FOURIER-BESSEL DOMAIN BASED BAND-LIMITED ENTROPIES FOR AUTOMATED DETECTION OF HUMAN EMOTIONS

M.Tech. Thesis

By

BETHAPUDI SHIRLY SUSAN

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A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **FOURIER-BESSEL DOMAIN BASED BAND-LIMITED ENTROPIES FOR AUTOMATED DETECTION OF HUMAN EMOTIONS** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DISCIPLINE OF ELECTRICAL ENGINEERING, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from August 2021 to June 2023 under the supervision of Prof. Ram Bilas Pachori.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

B.S.Susan 04-06-2023

Signature of the student with date
(BETHAPUDI SHIRLY SUSAN)

This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

09.06.2023

Signature of the Supervisor of M.Tech. thesis #1 (with date)
(PROF. RAM BILAS PACHORI)

Signature of the Supervisor of M.Tech. thesis #2 (with date)
(NAME OF SUPERVISOR)

BETHAPUDI SHIRLY SUSAN has successfully given his/her M.Tech. Oral Examination held on **May 12th, 2023**.

09.06.2023
Signature(s) of Supervisor(s) of M.Tech. thesis
Date:

Convener, DPGC
Date: 12/06/2023

Signature of PSPC Member #1
Date: 15/6/2023

Signature of PSPC Member #1
Date: 12/06/2023

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ABSTRACT

In various aspects of our daily lives, emotions play an important role in behavior, decision making, cognitive learning, perception and rational thinking. Therefore, analyzing emotions is key to understand human nature. Emotions can be recorded using facial expressions, galvanic skin response, speech signals, electroencephalogram (EEG) signals, etc., In this work, we use EEG signals for emotion detection for its advantages over the rest of the approaches especially in terms of signals getting changed when the person tries to hide his/her emotion. An EEG based dataset is used to validate the proposed approach. The emotions are evoked by showing the subjects videos which stimulate happy, sad and neutral emotions. We propose a Fourier-Bessel (FB) domain based band limited entropies for classifying emotions. We introduce a novel concept of band-limited entropies which is computed for each sub band without decomposing the signal completely. Once the entropies are obtained, they are used as features for classification. A few machine learning classifiers such as support vector machine (SVM), k-nearest neighbor (KNN) and their variants are used to perform classification. It is observed that fine KNN classifier has performed well with an accuracy of 92.3% with less computational complexity.

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CHAPTER 1

INTRODUCTION

In human daily life, emotions play a prominent role because they influence our routine in activities such as human cognition, perception, decision making, behavior, human intelligence etc [1]. Emotional intelligence is an integral part of human intelligence. Emotion research encompasses research in various fields like neuroscience, cognitive science, psychology, etc. A few mental disorders such as depression, autism, game-addiction, etc are linked directly to human emotions [4]. In this regard, affective computing (AC) has become an emerging research area, that focusses primarily on detection of human emotions and their modeling using several machine learning methods. The motive of AC is to narrow down the gap between a human and computer by developing computationally efficient systems so that they respond to human emotions by identifying them [48]. It can build artificial intelligence that humans are aware of. It is capable of recognizing, comprehending and controlling emotions.

In general, emotion recognition systems follow two approaches for recognizing emotions. They are, explicit and implicit approaches [3]. Explicit approach involves capturing emotions from external appearances such as facial expressions, speech signal, gestures, etc [3]. But these approaches are not very reliable because the participant or subject can conceal his/her emotion. They may not show the actual state of emotion of the subject. Implicit approach is a more reliable approach that captures emotions from different physiological signals such as electroencephalogram (EEG), galvanic skin response (GSR), electrocardiogram (ECG), etc. [3]. Such signals are generated by the autonomous nervous system (ANS) inside the body. They are not visible [49].

The brain is regarded as the central command center of the body. It fires neurons whenever it reacts to any external stimulus. This in turn causes changes in the autonomous nervous system (ANS) activity [3]. As a result, the activity of several organs varies. The emotions also get affected in addition to this. EEG signals are considered as the biopotential that reflects brain activities very effectively[3]. Thus, EEG signals are considered for this study.

1.1 Advantages of EEG signals:

1. They have low signal-to-noise ratio (SNR) and are usually noisy when captured. So, several preprocessing steps are performed to remove the effect of noise and artifacts.
2. EEG based human recognition gives high accuracy and objective evaluation.
3. They provide good time resolution which can be used effectively to measure the changes in the characteristics of the signal due to external stimuli.
4. In addition to that, EEG signals are non-invasive, inexpensive and fast providing the best means to record and analyze human emotions.

But unlike speech signals or two-dimensional signals like images, EEG signals are non-stationary and have temporal asymmetry. So, analyzing them is a challenging task. Therefore, extraction of features should be carried out using sophisticated signal processing approaches [64-69]. Emotion classification is performed after carrying out feature extraction using machine learning approaches.

In the recent days, the signals that are being recorded and analyzed frequently in the fields of neuroscience, biomedical engineering, etc., are multi-variate in nature. In order to analyze these signals, multivariate versions of several techniques are introduced in literature. EEG signals are recorded from an EEG cap which has multiple channels. So, it is multi-variate in nature. Thus, in this study, the multi-variate Fourier-Bessel series expansion based empirical wavelet transform (FBSE-EWT) is used to obtain different the signal spectrum at different scales [51-58].

Also, entropies have become increasingly important parameters to determine the randomness in a dynamically varying signals like physiological signals, etc., EEG signals are one of those physiological signals and are highly dynamic in nature. Therefore, in this work, three different entropies namely Shannon-spectral entropy, Wiener entropy, Log energy entropy and their combinations are computed to measure the uncertainty in these signals which are further used as features for classifying human emotions as happy, neutral and sad emotions using several machine learning classifiers such as support vector machine (SVM), k-nearest neighbour (KNN), and their variants.

1.2 Organization of thesis:

The remaining part of the thesis is organized as follows:

Chapter 2: This chapter explores the recent studies and the past work done in the field of emotion recognition using EEG signals in brief. A new method is proposed in this study which emphasizes the use of FBSE-EWT for defining band-limited entropies that are used for automated human emotion recognition using EEG signals.

Chapter 3: This chapter provides the motivation for employing Fourier-Bessel series expansion (FBSE) in the methodology. It explains the classification of signals followed by the limitations of Fourier transform especially for non-stationary signals like EEG signals. It then describes the mathematics of Fourier-Bessel series expansion followed by its advantages over Fourier transform.

Chapter 4: Chapter 4 explains the essence of the study which is FBSE-EWT in detail. It begins with the introduction of several time-frequency analysis methods used for non-stationary signal analysis like short-time Fourier transform (STFT), wavelet transform (WT), Wigner-Ville distribution (WVD), empirical mode decomposition (EMD), empirical wavelet transform (EWT) followed by each of their limitations. It then explains the disadvantages of the conventional empirical wavelet transform which are overcome FBSE-EWT. At last, the algorithm for the same is discussed in detail.

Chapter 5: This chapter explains the basics of entropy followed by the motivation to use entropies as features for human emotion classification. The concept of band-limited entropies is introduced. In addition, the advantages over full-scale entropies is discussed. Three different entropies namely, Shannon-spectral entropy, Wiener entropy, log energy entropy are discussed in detail.

Chapter 6: This chapter explains several machine learning classifiers that are used for human emotion recognition along with their specifications in detail. A few classifiers such as SVM, K-NN, and their variants are discussed in detail.

Chapter 7: This chapter provides insights on the proposed methodology. The EEG dataset that is used for the validation of the method along with its preprocessing is described in detail. In addition, the preprocessing involved is explained.

Chapter 8: The results obtained after validating the proposed method with the dataset are shown and discussed in this chapter. The performance analysis parameter, accuracy is computed to analyze the performance of the algorithm for different classifiers.

