..., $N-1$.

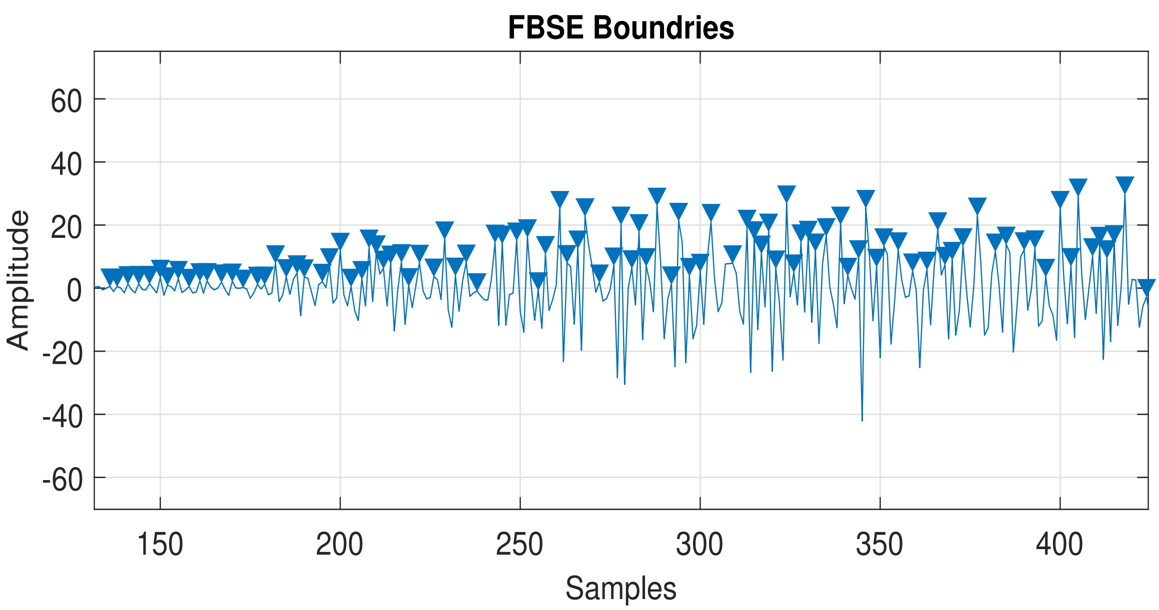


Figure 3.2: Boundary detection in the FBSE spectrum.

3.3.2 Feature extraction from MI-EEG rhythm

As discussed in above section the once boundary detection is done. We will calculate mean frequency based on mean frequency we are easy to categories them in bands and feature are extracted from respective band. Most widely used feature for MI-EEG signals is Hjorth which contain mobility and complexity and second one is band power. The details discussion is as fallows:
The process of feature extracting from a signal is an important step in reducing its dimensionality. And at the same time extract the important information. Feature extraction plays an great role in the future development and application of BCI. Here we have extracted two major features that are Hjorth (mobility, complexity) and band power which is most commonly used for classification of left hand, Right hand, both feet, tongue MI EEG signals for BCI applications. For MI EEG signal classification, both of these features were extracted from enhanced EEG signals. Hjorth parameters are defined in the time domain which shows the variation characteristics of EEG signals related to motor imagery tasks. Hjorth comprises three parameters namely activity, mobility, and complexity, which is defined as follows: The average power of signals is measured by activity, which is also known as signal variance, it can be expressed as [69]

$$Activity = \sum_{i=1}^{M_s} \frac{(p(i)-\mu)^2}{M_s} \tag{3.2}$$

Mobility, the second Hjorth parameter, is the estimate of the mean frequency. Given below is the formula for mobility [70]–[74]

$$Mobility = \sqrt{\frac{var(p')}{var(p)}} \tag{3.3}$$

Complexity, the third Hjorth parameter, is a measurement of the signal's bandwidth. Its definition is as follows [68]:

$$Complexity = \frac{mobility(p')}{mobility(p)} \tag{3.4}$$

Where p: Signals, p': First derivative of the signals, μ: The mean of the signals in the computation sampling window, M_s: The number of samples available in the window. Hjorth characteristics stated above were calculated using a first-second window of MI-EEG data from the one channel C_3. For a specific classifier, all three Hjorth parameters are employed as input features. Hjorth parameter calculated in the frequency band of 6-24 Hz. Next, the classification of several tasks is based on the power differences in each band [75]–[78]. Here we are using 2 bands based upon spectral power. The subject's task is simple to determine. Band power attributes that are calculated over a short time window were studied. Generally, the band powers of the two frequency bands, 8 - 12 Hz, and 16 -

24 Hz, are calculated and sent into the classifier for the categorization of EEG data corresponding to left-hand and right-hand MI tasks.

