Dark Sector as an Origin to Neutrino Mass and Matter-Antimatter Asymmetry

M.Sc. Thesis

By

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CANDIDATE's DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **DARK SECTOR AS THE ORIGIN OF NEUTRINO MASS AND MATTER ANTIMATTER ASYMMETRY** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF SCIENCE** and submitted in the **DISCIPLINE OF PHYSICS**, **Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July-2017 to June-2018 under the supervision of Dr. Subhendu Rakshit, Professor, IIT Indore.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

KAJAL SINGH

....

This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

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KAJAL SINGH has successfully given his/her M.Sc. Oral Examination held on 21-June-2018

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Date: Date:

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Abstract

Dark matter, Neutrino mass and Baryon asymmetric universe can only be explained by beyond Standard Model physics. We have been investigating the combined models for all three problems and concerned energy scales. Our main focus is on the inert doublet model with heavy Majorana neutrinos to explain dark matter, neutrino mass and baryon asymmetry simultaneously at TeV scales. Also, we have been investigating the flavor structure of Yukawa couplings to satisfy neutrino oscillation data.

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Chapter 1

Introduction

1.1 Matter-Antimatter Asymmetry

Recent developments in cosmology has enhanced our knowledge of the universe. But on other hand, it has introduced many other challenges to the basic understanding of laws of nature. The Standard Cosmological Model of the universe tells that dominant contents of the universe are still dark and mysterious to our present knowledge.

However, ordinary matter which makes barely 4% of the universe has been well understood by a combined model of fundamental theories so far. This model is known as the Big Bang Theory. It offers a good explanation for evolution of universe into what we see today. Despite of that it fails to explain the absence of antimatter in visible universe, which is observed in day to day experience and in high precision cosmological measurements as well.

In this section we will briefly discuss the matter-antimatter asymmetry of the universe, its evidences and ideas to resolve this problem. Since the ordinay matter is made of baryons, the problem is usually referred to as Baryon Asymmetric Universe (BAU). The parameter of this asymmetry is given as ratio of density of total baryons to photons i.e.,

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma}. \tag{1.1}$$

1.1.1 Evidence of Asymmetry

The evidence that the universe is entirely lacking of antimatter comes from different observations.

- In our everyday life, the absence of matter-antimatter annihilations indicates that there is no antimatter on the earth. Also beyond the earth, successful space missions are in favor that in nearby space, abundance of antimatter is negligible.

- The measurement of cosmic rays and its products at the top of the atomsphere provides sample of contents of entire galaxy and distant galaxies as well. However, the measurements of antiprotons to proton ratio[1] have been consistent with the fact that antiprotons are the secondary products of interaction of primary cosmic rays as $\frac{\bar{p}}{p} \sim 10^{-4}$.

- The presence of intra-cluster gas clouds (as known by X-ray emission) indicates that antibaryon cannot exist in galaxy cluster. Also the absence of strong γ-ray fluxes from particle-antiparticle annihilations is an evidence of non-existence of antimatter.

1.1.2 Measurements of Asymmetry

There are two indirect probes that give consistent estimate of total baryon density of the the universe as follows

- **CMB Anisotropy:** The Cosmic Microwave Background (CMB), the relic radiation from last scattering surface is an important source of information on early universe. It is almost isotropic throughout the space, however it has some small temperature anisotropies. These anisotropies of the cosmic microwave background radiation are usually analyzed in terms of special harmonics as given below

$$\frac{\Delta T}{T} = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi). \tag{1.2}$$

  The power spectrum of anisotropies are measured in terms of

$$C_l = l(l+1)\langle | \, a_{lm} \, |^2 \rangle. \tag{1.3}$$

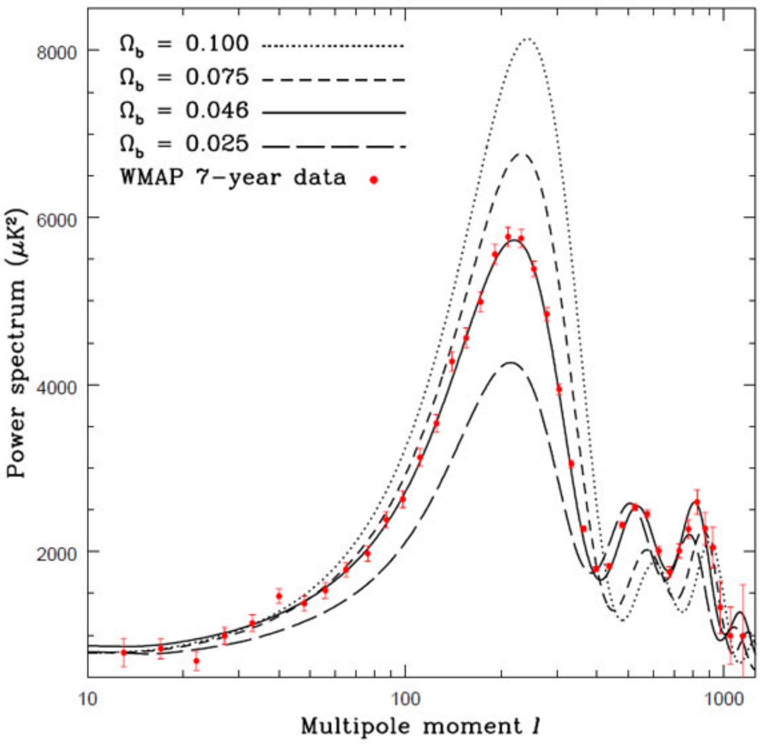

Figure 1.1: The power spectrum of CMB anisotropies as a function of the multipole moment l [5].

The amount of these anistropies depends on baryon density as illustrated in Figure (1.1). The best fit for WMAP (Wilkinson Microwave Anisotropy Probe)[2] data is for $\Omega_B \sim 0.046$ which corresponds to the baryon to photon ratio $\eta_B \sim 10^{-10}$.

- **Big Bang Nucleosynthesis:** The primordial abundances of light elements like He-4, He-3, Deuterium, Li-6, Li-7 etc. have been explained by Big Bang Nucleosynthesis (BBN) very successfully. The key parameter for determining the abundance of light elements from BBN is the baryon to photon ratio. The astrophysical observations suggest that $\eta_B \sim 6.10^{-10}$[3], which is consistent with CMB measurements.

1.1.3 Basic Elements for Baryogenesis

The Big Bang should have produced equal amounts of matter and antimatter as to preserve the assumption of neutral universe. Therefore, to produce a baryon asymmetric universe from a baryon symmetric universe, we must have some physical assumptions which differentiate between matter and antimatter. There are three necessary ingredients to produce a BAU.

- **Baryon Number Violation-** Provided the fact that the universe was symmetric intially, there must be a violation of baryon number to produce BAU.

- ***C* and *CP* violation-** The conservation of C and CP ensures that particles and antiparticles will behave in identical way in interactions. So, despite of having a B-nonconserving interaction, there will not be any final asymmetry unless both C and CP are violated. As under conserved C and CP, B-violating interaction will produce baryons and antibaryons at same rate resulting in net zero baryon number.

Channel	Branching ratio	B
$X \to qq$	r	$2/3$
$X \to \bar{q}l$	$1-r$	$-1/3$
$\bar{X} \to \bar{q}\bar{q}$	$\bar{r}$	$-2/3$
$\bar{X} \to q\bar{l}$	$1-\bar{r}$	$1/3$

  - Decay of X is B-violating.
  - $\Delta B_{total} = r - \bar{r}$, i.e., to produce a final non-zero baryon number $r \neq \bar{r} \to C$ and CP violation.

- **Out of Equilibrium Condition-** In equilibrium the phase space density of baryons and antibaryons will be identical i.e., $n_B - n_{\bar{B}} = 0$.