Chapter 9: The study concludes with this chapter focusing on the inferences drawn from the proposed method along with the future aspects of the same followed by references.

CHAPTER 2

LITERATURE REVIEW

In the literature, several methods based on the signal processing based EEG signal features along with classifiers were mentioned for automated human emotion classification [5]-[18].

In [5], discrete wavelet transform with multi- resolution analysis was employed on the preprocessed EEG signals for extracting features. Fuzzy c-mean clustering and fuzzy k-mean clustering methods are used for the classification of emotions. Both 63-channel and 24-channel EEG data were used to validate the method. A five level decomposition was done using db4 wavelet. The fourth level detailed coefficients are used as features. The results showed the possibility of discrete wavelet transform in the emotion detection. Fuzzy k-means clustering method performed well on 24-channels data than 63-channels data whereas fuzzy c-means performed well in classifying both 63-channel and 24-channel data. Hence, the wavelet transform based emotion detection was successful.

In [6] authors used EEG signals along with gaze distance and pupillary response for emotion recognition. The modality fusion strategy with SVM classifier were used. A one-participant out cross validation was performed to analyze the performance of classifiers.

In [7], emotions were classified into happy, fear, sad, and relax classes using flexible analytic wavelet transform (FAWT). The emotion specific information was extracted using FAWT. It was used to decompose the EEG signal into different sub bands. A few statistical measures that serve as features are computed from the sub bands for the extraction of emotion-specific information. Variants of K-NN were used for classification. Of all, weighted-KNN provided best accuracy for emotion classification.

In [8], the two class emotion recognition was performed using EEG signals. A method to search for frequency band was proposed to find an optimal band for filtering. Two different emotions namely, smiling and crying are classified. The authors used common spatial patterns (CSP) along with linear SVM classifier for the purpose of classification.

In [9], the authors propose a classification algorithm from low-amplitude EEG signals that are produced by remembering a very unpleasant odor. The signals were trained by a suitable classifier after finding features using wavelet analysis. Principal component analysis (PCA) is performed for reducing the dimensionality of the feature followed by classification using SVM. It was observed that right hemisphere of the brain is predominant in revealing the stimulus.

In [10], the authors explored time-frequency analysis techniques for discriminating musical appraisal EEG responses. Features are extracted from beta and gamma EEG bands in windows of various lengths using spectrogram, Hilbert- Huang spectrum and Zhao-Atlas-Marks transform and different time-frequency (TF) representations are obtained. Classification is performed using K-NN and SVM classifiers to classify feature vectors into like and dislike categories

In [11], emotions are represented on two dimensions, valence and arousal dimensions. The fast Fourier transform (FFT) was employed on EEG signals for feature extraction. Feature selection based on Pearson correlation coefficient was performed. The authors proposed a probabilistic classifier based on Baye’s theorem using a weighted-log-posterior function and a supervised algorithm using perceptron convergence algorithm for emotion recognition. An open dataset, DEAP was used to verify the proposed method. It was observed that the Bayesian approach was robust to the learning rate parameter. An emotion is defined as high and low level classes in valence dimension and as high, medium and low level classes in dominance dimension. The accuracy values obtained in valence dimension are higher than that of dominance dimension.

The authors proposed differential entropy as a feature for studying the characteristics of emotion EEG signals in [12]. It is used in combination with its symmetrical electrodes namely, differential and rational asymmetries. The performance is compared with that of energy spectrum. It was observed that differential entropy has given high accuracy that the rest of the parameters. They also concluded that emotional signals correspond to gamma frequency band. Linear dynamical system (LDS) along with the feature selection method, and minimal-redundancy maximum-relevance (MRMR) were used for improving the performance of the method.

In [13], Wei-Long and Bao-Liang proposed a deep belief networks (DBN) based investigation of critical frequency bands and critical channels of a 62-channel EEG cap for improving the emotion recognition accuracies for positive, negative and neutral emotions. SEED dataset was introduced. It was observed that the profile of 12 channels given by, FT7, FT8, T7, T8, C5, C6, TP7, TP8, CP5, CP6, P7 and P8 gives good mean accuracies and standard deviation among different group of electrodes. The performance was observed to be better than that of original full 62 channels.

In [14], the authors proposed a new deep learning framework for emotion classification. Since EEG signals have their common characteristics of spatial-temporal volumes, spatial-temporal recurrent neural network (STRNN) was introduced to combine into a spatial-temporal dependency model the learnings of two different signal sources. A multi-directional recurrent neural network (RNN) layer is employed in STRNN in order to capture long range contextual cues by moving along the spatial region from multiple angles. A bi-directional temporal RNN layer was further employed to learn discriminative temporal dependencies. The model discriminant ability was increased by the sparse projection onto the hidden states of both spatial and temporal domains. Public EEG emotion datasets and facial expressions were used to validate the method. It was proved that this method is more competitive.

The researchers in [15] explored the capability of different EEG features to detect cross-subject emotions. Eighteen kinds of both linear and non-linear EEG features were used. Two publicly available emotion EEG datasets namely, DEAP and SEED were used to verify the method. SVM classifier was adopted and leave-one-subject-out strategy was employed to evaluate performance. The authors have explored the significance of different EEG features for cross-subject emotion recognition from different perspectives. A pilot correlation analysis was performed to examine the highly correlated features.

In [16], the authors proposed a technique to recognize cross-subject emotions from EEG signals accurately using spatial correlation and time-series analysis. The spatial connectivity between the brain regions was represented by a channel-wise feature that handles correlation between different channels. Pearson correlation coefficient was computed to handle subject-specific variability between two-pair. A two-layered stack long short-term memory (LSTM) was used to extract time domain features to learn an emotional model. SEED and DEAP dataset demonstrated the effectiveness of the combination of both channel-wise features and LSTM. The method achieved good accuracies for both two-class and three-class emotion classification.

In a study [17], FAWT was used to decompose an EEG signal into its sub band signals in order to analyze cross-subject emotion recognition. The information potential (IP) was employed on the decomposed sub-band signals to extract features. Feature smoothing was further performed and they are fed to Random forest classifier and SVM classifier which were further used for emotion classification. This method has yielded channel specific subject classification when exposed to the same stimuli.

The researchers in [18] used advanced properties of empirical mode decomposition and multivariate empirical mode decomposition (MEMD) were employed for feature extraction to process multi-channel EEG signals for emotion detection. The multichannel intrinsic mode functions (IMFs) are extracted and analyzed using various parameters such as power ratio, power spectral density, correlation and Hjorth parameters. They were used as features of valence and arousal scales of subject. DEAP EEG dataset is used to verify the method. The features fed to an artificial neural network (ANN) for emotion recognition.

In this work, we propose multi-variate FBSE-EWT followed by band-limited entropy computations for emotion classification. Fourier-Bessel domain based band-limited entropies are computed. Feature smoothing is performed using moving average filter. An EEG emotion dataset is taken to validate the proposed method. The emotion classification is performed by using different machine learning classifiers.

CHAPTER 3

INTRODUCTION TO FOURIER-BESSEL SERIES EXPANSION

3.1 Types of signals:

Signals are broadly classified into two types, stationary and non-stationary signals. Stationary signals are those whose frequency or any of its spectral components doesn't vary with time. All periodic signals are stationary in nature. Non-stationary signals are those whose frequency or its spectral components vary with time. All real world signals and a few mathematical signals like chirp signals are the examples of non-stationary signals.

3.2 Signal representation using signal processing:

In the recent days, signal processing has become most widely used for signal analysis and representation. In order to represent any stationary signal in frequency domain, several transforms are used as mathematical tools. Transform converts signal from one domain to the another. Fourier transform is the most widely used transform to analyze and represent signals in frequency domain.