3.3.3 Feature Ranking

The feature ranking is done with the use of six feature selection strategies and majority vote. The techniques for feature selection are used to evaluate the features which are the most important. The process of feature selection makes the model simpler and improves classifier accuracy. In classical feature selection in classification, only an estimation criterion is used. This might cause a bias against that criterion. As a result, we used a novel feature selection which was based on multiple criteria in our research. Using a majority vote, several feature selection procedures are combined to extract the most significant feature. The following sections cover all of the feature selection strategies that are used for majority feature smoothing technique [79]:

- **Relief:** Relief is a feature selection technique based on a filter mechanism which is particularly interaction between features sensitive. It is mostly used to solve binary classification problems with feature vectors that are numerical. The main idea behind this approach is to forecast how effectively two occurrences of characteristics can be discriminated against when they are close enough. Each feature is given a score, and The features with the highest scores are selected as relevant features. [80] provides a detailed explanation of the algorithm.

- **Relief-F:** Relief-F is a more powerful version of relief that can deal with noisy and incomplete data. Feature relevance is used in this algorithm to estimate weights for certain features. In order to select random samples, the algorithm looks to determine neighbors of the same class that are known by nearest hits, and nearest misses among neighbors of other classes. The closer the distance between a randomly selected sample and the nearest miss is to the nearest hit, the greater the relevancy of the feature and vice versa. The more details about relief-F algorithm is found in [81].

- **Chi-square:** This is a hypothesis test which is also called as χ^2 test. It estimates characteristics that are reliant on each other [82], according to statistical theory. The Chi-square test is also used to determine the level of independence between classes and characteristics. It first guarantees that the feature vector is numeric and then discretized.

- **Student t-test:** The Student-t test is affected by the binary class mean. The algorithm uses the mean of binary classes to rank the features based on the acquired t-value; higher feature t-values indicate greater distinction between two classes. It is a statistical method for determining the statistical significance of a variation between two data sets [83].

- **Correlation Attribute (CA):** This is a strategy for feature selection which is correlation-based. It takes into account the amount of repetition among characteristics as well as their co-relation. The correlation coefficient describes the connection between classes and the inter-correlation between features [84]. The importance of characteristics increases as the correlation between them and the classes grows, however the relevance of features reduces as the correlation between features grow.

- **GainRatio Attribute (GA):** Information ratio has been reformed into gain ratio. When choosing characteristics [84], the size of the branches and and the number of the branches are taken into account. The intrinsic information from the attribute information is used to correct gain. Intrinsic information depends on where the entropy of accessible samples in that branch is.

3.4 Classification

Classification is one of the fundamental techniques in machine learning for data analysis, which classifies categorical labels. The process can be made into four unique stages. The system has been trained with the database tuples and their related class mark in the learning stage. In the subsequent stage, the system is practiced with test data for classification. The performance of the system is assessed by the percentage of test data that are precisely classified by the classifier. To categorise motor imagery EEG data, several machine learning methods have been applied [85] [66][86]. Features such as Hjorth and band power are fed to four distinct classifiers in order to identify motor imagery tasks from subjects and to compare the classification accuracy have been studied in this thesis report. Unlike conventional data sets, real-time data is exceedingly confusing and varies depending on a variety of things. In the last few years, traditional classifiers for brainwave classification have been able to produce good results for a wide range of data in the BCI field. Some of them are LDA, K-NN and ensemble K-NN. Thus, experimenting with ensemble learning gives us a different approach for learning from real-time data obtained. Classification in our model is Quaternary which means that our model predicts whether it is right hand, left hand, tongue, both feet. In our model, for classification, we have incorporated LDA, K-NN, ensemble K-NN classifiers.

3.4.1 Random Forest Classifier

Random forest (RF), designed by Leo Breiman in 2001, has shown to be a potent method with outstanding classification performance [85][66]. An ensemble of classification trees, based on the bootstrap sample of the data, is used for introducing both bagging and random variable selection for tree creation. The variable candidate set is randomly selected from the whole variable set at each split. By creating each tree on several random subsamples and deciding the splitter partially at random, randomness is incorporated. To achieve lower bias, each tree is completely matured. Individual trees have a low correlation because of both bagging and random variable selection. The algorithm generates an ensemble forest [85] by averaging across a wide ensemble of low-bias, high-variance, but low correlation tree.

3.4.2 K-nearest neighbors classifier

K-NN is a non-parametric learning algorithm that is based on two parameters: the number of closest neighbours and different distance measures. Euclidean, Minkowski, and Mahalanobis distance matrices are the most frequent distance matrices utilised by the K-NN algorithm to improve classifier performance. Because it just based on the worth of of k and the distance matrix [87], the K-NN is suited for EEG data because it is simple to handle noisy and big data. The K-NN algorithm predicts the values of new data points based on 'feature similarity,' which means that the new data point will be assigned a value based on how closely it matches the points in the training set. The following steps will help us understand how it works:

Step 1 We'll need a dataset to implement any algorithm. As a result, we must load both training and test data during the first step of KNN.

