# SEMI-DIRAC MATERIAL BASED N-S JUNCTION

**M.Sc. Thesis**

**By:**
**Lucky Raj**
**ID : 2103151017**

DEPARTMENT OF PHYSICS

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## A THESIS

*Submitted in partial fulfillment of the requirements for the award of the degree*

***of***

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***by***

**Lucky Raj**

DEPARTMENT OF PHYSICS

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# CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Semi-Dirac Material Based N-S Junction"** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF SCIENCE** and submitted in the **DEPARTMENT OF PHYSICS, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July, 2022 to June, 2023 under the supervision of **Dr. Alestin Mawrie, Asst. Professor at Indian Institute of Technology Indore**.
The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

Lucky Raj 07/06/2023

**Signature with date**
**(Lucky Raj)**

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Mawrie 07/06/2023

Signature of the Supervisor of
M.Sc. Thesis
**(DR.ALESTIN MAWRIE)**

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**Lucky Raj** has successfully given his M.Sc. Oral Examination held on **6$^{th}$June,2023**

Mawrie 07/06/2023

**Signature(s) of Supervisor(s) of M.Sc. Thesis, Date:**

**Convener, DPGC**
**Date:-** 8/6/2023

**Signature of PSPC Member 1**
**Date:-** 07/06/23

**Signature of PSPC Member 2**
**Date:-** 08/6/23

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# Abstract

We study the Andreev bound states and provide unique signatures of Andreev bound states using the Blonder-Tinkham-Klapwijk model for Normal-Superconductor (N-S) junction. Taking this forward, we have looked into the possibility of getting such Andreev-bound states in a semi-Dirac materials-based N-S junction. This report contains the solution of such Andreev bound states in such an N-S junction obtained by solving the Bogoliubov-de-Gennes equation. We find that for the case of the ideal N-S interface if the energy of an incident electron is such that $E < \Delta$ (where $\Delta$ is the superconducting gap), we have a perfect Andreev reflection. But as this ratio $\frac{E}{\Delta}$ become greater than 1, the probability of transmission of an electron transmitting into the S region increases since there are no allowed Cooper pair states at such energy. We also studied the differential conductance of such a setup and it shows an anomaly giving a very large conductance at a certain energy. It must be mentioned that the singularity of the differential conductance at $eV = \Delta$ also appears (as expected).

# Contents

# List of Figures

# Chapter 1

# Introduction

The *Bardeen-Cooper-Schrieffer* (BCS) theory is a fundamental theory of superconductivity in modern condensed matter physics. The key feature of the theory is the pair condensation of a Fermionic pair to form a condensate called Cooper pair. The binding of Fermions into Cooper pairs typically leads to an energy gap in the Fermionic excitation spectrum implying that the Fermionic excitations are no longer charge eigenstates, but each is a coherent superposition of a normal-state particle and hole, e.g. $\gamma^\dagger = uc^\dagger + vc$, where $u$ and $v$ are the particle and hole amplitudes defining the Bogoliubov quasiparticles. Charge conservation is maintained by an additional channel for charge transport via the coherent motion of the pair condensate.

## 1.1 Andreev Reflection

Many of the important properties of superconductors come from the coherent superposition of particle and hole states that defines the low-energy excitations of a superconductor, *i. e.* Bogolibov quasiparticles. The coherence amplitudes, talked about in the previous section $u$ and $v$ clearly depend on the pair potential, $\Delta$(r). In an N-S junction, it was identified by A. F. Andreev, that an incoming particle-like excitation from the N-region has a finite probability of converting to an outgoing hole-like excitation, a process called branch conversion scattering, or Andreev scattering thus creating a Cooper pair state in the S region. The entire process is shown in Fig. 1.1. Another condition, we must put forward is that the energy of the incident particle-like excitation should be such that $\epsilon$¡

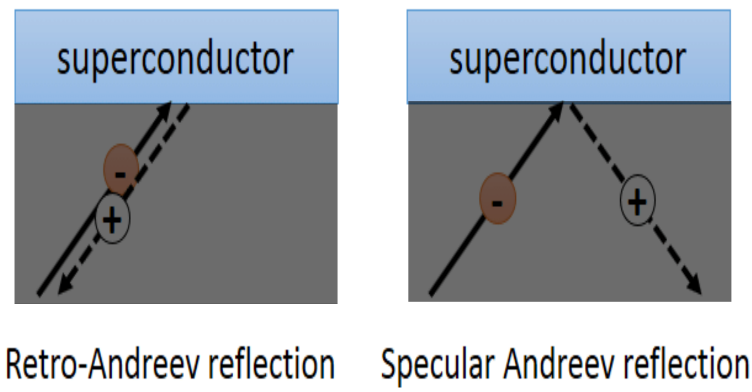


Figure 1.1: Left: Retro and Specular Andreev reflection at an N-S boundary.