The above conditions are known as Sakharov Conditions [4]. This problem is not explained by Standard Model (SM) as it treats matter and antimatter in same way and there is no B-violating interaction possible in SM. All these conditions were firstly implemented in GUT (Grand Unified Theory). In GUTs quarks and leptons are unified in the same irreducible presentations, therefore Baryon number violation occurs naturally as baryons can decay to leptons. Also due to unified representation there exists many possible complex phases to produce sufficient amount of CP violation. And at last out of equilibrium condition is achieved by slow decay rates of heavy gauge bosons. However, in GUTs to produce sufficient BAU requires high reheating temperatures which leads to over production of relics. Also it is hard to test GUTs in colliders as the scale is $M_{GUT} \sim 10^{16}$ GeV.

In SM baryon number(B) and lepton number(L) are accidental symmetries so it not possible to violate them at tree level. However, a non-perturbative process, named as "sphaleron" process, can violate $(B+L)$ but conserve $(B-L)$. Hence instead of directly producing baryon asymmetry we can at first produce lepton asymmetry to get BAU. We will be using this basic mechanism in our further work.

1.2 Neutrino Mass

In standard model, neutrinos are classified in terms of their leptonic flavor. These flavor eigenstates were supposed to have zero mixing as they form orthonormal basis for weak interactions of SM neutrinos. In late 1960's, neutrino oscillation was first observed by Ray Davis in Homestake experiments. After that, there were a number of experiments in favor of neutrino oscillations. Neutrino oscillations indicate that neutrinos have non-zero mixing and finite mass which was not in the case with SM. Since the mass eigenstates and flavor eigenstates are not same, one set can be written as the superposition of other set of eigenstates.

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i \tag{1.4}$$

where ν_α and ν_i are the flavor and mass eigenstates respectively. The matrix U is Pontecorvo-Maki-Nakagawa-Sakata(PMNS) matrix, which is parameterized by three mixing angles and one phase angle.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{13} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{13} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{13} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{13} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \tag{1.5}$$

where s_{ij} and c_{ij} are $\sin\theta_{ij}$ and $\cos\theta_{ij}$ respectively. The values of mixing angles and CP phase have been measured, which fits the neutrino oscillation data at best so far [5].

$$\theta_{12} = 33.62^{\circ}{}^{+0.78^{\circ}}_{-0.76^{\circ}},\ \ \theta_{23} = 47.2^{\circ}{}^{+1.9^{\circ}}_{-3.9^{\circ}},\ \ \theta_{13} = 8.54^{\circ}{}^{+0.15^{\circ}}_{-0.15^{\circ}},\ \ \delta_{CP} = 234^{\circ}{}^{+43^{\circ}}_{-31^{\circ}}. \tag{1.6}$$

Many extensions of standard models can explain the neutrino mass, though these models are yet to be verified.

In SM, masses are generated via Higgs mechanism. We can construct two types of mass for fermions - Dirac mass and Majorana mass. To construct Dirac mass term two independent chiral fields ψ_R and ψ_L with their conjugates are required. All fermions in SM have Dirac masses except neutrino. Since neutrinos do not have such chiral fields, we can not construct Dirac masses for them as there are only left-handed neutrinos and right-handed anti-neutrinos.

However we can construct Majorana masses for neutrinos with $(\bar{\nu}_L^c \nu_L)$. But this term is not invariant under weak isospin symmetry. Therefore it is not possible to construct neutrino mass in SM.

However the See-Saw models attempt to evade this problem. There are three types of such models. We have reviewed one of them called Type-I see-saw. In Type-I see-saw SM is extended by three right-handed singlet neutrinos. These right-handed neutrinos interact with left-handed neutrinos to generate masses. Also right-handed neutrinos are singlet under $SU(3)_C \times SU(2)_L$, therefore they do not interact via weak interactions as they have zero weak isospins.

The Lagrangian will contain mass terms given as

$$\mathcal{L}_M = \frac{1}{2} \begin{bmatrix} \nu_L \\ {\nu_R}^c \end{bmatrix}^T \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L \\ {\nu_R}^c \end{bmatrix}$$

where, (ν_L, ν_R^c) is a Weyl spinor containing left-handed SM neutrinos and right-handed sterile neutrinos, and m_D is Dirac mass, m_L is Majorana mass for left-handed neutrinos and m_R is Majorana mass for right-handed neutrinos. A very interesting case is $m_D \ll m_R$, and m_L=0 since Majorana mass for left-handed neutrino is forbidden. We obtain the eigenvalues of mass matrix :

$$\mid m_1 \mid \simeq \frac{{m^2}_D}{m_R}, \quad m_2 \simeq m_R. \tag{1.7}$$

Hence one of the mass eigenstates is as heavy as m_R and other one is light because its mass is supressed by a small ratio m_D/m_R. This is the well known see-saw mechanism of neutrino mass as the heaviness of one is responsible for lightness of other. The explanation of smallness of neutrino masses is given by this mechanism. The neutrino masses are supposed to be a mixed state of these two mass eigenstates. The see-saw models are very important to produce BAU as they play a viable role in leptogenesis which we will be discussing later.

1.3 Dark Matter

The motivation for existence of dark matter comes from mainly a few powerful astrophysical observations. The most famous evidence that suggests that there are non-luminous matter in our universe is the anomalous galactic rotation curves. To reproduce the curve, a large amount of mass is required other than that which are visible to us. The other evidence, which comes from the CMB power spectrum favors that five-sixths of total matter interacts significantly with ordinary matter by only gravitational interactions.

At first it was thought that dark matter is some non-luminous astronomical object referred to as MACHOs (MAssive Compact Halo Objects). But two collaborations, MACHO and EROS looking into galactic halo by gravitational microlensing, concluded that MACHOs can not make up 100% of the galactic halo.

So the paradigm of dark matter shifted to heavy elementary particles. The current paradigm for explaining dark matter is WIMPs (Weakly Interacting Massive Particles). WIMPs with masses $\geq 10^2$ GeV can explain the DM relic density. This theoretical speculation is broadly known as the 'WIMP miracle'. The Boltzmann equation for number density of dark matter particle say χ is [6]

$$\frac{dY}{dx} = \frac{-1}{x^2}\frac{s(m_\chi)}{H(m_\chi)}\langle\sigma_{ann}v\rangle(Y^2 - Y_0^2), \tag{1.8}$$

where $Y = n_\chi/s$, $s(m_\chi) = g_* m_\chi^3/x^3$ is entropy density, $H(m_\chi) = g_*\pi^2 m_\chi^4/90M_{Pl}^2 x^4$ is the Hubble parameter, $x = m_\chi/T$, equilibrium yield $Y_0 = 0.145x^{3/2}e^{-x}$ and

$\langle\sigma_{ann}v\rangle$ is annihilation cross section. One can analytically solve the equation(1.8) by further simplification with a new variable defined as

$$y = \frac{s(m_\chi)}{H(m_\chi)}\langle\sigma_{ann}v\rangle Y. \tag{1.9}$$

The Boltzmann equation will look like

$$\frac{dy}{dx} = \frac{-1}{x^2}(y^2 - y_0^2) \tag{1.10}$$

where, $y_0 = 0.192 g_*^{-1/2} M_{Pl} m_\chi x^{3/2} e^{-x}$. Since for $x < x_f$, the dark matter will be in equilibrium, hence Y tracks Y_0 and for $x > x_f$, $Y >> Y_0$ as Y_0 drops as e^{-x}, where $x_f = m_\chi/T_f$ corresponds to freeze-out temperature for dark matter. These assumptions give a simple solution to equation (1.8)[6]

$$y(\infty) \sim x_f. \tag{1.11}$$

The value of x_f can be found assuming that at $x = x_f$, $Y_0 \sim Y(\infty)$ as after this point dark matter interactions are out of equilibrium,

$$0.192 g_*^{-1/2} M_{Pl} m_\chi x_f^{3/2} e^{-x_f} \approx x_f \tag{1.12}$$

and hence

$$x_f \approx 24 + \frac{m_\chi}{100\,\mathrm{GeV}}. \tag{1.13}$$

The abundance of any particle species (say i) is defined as

$$\Omega_i = \frac{\rho_i}{s}\frac{s_0}{\rho_c}, \tag{1.14}$$

where $s_0 = 2890$ cm^{-3} is current entropy density of the universe, $\rho \approx 4.4*10^{-6}$ GeV cm^{-3}.