Step 2 Next, we must choose K's value, i.e. the data points that are closest to it. K can be any integer.

Step 3 We do the following for each point in the test data

- Calculate the distance between each row of test data and each row of training data using any of the following methods namely: Euclidean, Manhattan, or Hamming distance. And The Euclidean method is the most widely used method for calculating distance.
- Now, sort them in ascending order based on the distance value.
- Next, it will choose the top K rows from the sorted array.
- Now, it will assign a class to the test point based on the most frequent class of these rows.
- End

3.4.3 Ensemble K-NN classifier

The K-NN method is a rudimentary machine learning approach in which the features associated with different classes form separate clusters in the feature space. To categorise a test feature vector, this classifier employs the k distance metric between test sample features and those of the closest classes. The number of neighbours and the types of distance measurements are the most important elements in K-NN architecture. Because of its powerful generality and ease of implementation, In pattern recognition, the K-NN is extensively used. The great dimensionality of EEG, on the other hand, frequently hinders K-NN performance. Complexity increases exponentially with the number of features. A strategy that can take advantage of the benefits of a K-NN classifier while not being harmed by the sparsity of high-dimensional data would be highly preferred in this scenario, and the well-known ensemble learning technique efficiently takes advantage of high-dimensionality. Ensemble classifier improves overall classification efficiency by merging the results of weak or base classifiers to create a robust classifier. In terms of adjusting training datasets, K-NN is effective and reliable. However, it is susceptible to feature set fluctuation. Random subspace-based ensemble systems can improve the efficiency of single K-NN classifiers since K-NN is cognizant of input options. The most frequently used ensemble approach is random subspace which constucts separate classifiers from subspaces of data which are randomly selected. The final result is obtained by combining the findings of separate classifiers using a typical majority vote. Only the specified features are entered into the distance when a test sample is chosen as a prototype for a K-NN classifier. However, in subspace K-NN, it is the projection of all points to the specified subspace, and The distances are used to find the k nearest neighbours. A fresh set of k-nearest neighbors is determined once a random subspace is generated. A majority vote on the test sample's class membership is achieved by merging k near neighbours in each chosen subspace. The same training sample recurs in this ensemble if it is discovered in several selected subspaces in the center of adjacent k neighbors.

3.4.4 Linear discriminant analysis classifier

In general, classifying the derived characteristics for MI-EEG signals classification in BCI applications is tricky. Finding the best mix of traits that can improve discrimination is a critical challenge. The LDA classifier is a popular choice for BCI applications which are EEG-based. The LDA classifier attempts to minimize dimensionality while yet protecting the majority of class discriminating information. The linear discrimination analysis (LDA) has a rudimentary structure and is quick to compute. For instance, we have a collection of two classes designated by the letters W_1 and W_2. Then we categorize the n-dimensional sample points $x = x_1, x_2, x_3, ..., x_n$, N_1 samples into class W_1 and n_2

samples into class W_1. In this technique, we strive to choose a line $y = w^T x$ from a collection of all feasible line that maximizes the discrimination between the two classes. We must estimate the separation between the two classes chosen for the research in order to acquire an appropriate projection vector. The mean vector in x-space and y-space for each class is listed below equations [86].

$$\mu_i = \frac{1}{N_i} \sum_{x \varepsilon w_i} x \tag{3.5}$$

and

$$\mu_i = \frac{1}{N_i} \sum_{y \varepsilon w_i} x \tag{3.6}$$

$$y = \frac{1}{N_i} \sum_{y \varepsilon w_i} w^t x = w^t \mu_i \tag{3.7}$$

the distance between the two projected means is defined as the objective function. Its representation is given below [86].

$$J(w) = |v_1 - v_2| = \left| w^t (\mu_1 - \mu_2) \right| \tag{3.8}$$

But, because the standard deviation across classes has not been accounted, the distance calculated between projected means may not necessarily be a valid estimate. To alleviate the aforementioned problem, the Fishers LDA classifier was developed as an augmentation of LDA. To categorize features into discrete classes, it finds a decision boundary or, more likely, a hyperplane in the feature space. It calculates the separation border between two specified distributions using the ratio of two group variances, as mentioned underneath [88]:

$$J(w) = \frac{\sigma_2^{between}}{\sigma_2^{within}} = \frac{w^t (\mu_1 - \mu_2)^2}{w^t S_1 w + w^t S_2 w} \tag{3.9}$$

Where μ_1, μ_2 are the class means, and s1 and s2 are the feature distribution variations between two classes w1 and w2. Equation 3.9 describes how to compute the largest separation between two classes:

$$w^\star = (S_1 + S_2)^{-1} (\mu_1 - \mu_2) \tag{3.10}$$

The $w^\star$ is a weight vector that offers the best projection direction for the data. The decision boundary in Fisher's LDA employs the below equation to categorize $d(m)$ as [89], $d(m)$ here is the feature vector.

$$p(m) = d(m) w^t + b \tag{3.11}$$

b is the bias or threshold. Based on the sign of the $p(m)$, the features are allocated to one of the classes.