3.2.1 Fourier transform:

Fourier transform uses complex exponentials which are stationary in nature as basis functions to obtain spectrum. For any continuous-time signal $x(t)$, the Fourier transform $X(\omega)$ is defined as follows [21]:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

The above equation is the analysis equation. The synthesis equation is given by [21],

$$x(t) = \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

3.2.1.1 Shortcomings of Fourier transform:

The following are the shortcomings of Fourier transform:

1. It provides best performance only for stationary signals. For non-stationary signals, Fourier transform may produce undesired spurious harmonic components.
2. For periodic signals, Fourier transform uses window function to derive the spectrum of the signal.
3. It uses complex exponentials which are stationary in nature as basis functions [21].
4. It represents real signals in terms of complex exponentials.
5. It represents real signals in terms of both positive and negative frequencies [21].
6. The basis functions are periodic in nature and do not converge.
7. They cannot represent any modulations in the signal.
8. The number of frequency points obtained is equal to half of the length of the signal. As a result, the resolution obtained is very less [19].
9. Fourier spectrum doesn't provide any information about the time at which the frequency component is present.

3.2.2 Fourier-Bessel series expansion:

All the shortcomings of Fourier transform paved a way to analyze non-stationary signal using Fourier-Bessel series expansion. All the real world signals are in general non-stationary in nature. In order to analyze non-stationary signal, the mathematical tool should use some non-stationary function as basis. Fourier-Bessel analysis uses Bessel functions that are non-stationary in nature as bases.

3.2.2.1 Advantages of FBSE over FT:

The following are the advantages of FT:.

1. FBSE uses Bessel functions which are non-stationary in nature as basis making it suitable to represent all non-stationary signals.
2. Bessel functions are aperiodic in nature and converging [21].
3. FBSE doesn't require a window function in order to obtain the spectrum of the signal.
4. It provides more compact representation than FT [19].

5. Any real signal can be represented in terms of real Bessel basis functions using FBSE.
6. It provides representation of real signals in terms of only positive frequencies unlike FT.
7. The basis functions include amplitude modulation in the representation.
8. The spectrum obtained using FBSE has the number of frequency points equal to the length of the signal. As a result, the resolution obtained twice as that of Fourier transform [19].

3.2.2.2 Mathematical background:

Bessel functions are the solutions of the linear second order differential equation given by [21-24],

$$x^2\ddot{y} + x\dot{y} + (x^2 - v)y = 0$$

The solution to the above equation is given by [21][32],

$$y = AJ_v(x) + BY_v(x)$$

where A and B are the arbitrary constants, $J_v(x)$ and $Y_v(x)$ are the Bessel functions of first kind (order v) and second kind (order v) respectively. The Bessel functions of first kind and order v follows orthogonality principle [21][42-43].

$$\int_a^b xJ_v(\alpha x)J_v(\beta x)dx = 0$$

where α and β are the roots of $J_v(x) = 0$ [21]. Bessel functions of first kind and of orders zero and one were found to be very useful in representing a non-stationary signal.

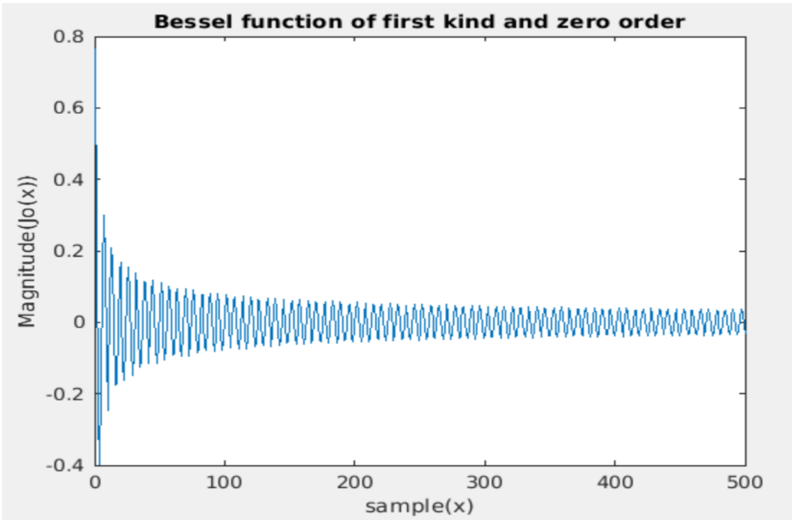

Fig 3.1a. MATLAB simulated Bessel function of first kind (zero order)

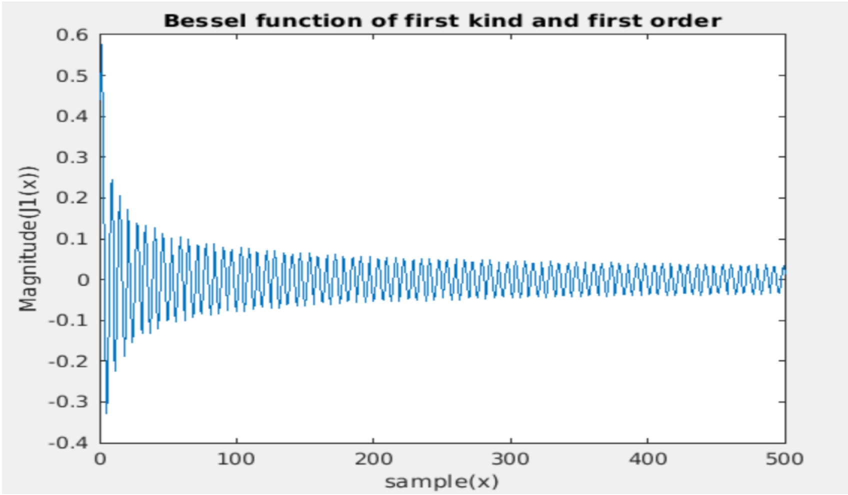

Fig 3.1b. MATLAB simulated Bessel function of first kind (first order)

The FBSE of a discrete time signal $y(n)$ using zero-order Bessel functions is expressed by its synthesis function given by [19][45-47]:

$$y(n) = \sum_{i=1}^{U} C_i J_0(\frac{\beta_i n}{U}), \quad n = 0,1, \dots, U-1$$

where, C_i are known as Fourier-Bessel (FB) series coefficients of $y(n)$. They are given by [19],

$$C_i = \frac{2}{U^2 (J_1(\beta_i))^2} \sum_{n=0}^{U-1} n y(n) J_0\left(\frac{\beta_i n}{U}\right), \ i = 1,2, \dots, U$$

where, $J_0(.)$ and $J_1(.)$ denote zero and first-order Bessel functions, respectively and U is the discrete-time signal length. The above equation is called analysis equation. Similarly, the FBSE expansion of $y(n)$ using first-order Bessel function is given by its synthesis equation [23],

$$y(n) = \sum_{i=1}^{U} D_i J_I\left(\frac{l_i n}{U}\right), \quad n = 0,1, \dots, U-1$$

where, D_i are known as first order Fourier-Bessel series coefficients of $y(n)$. They are given by [23],

$$D_i = \frac{2}{U^2 (J_0(l_i))^2} \sum_{n=0}^{U-1} n y(n) J_1\left(\frac{l_i n}{U}\right), \ i = 1,2, \dots, U$$

This equation is called analysis equation. β_i and l_i are positive roots of zero order and first order . Bessel functions respectively with $i = 1,2, \dots, U$ in ascending order.