Using the solution of the Boltzmann equation in equation (1.14)

$$\Omega_\chi h^2 \sim g_*^{1/2} x_f \frac{1.12 * 10^{-10}\mathrm{GeV}^2}{\langle\sigma_{ann}\rangle} \tag{1.15}$$

where $h \sim 0.65$. Since the relic density of dark matter is $\Omega_\chi h^2 \sim 0.12$, it gives

$$\langle\sigma_{ann}v\rangle \sim 10^{-9}\mathrm{GeV}^{-2}. \tag{1.16}$$

If we take the annihilation cross section $\langle\sigma_{ann}v\rangle = \pi\alpha^2/m_\chi^2$, where $\alpha = 1/137$, it gives $m_\chi \sim 300$ GeV, which provides an interesting scenario in the light of collider searches.

The hypothesis of dark matter particles helps us in understanding many other problems related to baryon acoustic oscillation, red shift distortions etc. However we have null results in direct, indirect, as well as collider searches of WIMPs till now.

Chapter 2

Unified Models of Dark Matter, Neutrino Mass and Matter-Antimatter Asymmetry

The existence of dark matter, neutrino masses and matter-antimatter asymmetry, all three problems are beyond the scope of the standard model of particle physics. The see-saw mechanism to generate neutrino mass gives rise to a lepton violating term which can solve the problem of matter-antimatter asymmetry. Also the densities of dark matter and baryons are of the same order i.e.,

$$\Omega_{DM} \sim 5\Omega_B, \tag{2.1}$$

which suggests that dark matter and baryonic matter can have same origin. Hence, it is possible that these problems can be explained by a single combined model.

2.1 Leptogenesis

Leptogenesis [7] is a mechanism which connects the cosmological matter-antimatter asymmetry with the neutrino properties. In a see-saw model, the extension of SM with three right-handed neutrinos gives rise to lepton number violating interactions. Also, complex Yukawa couplings can produce a finite CP asymmetry. Therefore, the Sakharov's conditions can be satisfied

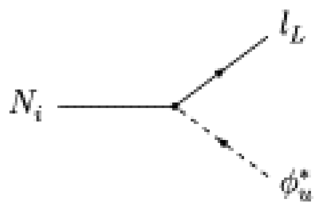

Figure 2.1: L-violating interaction [8].

by the decay of see-saw partners of light neutrinos in leptogenesis. At last the asymmetry produced in lepton sector can be transferred to baryon sector via sphaleron process as

$$\Delta B = \frac{8}{28}\Delta(B-L). \tag{2.2}$$

2.1.1 Toy Model for Leptogenesis

For better understanding of underlying concepts, one can simplify the leptogenesis by following assumptions

- The lepton asymmetry is produced in only one leptonic flavor.
- The mass spectrum of right-handed neutrinos is hierarchical i.e., $M_{N_1} << M_{N_{2,3}}$.
- Thermal production of N_1.

The Lagrangian for leptogenesis will be

$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} - h_{1j}\bar{N}_1\ell_j\phi - (M_1/2)\bar{N_1}^c N_1. \tag{2.3}$$

The relevant processes to satisfy the Sakharov conditions are as follows

- **Lepton number violation:** The decay of right-handed neutrinos into lepton and Higgs violates the lepton number by $\Delta L = 1$ (Figure (2.1)). The decay width of right-handed neutrinos N_i at tree level is given as

$$\Gamma_{Di} = \frac{(hh^\dagger)_{ii}M_i}{8\pi}. \tag{2.4}$$

  For non-degenerate masses of heavy right-handed neutrinos, only decay of the least massive one (say N_1) will be relevant for leptogenesis. The asymmetry produced by $N_{2,3}$ will be erased by N_1 as for $T > M_1$ interactions are in equilibrium.

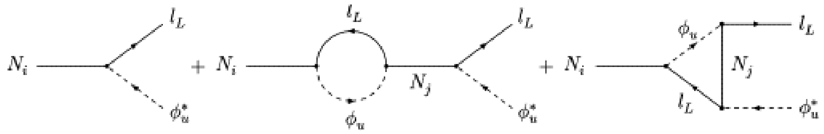

Figure 2.2: Diagrams contributing to CP asymmetry[8].

- **CP violation:** The CP asymmetry parameter is defined as

$$\epsilon_i = \frac{\Gamma_{Di} - \bar{\Gamma}_{Di}}{\Gamma_{Di} + \bar{\Gamma}_{Di}}, \tag{2.5}$$

where $\Gamma_{Di} = \Sigma_\alpha \Gamma(N_i \to l_\alpha \phi^\dagger)$ is decay width and $\bar{\Gamma}_{Di} = \Sigma_\alpha \Gamma(N_i \to \bar{l}_\alpha \phi)$ is inverse decay width of N_i.

The decay of N_1 is not CP invariant as the interference between tree-level and one-loop level diagrams (as shown in Figure (2.2)) is non-zero.

For $M_1 << M_{2,3}$ the amount of CP asymmetry is calculated by integrating out the $N_{2,3}$,

$$\epsilon_1 = -\frac{3}{16\pi} \frac{M_1}{(hh^\dagger)_{11} \langle \phi \rangle^2} Im(h^* m_\nu h^\dagger), \tag{2.6}$$

where $\langle \phi \rangle = 174\text{GeV}$, is vacuum expectation value of SM higgs and m_ν is mass matrix of SM neutrinos. In a basis where heavy mneutrino masses are diagonalized,

$$\epsilon_1 = -\frac{3}{16\pi} \frac{1}{(hh^\dagger)_{11}} \sum_{i=2,3} Im[(hh^\dagger)_{1i}{}^2] \frac{M_1}{M_i}. \tag{2.7}$$

- **Out of equilibrium:** The expansion of universe provides the out of equilibrium conditions. For a decay width small compared to the Hubble parameter i.e., $\Gamma_{Di}(T) < H(T)$ at $T \sim M_i$, the heavy neutrinos are out of thermal equilibrium, otherwise they are in thermal equilibrium. The convenient parameter to describe when the decays are out of equilibrium is,

$$K_i = \frac{\Gamma_{Di}(T=0)}{H(T=M_i)} = \frac{\tilde{m}}{m^*}, \tag{2.8}$$

where, $\tilde{m}$ is effective neutrino mass and m^* is equilibrium neutrino mass. For M_1 being the least massive, $K_1 = 1$ will be the boundary between two phases which is equivalent to condition $\tilde{m} = m^*$. It is obvious that for $K_1 < 1$ decay of N_1 will be out of equilibrium and vice versa.

2.1.2 The Boltzmann Equation

In the simplest form of leptogenesis only the lightest right-handed neutrino decay contributes to lepton asymmetry and hence baryon asymmetry. Also we do not consider the flavor pattern of neutrinos.