3.5 Classification evaluation index

The performance of the studied classifiers was evaluated with three evaluation indexes, namely accuracy, sensitivity, and specificity. These parameters' mathematical expressions are listed underneath:
Accuracy: It is the ratio of correctly identified samples by the classifier to total number of samples in test set. It can be formulated as follows:

$$\text{Accuracy} = \frac{T_p + T_n}{T_p + T_n + F_p + F_n} \tag{3.12}$$

Sensitivity: This parameter measures ratio of correctly identified positive samples present in test set to the total number of positive samples in the data set, which can be expressed in the following way:

$$\text{Sensitivity} = \frac{T_p}{T_p + T_n} \tag{3.13}$$

Specificity: Parameter evaluates classifier on the basis of ratio correctly identified negative samples present in test set to the total number of negative samples in the data set which can formulated as follows:

$$\text{Specificity} = \frac{T_n}{T_p + T_n} \tag{3.14}$$

Where T_p represents the rate of true positive, T_n represent rate of true negative, F_p represent rate of false positive, and F_n represent false negative rate. Above mentioned parameter, has been taken into consideration for performance evaluation of classifier. Development of robust classifier, which performs well with new data samples, is done using ten-fold cross-validation technique, a data resample for training and testing purpose. In this re-sampling technique, ten-part splitting of data is done with approximately same proportion of date in each part result obtained in each fold are used for calculation of sensitivity, specificity, accuracy and classification. Performance of classifier is estimated by averaging the value of performance measure over these ten folds.

3.6 K-fold cross-validation

k-fold cross validation was used to reduce the influence of the chosen training and test data on the model evaluation. This involves the training data being divided into subsets without repetition.

$$(V_1, V_2, ..., V_k)(V_i, V_j = \phi) \tag{3.15}$$

for training $k-1$ subsets were used, and the remaining subset was used for testing. To obtain k accuracy values This process was repeated k times, which were averaged to provide a mean value for the evaluation.

3.7 Conclusions

In this chapter, the frameworks proposed for MI-EEG signals classification were discussed for motor imagery- EEG signals classification only 3 of 22 channels used, it is found that these channels were more important in motor imagery classification scenario. Hjorth (mobility, complexity) band power was the feature extracted here and this was given as input to various classifier models we have also discussed feature ranking method which help us to choose best feature. Different machine learning based classifier were tried. Classification evaluation index help us to find performance with help of evaluation indexes namely accuracy, sensitivity, specificity. Further we discussed k-fold cross-validation.

Chapter 4

Results and discussions

4.1 Introduction

In the previous chapters, the features extracted and the frameworks proposed for BCI motor imagery task classification were discussed. Here, the classification results obtained using these frameworks are presented. For the BCI motor imagery task classification problem, different classifier models are used and the accuracies obtained are discussed and compared. The best accuracy obtained and then compared with existing literature to get an idea about the effectiveness of the proposed framework. For the MI task classification problem, a different classifier frameworks were used on obtained features (Hjorth, band power) on the EEG sample size of 4716 and then performance metrics were compared. Further, the best performance metric obtained was compared with existing literature. The results obtained for both the use cases are then discussed and the reasons for the same are explored.

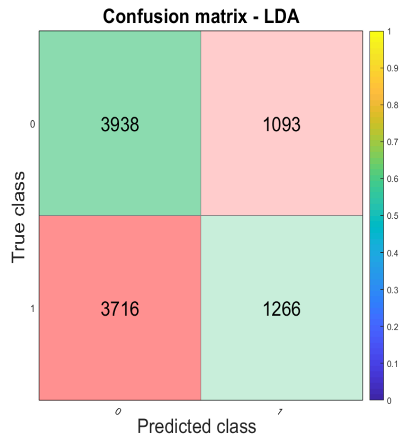
4.2 Organisation of chapter

The rest of the chapter is organized as follows. The results obtained for the BCI motor imagery task classification problem are discussed in section 4.3. Section 4.4 provides the results obtained for the BCI motor imagery task classification problem. The concluding remarks are given in Section 4.5.

