B. TECH. PROJECT REPORT

On

Game Theoretic Approaches in Lending to the Poor

BY

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DISCIPLINE OF COMPUTER SCIENCE AND ENGINEERING

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Game Theoretic Approaches in Lending to the Poor

A PROJECT REPORT

Submitted in partial fulfillment of the requirements for the award of the degrees

of
BACHELOR OF TECHNOLOGY
in

COMPUTER SCIENCE AND ENGINEERING

Submitted by:
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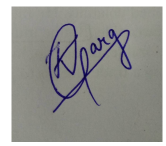
Guided by:
Dr. Kapil Ahuja

INDIAN INSTITUTE OF TECHNOLOGY INDORE
May 2022

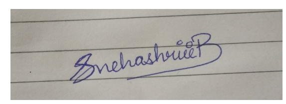
CANDIDATE'S DECLARATION

We hereby declare that the project entitled **"Game Theoretic Approaches in Lending to the Poor"** submitted in partial fulfillment for the award of the degree of Bachelor of Technology in 'Computer Science and Engineering' completed under the supervision of **Dr. Kapil Ahuja** (Professor, Computer Science and Engineering)**,** IIT Indore is an authentic work.

Further, we declare that we have not submitted this work for the award of any other degree elsewhere.



Kartik Garg



Snehashriie Bhukya

CERTIFICATE by BTP Guide(s)

It is certified that the above statement made by the students is correct to the best of my/our knowledge.

Dr. Kapil Ahuja

Professor

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<u>Preface</u>

This report on “**Game Theoretic Approaches in Lending to the Poor**" is prepared under the guidance of **Dr. Kapil Ahuja**.

Through this report we have tried to give a mathematical formulation of an innovative model on lending to the poor under group liability and also tried to find the optimal group size so that the social welfare is maximized.

We have tried to the best of my ability and knowledge to explain the content in a succinct and comprehensible manner. We have also added visual representations wherever possible to make the report more illustrative.

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Acknowledgements

We wish to thank Dr. Kapil Ahuja for his kind support and valuable guidance. He helped mold the direction of the project while constantly encouraging us to think outside the box.

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ABSTRACT

Abhijit Banerjee and Esther Duflo were jointly awarded the Nobel Memorial Prize in Economic Sciences in 2019 for their experimental approach to alleviating global poverty. Their works in this field have been summarized in the book named "Poor Economics" authored by them. One of the areas of focus in their work was their empirical study on how lending happens to poor borrowers. One of the methods discussed by them was group lending through various microfinance institutions. This method has become a popular practice in many developing countries to address the issue of lending for poor borrowers. Instead of attempting to borrow individually, people organize into self-help groups that collectively take out loans; all group members are responsible for paying them back. This arrangement allows financial institutions to rely on information advantages among group members, rather than on financial collateral, to mitigate information asymmetries between lenders and borrowers.

In our project, using the empirical study done by Banerjee and Duflo in their book "Poor Economics", we have studied and formed a game theoretic model of lending. For that, we first understood the setting and formed variables accordingly. Using those variables, we formulated our game between borrower and lender and computed the payoff of the borrowers in case of individual lending and group liability lending. We then compared payoff of borrowers under individual lending and group liability lending when using multiple strategies. Finally, using the game theoretic model which we had created for group liability lending, we found the optimal size of group which maximizes the social payoff of the members present in the group

Keywords: microfinance, group lending, group liability, strategies, social payoff

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Chapter 1

Introduction

Game theory is a theoretical framework for conceiving social situations among competing players. In some respects, game theory is the science of strategy, or at least the optimal decision-making of independent and competing actors in a strategic setting. (1)

Consider the social situation of poor people, who would need to lend money to start small businesses (selling flowers etc.) or improve their living. Poor people have no assets like lands or vehicles that they can use as a collateral and take loans. Moreover, the amount they take as loans are small but nevertheless important to sustain their day-to-day business.

Lending Institutes like Banks (private and government) become hesitant when it comes to giving loans to the poor people as giving loans without assets is seen as high risk. The returns they get (in the form of interest on the loans they lend) are also low due to the loan amount itself being small. (2) Since the poor people depend on their daily earnings to sustain themselves as well as repay the loans, if their business doesn't succeed or doesn't do well, they would not be able to repay the loans back. More often than not, they are more likely to default - i.e. fail to repay the debt.