The roots of a Bessel function $J_0(a) = 0$, 'a' are determined using Newton-Raphson's method which is derived using Taylor's series approach [21]. Based on Taylor's series, the roots β_i are approximated as [19][47]

$$\beta_i \approx \beta_{i-1} + \pi \approx i\pi$$

The order i of FBSE coefficients and its relevant continuous-time frequencies f_i (in Hz) are related to each other as [19][47-60]

$$\beta_i = \frac{2\pi f_i U}{f_s}$$

$$\text{or} \quad i \approx \frac{2 f_i U}{f_s}$$

where f_s is the sampling frequency. The order of FBSE coefficients can be converted into its corresponding frequency using this expression [19]. When FBSE coefficients are plotted, they are obtained in Fourier-Bessel domain which is with respect to the order. The spectrum of the signal is obtained by plotting FBSE coefficients with respect to frequency.

CHAPTER 4

FOURIER-BESSEL SERIES EXPANSION BASED EMPIRICAL WAVELET TRANSFORM

It is understood from section 3.2 that Fourier transform cannot be used to analyze any non-stationary signal. It introduces spurious harmonic frequencies which do not provide compact representation of the signal. Several non-stationary signal analysis methods were introduced in the literature.

4.1 Time-frequency analysis techniques:

Time-frequency analysis methods are the most widely used for non-stationary signal analysis because of their ability to provide analysis of the signal both in time and frequency domains simultaneously. Several time-frequency techniques exist in the literature. They are classified based on the type of basis functions used i.e., pre-fixed basis and adaptive basis [20].

1. Pre-fixed basis:
   - Short time Fourier transform (STFT)
   - Wavelet Transform (WT)
2. Adaptive basis:
   - Wigner-Ville distribution (WVD)
   - Empirical mode decomposition (EMD)
   - Empirical wavelet transform (EWT), etc.

4.1.1 Short-time Fourier transform:

As the name suggests, the main idea of STFT is to slice a non-stationary signal into different segments (with possible overlaps) for better time localization. Slicing is nothing but windowing a signal with a finite width window function, $w(t)$ i.e., [20]

$$x(\tau, t) = x(t).w(t-\tau)$$

At the beginning, τ is at the start of the signal. For more dense representation, τ increments itself by 1 to traverse along the length of the signal. STFT of a signal $x(t)$ is given by [20],

$$X(\tau,\xi) = \int_{-\infty}^{\infty} x(t).w(t-\tau)e^{-j\xi t}dt$$

where ξ is the center frequency in the frequency domain and τ is the mean time in time domain. The perfect recovery of the signal is possible with STFT. The reconstructed signal, $x(t)$ is given by [20],

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(\tau,\xi)e^{j\xi t}d\xi d\tau$$

STFT can also be viewed as Fourier transform of $x(t)$ with clipped or amplitude modulated sinusoids. The window function must have compact support with unit energy to preserve the energy of the signal [20]. The length of the window should be much lesser than the length of the signal. A spectrogram, in general, is defined as the squared magnitude of STFT. It gives energy decomposition of the signal. The maxima of the spectrogram gives a good estimate of instantaneous frequency [20].

4.1.1.1 Limitations of STFT:

1. Once a window function and its length are fixed, it cannot be changed.
2. As a result, there is no good energy localization in both time and frequency domains due to duration-bandwidth principle.

4.1.2 Wavelet transform:

Wavelet transform is used to overcome the limitations of STFT. In WT, wavelets are used as basis functions.

The wavelets are associated with a scale parameter which can be used to adjust duration and bandwidth of the wavelet [20]. If the scale is large, wide window is obtained. As a result, poor time resolution and good frequency resolution are obtained. On the other hand, if the scale is small, narrow window is obtained [20]. As a result, good time domain localization and poor frequency domain localization are obtained. Translation parameter is used to traverse along the entire signal.

Wavelet transform is classified as follows:

1. Continuous wavelet transform (CWT)
2. Discrete wavelet transform (DWT)

4.1.2.1 Continuous wavelet transform:

In CWT, the scale and translation parameters take continuous values but the signal to be analyzed is a continuous signal. CWT for an input signal $x(t)$ is given by [20],

$$T_x(\tau, s) = \int_{-\infty}^{\infty} x(t)\Psi^*_{\tau,s}(t)dt$$

where τ is the translation parameter, s is the scaling parameter and $\Psi_{\tau,s}(t)$ is the daughter wavelet specified as [20],

$$\Psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\Psi^*_{\tau,s}\left(\frac{t-\tau}{s}\right)dt$$

4.1.2.2 Discrete Wavelet transform (DWT):

CWT has a highly redundant representation i.e., it produces may more coefficients than necessary to represent the signal. DWT gives a compact representation. DWT is nothing but CWT evaluated at discrete values of scale and translation parameters given by [20],

$$s = {a_0}^m \text{ and } \tau = nb_o{a_0}^m$$

It is clear that the translation parameter is proportional to the scaling parameter. So, for a large value of s, τ takes large steps to traverse along the signal and similarly for the smaller values of s. It is given by [20],

$$T(m,n) = \int_{-\infty}^{\infty} x(t)\Psi^*_{m,n}(t)dt$$

where,

$$\Psi_{m,n}(t) = \frac{1}{\sqrt{{a_0}^m}}\Psi^*_{\tau,s}\left(\frac{t-nb_o{a_0}^m}{{a_0}^m}\right) = {a_0}^{-\frac{m}{2}}\Psi({a_0}^{-m}t - nb_o) \text{ and } a_o,\ b_o \epsilon\, R^+,\ \ m,\ n\ \epsilon\, Z.$$

4.1.2.3 Limitations of WT:

1. The wavelet approach can be described as an adjustable window Fourier spectral analysis which uses wavelet as window.
2. It is non-adaptive. Once the mother wavelet is chosen, it must be used to analyze all the data.

4.1.3 Wigner-Ville distribution:

Wigner-Ville distribution of a signal $x(t)$ is nothing but Fourier transform of the local auto correlation function. It is defined as [20],

$$W_{x,x}(\tau,\ \xi) = \int_{-\infty}^{\infty} x^*\left(\tau - \frac{t}{2}\right) x\left(\tau + \frac{t}{2}\right) e^{-jt\xi} dt$$

4.1.3.1 Physical Interpretation:

For real signals, we look at the left and right of the signal over the same duration at that particular time instant and then compute the extent of overlap between both the segments. This is done for all possible segments. Since there is no windowing, WVD has excellent time localization. WVD uses adaptive basis as the basis depends on one of the signal terms. Interference is observed when any two signals are added exactly at the midway of the frequencies. This is because of the cross terms obtained after applying WVD. In order to reduce the cross-terms, usually WVD is computed on the analytic form of the input signal. Signal recovery is not possible from WVD. This is because WVD represents the energy distribution form which it is not possible to recover the signal unless there is information about the phase.

4.1.3.2 Drawbacks of WVD:

1. Non-local nature: WVD gives equal importance both to nearby and far off past and future samples.
2. Interference: For multi-component signals, interference is obtained due to cross-terms.
3. Windowing (smoothing) gives more localization by giving more weightage to the nearby samples but at the expense of poor time-frequency localization.