The Boltzmann equations for unflavored leptogenesis are

$$\frac{dN_{N_1}}{dz} = -D(N_{N_1} - N_{N_1}^{eq}), \tag{2.9}$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon D(N_{N_1} - N_{N_1}^{eq}) - W_{ID} N_{B-L}. \tag{2.10}$$

In the equations (2.9) and (2.10), N_{N_1} is the density of N_1, $N_{N_1}^{eq}$ is equilibrium density of N_1, N_{B-L} is density of asymmetry $(B - L)$, and $z = M_1/T$, D accounts for decays which contribute to L-asymmetry.

$$D(z) = \frac{\Gamma_{D1}(z)}{Hz} = \frac{\Gamma_{D1}}{Hz}\langle\frac{1}{\gamma}\rangle. \tag{2.11}$$

W_{ID} is the washout term which is contributed only by inverse decays, as scattering processes will cease for $K << 1$:

$$W_{ID} = \frac{\Gamma_{ID}(z)}{Hz} = \frac{\Gamma_{D1}(z)}{Hz}\frac{N_{N_1}^{eq}}{N_l^{eq}}. \tag{2.12}$$

Solution to equation (2.10) is

$$N_{B-L}(z) = N_{B-L}^{i} e^{\int_{z_i}^{z} W_{ID}(z')dz'} - \frac{3}{4}\epsilon_1 \kappa(z). \tag{2.13}$$

The first term in (2.13) is proportional to initial abundance of $(B - L)$ asymmetry along with an exponentially decaying factor. This accounts for washout effects. The second term in (2.13) describes the portion of $(B - L)$ asymmetry produced from decay of N_1 itself. $\kappa(z)$ is called efficiency factor as it measures the efficiency of decays to produce $(B - L)$ asymmetry and given by

$$\kappa(z) = \frac{4}{3}\int_{z_i}^{z} D(N_{N_1} - N_{N_1}^{eq}) e^{\int_{z'}^{z} W_{ID}(z'')dz''} dz'. \tag{2.14}$$

By substituting $\langle\frac{1}{\gamma}\rangle = \frac{K_1(z)}{K_2(z)}$, which is thermally averaged dilation factor in equations (2.11) and (2.12) and using them in (2.14), we get

$$\kappa(z) \simeq \frac{4}{3}(N_{N_1}^{i} - N_{N_1}(z)), \tag{2.15}$$

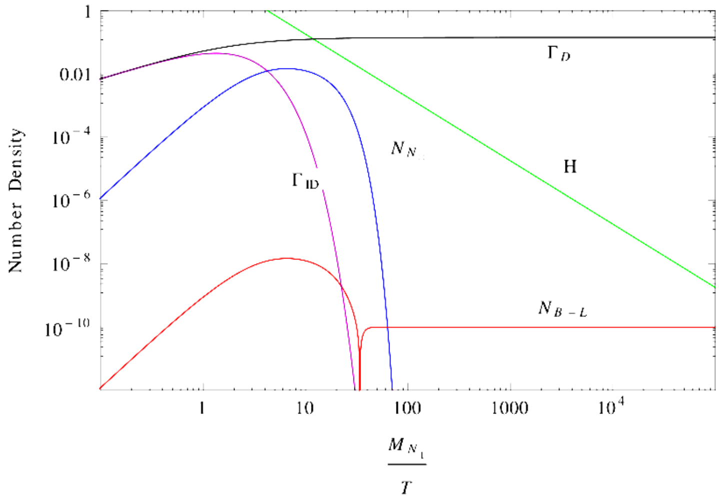

Figure 2.3: Plot of N_1 number density with final N_{B-L} number density vs. $\frac{M_{N_1}}{T}$ along with other parameters [8].

where $N_{N_1^i}$ is initial abundance of N_1. For $z = \infty$, $N_{N_1}(z) = 0$ which implies $\kappa(z) = \frac{4}{3}N^i_{N_1}$. Since efficiency factor is proportional to initial abundance of N_1, therefore if $N^i_{N_1} = 0$, no asymmetry will be produced.

In simplest case, when there is some initial abundance one can easily see the evolution of N_{N_1} and N_{B-L} as well [8].

In Figure(2.3) we have reproduced the results for the unflavored leptogenesis [8]. The amount of Baryon asymmetry is given by

$$\Delta B = \frac{8}{28}\Delta\left(B - L\right). \tag{2.16}$$

In above plot $\Delta(B-L) = 9.602 \times 10^{-11}$ which implies that $\Delta B = 3.44 * 10^{-11}$ can said to be consistent with observations. Hence, leptogenesis can be very successful for explaining BAU. However the only problem with Leptogenesis is that its scale is very high $\geqslant 10^{10}$ GeV which is not desirable from both experimental as well as theoretical point of view which we will discuss in last section of this chapter.

2.2 Combined Models for NM, BAU and DM

Leptogenesis describes a potential resolution to BAU and NM qualitatively. There have been many independent models for dark matter as well. For

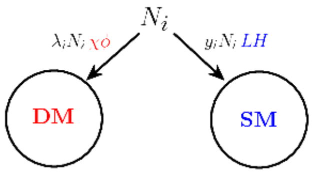

Figure 2.4: Two sector Leptogenesis [9].

example, as we have already discussed, Weakly Interacting Massive Particles (WIMP) is one of them.

However, there are many alternatives which connect dark matter with baryon asymmetry and neutrino mass.

2.2.1 Two Sector Leptogenesis

Two sector leptogenesis is a simple extension of leptogenesis model with chiral fermion χ and a scalar ϕ_χ which represents dark sector. In this model, which is also referred as Asymmetric Dark Matter(ADM), asymmetry is produced in both sectors simultaneously by the decay of heavy neutrinos [9].

The Yukawa Lagrangian for two sector leptogenesis is

$$-\mathcal{L} \supset h_i \bar{N}_i \ell \phi + \lambda_i \bar{N}_i \chi \phi_\chi. \tag{2.17}$$

The key points of this model are as follows

- It is assumed that l, χ and N_i's are charged under an approximate global lepton symmetry with charges $+1, +1$ and -1 respectively.
- χ gets mass through Dirac mass term $m_\chi \chi \bar{\chi}$ with a fermion $\bar{\chi}$ which is oppositely charged under the approximate symmetry.
- Dark matter is stable as $m_\chi < m_{\phi_\chi}$.
- The N_i's are hierarchical, decay of lightest right-handed neutrino i.e., N_1 produces asymmetry in both sectors simultaneously.
- The symmetric part in both sectors i.e., lepton and dark sectors are annihilated, which only leaves lepton asymmetry and asymmetric dark matter.

CP **asymmetry**

The amount of CP asymmetries produced in the decay of N_1 are

$$\varepsilon_\chi \simeq \frac{M_1}{M_2}\frac{1}{16\pi(h_1^2+\lambda_1^2)}\left[2\lambda_1^2|\lambda_2|^2\sin 2\psi_\chi + \lambda_1\lambda_2 h_1|h_2|\sin(\psi_l+\psi_\chi)\right], \quad (2.18)$$

$$\varepsilon_l \simeq \frac{M_1}{M_2}\frac{1}{16\pi(h_1^2+\lambda_1^2)}\left[2h_1^2|h_2|^2\sin 2\psi_l + \lambda_1\lambda_2 h_1|h_2|\sin(\psi_l+\psi_\chi)\right], \quad (2.19)$$

where h_1, λ_1 are real and positive but $h_2 = |h_2|e^{i\psi_l}$ and $\lambda_2 = |\lambda_2|e^{i\psi_\chi}$. The ratio of asymmetries for some specific phases can be given as

$$\frac{\varepsilon_l}{\varepsilon_\chi} \simeq \frac{h_1|h_2|}{\lambda_1|\lambda_2|}. \quad (2.20)$$

It is obvious from equation (2.20) that the asymmetry produced in both sectors are not the same.