Poor people, when taking loans individually, can turn to money lenders for their loans. However, the rates of interest that they charge is substantially higher than those of banks. And since these money lenders lend only to people they know or are of the same locality, it is difficult for the borrowers to default. (3) Although this sounds like a plausible solution for the poor people, the inability to repay the loan amount at such high interest rates presents itself as a major disadvantage.

Microfinance institutions give loans to a group of borrowers instead of individuals. In this group, every borrower is liable for each other's loans. This serves as an incentive to make sure others repay and makes it easier for MFIs to collect the repayment. (4) Some groups consist of people who know each other even before they come to take loans. In other cases, the borrowers in the group are brought together with the help of weekly meetings that allows them to get to know one another better. (2)

This innovative model of group liability is what we will be formulating, from a mathematical perspective. As a part of this, we design variables and payoff for borrowers under group liability loans in case of different strategies. We also find optimal size of groups which maximizes social welfare. Towards the end, we compare lending under group liability with lending to individual borrowers.

Chapter 2

Literature Review

2.1 What is Microfinance

Microfinance is a type of financial service aimed for people and small enterprises that do not have access to traditional banking and associated services. Microcredit, or the provision of small loans to poor clients, savings and checking accounts, micro - insurance, and payment systems are all examples of microfinance services. Microfinance services are intended to reach out to underprivileged clients, mainly from lower socioeconomic groups that are socially marginalised or geographically isolated, and assist them in becoming self-sufficient (2).

2.2 Group Liability Lending

Under group liability lending scheme, bank lends to each individual member in the group. However, now group members are not only responsible for their own repayment, but also the repayments of other members of the group. If the group members are willing to cover for each other when they are unable to repay, then group liability lending can improve social welfare.(5) If, the group completely repays the amount of money needed to be repaid back by all its members, then the loan continues, otherwise all group members will lose access to future loans (6).

Chapter 3

Methodology

3.1 Setting

Our model consists of two important entities that are interacting with each other in the game, lenders and borrowers.

3.1.1 Lenders

There is only one single lender, the MFI, who issues loans to borrowers under both individual liability lending and group liability lending. The objective of the lender is to recover the lending costs of the loan.

The cost incurred by the lenders (lending cost) is same for each borrower and is equal to c. Since c is the lending cost per borrower, individual borrowers are required to pay atleast c. For a group of borrowers, the lending cost becomes $\eta * c$, where η is the number of members in the group.

Borrowers who had previously defaulted on their payments do not get loans from lenders. Whereas the credit is renewed for borrowers who paid on-time.

3.1.2 Borrowers

All the borrowers could take the loans under two schemes, individual liability lending and group liability lending. We assume that the probability of success is independent across projects and that borrowers are risk neutral. To focus on the strategic default problem, we also assume that our borrower's only source of income is the return on their investment and they spend their whole income at the end of each time period, so they don't acquire assets over time.

3.1.3 Assumptions on Borrowers under Group Liability

Borrowers who are part of a group are equally accountable for the loans of everyone else. They all are identical, having the same project, capital requirement, and earning potential. Income from the investment in a project is Π if successful and 0 if unsuccessful. In order to demonstrate the insurance effect (7), we assume that even if only a single project is successful in the group, the resulting return will be sufficient to cover for all other members of the group i.e. $\Pi > \eta * c$. The groups are completely uniform and combined payoff of the whole group is equally divided amongst all its members.

3.2 Game Interaction between borrower and bank

1. Bank lends to the borrower
2. Borrower invest in projects with some success probability p
3. Nature independently draws the project outcomes. The borrowers observe the project outcome.
4. Borrowers then make their repayment decision.
5. If the repayment is sufficient to cover the bank's cost of funds, then the bank continues to lend and the process returns to 1. If some borrower hasn't repaid, then the bank asks all its neighbor borrowers to cover for him. If the amount repaid is still insufficient to cover the debt, lending ceases for all members in the group and the game ends.