4.1.4 Empirical mode decomposition:

Empirical Mode Decomposition (EMD) method is an adaptive and data dependent method. It doesn't require any condition about linearity of the signal. It decomposes a non-stationary and non-linear signal into finite set of narrow band signals called Intrinsic mode functions (IMFs) [20]. IMFs are obtained in the decreasing order of frequencies. The first IMF contains the finest scale or the shortest period component whereas the last IMF contains the longest period component. Usually, the first IMF will have the higher frequency which is dictated by the time lapse between the successive extrema of the original signal. The first IMF is derived directly from the extrema of the original signal. Hence, it contains the high frequency components. This principle is applied for the rest of the IMFs as well. The algorithm for the sifting process is defined as follows [20]:

1. Identify local extrema for any given non-stationary signal $x(t)$
2. Separately connect all extrema with the help of cubic spline lines to form upper envelope and lower envelope, $u(t)$ and $l(t)$ respectively.
3. Find mean as, $m(t) = [u(t) + l(t)]/2$.
4. Obtain the mean subtracted data from the signal data, $h_1(t) = x(t) - m(t)$.
5. Check if $h_1(t)$ is satisfying the conditions of IMF.
6. If not, repeat the same procedure on $h_1(t)$ till it satisfies the definitions of being an IMF.
7. If yes, then $h_1(t)$ is considered as the first IMF, c_1 i.e., $c_1 = h_1(t)$.
8. The residue, $r_1 = x(t) - c_1$ is obtained by separating c_1 from the rest of the data
9. This procedure is repeated on all the subsequent r_js, and the result is

$$r_1 - c_2 = r_2 - c_3, \ldots, r_{n-1} - c_n = r_n$$

To guarantee that the IMF components contain significantly enough information of amplitude modulation as well as frequency modulations, sifting process should stop after certain point. This is accomplished by limiting the standard deviation, SD that is computed from the two consecutive decomposition results as [20]

$$SD = \sum_{t=0}^{T} \left[\frac{\left| h_{1(k-1)}(t) - h_{1k}(t) \right|^2}{h_{1(k-1)}^2(t)} \right]$$

4.1.4.1 Drawbacks of EMD:

1. Empirical mode decomposition is an algorithmic approach
2. The problem of mode-mixing is observed for some signals in its IMFs.

4.1.5 Empirical wavelet transform:

Empirical wavelet transform is an adaptive signal decomposition method. Based on the frequency information content in the spectrum It extracts narrow band frequency components from the analyzed signal based on the frequency information content in the spectrum. The conventional EWT finds boundary frequencies and then decomposes signals using adaptive wavelet filters in the Fourier transform based spectrum [35].

4.1.5.1 Drawbacks of conventional EWT:

1. The closely spaced frequency components can be properly represented in the time-frequency plane [19].
2. Also, the difficulty in estimating the frequency components accurately for the short duration signals is attributed to the use of Fourier transform.

4.1.5.2 Fourier-Bessel series expansion based empirical wavelet transform:

The Fourier-Bessel series expansion based empirical wavelet transform (FBSE-EWT) uses FBSE to compute spectrum. It is an improvisation of conventional EWT which was applied earlier in the literature for the analysis of biomedical signals. The following are the steps to compute FBSE-EWT [19].

Step 1: Computation of multi-variate FBSE spectrum

For any signal $x(t)$, FBSE spectrum is computed by applying FBSE to the input signal as described in section 3.2.2.1. For a given multi-variate signal given by, $x(t) = [x_1(t), x_2(t), x_3(t), \dots, x_M(t)]$, the mean FBSE spectrum is defined as follows [2],

$$K(\Omega) = \frac{1}{M}\sum_{m=1}^{M}|C_i|$$

where M is the number of channels, $|C_i|$ is the FBSE spectrum of individual channels and Ω is the frequency of the signal. The magnitude spectrum obtained is considered for the further steps.

Step 2: Scale-space based boundary detection

The Fourier-Bessel spectrum is segmented into N number of contiguous segments and the optimal set of boundary frequencies ω_i using scale-space based representation[33]. The frequencies of the first and the last boundaries are set to 0 and π respectively[19][33]. A total of $N-1$ intermediate boundary frequencies are to be detected. The scale-space representation of any discrete time signal $p(n)$ is obtained by convolving the signal with the Gaussian kernel which is given by [19],

$$\Upsilon(m,s) = \sum_{n=-M}^{M} p(m-n)q(n;s), \quad q(n;s) = \frac{1}{\sqrt{2\pi s}} e^{\frac{-n^2}{2s}}$$

where $M = B\sqrt{s} + 1$ with $3 \le B \le 6$ and s is known as scale parameter. As the scale-space parameter, $\epsilon = \sqrt{\frac{s}{s_o}}$, $\epsilon = 1, 2, 3, \ldots, \epsilon_{max}$, increases, the decrease in the number of minima is observed, and no other minima will show up in the scale space plane [19]. The FBSE spectrum generated in the above step is segmented using this boundary detection method. It varies from 0 to π. The segments obtained are denoted by, $[0, \omega_1], [\omega_1, \omega_2], [\omega_2, \omega_3], \ldots, [\omega_{i-1}, \pi]$ [19]. Typically, the boundaries are defined between any two local minima, that are obtained by the two curves whose length is greater than the threshold obtained using otsu's method[34].

Step 3: Generation of modes

The empirical scaling function and wavelet functions are defined in every segment as aset of band-pass filters. Littlewood–Paley and Meyer's wavelets are used for the construction of wavelet based filters. The mathematical expressions of empirical scaling function and wavelet functions are defined as [19][28-31],

Scaling function:

$$\Lambda_i(\omega) = \begin{cases} 1, & if\ |\omega| \le (1-\xi)\omega_i \\ \cos\left(\dfrac{\pi\eta(\xi,\omega_i)}{2}\right), & if\ \ (1-\xi)\omega_i \le |\omega| \le (1+\xi)\omega_i \\ 0, & otherwise \end{cases}$$

Wavelet function:

$$\Theta_i(\omega) = \begin{cases} 1, & if\ (1+\xi)\omega_i \le |\omega| \le (1-\xi)\omega_{i+1} \\ \cos\left(\frac{\pi\eta(\xi,\omega_{i+1})}{2}\right), & if\ (1-\xi)\omega_{i+1} \le |\omega| \le (1+\xi)\omega_{i+1} \\ \sin\left(\frac{\pi\eta(\xi,\omega_i)}{2}\right), & if\ (1-\xi)\omega_i \le |\omega| \le (1+\xi)\omega_{i+1} \\ 0, & otherwise \end{cases}$$

where the function $\eta(\xi, \omega_i)$ is given by [19],

$$\eta(\xi, \omega_i) = \psi\left(\frac{(|\omega| - (1-\xi)\omega_i)}{2\xi\omega_i}\right)$$

where $\psi(z)$ is an arbitrary function given by [19],

$$\psi(z) = \begin{cases} 0, & if\ z\ \ge 0 \\ \psi(z) + \psi(1-z) = 1, & \forall\ z \epsilon\ [0,1] \\ 1, & if\ z \ge 1 \end{cases}$$

The parameter ξ ensures that the empirical wavelets and scaling function are in the tight frame of $L_2(R)$. The expression for tight frame is given by [19],

$$\xi < \min\left(\frac{\omega_{i+1} - \omega_i}{\omega_{i+1} + \omega_i}\right)$$

The approximation coefficients and detailed coefficients are determined by the inner product of the analyzed signal with scaling and wavelet functions. The reconstructed sub-band signals can be obtained by convolving approximation and detailed coefficients with scaling and wavelet functions respectively [19][36-41].

$$f_0(t) = V_{y,\Lambda}(0,t) * \Lambda_1(t),$$

$$f_i(t) = V_{y,\Theta}(i,t) * \Theta_1(t)$$

where $f_0(t)$ is the approximation sub-band signal and $f_i(t)$ denotes detailed sub-band signal of i^{th} level. Hilbert transform is applied to obtain time-frequency representation. Instantaneous frequency and amplitude are obtained from the Hilbert spectrum[19][59].

Chapter 5

BAND-LIMITED ENTROPIES

5.1 Advantages of using band-limited entropies:

Entropies are the most widely used parameters used for understanding and quantifying the information associated with any dynamically changing phenomenon. Physiological signals like speech signals, EEG signals, ECG signals, etc., are dynamically varying signals. Therefore, in order to analyze these signals, entropies are considered. Entropy is the uncertainty present in the information [36][44]. There is a lot of computational complexity involved in finding entropies for all the sub-bands obtained after decomposing the signal. Instead, entropies can be computed right away after obtaining boundaries using multi-variate FBSE-EWT based boundary detection method. These entropies obtained for each segment just after identifying boundaries are called 'band-limited entropies'. Band-limited entropies serve as best features for human emotion identification.