$\Delta$, since there are no Cooper pairs excitation in the superconducting side for $\epsilon > \Delta$. There are two types of Andreev reflection as shown in Fig. 1.1. The condition for these two types of reflection depends on whether the excitons lies in the conduction or valence bands as we shall see in the subsequent sections.

## 1.2 Dirac materials

Dirac materials are materials that exhibit a linear $E(k)$ vs $k$ relation about the low energy spectrum making them massless relativistic particles. In Dirac materials the low-energy fermionic excitations or quasiparticles do not obey the Schrodinger Hamiltonian $H_S$ but rather a Dirac Hamiltonian (Eqn. (1.1)) along with the relativistic effect. In two spatial dimensions, this Hamiltonian has the form

$$H_D = v_F\,\sigma.p \tag{1.1}$$

where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the usual Pauli matrices and $v_F$ denotes the Fermi velocity. Understandably, $v_F$ is huge since they are quasi relativistic particles. One of the very good examples of Dirac materials are that of graphene which shows such Dirac like dispersion at all the valley points in the Brillouin zone. (Refer to the figure below)

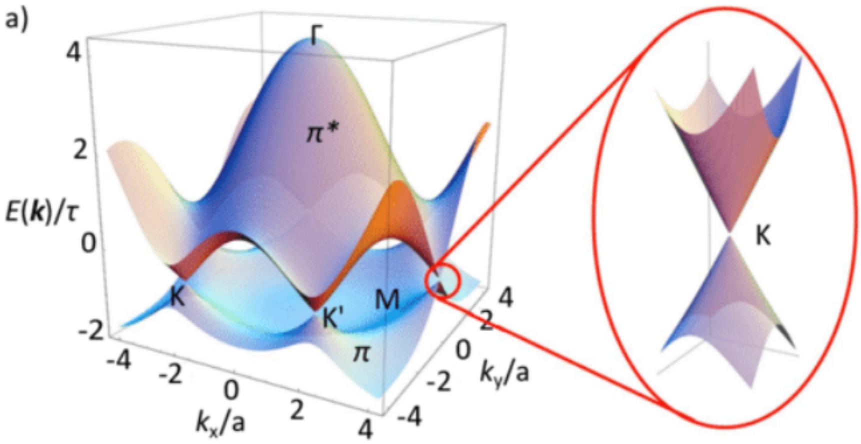


Figure 1.2: Energy dispersion of electrons in graphene which displays the Dirac cones at each valley point.

# Chapter 2

# The Blonder-Tinkham-Klapwijk model for N-S junction

We consider the following N-S junction (figure 2.1) and the calculations involved with it as a benchmark for our study. We denote by $x$ the longitudinal coordinate and by $(y, z)$ the transversal coordinates. The junction is located at the coordinate $x = 0$. We model the system with the Bogolubov de Gennes Equation

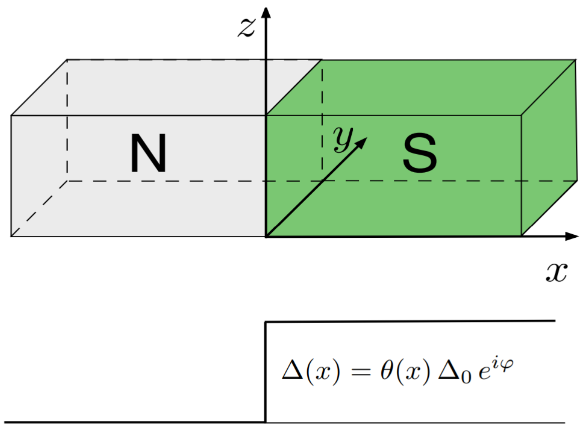


Figure 2.1: N-S junction

$$\begin{bmatrix} H_e & \Delta \\ \Delta^* & -H_e^* \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = E \begin{bmatrix} u \\ v \end{bmatrix} \tag{2.1}$$

We describe the junction with a step-like order parameter

$$\Delta(x) = \theta(x)\Delta_0 \exp(\iota\phi) \tag{2.2}$$

Assume that the system is separable so that we can factorize the wavefunction into

$$\Psi(x, y, z) = \psi(x)\Phi(y, z) \tag{2.3}$$

$$[\frac{-\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) + V(y,z)]\Phi(y,z) = E\Phi(y,z) \tag{2.4}$$

In conclusion, the system is described by the following effective 1D BdG Hamiltonian

$$\begin{bmatrix} \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \epsilon_f & \Delta(x) \\ \Delta^*(x) & \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \epsilon_f \end{bmatrix} \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} = E \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} \tag{2.5}$$

is called the Blonder-Tinkham-Klapwijk (BTK) Model.