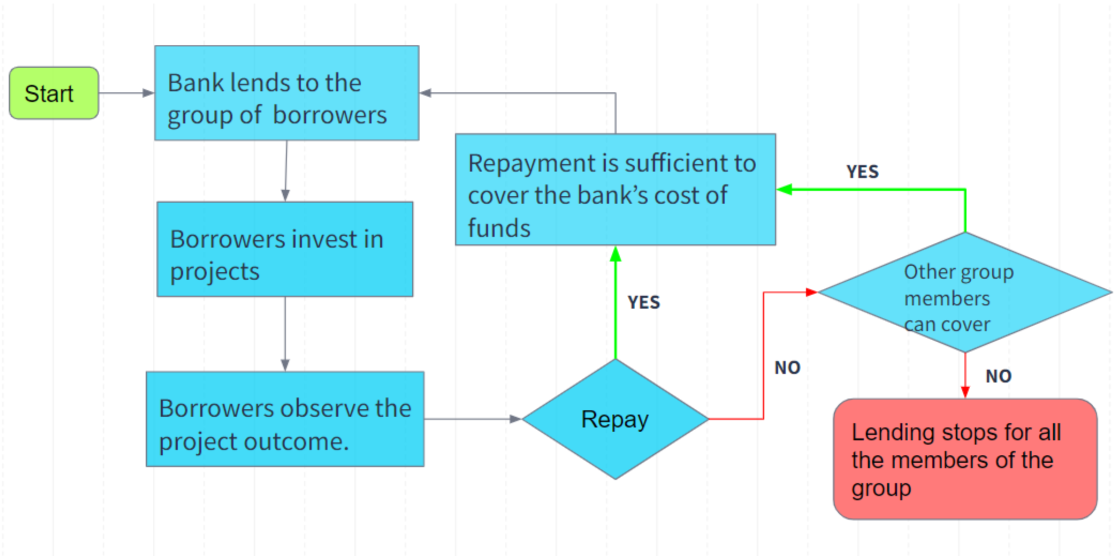

Figure 3.1: Game Interaction between borrower and bank

3.3 Variables

Table 3.1: List of important variables

Variable	Description
η	Number of members in the group of borrowers
c	Cost of lending incurred by the bank for one borrower
Π	Income from successful project (not fixed)
p	Success probability of project
V	Probability of borrower reaching new period
S	Total payoff of the group
W	Individual payoff of a borrower
W_I	W in case of Individual Lending, Repayment Strategy
W_I'	W in case of Individual Lending, Default Strategy
W_G	W in case of Group Lending, Repayment Strategy
W_G'	W in case of Group Lending, Default Strategy

3.4 Strategies

We will be modelling both individual lending scenarios as well as group liability lending scenarios mathematically.
For both scenarios, we will be discussing two of the strategies which borrowers can follow:

1. **Repayment strategy** : Borrowers repay back the loans in time to the lenders whenever it is possible to do so. This ensures continuation of the loans for the next iterations.

2. **Default strategy** : Borrowers always doesn't repay the loans back to lenders and always defaults in the first iteration itself. This leads to termination of the loan.

Chapter 4

Individual Lending

As our objective is to determine if group liability lending can increase social surplus, we compare joint liability lending to the conventional individual liability lending. Under individual liability lending scheme, each individual borrower is responsible for her own repayment. The bank continues to fund the loans only if the borrowers repay at least c. The bank deters delinqent borrowers from voluntary default by denying credit to those who have defaulted previously (8).

4.1 Repayment Payoff

The bank continues to fund the loans only if the borrowers repay at least c. We look at the case where an individual borrower is trying to keep getting loans for as long as they can.
Let, X_0 and X_1 be the group payoff at $t = 0$ and $t = 1$ respectively.
Then total payoff till $t = 1$ would be X_0 + (probability of reaching $t = 1) \times X1$.
Similarly, if X_2 is payoff at $t = 2$,
then total payoff till $t = 2$ would be earlier part + (probability of reaching $t = 2) \times X2$.
Hence the total payoff across all time periods would be:

$$W_I = X_{t=0} + VX_{t=1} + V^2X_{t=2} + \dots$$

Here V is the probability of reaching a new time period (i.e. $0 < V < 1$). We are assuming that payoff is time independent, i.e.