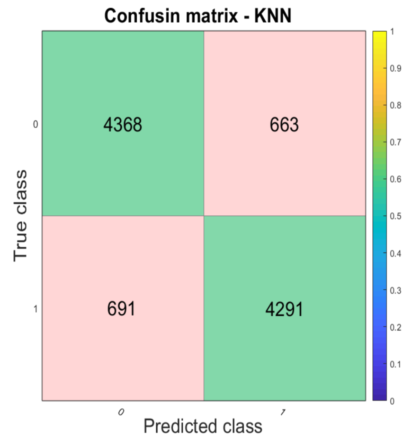
4.3 MI-EEG task classification

4.3.1 Result obtained

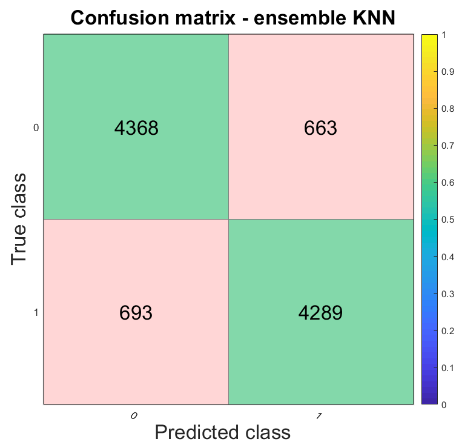
In this work, MI task are extracted from the EEG signals with different classifiers namely LDA, K-NN, Ensemble K-NN, and random forest respectively. The Hjorth, band power is then evaluated for each event for each scenario. The features (Hjorth and band power) was obtained by using the images function in MATLAB. These features input to the classifier. The training set had 480 samples with 4096 sample length each for BCI motor imagery task events. The validation test set had 480 samples, 240 each for both classes. Each event in the EEG signals has 4716 samples length. To reduce complexity, feature smoothing has been done using t-test feature ranking method. The obtained significant features were applied to different classification framework namely random forest, K-NN, ensemble k-NN, and LDA. Fig. 4.5 shows confusion matrices obtained for MI-EEG classification when EEG signals with 480 samples.



(a) Confusion matrix obtained for the LDA classifier



(b) Confusion matrix obtained for the KNN classifier



(c) Confusion matrix obtained for the ensemble KNN classifier

Figure 4.1: Confusion matrices obtained for various classifiers when of EEG signals are considered.

with the help of confusion matrix we are able to check the performance of classification model. As we know confusion matrix is a very popular measure used while solving classification problems. Confusion matrices represent counts from predicted and actual values. The confusion matrix is made up of four basic attributes (numbers) that are used to define the classifier's measurement metrics.

These are the four numbers is given as fallows:

1. True Positive (T_P): T_P denote the number of who have been properly classified to have malignant nodes, meaning they have the disease.

2. True Negative (T_N): T_N denote the number of correctly classified patients who are healthy.

3. False Positive (F_P): F_P denote the number of misclassified patients with the disease but actually they are healthy. FP is also known as a Type I error.

4. False Negative (F_N): F_N denote the number of patients misclassified as healthy but actually they are suffering from the disease. FN is also known as a Type II error.

Accuracy of an algorithm is represented as the ratio of correctly classified patients (T_P+T_N) to the total number of patients (T_P+T_N+F_P+F_N).

$$\text{Accuracy} = \frac{T_P + T_N}{T_P + T_N + F_P + F_N} \tag{4.1}$$

It can be applied to binary classification as well as for multiclass classification problems. The accuracy obtained for various classification framework i.e LDA, KNN, EKNN is described in Figure 4.1. The KNN classifier give good accuracy compare to othe classifier.

as mention in above Table 4.1 it shows the performance of FBSE method using different classifier. Here we have 9 subject of training and 9 subject of evaluation. FOR random forest classifier average value of training is 79.39% and average value of evaluation is 78.96%. Average value of both training and evaluation is 79.19%. The highest Accuracy obtain maxing value obtain in training is 87.39% and maximum value obtain for evaluation is 81.29% maximum of both training and evaluation is 87.39%.

for KNN classifier average value of training is 82.63% and average value of evaluation is 87.34%. Average value of both training and evaluation is 84.98%. The highest accuracy obtain in training is 87.4% and maximum value obtain for evaluation is 98.6% maximum of both training and evaluation is 98.6%.

for ensemble KNN classifier average value of training is 82.44% and average value of evaluation is 85.63%. Average value of both training and evaluation is 84.70%. The highest accuracy obtain. Maxing value obtain in training is 87.4% and maximum value obtain for evaluation is 97.6% maximum of both training and evaluation is 97.6%.

for LDA classifier average value of training is 60.8% and average value of evaluation is 72.77%. average value of both training and evaluation is 66.78%. the highest accuracy obtain.maxing value obtain in training is 73.1% and maximum value obtain for evaluation is 97.3% maximum of both training and evaluation is 97.3%.

As mention in above table 4.2 it shows the performance of EMD method using different classi-

Table 4.1: Classification performance obtained using the FBSE method.