## 2.1 Solution in N

$$\begin{bmatrix} \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \epsilon_f & 0 \\ 0 & \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \epsilon_f \end{bmatrix} \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} = E \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} \tag{2.6}$$

$$[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \epsilon_f]u(x) = Eu(x) \tag{2.7}$$

$$[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \epsilon_f]v(x) = Ev(x) \tag{2.8}$$

which exhibit two particle solutons and two hole solutions

$$\Psi_e(\pm x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \exp(\pm \imath k_e x) \tag{2.9}$$

$$\Psi_h(\pm x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \exp(\pm \imath k_h x) \tag{2.10}$$

where,

$$k_e = k_f\sqrt{1 + \frac{E}{\epsilon_f}} \tag{2.11}$$

$$k_h = k_f\sqrt{1 - \frac{E}{\epsilon_f}} \tag{2.12}$$

$$k_f = \frac{\sqrt{2m\epsilon_f}}{\hbar} \tag{2.13}$$

## 2.2 Solutions in S $(E > \Delta_0)$

In the superconductor side S the equation (2.5) reduces to

$$\begin{bmatrix} \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \epsilon_f & \Delta_0 \exp{(\iota\phi)} \\ \Delta_0 \exp{(-\iota\phi)} & \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \epsilon_f \end{bmatrix} \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} = E \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} \tag{2.14}$$

Here there are two particle like solutions and two hole like solutions

$$\Psi_e(\pm x) = \begin{bmatrix} u_0 \exp{(\frac{\iota\phi}{2})} \\ v_0 \exp{(\frac{-\iota\phi}{2})} \end{bmatrix} \exp{(\pm\iota q_e x)} \tag{2.15}$$

$$\Psi_h(\pm x) = \begin{bmatrix} v_0 \exp{(\frac{\iota\phi}{2})} \\ u_0 \exp{(\frac{-\iota\phi}{2})} \end{bmatrix} \exp{(\pm\iota q_h x)} \tag{2.16}$$

where,

$$q_e = k_f \sqrt{1 + \sqrt{\frac{E^2 - \Delta_0^2}{\epsilon_f^2}}} \tag{2.17}$$

$$q_h = k_f \sqrt{1 - \sqrt{\frac{E^2 - \Delta_0^2}{\epsilon_f^2}}} \tag{2.18}$$

$$k_f = \frac{\sqrt{2m\epsilon_f}}{\hbar} \tag{2.19}$$

$$u_0^2 + v_0^2 = 1 \tag{2.20}$$

using eq(2.20) quantities $u_0$ and $v_0$

$$u_0 = \sqrt{\frac{\Delta_0}{2E}} \exp{(\frac{1}{2} arccosh(\frac{E}{\Delta_0}))} \tag{2.21}$$

$$v_0 = \sqrt{\frac{\Delta_0}{2E}} \exp{(-\frac{1}{2} arccosh(\frac{E}{\Delta_0}))} \tag{2.22}$$

i.e. the final form of wavefunctions (2.15) and (2.16)are

$$\Psi_e(\pm x) = \sqrt{\frac{\Delta_0}{2E}} \begin{bmatrix} \exp\left(\frac{1}{2} arc\cosh(\frac{E}{\Delta_0})\right) \exp\left(\frac{\iota\phi}{2}\right) \\ \exp\left(-\frac{1}{2} arc\cosh(\frac{E}{\Delta_0})\right) \exp\left(\frac{-\iota\phi}{2}\right) \end{bmatrix} \exp\left(\pm\iota q_e x\right) \tag{2.23}$$

$$\Psi_h(\pm x) = \sqrt{\frac{\Delta_0}{2E}} \begin{bmatrix} \exp\left(-\frac{1}{2} arc\cosh(\frac{E}{\Delta_0})\right) \exp\left(\frac{\iota\phi}{2}\right) \\ \exp\left(\frac{1}{2} arc\cosh(\frac{E}{\Delta_0})\right) \exp\left(\frac{-\iota\phi}{2}\right) \end{bmatrix} \exp\left(\pm\iota q_h x\right) \tag{2.24}$$

The transmission and reflection amplitudes

$$r_{he} = \frac{u_0 v_0}{u_0^2 + z^2(u_0^2 - v_0^2} e^{-\iota\phi} \tag{2.25}$$

$$r_{ee} = \frac{(z^2 + \iota z)(v_0^2 - u_0^2)}{u_0^2 + z^2(u_0^2 - v_0^2} \tag{2.26}$$

$$t_{ee} = \frac{(1 - \iota z) u_0 \sqrt{u_0^2 - v_0^2}}{u_0^2 + z^2(u_0^2 - v_0^2)} e^{-\iota\phi/2} \tag{2.27}$$

$$t_{he} = \frac{\iota z v_0 \sqrt{u_0^2 - v_0^2}}{u_0^2 + z^2(u_0^2 - v_0^2)} e^{-\iota\phi/2} \tag{2.28}$$

where, $z = \frac{m}{\hbar^2 k_f}$
is the BTK dimensionless parameter of the interface transparency.