$$X = X_{t=0} = X_{t=1} = X_{t=2} = \dots$$

$$\Rightarrow W_I = \frac{X}{1-V}$$

An individual borrower will reach a new time period only in case of having successful project and hence $V = p$.
For the initial time period, we will assume that the project taken up by the borrower is successful because if the project is not successful, then the borrower cannot repay and hence would stop getting loans from the lender. The payoff of an individual borrower for the initial time period is therefore $\Pi - c$.
Hence,

$$W_I = \frac{\Pi - c}{1-p}$$

4.2 Default Payoff

If the borrower decides to default during the initial time period itself, the payoff would only be the returns which he will be getting from the successful project, or Π.
Hence,

$$W_I' = \Pi$$

4.3 Comparison between Repayment and Default strategy

Table 4.1: Individual Lending Payoff

Repayment Strategy	**Default Strategy**
$\mathrm{W}_I = \frac{\Pi - c}{1-V}$	$\mathrm{W}_I' = \Pi$

Proposition 1 *Under individual liability loans, repayment strategy is the optimal strategy whenever* $\frac{c}{\Pi} \leq p$.

Proof: In case of non-success of the project, the borrower has no means to repay and will default. Hence, we only need to check for the case when her project is successful. If her project is success, then she has two strategies to follow, repayment and default. To find conditions for the case when repayment strategy would be suitable, we would consider

$$W_I \geq W_I^D$$

$$\implies \frac{\Pi - c}{1 - p} \geq \Pi$$

$$\implies \Pi - c \geq \Pi - \Pi p$$

or

$$\implies \frac{c}{\Pi} \leq p$$

Hence, repayment strategy is the optimal strategy whenever $\frac{c}{\Pi} \leq p$.

This graph would help you get a logical intuition of it. X axis denotes p and y axis denotes $\frac{c}{\Pi}$

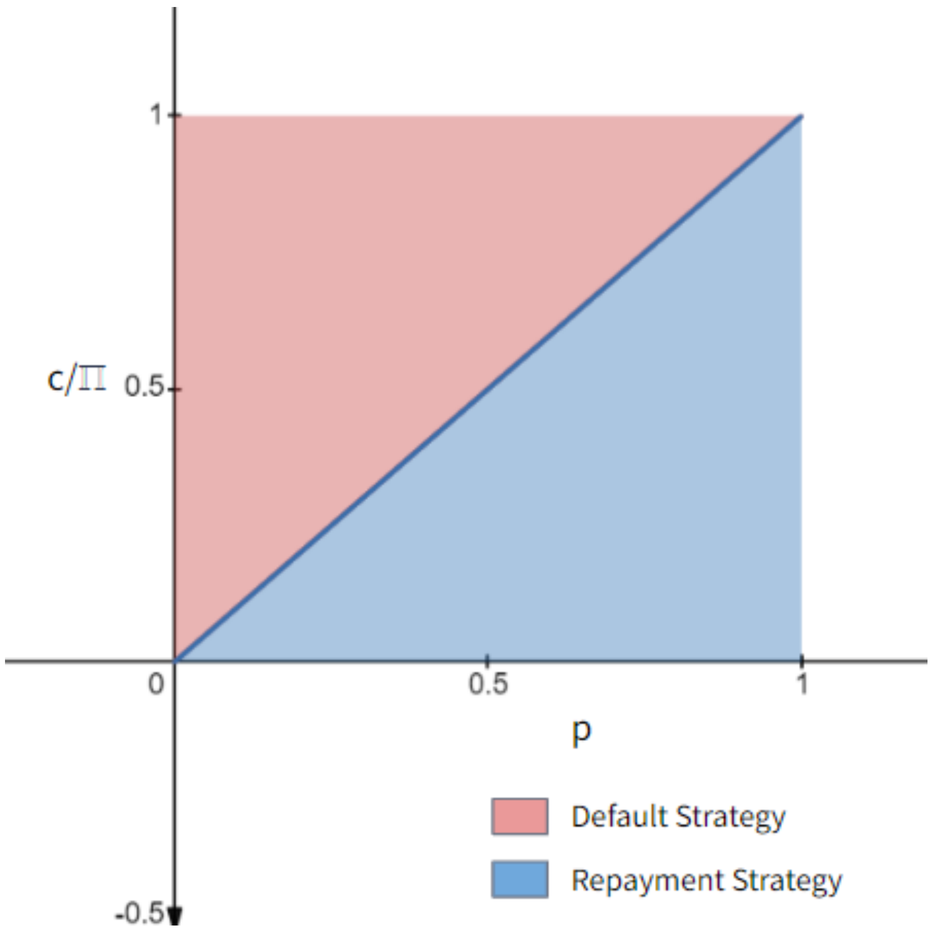

Figure 4.1: Individual Lending Comparison of Default and Repayment strategies

Whenever Π or returns from project and probability of its success both are high, it is better to continue taking loans and thus make timely repayments. This is denoted by the blue region where repayment strategy is better. If c or amount of money needed to pay back is very high, it might not be feasible to repay and thus defaulting on the loan is a better option. This is denoted by the red region where default strategy is better.