5.2 Shannon-spectrum entropy:

The uniformity in the distribution of the energy of a signal, in general, can be measured using spectral entropy. Entropy derived from Shannon's expression is used for analysis. It gives the information about the spectral properties of the signal. The Shannon-spectral entropy (SHE) is defined as follows [61],

$$SHE = -\sum_{n=i_j}^{i_{j+1}} q(n) log_2(q(n))$$

where i_j and i_{j+1} are the boundaries for a particular order n and $p(n)$ is the energy distribution normalized over order n. It is similar to histogram. It is defined mathematically as [61],

$$q(n) = \frac{ES_n}{\sum_{n=1}^{k} ES_n}$$

where k is the length of the signal and ES_n is the energy spectrum corresponding to the Fourier-Bessel coefficient of order n. It is expressed as [61],

$$ES_n = \frac{{X_n}^2 k^2 [J_1(\xi_n)]^2}{2}$$

where X_n are the FB coefficients and $J_1(\xi_n)$ is the first order Bessel function of order n.

5.3 Wiener entropy:

Wiener entropy (WIE) is also considered to measure the flatness or unfiromity in the distribution of the spectral power of the signal. WE is also called “the measure of spectral flatness”. It is defined as the ratio of geometric mean and arithmetic mean of the energy spectrum (ES_n) of the Fourier-Bessel coefficient of order n. WIE is a unitless quantity. Wiener entropy is mathematically defined as [62],

$$WIE = m \frac{\sqrt[m]{\prod_{n=i_j}^{i_{j+1}} ES_n}}{\sum_{n=i_j}^{i_{j+1}} ES_n}$$

where $m = i_{j+1} - i_j + 1$ which is nothing but the length of the segment between the boundaries i_{j+1} and i_j. H_{WE} takes values between 0 and 1 where 0 indicates a pure tone and 1 indicates a uniform spectrum.

5.4 Log energy entropy:

Log energy entropy (LOE) is another variant to measure information. It is defined as the logarithm of normalized energy distribution in Fourier-Bessel domain. LOE is expressed mathematically as [63],

$$LOE = -\sum_{n=i_j}^{i_{j+1}} log_2(q(n))$$

where n is the order, i_j and i_{j+1} are the boundaries for order n.

CHAPTER 6

MACHINE LEARNING CLASSIFIERS

6.1 Support Vector Machine:

Support vector machine (SVM) comes under supervised machine learning algorithm that performs classification of data by learning from an already labelled data of various classes [4]. The principle on which SVM works is to find decision boundaries that can be obtained by constructing a hyperplane obtained by training the classifier with training data. So, it acts as a decision boundary [4]. The hyperplane separates the data into classes. The support vectors obtained determine the optimum location of the decision boundary. Support vectors are the points that are nearest to the obtained decision boundary. The location of the decision boundary is optimized iteratively by SVM so that it can maximize the margin. A margin is defined as the total separation between the two classes [4][21][25-26].

Several kernels are used to perform classification. Depending on the type of kernel use, SVM is classified as a linear or nonlinear classifier. In linear SVM, either flat plane, straight line or an N-dimensional hyperplane are used as they provide the simplest way of separation of data into two groups [4]. In case of nonlinear SVM, different kernels like polynomial, hyperbolic tangent curve, Gaussian radial function, etc., are used [4]. Sometimes, it is observed that boundary obtained by nonlinear kernels separates the data more efficiently. A hyperplane is defined by its mathematical expression given by [4],

$$f(x) = sign\left[\sum_{i=1}^{R} b_i f_i K(x, x_i)\right] + c$$

where R is the total number of observations, b_i is a positive real constant, c is a real constant, $K(x, x_i)$ is a kernel, x_i is the input vector and f_i is the output vector [4]. Linear SVM and median Gaussian SVM are employed in this work for classification.

6.2 K-nearest neighbour:

K-nearest neighbour (KNN) is also a supervised machine learning algorithm and it is non-parametric in nature [4]. KNN categorizes the data samples into different groups of data based on the distance from a few nearest neighbours. It can be used for both classification and regression.

In this work, median KNN and weighted KNN are used for classification. The following are the steps involved for classifying data using KNN [4].

Step 1: Find the distance between the current sample to the other sample using any distance metric like Mahalanobis distance, Euclidean distance or Minkowski distance.

Step 2: Now, rearrange the distance metric obtained in the above step in ascending order. The top k values indicate that the distance with the current sample is minimum.

Step 3: Now, depending on the maximum number of classes of nearest neighbors, class is assigned to the sample data.

CHAPTER 7

METHODOLOGY

7.1 Block diagram:

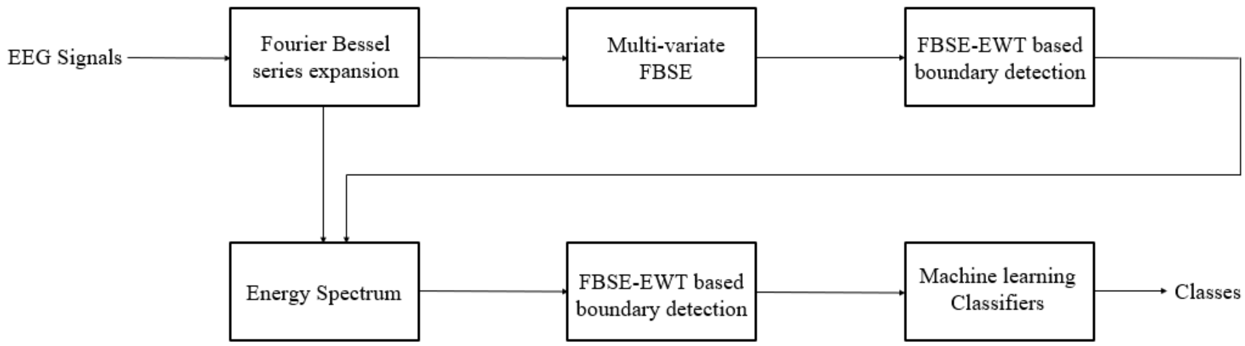

Fig 7.1 Block diagram of the proposed method

7.2 Dataset description:

The SEED EEG dataset is used for the validation of the proposed methodology. The SEED dataset includes EEG signals recorded from 7 males and 8 females, so a total of 15 subjects [13]. The EEG cap used is according to the international 10 - 20 system for 62 channels.

Fifteen film clips which stimulate positive, neutral and negative emotions were chosen as stimuli for the experiment. The duration of each film clip is approximately 4 minutes. There was a 5 seconds hint before each clip, 45 seconds for self-assessment and 15 seconds to rest after each video in one session [13].

There are a total of 15 trials in each experiment. The order of presentation of video clips is in such a way that two clips that target the same type of emotion are not shown to the subject consecutively. Each subject performed the experiment thrice with an interval of approximately 1 week. The data obtained is down sampled to 200 Hz [13]. A band pass filter of 0-75 Hz is applied to remove the interference caused due to power line noise [13]. Researchers investigated the critical frequency bands and critical channels for better emotion recognition. It was observed that the profile of 12 channels, FT7, FT8, T7, T8, C5, C6, TP7, TP8, CP5, CP6, P7 and P8 gives good mean accuracies and standard deviation among different pool of electrodes, even better than that of full 62 channels [13]. These 12 channels were considered for analysis on the basis of the results obtained in the previous study. The last 30 seconds of one second epoch length of all the 12 channels are considered. Each epoch has a length of 200 samples. Therefore, there are a total of 6000 samples that are considered for analysis.