We have denoted
$r_{ee}$ = reflection coefficient e $\rightarrow$ e
$r_{he}$ = reflection coefficient e $\rightarrow$ h
$t_{ee}$ = transmission coefficient e $\rightarrow$ e
$t_{he}$ = transmission coefficient e $\rightarrow$ h
The corresponding transmission and reflection coefficients
$A = |r_{he}|^2$
$B = |r_{ee}|^2$
$C = |t_{ee}|^2$
$D = |t_{he}|^2$

in Supra-gap $(E > \Delta_0)$

$$A(E) = \frac{\Delta^2}{(E + (1 + 2z^2)\sqrt{E^2 - \Delta^2})^2} \tag{2.29}$$

$$B(E) = \frac{4z^2(1 + z^2)(E^2 - \Delta^2)}{(E + (1 + 2z^2)\sqrt{E^2 - \Delta^2})^2} \tag{2.30}$$

$$C(E) = \frac{2(1 + z^2)\sqrt{E^2 - \Delta^2}(E + \sqrt{E^2 - \Delta^2})}{(E + (1 + 2z^2)\sqrt{E^2 - \Delta^2})^2} \tag{2.31}$$

$$D(E) = \frac{2z^2\sqrt{E^2 - \Delta^2}(E - \sqrt{E^2 - \Delta^2})}{(E + (1 + 2z^2)\sqrt{E^2 - \Delta^2})^2} \tag{2.32}$$

$$A + B + C + D = 1 \tag{2.33}$$

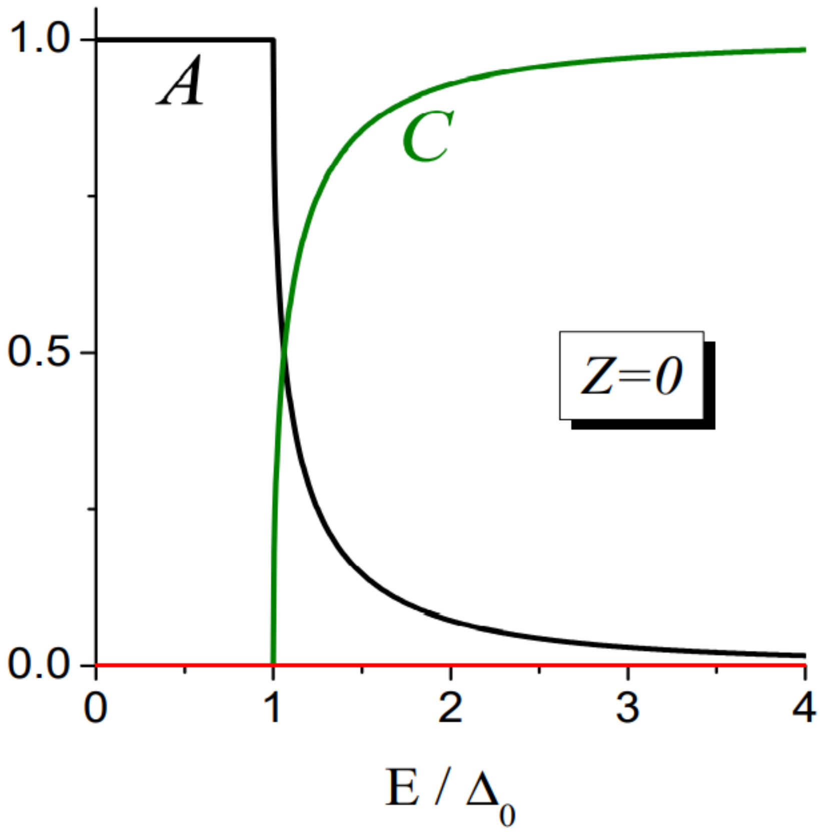


Figure 2.2: The case of ideal interface Z = 0: The Coefficients A, and C are plotted as a function of energy. The coefficients B and D are zero.