Chapter 5

Group Liability Lending

5.1 Group Payoff

Firstly, we will try to model the payoff for our group. In its calculation, we are considering 3 factors,

1. The returns which borrowers will receive from their projects
2. Money they need to repay back to the lending institutions
3. Costs of maintaining the group

In a group there are η members out of which $p \times \eta$ is the expected number of members with successful project. Hence, returns from those successful projects would be $\eta \times \Pi \times p$. Also, the money that is needed to be repaid is simply $\eta \times c$.

Lastly we will factor in the cost of maintaining the group. This cost arises from the need to monitor all members of the group so as to check that they are not wasting the loan money and investing it in the right manner and are not risking other members of the group for default (9).

We have considered it as

$$f(\eta) = b * \eta^3$$

Behind the reasoning of choosing this function, we wanted our cost function to be a polynomial for easy calculations and a simpler model. Linear function would have made everything way too simple. We chose cubic function because dividing it with eta as required in one of the later steps would yield quadratic function which suits our needs.

The returns from the project form the positive part while repayment costs and cost of maintaining the group forms negative part of our payoff function.

Combining all these together yields this as the total group payoff in a single time period.Hence, total payoff group payoff for a single period becomes:

$$X = \eta \Pi p - \eta c - b\eta^3$$

Similar to the calculations we have done for individual lending case, we are given group payoff for a single time period and using that we calculated the total payoff for all the time periods. Total group payoff across all time periods is given by the following infinite series.

$$S = X_{t=0} + VX_{t=1} + V^2 X_{t=2} + \ldots\ldots$$

$$\implies S = \frac{X}{1-V}$$

where V is the probability of reaching a new time period. Now, V is the probability that one of the projects succeeds, which is 1 - none of the projects succeed. Hence

$$V = 1 - (1-p)^\eta$$

Therefore, the total payoff of the group is

$$S = \frac{\eta(\Pi p - c - b\eta^3)}{(1-p)^\eta}$$

And the payoff of each member of the group is $W_G = \frac{S}{\eta}$.
Or,

$$W_G = \frac{\Pi p - c - b\eta^3}{(1-p)^\eta}$$

5.2 Optimal Size of Group

From Group size vs. Payoff graph, we can see that the payoff increases for a while till a certain η after which, it starts to decline sharply. The reason for sharp decline in the payoffs is the increasing costs to maintain the group which exceeds even the positive returns from the successful projects.