7.3 Multi-variate FBSE:

The preprocessed dataset is now used to compute FB coefficients. The EEG data from the 12 channels specified in the section 7.2 are applied as input to FBSE which gives FB coefficients. For each epoch of a particular channel, 200 FB coefficients are obtained. So, the number of FB coefficients is same as the signal length. Now, since the EEG data is multi-variate in nature, a multivariate FBSE is evaluated from the obtained coefficients by taking the mean of the magnitude of all the FB coefficients as mentioned in section 4.1.3.2. This is used to find the boundaries of the FBSE spectrum using FBSE-EWT based boundary detection method.

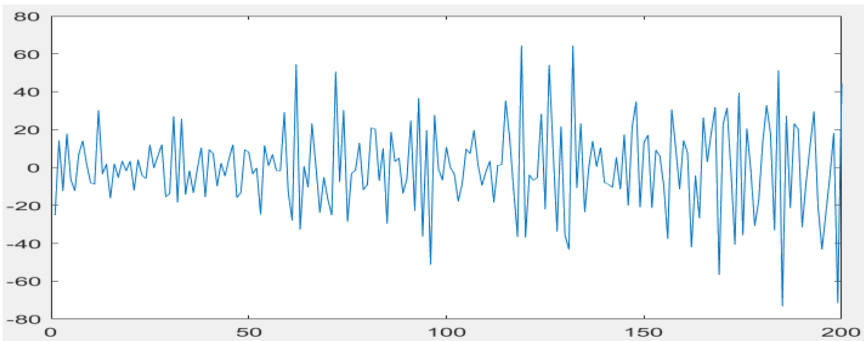

Fig 7.3.1a MATLAB simulated FB coefficients of EEG signal with label 1 (happy emotion)

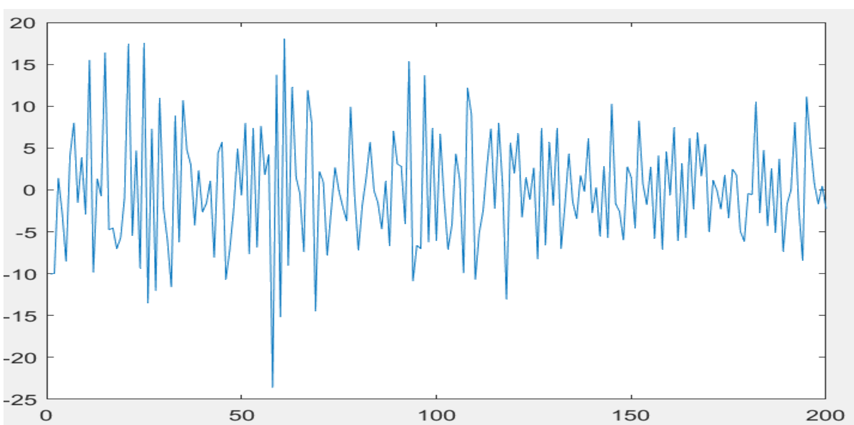

Fig 7.3.1b MATLAB simulated FB coefficients of EEG signal with label 0 (neutral emotion)

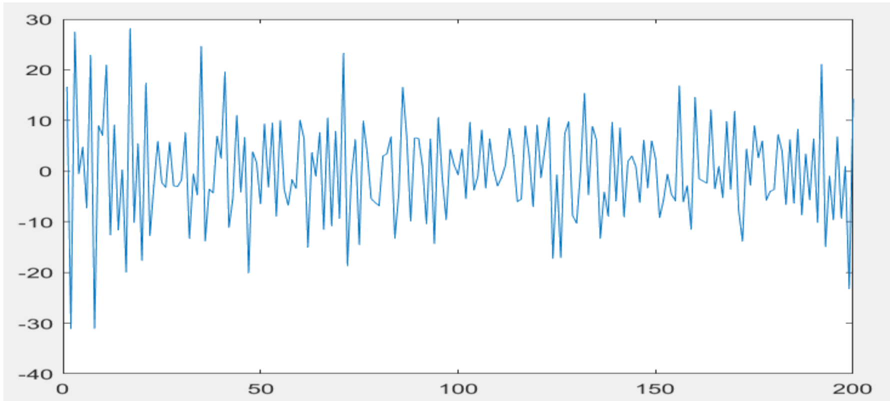

Fig 7.3.1c MATLAB simulated FB coefficients of EEG signal with label -1 (sad emotion)

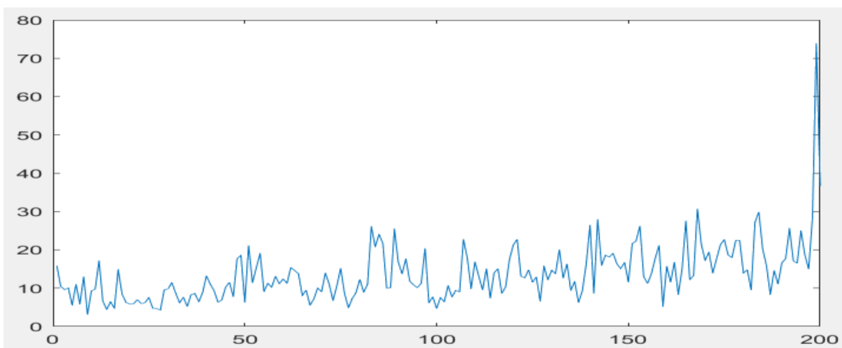

Fig 7.3.2a MATLAB simulated multi-variate FB coefficients of EEG signal with label 1 (happy emotion)

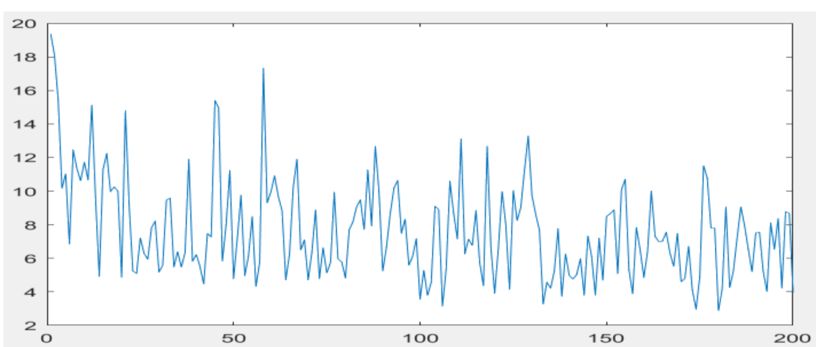

Fig 7.3.2b MATLAB simulated multi-variate FB coefficients of EEG signal with label 0 (neutral emotion)

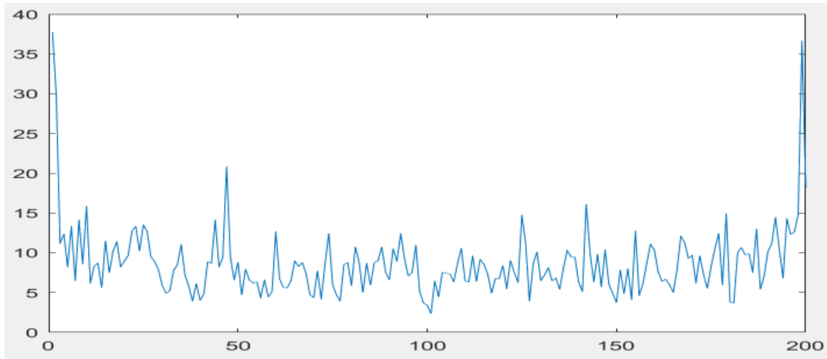

Fig 7.3.2c MATLAB simulated multi-variate FB coefficients of EEG signal with label -1 (sad emotion)

7.4 FBSE-EWT based boundary detection:

Once the multi-variate FB spectrum is obtained, we perform boundary detection using scale-space based boundary detection method described in section 4.1.3.2. The number of sub bands are set to 14 to maintain uniformity. The boundaries are obtained with respect to order.