Subject No.	Random forest (%)	K-nearest neighbors (%)	Ensemble KNN (%)	Linear discriminant analysis (%)
A01T	78.58	87.4	87.4	73.1
A02T	79.64	86.4	86.4	70.0
A03T	77.62	83.6	83.6	64.9
A04T	78.14	82.5	82.5	63.0
A05T	78.44	81.6	80.8	59.7
A06T	77.96	80.0	79.4	56.6
A07T	87.39	87.4	87.4	56.7
A08T	78.28	78.1	77.9	52.3
A09T	78.46	76.7	76.6	50.9
A10E	77.58	98.6	97.6	97.3
A11E	81.29	95.3	95.00	87.6
A12E	79.12	91.6	91.5	79.4
A13E	77.70	88.5	88.6	75.3
A14E	81.26	87.7	87.6	71.0
A15E	77.57	83.5	82.9	65.4
A16E	78.94	82.9	82.52	62.3
A17E	78.67	81.00	80.08	65.9
A18E	78.51	77.00	76.9	50.8
Avg all	79.19	84.98	84.70	66.78
Avg training	79.39	82.63	82.44	60.8
Avg evaluation	78.96	87.34	85.63	72.77
Max training	87.39	87.4	87.4	73.1
Max evaluation	81.29	98.6	97.6	97.3
Max all	87.39	98.6	97.6	97.3

Table 4.2: Classification performance obtained using the EMD method.

Subject No.	Random forest (%)	K-nearest neighbors (%)	Ensemble KNN (%)	Linear discriminant analysis (%)
A01T	52.18	72.4	72.6	71.3
A02T	52.70	70.4	70.5	68.9
A03T	51.92	67.2	67.4	66.3
A04T	52.19	64.4	64.5	63.0
A05T	52.72	63.6	63.6	62.4
A06T	52.25	60.3	60.5	58.8
A07T	52.84	58.4	56.9	56.3
A08T	52.70	55.2	49.5	53.4
A09T	51.84	51.8	50.8	50.6
A10E	52.21	97.1	97.1	97.0
A11E	51.91	89.3	88.6	88.6
A12E	52.45	84.5	84.5	83.5
A13E	52.70	79.1	78.8	78.0
A14E	52.04	73.4	72.7	72.4
A15E	52.60	68.5	68.4	66.1
A16E	52.49	62.9	61.3	61.3
A17E	52.24	57.5	57.2	55.1
A18E	52.26	52.1	50.9	50.7
Avg all	52.34	68.22	67.54	66.87
Avg training	52.37	62.63	61.81	61.22
Avg evaluation	52.32	73.82	73.27	72.52
Max training	52.84	72.4	72.6	71.3
Max evaluation	52.7	97.1	97.1	97
Max all	52.84	97.1	97.1	97

fier.here we have 9 subject of training and 9 subject of evaluation. For random forest classifier average value of training is 52.37% and average value of evaluation is 52.32%. Average value of both training and evaluation is 52.34%. The highest accuracy obtain.maxing value obtain in training is 52.84% and maximum value obtain for evaluation is 52.7% maximum of both training and evaluation is 52.84%

for KNN classifier average value of training is 62.63% and Average value of evaluation is 73.82%. average value of both training and evaluation is 68.22%. The highest accuracy obtain.maxing value obtain in training is 72.4% and maximum value obtain for evaluation is 97.1% maximum of both training and evaluation is 97.1%.

For ensemble KNN classifier average value of training is 61.81% and Average value of evaluation is 73.27%. average value of both training and evaluation is 67.54%. The highest accuracy obtain in training is 97.1% and maximum value obtain for evaluation is 97.1% maximum of both training and evaluation is 97.1%.

For LDA classifier average value of training is 61.22% and average value of evaluation is 72.52%. average value of both training and evaluation is 66.87%. The highest accuracy obtain in training is 71.3% and maximum value obtain for evaluation is 97% maximum of both training and evaluation is 97%.

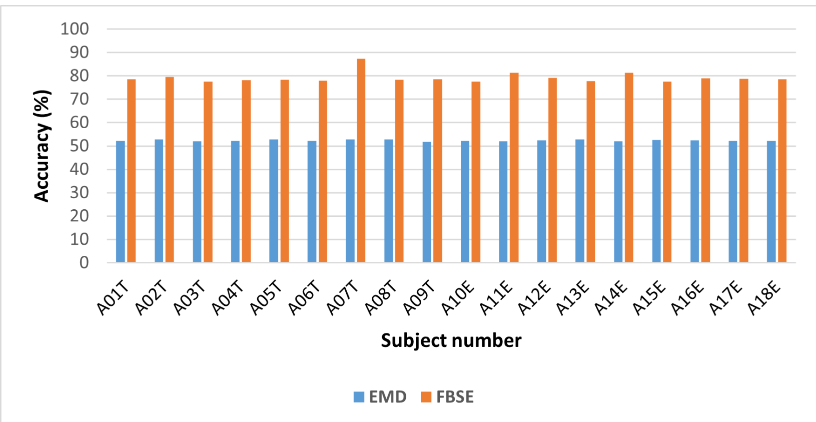


Figure 4.2: Comparison of FBSE and EMD based classification using RF classifier.