# Chapter 3

# Semi Dirac materials

A semi-Dirac material is a material that exhibits linear band dispersion in a given direction and quadratic band dispersion in a direction orthogonal to the former. We can thus say that it hosts massless and massive fermions at the same point in the Brillouin zone which in this case is the $M$ point. A DFT proposal of such material is given in Fig. (3.1), where there is oxygen absorbed in between two silicon atoms of a silicene. We adopt the tight-binding Hamiltonian as given below to describe the Hamiltonian of the semi-Dirac material

$$H = \sum_{P} [t_2 \hat{C}^{\dagger}_{B,P} \hat{C}_{A,P} + t_1 \hat{C}^{\dagger}_{B,P} \hat{C}_{A,P+A_1} + t_1 \hat{C}^{\dagger}_{B,P} \hat{C}_{A,P+A_2}] \tag{3.1}$$

where $t_1$ is the hopping parameter between the Si-Si bond [i.e., in between the Si atoms at $(0,0)$ and at $(\frac{\sqrt{3}}{2}, \frac{-1}{2})$ ; setting the Si-Si bond distance to be $a = 1$], t2 is the hopping parameter between the oxygen adsorped Si-Si bond [i.e., between the Si atoms at$(0,0)$ and at $(0,1)$], and $\hat{C}^{\dagger}_{A/B,P}/\hat{C}_{A/B,P}$ are the creation/annihilation operators at the site A/B is given by the green/red sphere in Fig. (3.1)

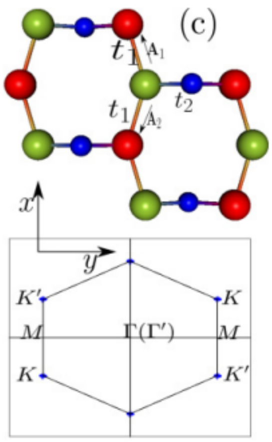


Figure 3.1: The nearest hopping parameters in the honeycomb lattice and the first Brillouin zone with different high-symmetry points. [1]

The dispersion relation corresponding to the Hamiltonian in Eq. (3.1) is

$$H(k) = t_2[e^{-\iota k_y}] + t_1[e^{\iota(\frac{\sqrt{3}k_x}{2}+\frac{k_y}{2})} + e^{-\iota(\frac{\sqrt{3}k_x}{2}-\frac{k_y}{2})}] \tag{3.2}$$

$$\epsilon(k) = |H(k)| \tag{3.3}$$

$$\epsilon(k) = \sqrt{2t_1^2 + t_2^2 + 2t_1^2\cos(\sqrt{3}k_x) + 4t_1t_2\cos(\frac{\sqrt{3}k_x}{2})\cos[\frac{3}{2}(k_y - \frac{2\pi}{3})]} \tag{3.4}$$

take , $t_2 = 2t_1$

$$\epsilon(k) = t_1\sqrt{\frac{9k_x^4}{16} + 9k_y^2}. \tag{3.5}$$

Clearly, the dispersion is massive in the direction $K^{'} \rightarrow$ M $\rightarrow$ K and massless along $\Gamma^{'}\rightarrow$ M $\rightarrow\Gamma$. Thus the model provided by the Hamiltonian in Eq. (34) garners a full description of the nature of the dispersion in semi-Dirac materials. In the proximity of the M point, the Hamiltonian can be written as

$$H = \mathbf{g}\cdot\sigma \tag{3.6}$$

where $\mathbf{g} = (\alpha k_x^2 x - \delta, vk_y, 0)$ and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices, with $\alpha$, $\delta$, and $v$ representing the inverse of the quasiparticle mass along the $x$ direction, the system gap parameter, and the Dirac quasiparticle velocity along the $y$ direction, respectively. The information on the hopping strength between the corresponding nearest atoms is well contained in the gap parameter $\delta$. It is zero for$t_2 = 2t_1$, positive for $t_2 < 2t_1$, and negative for $t_2 > 2t_1$.

### 3.0.1 Semi-Dirac material based N-S junction

Let us consider a normal(N) - superconductor(S) junction, as shown in Fig.6.Model the system with the equation

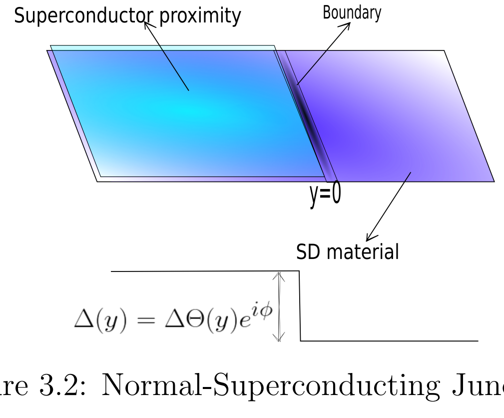


Figure 3.2: Normal-Superconducting Junction

$$H\Psi - \epsilon\Psi = 0 \tag{3.7}$$

$$\begin{bmatrix} (\mu-\epsilon) & [(\alpha p_x^2-\delta)+\iota v p_y] & \Delta e^{\iota\phi} & 0 \\ [(\alpha p_x^2-\delta)-\iota v p_y] & (\mu-\epsilon) & 0 & \Delta e^{\iota\phi} \\ \Delta e^{-\iota\phi} & 0 & -(\mu+\epsilon) & [-(\alpha p_x^2-\delta)-\iota v p_y] \\ 0 & \Delta e^{-\iota\phi} & [-(\alpha p_x^2-\delta)+\iota v p_y] & -(\mu+\epsilon) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{bmatrix} = 0 \tag{3.8}$$