The Boltzmann equation for evolution of densities are given as

$$\frac{sH_1}{z}\frac{dY_{N_1}}{dz} = -\gamma_D\left(\frac{Y_{N_1}}{Y_{N_1}{}^{eq}}-1\right) + (2\leftrightarrow 2 \text{ processes}), \quad (2.21)$$

$$\frac{sH_1}{z}\frac{dY_{\Delta\chi}}{dz} = \gamma_D\left(\varepsilon_\chi\left(\frac{Y_{N_1}}{Y_{N_1}{}^{eq}}-1\right) - \frac{Y_{\Delta\chi}}{Y_\chi{}^{eq}}Br_\chi\right) + (2\leftrightarrow 2 \text{ processes}), \quad (2.22)$$

$$\frac{sH_1}{z}\frac{dY_{\Delta\ell}}{dz} = \gamma_D\left(\varepsilon_\ell\left(\frac{Y_{N_1}}{Y_{N_1}{}^{eq}}-1\right) - \frac{Y_{\Delta\ell}}{Y_\ell^{eq}}Br_\ell\right) + (2\leftrightarrow 2 \text{ processes}), \quad (2.23)$$

where $Y_{N_1}{=}n_1/s$, $z = M_1/T$, $\gamma_D = m_{N_1}^3 K_1(z)/z$, $Y_{N_1}^{eq} = \frac{3}{8}z^2K_2(z)$, Br_χ and Br_l are branching ratios of respective decay channels. The first equation explains the N_1 abundance due to decays and inverse decays. The second and third equations describe the evolution of asymmetry in respective sectors. The first term is source of asymmetry and is proportional to ε's and the second term takes care of inverse decays. The $2 \leftrightarrow 2$ processes as shown in Figure (2.5), describe the scattering processes which can transfer one sector's asymmetry to the other.

The washouts are negligible if $\Gamma_1^2/M_1H_1 << 1$ and N_1 have initial abundance. If $Y_{N_1}(0) = 0$, then there will be negative asymmetry at $z << 1$ as the inverse decays will be more effective than decays. Hence the positive asymmetry produced at $z > 1$ can be cancelled completely. But a small asymmetry

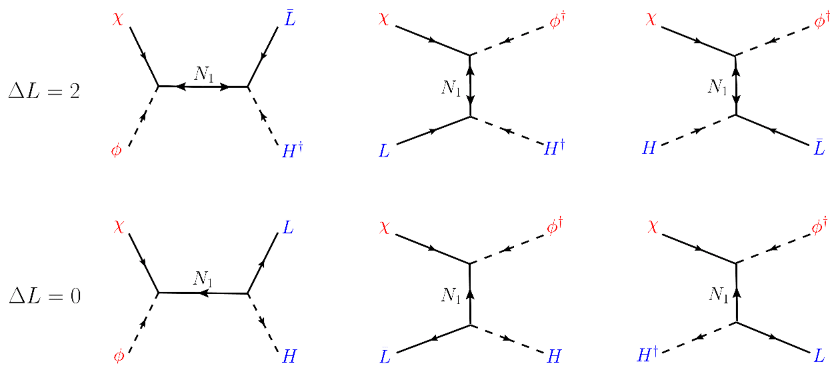

Figure 2.5: Feynman diagrams of processes which transfer the lepton asymmetry between the two sectors [9].

survives because of the fact that washouts are different at different z. The efficiency coefficients are estimated as (where $j = l, \chi$)

$$\eta_j \simeq \frac{\Gamma_1^2}{H_1^2} Br_j. \tag{2.24}$$

The asymptotic solution to Boltzmann equations are

$$Y_{\Delta\chi}^{\infty} = \eta_\chi \varepsilon_\chi Y_{N_1}^{eq},$$
$$\text{and} \quad Y_{\Delta\ell}^{\infty} = \eta_\ell \varepsilon_\ell Y_{N_1}^{eq}.$$

Now using equation (2.1), it can be easily seen that

$$m_{DM} n_{DM} = 5 m_b n_b$$

$$m_{DM} = 5 m_b \frac{\varepsilon_\ell}{\varepsilon_\chi} \frac{\eta_\ell}{\eta_\chi}. \tag{2.25}$$

Therefore, depending on branching fractions and washout regimes one can get a wide range of masses unlike other ADM models. In two sector leptogenesis the masses for dark matter can vary from a keV to 10 TeV [9].

2.2.2 WIMPy Baryogenesis

In this model both dark sector and baryons have common origin of asymmetry [10]. Here, the Sakharov's conditions are satisfied by annihilation of WIMPs to SM baryon and some exotic particles (say exotic baryons) (Figure (2.6)). The SM baryon and exotic particle are oppositely charged under a $U(1)$ gauge symmetry. Hence, the annihilations create asymmetry in both sectors but preserve the global symmetry.

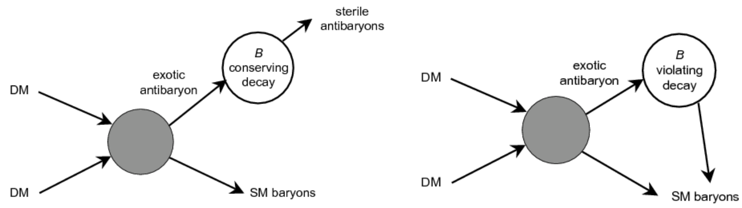

Figure 2.6: WIMPy Baryogenesis [10].

To generate the sufficient asymmetry the inverse annihilations i.e., washout processes must be suppressed. The kinematical condition for that is given as

$$m_{\rm DM} \leqslant m_{\rm exoticbaryon} \leqslant 2m_{\rm DM}.$$

Solving boltzmann equations we get

$$Y_{\Delta B} = \varepsilon Y_\chi(x_{washout}) - Y_\chi(\infty),$$

i.e., to get the required baryon asymmetry, washout must freeze out before WIMP anihilations.

2.2.3 Asymmetric Dark Matter from higher dimensional operators

In this model asymmetry generated in one sector at high temperatures and later on transferred to other sector by some higher dimensional operators[11]. The interactions responsible for transfer of asymmetry freeze out and fix the relic density of dark matter. In ADM, the number densities of dark matter and baryons are of same order i.e., $n_{DM} \sim n_b$. Therefore to explain dark matter relic density $m_{DM} \simeq 5m_b$ i.e., $m_{DM} \simeq 5$ GeV.

2.3 Bounds on RH Neutrino Mass from Thermal Leptogenesis

In any leptogenesis model, the amount of final asymmetry produced is always proportional to CP asymmetry,

$$\frac{n_{B-L}}{s} = \varepsilon\eta\frac{n^{eq}_{N_i}}{s}, \tag{2.26}$$

where N_i is right-handed neutrino whose decay is producing the asymmetry and $\eta \leqslant 1$ is the efficiency factor which is a model dependent parameter. In thermal leptogenesis, where N_i's are produced by scattering in thermal bath, the equilibrium density is estimated as $n^{eq}_{N_i}/s \sim 0.4/g_*$, where $g_* \simeq 230$ is number of propagating states in thermal bath.