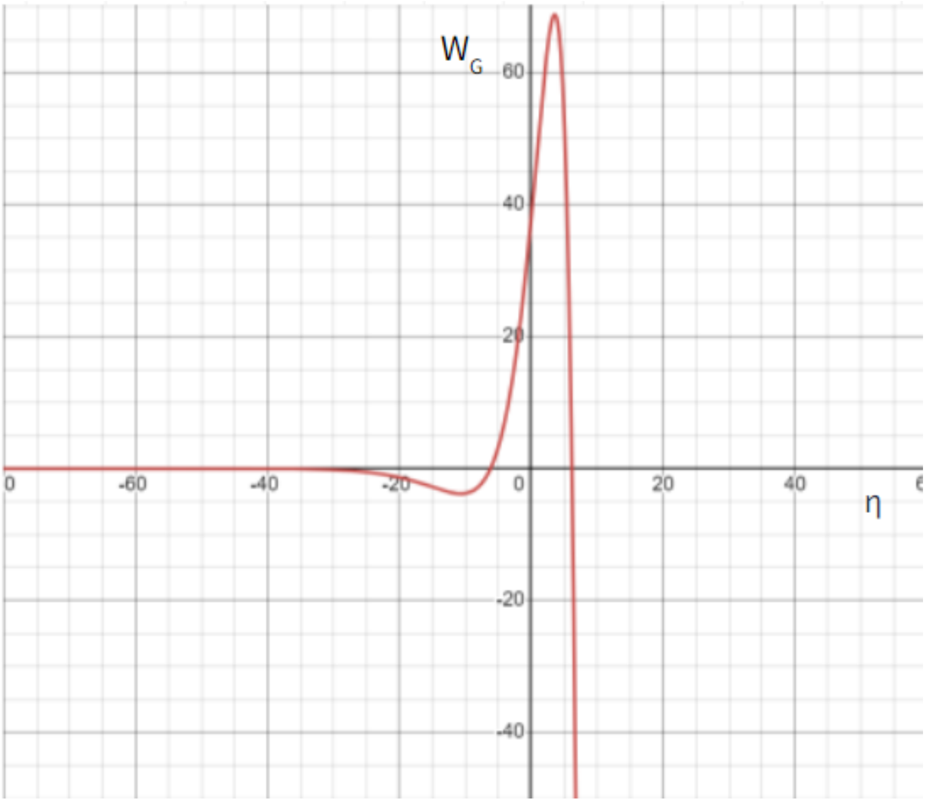

Figure 5.1: Group size Vs Payoff of a borrower in the group

Table 5.1: Parameters for Fig 5.1

Input	**Output**
$c = 40$ $b = 1$ $p = 0.244$ $\Pi = 320$	$\eta' = 3.557$ W_G at $\eta' = 68.771$

Proposition 2 *The optimal size of the group which maximizes the payoff for its member is*

$$\eta = \frac{b - \sqrt{bp\Pi \ln^2(1-p) - bc \ln^2(1-p) + b^2}}{b \ln(1-p)}$$

Proof:

Now, we have

$$W_G = \frac{\Pi p - c - b\eta^2}{(1-p)^\eta}$$

For simplification, let's consider $x = \frac{p\pi - c}{b}$ and $y = 1 - p$.

Then we have

$$W_G = \frac{b(x-\eta^2)}{y^\eta}$$

To find the η which maximizes each members payoff, we differentiate W_G wrt η.

Then we get,

$$\frac{dW_G}{\eta} = \frac{\frac{db(x-\eta^2)*y^\eta}{d\eta} - b(x-\eta^2)*\frac{dy^\eta}{d\eta}}{y^{2\eta}}$$

$$\implies \frac{dW_G}{\eta} = \frac{-2b\eta - b(x-\eta^2)*y}{y^\eta}$$

Now, to get η which maximizes W_G, we will take the roots of $\frac{W_G}{\eta}$.

Hence,

$$\frac{W_G}{\eta} = 0$$

$$\implies b\eta^2 ln(y) - 2b\eta + bxln(y) = 0$$

This is a quadratic in η. Hence, we will get two roots

$$roots = \frac{b \pm \sqrt{b^2 + b^2 ln^2(y)x}}{bln(y)}$$

$$\implies roots = \frac{1 \pm \sqrt{1 + ln^2(y)x}}{ln(y)}$$

To check for maxima, we will doubly differentiate the W_G wrt η and put the roots in that

and check whether the output is positive or negative. If the output is negative, then that root forms a local minima, else if its positive, then that root forms a local maxima.
Re-differentiating W_G wrt η

$$\frac{d^2W}{d\eta^2} = \frac{\frac{d(-2b\eta - b(x-\eta^2)ln(y))y}{d\eta}y^{\eta} - (-2b\eta - b(x-\eta^2)ln(y))\frac{dy^{\eta}}{\eta}}{y^{2\eta}}$$

$$\implies \frac{d^2W}{d\eta^2} = \frac{(-2b + 2b\eta ln(y)) + 2b\eta ln(y) + b(x-\eta^2)ln^2(y)}{y^{\eta}}$$

$$\implies \frac{d^2W}{d\eta^2} = \frac{(-2b + 2b\eta ln(y)) + 2b\eta ln(y) + b(x-\eta^2)ln^2(y)}{y^{\eta}}$$

$$\implies \frac{d^2W}{d\eta^2} = \frac{-bln^2(y)\eta^2 + 2bln(y)\eta + 4bln(y)\eta - 2b + bxln^2(y)}{y^{\eta}}$$

Now, we need to put roots in the equation above to check the sign of the output.
Since, $y > 0$, denominator is always going to be positive.
Hence, we will only check the sign of the numerator. Let $N(W_G)$ be numerator of the above equation.