for superconducting region eq(3.8) gives us two relations
for electronic state

$$\mu + \sqrt{\epsilon^2 - \Delta^2} = \sqrt{(\alpha p_x^2 - \delta)^2 + v^2 p_y^2} \tag{3.9}$$

for hole state

$$-\mu + \sqrt{\epsilon^2 - \Delta^2} = \sqrt{(\alpha p_x^2 - \delta)^2 + v^2 p_y^2} \tag{3.10}$$

after solving wavefunctions for left movers are given as,

$$\Psi_e = \frac{1}{2} \begin{bmatrix} e^{\iota\beta} \\ -e^{\iota\beta} e^{-\iota\gamma} \\ e^{-\iota\phi} \\ -e^{-\iota\phi} e^{-\iota\gamma} \end{bmatrix} e^{\iota(p_x x - p_y y)} \tag{3.11}$$

$$\Psi_h = \frac{1}{2} \begin{bmatrix} e^{-\iota\beta} \\ e^{-\iota\beta} e^{\iota\gamma} \\ e^{-\iota\phi} \\ e^{-\iota\phi} e^{\iota\gamma} \end{bmatrix} e^{\iota(p_x x - p_y y)} \tag{3.12}$$

$$e^{\iota\beta} = \frac{\epsilon - \sqrt{\epsilon^2 - \Delta^2}}{\Delta} \tag{3.13}$$

$$e^{-\iota\gamma} = \frac{(\alpha p_x^2 - \delta) - \iota v p_y}{\sqrt{(\alpha p_x^2 - \delta)^2 + v^2 p_y^2}} \tag{3.14}$$

if $\epsilon < \Delta$

$$\beta = arc\cos[\frac{\epsilon}{\Delta}] \tag{3.15}$$

if $\epsilon > \Delta$

$$\beta = -\iota arc\cosh[\frac{\epsilon}{\Delta}] \tag{3.16}$$

in normal region for electronic state

$$\begin{bmatrix} (\mu-\epsilon) & (\alpha k_x^2-\delta)-\iota v k_y \\ (\alpha k_x^2-\delta)+\iota v k_y & (\mu-\epsilon) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0 \tag{3.17}$$

energy dispersion relation for electronic state

$$\epsilon = \mu \pm \sqrt{(\alpha k_x^2-\delta)^2+v^2k_y^2} \tag{3.18}$$

in normal region for hole state

$$\begin{bmatrix} -(\mu+\epsilon) & -[(\alpha k_x^2-\delta)-\iota v k_y] \\ -[(\alpha k_x^2-\delta)+\iota v k_y] & -(\mu+\epsilon) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \tag{3.19}$$

energy dispersion relation for hole state,

$$\epsilon = -\mu \pm \sqrt{(\alpha k_x^2-\delta)^2+v^2k_y^2} \tag{3.20}$$

so wave functions in normal region are

$$\Psi_e = \frac{1}{\sqrt{\cos\Omega}} \begin{bmatrix} e^{-\iota\Omega/2} \\ e^{\iota\Omega/2} \\ 0 \\ 0 \end{bmatrix} e^{\iota(k_x x+k_y y)} \tag{3.21}$$

$$\Psi_h = \frac{1}{\sqrt{\cos\Omega'}} \begin{bmatrix} 0 \\ 0 \\ e^{\iota\Omega'/2} \\ e^{-\iota\Omega'/2} \end{bmatrix} e^{\iota(k_x x-k_y y)} \tag{3.22}$$

$$\Omega = \arcsin[\frac{v k_y}{\epsilon+\mu}] \tag{3.23}$$

$$\Omega' = \arcsin[\frac{v k_y}{\epsilon-\mu}] \tag{3.24}$$

The energy dispersion relation derived above is plotted as

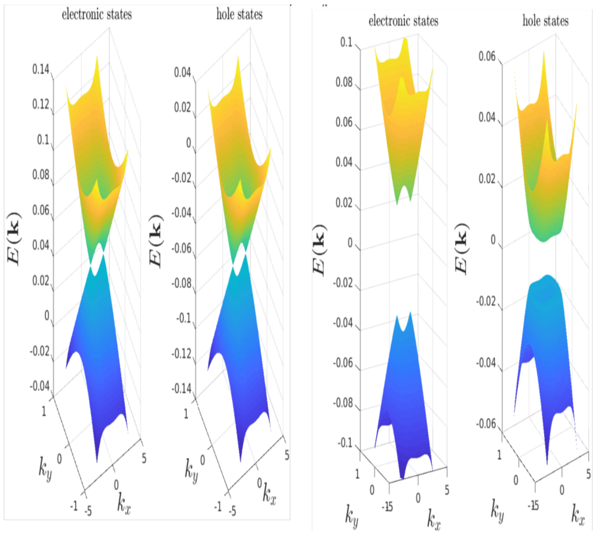

Figure 3.3: Dispersion in the normal region (Left) and in the superconducting region (right)