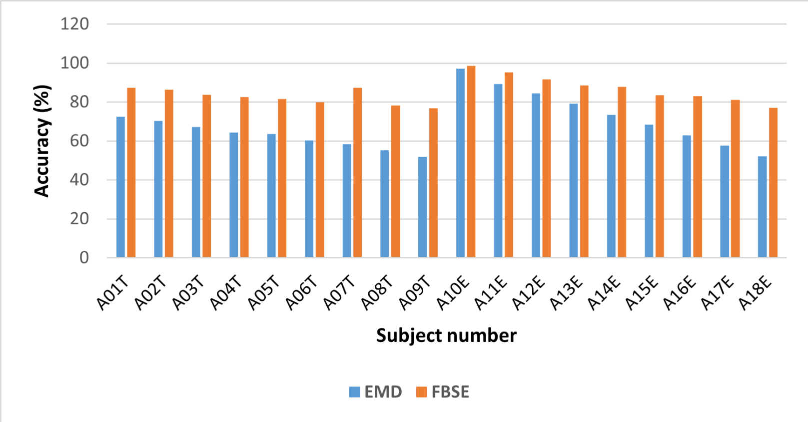


Figure 4.3: Comparison of FBSE and EMD based classification using ensemble KNN classifier.

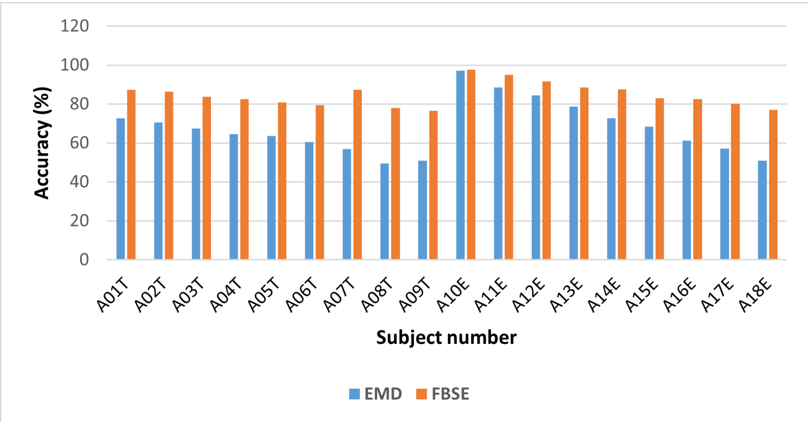


Figure 4.4: Comparison of FBSE and EMD based classification using KNN classifier.

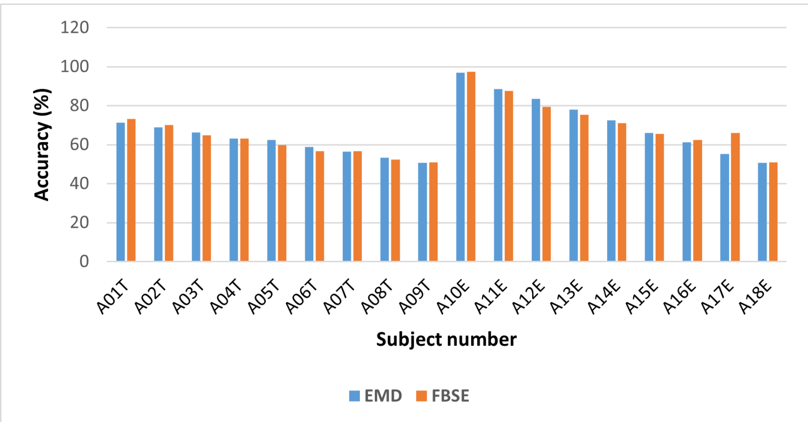


Figure 4.5: Comparison of FBSE and EMD based classification using LDA classifier.

figure 4.2, 4.3, 4.4,and 4.5 show the comparison of accuracy of two method i.e FBSE and EMD. figure 4.3 The highest accuracy using FBSE method with significant selection of feature and classifier. We are able to obtain maximum classification accuracy for MI-EEG signal is 86.46% When signals with 480 samples were considered, the accuracy's obtained for various models were as follows:

- For a classification model with random forest as a maximum accuracy of 87.39% was obtained on the test/validation set.

- For a classification model with KNN as the transfer learning block, a maximum accuracy of 98.6% was obtained on the test/validation set.

- For a classification model with Ensemble as the transfer learning block, a maximum accuracy of 97.6% was obtained on the test/validation set.

- For a classification model with Ensemble as the transfer learning block, a maximum accuracy of 97.3% was obtained on the test/validation set.