The amount of CP asymmetry produced in leptogenesis with $M_{N_1} << M_{N_{2,3}}$ is given as

$$\varepsilon_1 = \frac{-3}{8\pi}\frac{M_{N_1}}{\langle\phi\rangle^2}\frac{1}{[h_\nu h_\nu^\dagger]_{11}} Im([h_\nu M_\nu h_\nu^\dagger]). \tag{2.27}$$

In see-saw mechanism the most general Yukawa coupling is given as,

$$h_\nu = \frac{1}{\langle\phi\rangle} D_{\sqrt{M}} R D_{\sqrt{m}} U^\dagger, \tag{2.28}$$

where R is an orthogonal matrix, U is PMNS matrix and $D_{\sqrt{A}} = +\sqrt{D_A}$ where D is a diagonal matrix,

$$D_M = diag(M_{N_1}, M_{N_2}, M_{N_3}), \tag{2.29}$$

$$D_m = diag(m_1, m_2, m_3) = U^\dagger M_\nu U. \tag{2.30}$$

Now, substituting all these expressions in equation (2.27) and using the orthogonality condition for matrix R, we get

$$\varepsilon_1 \leqslant \frac{3}{8\pi}\frac{M_{N_1}}{\langle\phi\rangle^2}(m_3 - m_1). \tag{2.31}$$

From equation (2.26) and equation (2.31), we get

$$M \geqslant \frac{8\pi}{3}\eta\left(\frac{n_{B-L}}{n^{eq}_{N_1}}\frac{\langle\phi\rangle^2}{(m_3 - m_1)}\right). \tag{2.32}$$

Since $n_{B-L}/s \sim 10^{-10}$ and $(m_3 - m_1) \sim \sqrt{\Delta m^2_{atm}}$ which have value in the range 0.4–0.8 eV, we get a lower bound on $M_{N_1} \geqslant 10^9$ GeV. This bound is well known in literature as 'Davidson Ibarra bound' [12] on masses of right-handed neutrinos.

However, for thermal production of right-handed neutrinos, the reheat temperature must be larger than M_{N_1} which is not favorable by various cosmological models as it gives rise to the problem of overproduction of gravitinos. Also very high masses are out of scope of present and near future experiments. To relax the bound on masses, one can use non-thermal or resonant leptogenesis mechanism which can successfully explain leptogenesis with masses of order of TeV.

Chapter 3

Dark Matter and Leptogenesis in the Inert Doublet Model

As we know, in the SM Higgs resides in a $SU(2)$ doublet and a particular vacuum is chosen after symmetry breaking. A widely used parameter ρ is an important check for SM Higgs isospin and is defined as

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \tag{3.1}$$

where m_W and m_Z are masses of W and Z bosons respectively and θ_W is Weinberg angle. For SM, at leading order $\rho = 1$ which perfectly agrees with experimental value $\rho = 0.9998^{+0.0008}_{-0.0005}$. However besides of Higgs doublet, SM can be extended with more multiplets. Say ϕ_k are Higgs multiplets with weak isospins I^k and vacuum expectation values v_k. Then at the tree level ρ is given by,

$$\rho = \frac{\sum_k \left[I^k(I^k+1) - I_3^k\right] v_k^2}{2\sum_k (I_3^k) v_k^2}. \tag{3.2}$$

The equation (3.2) implies that $\rho = 1$ for any number of Higgs doublets. Therefore, one can extend SM with any number of doublets in principle.

In this chapter, we will investigate the possibilty that whether the extension of SM with inert Higgs doublet is a good choice to build combined model for dark matter and leptogenesis.

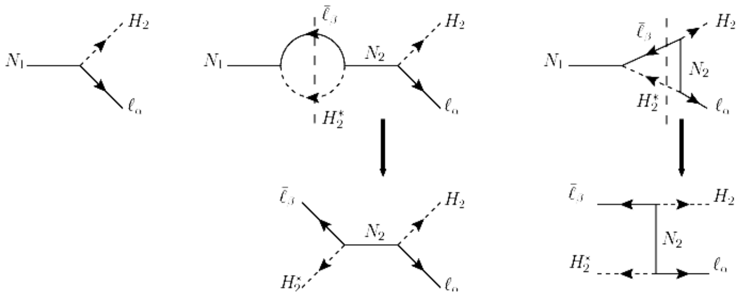

Figure 3.1: Feynman diagrams involved in leptogenesis with inert Higgs H_2 [13].

3.1 Inert Doublet Model

The Inert Doublet Model(IDM) is a very simple extension of SM by three right-handed neutrinos and one doublet which is an inert Higgs doublet. To have a stable dark matter an additional Z_2 symmetry has been introduced under which new particles have odd parity and SM particles have even parity. This forbids the generation of neutrino mass at tree level as the terms containing right-handed neutrino, left-handed neutrino and SM Higgs is not allowed at tree level.

The Lagrangian for model is [16]

$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} + (D_\mu \eta)^\dagger (D^\mu \eta) - V(\phi, \eta) - \frac{M_i}{2} \bar{N}_i N_i^c - (h_{\alpha i} \bar{N}_i \eta^\dagger \ell_\alpha + h.c.), \quad (3.3)$$

where ϕ is SM Higgs doublet and η is an inert Higgs doublet with

$$V(\phi, \eta) = m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\phi^\dagger \phi)(\eta^\dagger \eta)$$
$$+ \lambda_4 (\eta^\dagger \phi)(\phi^\dagger \eta) + [\frac{\lambda_5}{2} (\phi^\dagger \eta)^2 + h.c.]. \quad (3.4)$$

The key points of model can be summarized as follows

- The mass spectrum of right-handed neutrinos N_i's is assumed as $M_{N_1} < M_{N_2} < M_{N_3}$.
- Unlike Leptogenesis, here interactions of Majorana neutrinos are mediated by inert Higgs say H_2 instead of SM Higgs. Therefore, the decay process $N_i \to H_2 + l_\alpha$ will be lepton number violating process with $\Delta L = 1$ for each decay (Figure (3.1)).

- The lightest component of inert doublet η (say H_2) will be the stable dark matter.

- The quartic couplings will play an important role in dark matter relic density as the components of η can effectively coannihilate via quartic interactions.

- Neutrino masses are generated at the loop level.

- The source of CP violation and out of equilibrium conditions will be same as it was in leptogenesis, provided SM Higgs is replaced by the inert Higgs.

3.1.1 Dark Matter

In unitary gauge the SM Higgs is $\phi^T = (0, \langle\phi\rangle + \frac{h}{\sqrt{2}})$ and the inert Higgs doublet is $\eta^T = (\eta_+, \frac{1}{\sqrt{2}}(\eta_R + i\eta_I))$, therefore equation (3.4) can be written as

$$\begin{aligned} V(\phi,\eta) = & \left[\tfrac{m_\phi^2}{2} + \lambda_1\langle\phi\rangle^2\right](h^\dagger h) + \left[m_\eta^2 + \lambda_3\langle\phi\rangle^2\right]\eta^+\eta^- \\ & + \tfrac{1}{2}\left[m_\eta^2 + \lambda_+\langle\phi\rangle^2\right]\eta_R^2 + \tfrac{1}{2}\left[m_\eta^2 + \lambda_-\langle\phi\rangle^2\right]\eta_I^2 \\ & + \sqrt{2\lambda_1}\langle\phi\rangle h^3 + \tfrac{1}{4}\left[\sqrt{\lambda_1}h^2 - \sqrt{\lambda_2}(\eta^+\eta^- + \eta_R^2 + \eta_I^2)\right]^2 \\ & + \tfrac{1}{2}h^2\left[\eta^+\eta^-(\lambda_3 + \sqrt{\lambda_1\lambda_2}) + \eta_R^2(\lambda_+ + \sqrt{\lambda_1\lambda_2})\right] \\ & + \frac{1}{2}h^2\left[\eta_I^2(\lambda_- + \sqrt{\lambda_1\lambda_2})\right], \end{aligned} \tag{3.5}$$

where $\lambda_\pm = \lambda_3 + \lambda_4 \pm \lambda_5$ and masses of component fields of η are $M_{\eta_c} = \left[m_\eta^2 + \lambda_3\langle\phi\rangle^2\right]$, $M_{\eta_R} = \left[m_\eta^2 + \lambda_+\langle\phi\rangle^2\right]$, $M_{\eta_I} = \left[m_\eta^2 + \lambda_-\langle\phi\rangle^2\right]$. It is assumed that vacuum is stable i.e., V is bounded from below, which suggests

$$\lambda_1, \lambda_2 > 0; \qquad \lambda_3, \lambda\pm > -\sqrt{\lambda_1\lambda_2}. \tag{3.6}$$