Case 1: $\eta = \frac{1+\sqrt{1+ln^2(y)x}}{ln(y)}$

Then,

$$N(W_G) = -bln^2y\frac{1+1+ln^2(y)x+2\sqrt{1+ln^2(y)x}}{ln^2y} + 4blny\frac{1+\sqrt{1+ln^2(y)x}}{lny} - 2b + bxln^2y$$

$$\implies N(W_G) = -b(2+ln^2yx+2\sqrt{1+ln^2yx}) + 4b(1+\sqrt{1+ln^2(y)x}) - 2b + bxln^2y$$

$$\implies N(W_G) = 2b\sqrt{1+ln^2yx} > 0$$

Hence, $\eta = \frac{1+\sqrt{1+ln^2(y)x}}{ln(y)}$ forms a local minima.

Case 2: $\eta = \frac{1-\sqrt{1+ln^2(y)x}}{ln(y)}$

Then,

$$N(W_G) = -bln^2y\frac{1+1+ln^2(y)x-2\sqrt{1+ln^2(y)x}}{ln^2y} + 4blny\frac{1-\sqrt{1+ln^2(y)x}}{lny} - 2b + bxln^2y$$

$$\implies N(W_G) = -b(2+ln^2yx-2\sqrt{1+ln^2yx}) + 4b(1+\sqrt{1-ln^2(y)x}) - 2b + bxln^2y$$

$$\implies N(W_G) = -2b\sqrt{1+ln^2yx} > 0$$

Hence, $\eta = \frac{1-\sqrt{1+ln^2(y)x}}{ln(y)}$ forms a local maxima.
As it is the only maxima, therefore, $\eta = \frac{1-\sqrt{1+ln^2(y)x}}{ln(y)}$ maximizes W_G or payoff of all the members in the group.

5.3 Default Strategy Payoff

If all members of the group decide to default, their payoff would be the amount earned by them from successful projects - cost to maintain the group. If W'_G is the payoff of group members when using defaulting strategy,

$$W'_G = p\Pi - b\eta^2$$

Table 5.2: Group Liability Lending payoff of borrower of a group

Repayment Strategy	Default Strategy
$\mathrm{W}_G = \frac{\Pi p-c-b\eta^3}{(1-p)^\eta}$	$\mathrm{W}'_G = p\Pi - b\eta^2$

The following graph shows shows the comparison of the dominant strategy between repayment and defaulting in case of group lending. X-axis represents η and Y-axis represents p.

With increase in probability of success of projects, continuing the loan and thus timely repayment of the loans become better. Only when the probability of success is extremely low, defaulting on the loans is more feasible.

When the number of members in the group increases, they start covering for each other in case of failure of a project and hence it becomes very less likely that all the projects will fail at the same time. Hence repayment is more viable. However, when a group becomes very large, the cost of maintaining this group becomes very high and exceeds their earnings from successful projects, and hence defaulting becomes a better option.