### 3.0.2 Reflection coefficients

Applying boundary continuity condition at the N-S junction (referring to Fig. [??] at $y = 0$ ) the reflection coefficients for an incident electron is obtained by the following relation

$$\psi_e^- + r\psi_e^+ + r_a\psi_h^+ = a\psi_s^+ + b\psi_s^- \tag{3.25}$$

The reflection coefficients for an incident hole at $y = 0$ are obtained by the following relation

$$\psi_h^- + r'\psi_h^+ + r'_a\psi_e^+ = a'\psi_s^+ + b'\psi_s^- \tag{3.26}$$

$$r = -\frac{\cos\beta\sin\frac{(\Omega'+\Omega)}{2} + \iota\sin\beta\sin\frac{(\Omega'-\Omega)}{2}}{\iota\cos\beta\cos\frac{(\Omega'-\Omega)}{2} + \cos\frac{(\Omega'+\Omega)}{2}\sin\beta} \tag{3.27}$$

$$r_a = \frac{e^{-\iota\phi}\sqrt{\cos\Omega\cos\Omega'}}{\cos\beta\cos\frac{(\Omega'-\Omega)}{2} - \iota\cos\frac{(\Omega'+\Omega)}{2}\sin\beta} \tag{3.28}$$

$$r' = -\frac{\cos\beta\sin\frac{(\Omega'+\Omega)}{2} - \iota\sin\beta\sin\frac{(\Omega'-\Omega)}{2}}{\iota\cos\beta\cos\frac{(\Omega'-\Omega)}{2} + \cos\frac{(\Omega'+\Omega)}{2}\sin\beta} \tag{3.29}$$

$$r'_a = \frac{e^{\iota\phi}\sqrt{\cos\Omega\cos\Omega'}}{\cos\beta\cos\frac{(\Omega'-\Omega)}{2} - \iota\cos\frac{(\Omega'+\Omega)}{2}\sin\beta} \tag{3.30}$$

Case:1 $\mu << \epsilon$

$$\Omega = \Omega' \text{ (Retro reflection dominates)} \tag{3.31}$$

$$r = \frac{\iota \frac{\epsilon}{\Delta} \sin\Omega}{\frac{\epsilon}{\Delta} - \zeta\cos\Omega} \tag{3.32}$$

$$r_a = \frac{e^{-\iota\phi}\cos\Omega}{\frac{\epsilon}{\Delta} - \zeta\cos\Omega} \tag{3.33}$$

$$r' = \frac{\iota \frac{\epsilon}{\Delta} \sin\Omega}{\frac{\epsilon}{\Delta} - \zeta\cos\Omega} \tag{3.34}$$

$$r'_a = \frac{e^{\iota\phi}\cos\Omega}{\frac{\epsilon}{\Delta} - \zeta\cos\Omega} \tag{3.35}$$

Case:2 $\mu >> \epsilon$ for

$$\Omega = -\Omega' \text{ (Specular reflection dominates)} \tag{3.36}$$

$$r = \frac{-\iota\zeta\sin\Omega}{\frac{\epsilon}{\Delta}\cos\Omega - \zeta} \tag{3.37}$$

$$r' = \frac{\iota\zeta\sin\Omega}{\frac{\epsilon}{\Delta}\cos\Omega - \zeta} \tag{3.38}$$

$$r_a = \frac{e^{-\iota\phi}\cos\Omega}{\frac{\epsilon}{\Delta}\cos\Omega - \zeta} \tag{3.39}$$

$$r'_a = \frac{e^{\iota\phi}\cos\Omega}{\frac{\epsilon}{\Delta}\cos\Omega - \zeta} \tag{3.40}$$

where

$$\zeta = \iota\sqrt{1 - \frac{\epsilon^2}{\Delta^2}} \tag{3.41}$$

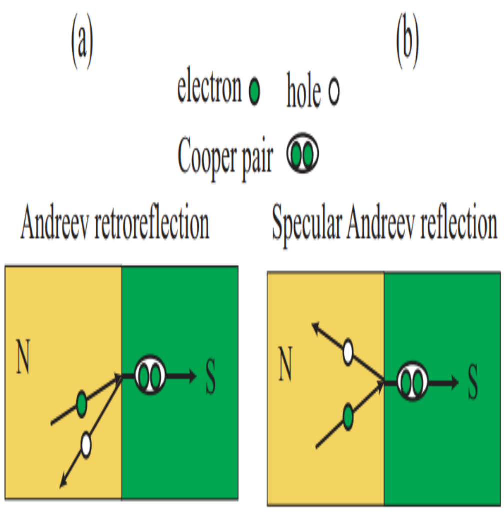


Figure 3.4: Schematic diagram (a) Andreev retroreflection (b) specular Andreev reflection