Since the vacuum expectation value of new doublet η is zero, Z_2 is unbroken, hence the lightest neutral component of η behaves as DM. If η_R is lightest, we have

$$M_{\eta_I}, M_{\eta_c} > M_{\eta_R} \quad \Rightarrow \quad \lambda_5 < 0, \quad \lambda_4 + \lambda_5 < 0. \tag{3.7}$$

The mass splitting among the component fields of η are given as

$$\frac{M_{\eta_I} - M_{\eta_R}}{M_{\eta_R}} \simeq \frac{\mid \lambda_5 \mid \langle \phi \rangle^2}{M^2_{\eta_R}}, \tag{3.8}$$

$$\frac{M_{\eta_c} - M_{\eta_R}}{M_{\eta_R}} \simeq \frac{\mid \lambda_4 + \lambda_5 \mid \langle \phi \rangle^2}{M^2_{\eta_R}}. \tag{3.9}$$

Dark Matter abundance

The Boltzmann equation for total density of dark matter i.e., $n = \sum_i n_i$ is given as follows, where $i = 1$ is for the lightest of dark matter η_R

$$\frac{dn}{dx} = -\langle \sigma_{eff} v \rangle (n^2 - (n^{eq})^2), \tag{3.10}$$

where $x = m_i/T$, $\langle \sigma_{eff} v \rangle$ is averaged effective coannihilation cross-section, $n^{eq} = m_i^3/(2\pi x)^{3/2} e^{-x}$. The relic abundance is estimated as

$$\Omega_{\eta_R} h^2 \simeq \frac{1.07 * 10^9 \mathrm{GeV}}{J(x_F)\sqrt{g_*} M_{Pl}}, \tag{3.11}$$

where $M_{Pl} \sim 10^{19}$ GeV is the Planck mass, $J(x_F)$ is given as

$$J(x_F) = \int_{x_F}^{\infty} \frac{\langle \sigma_{eff} v \rangle}{x^2} dx, \tag{3.12}$$

where, $x_F = x(T_F) \sim 25$ which we have seen in equation (1.13). T_F is the freeze-out temperature of DM, and

$$\langle \sigma_{eff} v \rangle = \frac{1}{g^2_{eff}} \sum_{i,j=1}^{4} \langle \sigma_{ij} \rangle \frac{n_i^{eq} n_j^{eq}}{(n_1^{eq})^2}, \tag{3.13}$$

where g_{eff} is effective number of degrees of freedom and $\langle \sigma_{ij} \rangle = a_{ij} + b_{ij} \langle v^2 \rangle$ is thermally averaged co-annihilation cross section between species i and j, which is given in terms of averaged relative velocity. For cold dark matter $\langle v^2 \rangle$ is very small. Therefore, the relic density is dominated by a_{ij} which is given as [15]

$$a_{eff} = \frac{A}{m_1^2}(N_{11} + N_{22} + 2N_{34}) + \frac{B}{m_1^2}(N_{13} + N_{14} + N_{23} + N_{24})$$
$$+\frac{C}{m_1^2}\left[(\lambda_+^2 + \lambda_-^2 + 2\lambda_3^2)(N_{11} + N_{22}) + (2\lambda_5)^2(N_{33} + N_{44} + N_{12})\right]$$
$$+\frac{C}{m_1^2}[(\lambda_4 + \lambda_5)^2(\sum_{i=3,4}(N_{1i} + N_{2i})) + ((\lambda_3 + \lambda_4)^2 + 4\lambda_3^2)N_{34}],$$

where N_{ij},

$$N_{ij} = \frac{1}{g_{eff}^2}\frac{n_i^{eq} n_j^{eq}}{(n_1^{eq})^2}. \tag{3.14}$$

It is clear from equation(3.14) that the relic density of dark matter gives weak bounds on quartic coupling but gives no bounds on Yukawa couplings.

However, λ_5 is constrained by direct searches of DM. The direct detection experiments aim to observe low-energy recoils of nuclei induced by interactions with particles of dark matter. After such a recoil the nucleus will emit energy as, e.g., scintillation light or phonons, which is then detected by sensitive apparatus. The kinematical condition for a interaction like η_R + Nuclei $\to \eta_I$ + Nuclei is,

$$\gamma M_{\eta_R} + M_N = E_N + M_{\eta_I}, \tag{3.15}$$

where M_N is rest mass of nuclei and E_N is the total energy of nuclei after scattering, $\gamma = 1/\sqrt{1-v^2}$, where v is the velocity of η_R (in units of c). Hence the equation (3.16) can be written as

$$M_{\eta_R} - M_{\eta_I} + v^2 M_{\eta_R} = E_N - M_N \simeq E_{recoil}, \tag{3.16}$$

where E_{recoil} is the recoil energy of nuclei. Since η_R is stable dark matter, i.e., $M_{\eta_R} < M_{\eta_I}$. Therefore, an inelastic scattering of DM with nuclei can occur if the dark matter velocity is

$$v_{DM}^2 \geqslant \frac{M_{\eta_I} - M_{\eta_R}}{M_{\eta_R}}. \tag{3.17}$$

If the scattering is mediated by Z boson, the estimated value of mass splitting for $M_{\eta_I}, M_{\eta_R} >> m_Z$ is [14]

$\mid M_{\eta_I} - M_{\eta_R} \mid > 110$ keV, which gives a bound on λ_5 through equation (3.8) for $M_{\eta_R} \sim O(1)$ TeV

$$\lambda_5 \geqslant 10^{-5}. \tag{3.18}$$

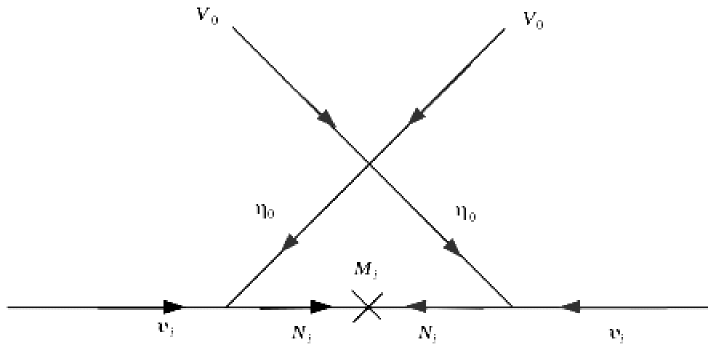

Figure 3.2: Feynman diagram for generation of neutrino mass at loop level [15].

This bound on λ_5 plays an important role in neutrino masses as explained in the following section.

3.1.2 Neutrino Masses

Since N_i's are odd under Z_2, the tree level interaction of N_i and SM Higgs ϕ is forbidden. Neutrino masses are generated by one loop diagrams as shown in Figure (3.2).