4.3.2 Discussions

Another major issue in any BCI system is subject dependency. or any approach, It is expected that the same group of subjects will perform similarly. The proposed method has been evaluated on BCI competition IV dataset 2A, which have nine subjects. The table 4.3 has listed the summary of some recent studies, where BCI competition dataset IVa, has been classified using different methods. Author Mensh et al. [18], 2004 proposed gamma-band feature as input to the multitaper classifier which gives an accuracy of 88.7%. Sun et al. [27], 2005 implemented considering feature of spectral centroid using the Bayesian as a classifier which gives an accuracy of 90.44%. Further to increase accuracy, Wang et al. [30], 2005 introduced a new method of wavelet packet transform. In this, a feature that is input to the neural network gives an accuracy of 91.47% on dataset Ia. Yi et al. [13], 2005 used a data set of BCI Ia for MI-EEG classification which gives an accuracy of 90.3% for the k-NN classifier used features are Kolmogorov and complexity. Duan et al. [12], 2014 tried to improve accuracy and they obtained an accuracy of 93.52% using the very extreme learning machine (V-ELM) classifier on dataset Ia. Duan et al. [61], 2016 apply neural network OS-ELM and feature applied is PCA and LDA which gives the accuracy of 94.2% and this is our reference value for accuracy. We proposed FBSE method and used Hjorth and band power as features to a different classifier framework using data set of BCI competition IV gives an accuracy of 98.6%. It is observed that, the proposed FBSE based classifier framework delivers best classification accuracy as compared to existing state-of-the-art methods that addressed the same classification problem on the same dataset. Therefore, we can state that our method performs well ,out of all other methods with a clear margin.

Table 4.3: Comparison of proposed method with the existing methods.

Author, reference, and year	**Dataset**	**Methodology (feature, classifier)**	**Accuracy(%)**
Mensh et al. [90], 2004	BCI dataset Ia	Gamma band power, multitaper	88.70
Sun et al. [91], 2005	BCI dataset Ia	Spectral centroid, bayesian classifier	90.44
Wang et al. [92], 2005	BCI dataset Ia	wavelet packet transform, neural networks	91.47
Yi et al. [93], 2005	BCI dataset Ia	Kolmogorov and complexity Fisher's linear discriminant, *k*-NN	90.3
Hu et.al. [94], 2011	BCI dataset Ia	Wavelet packet decomposition, *k*-NN	90.1
Duan et al. [95], 2014	BCI dataset Ia	LDA after PCA, V-ELM	93.52
Nguyen et al. [96], 2015	BCI dataset Ia	Wavelet coefficients, Tabu-FSAM	90.2
Duan et al. [97], 2016	BCI dataset Ia	PCA and LDA, neural network OS-ELM	94.2
Proposed work	**BCI dataset 2A**	**FBSE- *k*-NN**	**98.6**

Chapter 5

Conclusions and future works

This thesis examines the usage of EEG signals for MI-EEG signal classification. Fourier Bessel series expansion method was proposed and feature (Hjorth and band power) were extracted and used as input for classifier model framework. Various performance matrix was evaluated to check the performance of the model and these produce encouraging results.

5.1 Conclusions

In chapter 2, the databases that were used in this project i.e., the BCI competition dataset 2A. EEG signals were extracted from these datasets and were used for subsequent steps. In this work,initialLy a band-pass filter is also introduced here to remove any artifact present. In chapter 3, proposed FBSE based classification frameworks were proposed to complete the task of classification. For MI-EEG signals classification problems, different machine learning-based classification models are used to evaluate accuracy. In chapter 4 results obtained from various frameworks were presented and discussed. The accuracy obtained for various machine learning classifier for MI-EEG classification were compared and the reasons for the same were discussed.

Hence we have explored an application of the The FBSE method for improving the motor imagery brain-computer interface's performance. Here FFT spectrum has been replaced with the FBSE spectrum for estimation of optimal boundary frequency. The proposed method has been applied to the multichannel component of EEG signals. A set of sub-bands with a mean frequency that falls within the frequency range of 6-24 Hz (for Hjorth) and 8-12, 16-24 Hz (band power) rhythm has been provided. Improvement in the accuracy of classifying right and left hand, both feet, and tongue MI-EEG signals as compared to other methods. Band power and Hjorth features are most significant than other features for MI-EEG signals classification. The random forest along with three different classifiers such as LDA, k-NN, and ensemble k-NN, are employed in the testing and training, k-NN acquires the highest accuracy 87.4% for training and 98.6%for evaluation compared to other classifiers and other decomposition methods.

5.2 Future works

The scope for the future work can be summarized as follows:

- Fourier Bessel series expansion-based MI-EEG signals classification is not a field that is much explored in the literature. It can be one of the key areas which can be thoroughly explored.
- In the It would be in the future be interesting to create a new feature based on the existing one FBSE method for the classification of MI -EEG signals.
- In our work, we have considered EEG signal solely to classify MI-EEG task.Therefore there is scope to use hybrid modality including biomedical signals such as EEG, ECG, EMG combinaly, to get improved classification accuracy performance using proposed method. .
- To increase classification accuracy, we will also hybridize our methodology with empirical wavelet transform and discrete wavelet transform.
- We will implement a more practical classifier in our future work and classifier accuracy compared with the most recent developed approach. So that proposed classifier framework can be accommodate with existing clinical automated device.

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