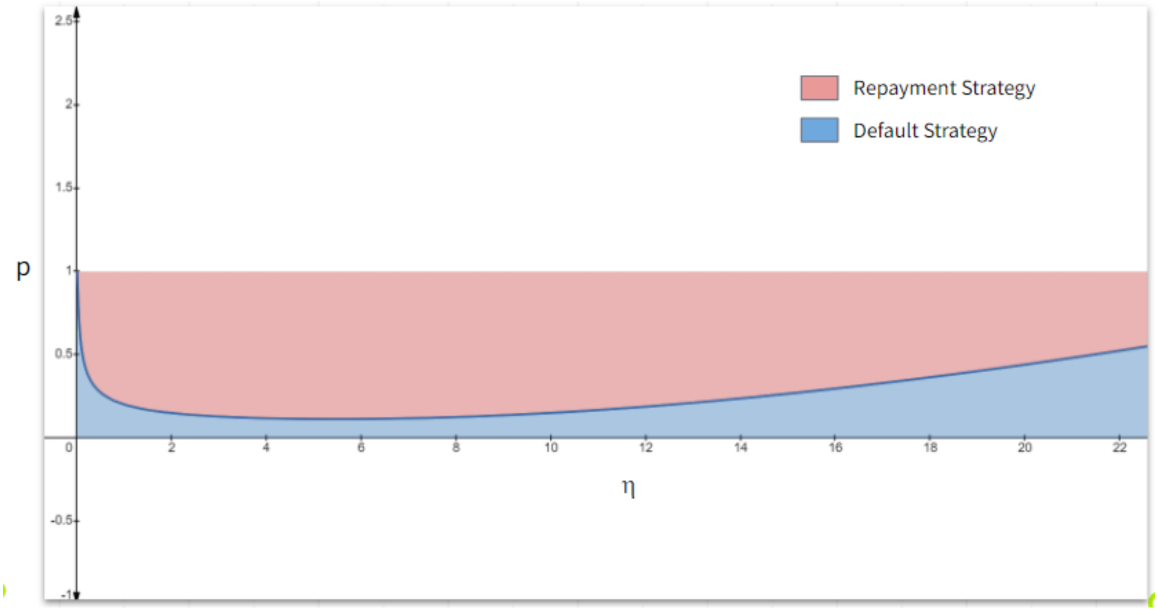

Figure 5.2: Group Liability Lending - Comparison of Default and Repayment strategies

The following graph shows comparison of both defaulting and repayment strategies under both individual and group liability lending. From the graph, it is evident that the group repayment strategy dominates for most of the region showing that it is a better strategy. The reason for its decline with increase in eta is simply due to increasing cost of maintenance of the group.

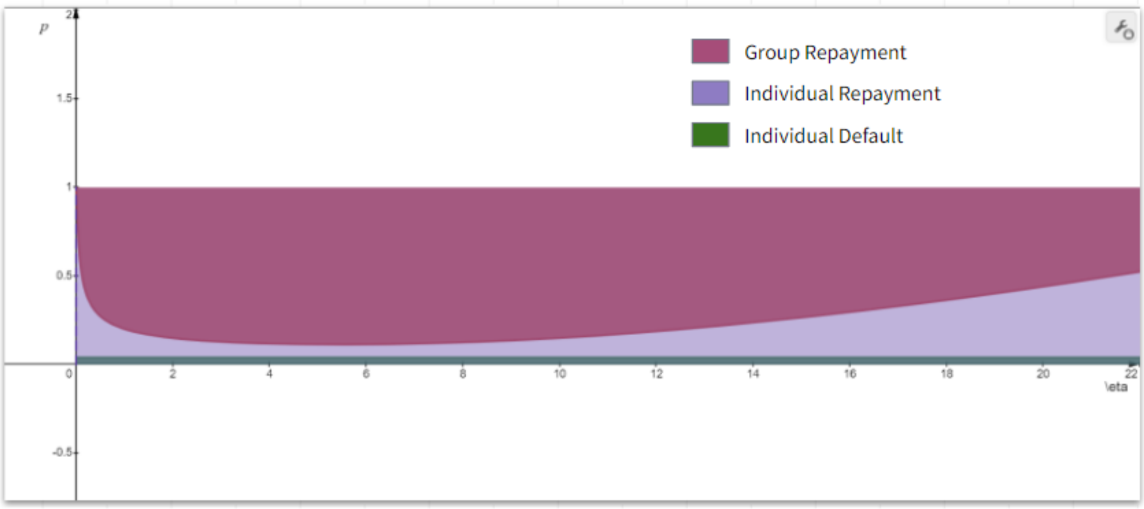

Figure 5.3: Comparison of all strategies

Chapter 6

Future Works

In this work, we have studied and modeled individual and group liability lending using game theoretic approaches and compared both of them in regards of various strategies that the borrowers can adopt. We have also formulated the optimal size of group which would maximize the social payoff of its members.

Future works involve studying more scenarios and including them in our model to make it more real world. An example could be to include the riskiness of the projects in which the borrowers invest into play.

Till now, we have only modeled the payoff of the borrowers. Studying and modeling institutional payoff (of lenders) is another important thing which we would like to do in the future. This would help greatly in policy making.

Right now, to make the calculations easier, we have considered the value of money as same over time. However, as we are playing a dynamic game, doing dynamic game calculations for better payoff calculations is another exciting thing that we would like to do in the future.

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