The elements of mass matrix is given as

$$M^{\nu}_{\alpha\beta} = \sum_i h_{\alpha i} h_{\beta i} \frac{\lambda_5 \langle\phi\rangle^2}{8\pi^2} \frac{M_i}{\left(M_\eta^2 - M_i^2\right)} \left(1 + \frac{M_i^2}{\left(M_\eta^2 - M_i^2\right)} \ln \frac{M_i^2}{M_\eta^2}\right), \quad (3.19)$$

$$M^{\nu}_{\alpha\beta} = \sum_i h_{\alpha i} h_{\beta i} \Lambda_i \ , \quad (3.20)$$

where Λ_i describes the scale of neutrino mass, $M_\eta^2 = [m_\eta^2 + (\lambda_3 + \lambda_4)\langle\phi\rangle^2]$, since λ_5 is very small and $m_\eta >> \langle\phi\rangle$, the difference in masses of components of η can be neglected for this analysis. To understand the neutrino mixing patterns one can assume the flavor pattern of Yukawa couplings as follows [15]

$$h_{ei} = 0, \quad h_{\mu i} = h_i, \quad h_{\tau i} = q_1 h_i, \quad (3.21)$$

$$h_{e3} = h_3, \quad h_{\mu 3} = q_2 h_3, \quad h_{\tau i} = q_3 h_3. \quad (3.22)$$

The mass matrix of neutrino will take a simple form as follows

$$\mathcal{M}^{\nu} = h_i^2 \Lambda_i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & q_1 \\ 0 & q_1 & q_1^2 \end{pmatrix} + h_3^3 \Lambda_3 \begin{pmatrix} 1 & q_2 & -q_3 \\ q_2 & q_2^2 & -q_2 q_3 \\ -q_3 & -q_2 q_3 & q_3^2 \end{pmatrix}. \tag{3.23}$$

For $q_1, q_2, q_3 = 1$ the PMNS matrix have tribimaximal form

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.24}$$

In this case one can easily diagonalize the mass matrix and can find the eigenvalues

$$m_1^{\nu} = 0, \quad m_2^{\nu} = h_i^2 \Lambda_i, \quad m_3^{\nu} = h_3^2 \Lambda_3, \quad \text{where } i = 1 \,\text{or}\, 2. \tag{3.25}$$

Since one of the eigenvalues is zero, one can directly compare the eigenvalues with $\sqrt{\Delta m_{atm}^2}$ and $\sqrt{\Delta m_{sol}^2}$. However, the mixing matrix will no longer be tribimaximal if we consider θ_{13} to non-zero. Therefore one can numerically diagonalize the matrix to impose the constraints on q_i's and can predict the value of θ_{13}. For example, for an arbitrary value of $q_1 = 0.85$, other parameters are calculated to be $(q_2, q_3) = (-0.27, 2.4)$ with $\sin^2(2\theta_{13}) = 0.085$ [15].

3.1.3 Baryon asymmetry

Since the masses of N_i's are hierarchical, the lightest of right-handed neutrinos will satisfy the Sakharov's conditions. The CP symmetry is produced by interference of one loop level to tree level diagrams (Figure (3.1)). The out of equilibrium decay of N_1 gives the condition $\Gamma_{N_1}^D < H(T = M_{N_1})$, where $\Gamma_{N_1}^D = \frac{|h_1|^2}{8\pi}(1+q_1)^2 M_{N_1}(1 - \frac{M_\eta^2}{M_{N_1}^2})^2$ and hence

$$\mid h_1 \mid^2 < 2 * 10^{-8}(1+q_1^2)^{-1/2} \left(\frac{M_{N_1}}{1\ \text{TeV}}\right)^{1/2}. \tag{3.26}$$

The generated asymmetry can be washed out by inverse decay and $2 \leftrightarrow 2$ scatterings like $\eta\eta \to l_\alpha l_\beta$ and $\eta \bar{l}_\alpha \to \eta^\dagger l_\beta$. Hence, for asymmetry to survive, the washout should freeze out before the freeze out of N_1 decay. The

Boltzmann equations for evolution of densities are given as

$$\frac{dY_{N_1}}{dz} = \frac{-z}{sH(M_{N_1})}\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right)\left[\gamma_D^{N_1} + \sum_{i=2,3}\left(\gamma^{(2)}_{N_1 N_i} + \gamma^{(3)}_{N_1 N_i}\right)\right], \quad (3.27)$$

$$\frac{dY_l}{dz} = \frac{z}{sH(M_{N_1})}\left[\varepsilon\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right)\gamma_D^{N_1} - \frac{2Y_l}{Y^{eq}_l}\left(\gamma^{(2)}_N + \gamma^{(3)}_N\right)\right], \quad (3.28)$$

where $z = M_{N_1}/T$, $H(M_{N_1}) = 1.66 M_{N_1}/\sqrt{g_*}M_{Pl}$ is Hubble parameter, $Y_x = n_x/s$, $s = 0.37 g_* T^3$ is the entropy density of universe, the equilibrium densities are $Y^{eq}_{N_1} = 45 z^2 K_2(z)/2\pi^4 g_*$, $Y^{eq}_l = 45/\pi^2 g_*$. Later, when universe cools down to the sphaleron decoupling temperature, the asymmetry is transferred to the baryonic sector,

$$Y_B = \frac{-8}{28} Y_l(z_{sphl}). \quad (3.29)$$

For a set of values of parameters i.e., $q_1 = 0.85$, $(q_2, q_3) = (-0.27, 2.4)$, $M_\eta = 1$ TeV, $M_{N_1} = 2$ TeV, $M_{N_2} = 6$ TeV, $M_{N_3} = 10$ TeV, $\lambda_5 = 10^{-5}$ and $(|h_1|, |h_2|, |h_3|) = (3\times10^{-8}, 3.45\times10^{-3}, 1.5\times10^{-3})$ as given in [15], the amount of final baryon asymmetry produced is

$$Y_B = 2.7 * 10^{-12}. \quad (3.30)$$

Hence, the combined solution of dark matter, neutrino mass and baryon asymmetry in inert doublet model is possible. However, the parameter space has not been searched completely as we have not taken $2 \leftrightarrow 2$ processes into account.

Chapter 4

Discussion and Future Scope

The combined models are better choice for explaining the dark matter, neutrino mass and baryon asymmetric universe as they require a small extension of standard model. Also the parameters in combined models can be well constrained because they need to satisfy the experimental observations for all three problems simultaneously, this leads to promising parameter space for experimental searches.

The inert doublet model with three right-handed neutrinos is an interesting framework for both leptogenesis and dark matter. The lightest component of inert doublet is identified with dark matter, which gives a bound on quartic couplings leaving the Yukawa couplings to be determined from the conditions for successful leptogenesis. The neutrino masses are generated at loop level for a particular flavor structure of Yukawa couplings. In equation (3.24), the mass matrix is diagonalized for arbitary values of q_1 to get the contours of (q_2, q_3) and predict the value of θ_{13}. However, one can take a more general form of Yukawa couplings and diagonalize the mass matrix to get the contours for parameters instead of some arbitary inputs. For example [18],

$$\frac{h_{e1}}{p_1} = \frac{-2h_{\mu 1}}{q_1} = 2h_{\tau 1} = 2h_1, \tag{4.1}$$

$$\frac{h_{e2}}{p_2} = \frac{h_{\mu 2}}{q_2} = -h_{\tau 2} = h_2, \tag{4.2}$$

which gives the mass matrix of form

$$\mathcal{M}^{\nu} = h_1^2\Lambda_1 \begin{pmatrix} 4p_2^2 & -2p_1q_1 & 2p_1 \\ -2p_1q_1 & q_1^2 & -q_1 \\ 2p_1 & -q_1 & 1 \end{pmatrix} + h_2^2\Lambda_2 \begin{pmatrix} p_2^2 & p_2q_2 & -p_2 \\ p_2q_2 & q_2^2 & -q_2 \\ -p_2 & -q_2 & 1 \end{pmatrix}. \quad (4.3)$$

Hence one can find the diagonalizing matrix for M_{ν} and the values of mixing angles. By comparing difference of square of eigenvalues with the observed values of atomspheric neutrinos Δm^2_{atm} and solar neutrinos Δm^2_{sol}, one can get a parameter space for (p_1, p_2) and (q_1, q_2).

Also it is obvious from the equation (3.31) that the amount of asymmetry produced differs by two orders from observed values. To enhance the asymmetry one can impose degenerate masses to right-handed neutrinos, which will enhance the CP asymmetry without increasing the washouts [13][19].

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