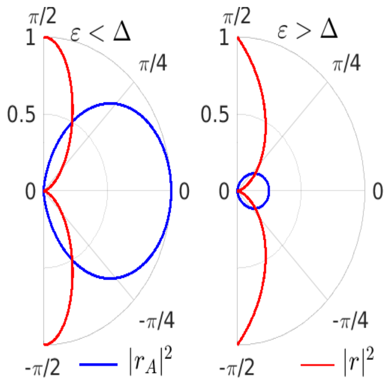

Figure 3.5: Polar Plots for Normal reflection and Andreev reflection for two cases Case(1):$\epsilon < \Delta$ Case(2):$\epsilon > \Delta$

The transmission probability to the superconducting side can be written as $|t|^2 = 1 - |r|^2 + |r_A|^2$. This concept is useful in calculating the differential conductance in the next section.

### 3.0.3 Differential Conductance

Differential conductance refers to the rate of change of electrical conductance with respect to a change in voltage or current. Here we calculate the differential Conductance of the interface b/w normal and superconducting junction for the case of small and large Fermi wavelength $\lambda_\mu$ in the normal region relative to the coherence length $\xi = \frac{\hbar v}{\Delta}$ in the superconductor.
The differential conductance of N-S junction using Blonder-Tinkham-Klapwijk formula is

$$\frac{\partial I}{\partial V} = g_0(V) \int_0^{\pi/2} (1 - |r|^2 + |r_A|^2) \cos\Omega \, d\Omega \tag{3.42}$$

where $g_0(V) = \frac{4e^2}{h}\rho(eV)$ and $\rho(eV)$ is the density if states at the given biasing $V$. The calculation of the density of states is based on the equation below

$$\rho(\epsilon) = \frac{1}{(2\pi)^2} \sum_{\lambda=-1}^{1} \iint \delta(\epsilon + \mu - \lambda\sqrt{(\alpha k_x^2 - \delta 1)^2 + v^2 k_y^2}) d^2k \tag{3.43}$$

We observe the following

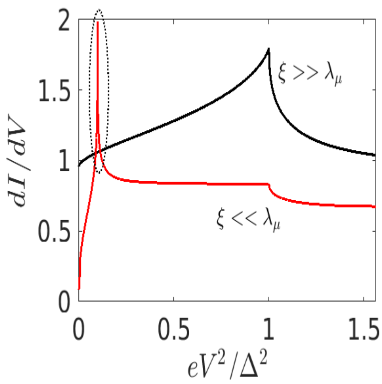


Figure 3.6: Plot of Differential conductance normalized by the Ballistic value $g_0(V)$

- The electron-hole conversion is predominantly retro reflection for $\lambda_\mu << \xi$ (red curve)

- The electron hole conversion is predominantly specular reflection for $\lambda_\mu >> \xi$ (black curve)

# Chapter 4

# Summary

In this thesis, we have taken the effective $1 - D$ system of N-S junction using B-T-K model that enable us to find the amplitude of the Andreev reflection in such a system as a benchmark. We carried out the same study using semi-Dirac materials which display a quadratic band dispersion along a particular direction and a linear dispersion in a direction normal to the former. We find that for the case of the ideal interface ($Z = 0$) if the energy of an incident electron is such that $E < \Delta$ (where $\Delta$ is the superconducting gap), the probability of reflection of an electron as a hole is 1. But as this ratio $\frac{E}{\Delta}$ become greater than 1 then the probability of transmission of an electron as an electron is increasing and the probability of reflection of an electron as a hole is decreasing since there are no allowed Cooper pair states at such energy. We also studied the differential conductance of such a setup and it shows an anomaly giving a very large conductance at a certain energy. It must be mentioned that the singularity of the differential conductance at $eV = \Delta$ also appears (as expected).

The above conclusion was made when we chose the transport direction along the direction where the dispersion is linear. The same study can be done when choosing a transport direction along a direction where the dispersion is quadratic. We do not expect a large conductance (that was observed when the transport direction is along the former) when the transport direction is along the latter. This is the future plan to be carried out in the next few months.

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