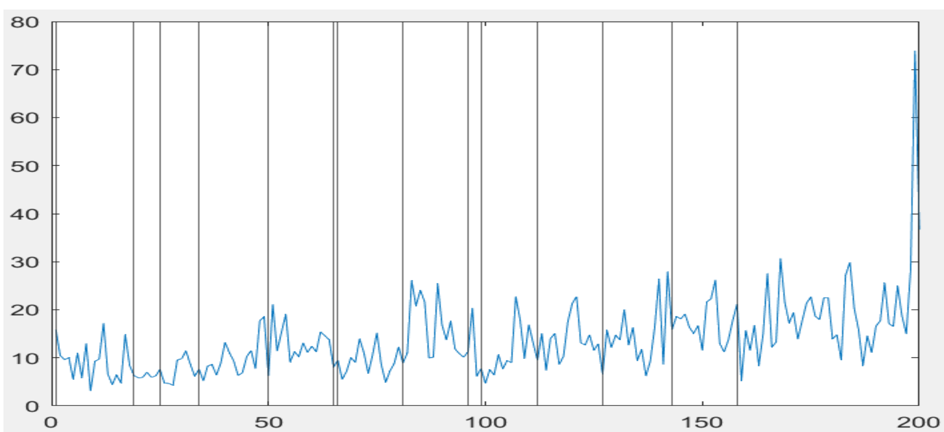

Fig 7.4a MATLAB simulated multi-variate FB coefficients and boundaries of EEG signal with label 1 (happy emotion)

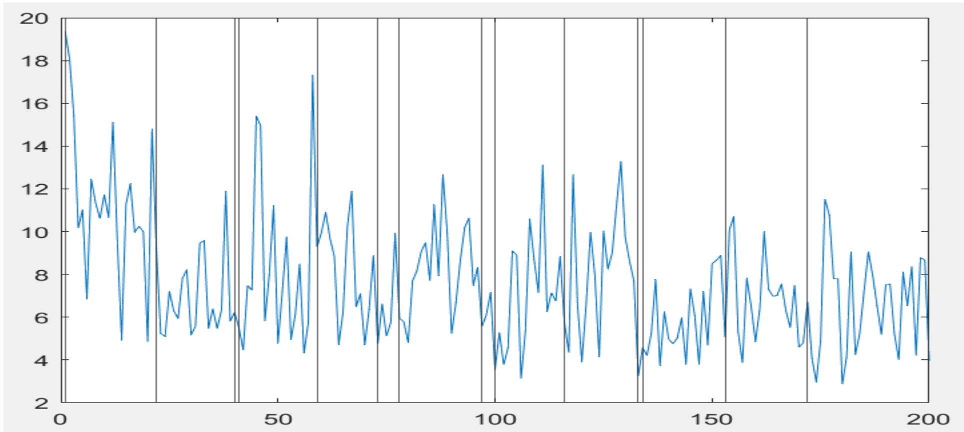

Fig 7.4b MATLAB simulated multi-variate Fourier-Bessel coefficients and boundaries of EEG signal with label 0 (neutral emotion)

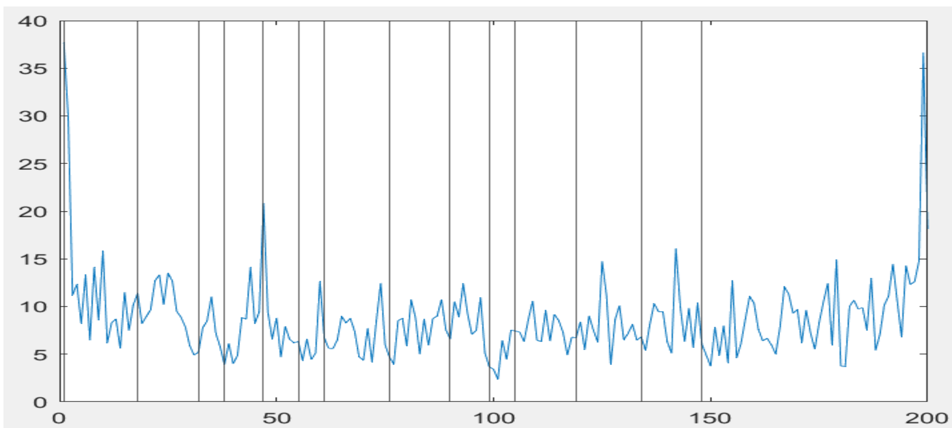

Fig 7.4c MATLAB simulated multi-variate Fourier-Bessel coefficients and boundaries of EEG signal with label -1 (sad emotion)

7.5 Energy spectrum:

Once the boundaries are obtained using multi-variate FBSE based EWT based boundary detection method, scale-space method, they are used along with the FB coefficients obtained in section 7.3 to compute energy spectrum as mentioned earlier in section 5.2. The length of the energy spectrum is equal to the signal length. Therefore, for one epoch, the length of energy spectrum is 200 samples. Now, the boundaries are identified on the energy spectrum.

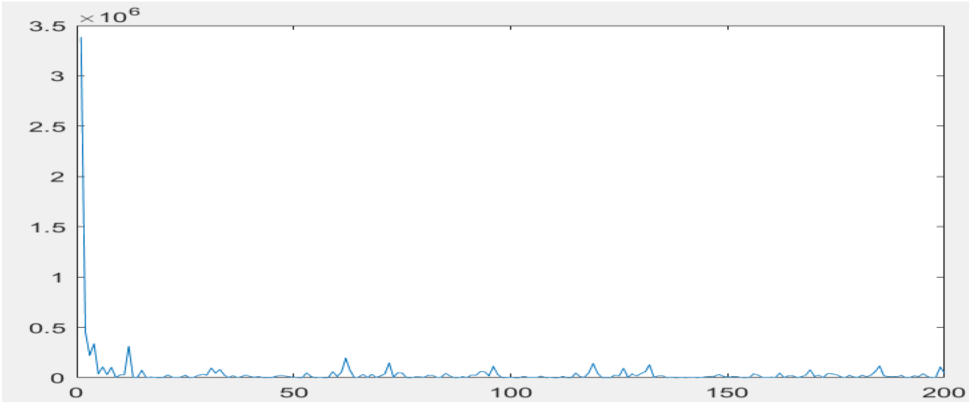

Fig 7.5a MATLAB simulated energy spectrum of EEG signal with label 1 (happy emotion)

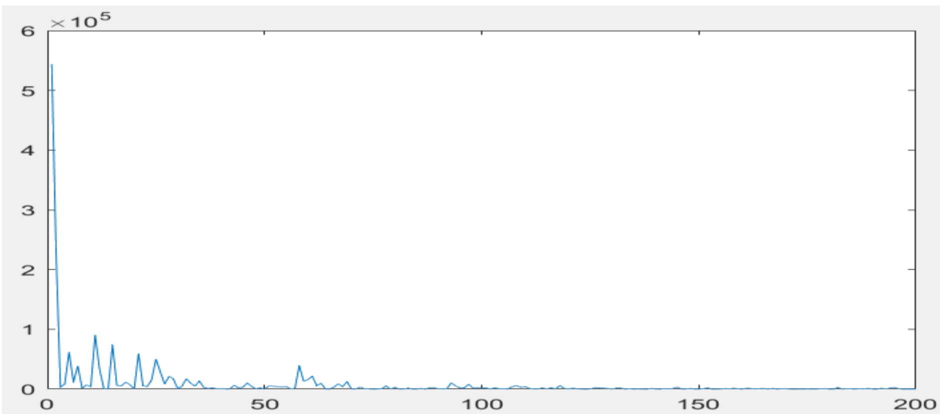

Fig 7.5b MATLAB simulated energy spectrum of EEG signal with label 0 (neutral emotion)

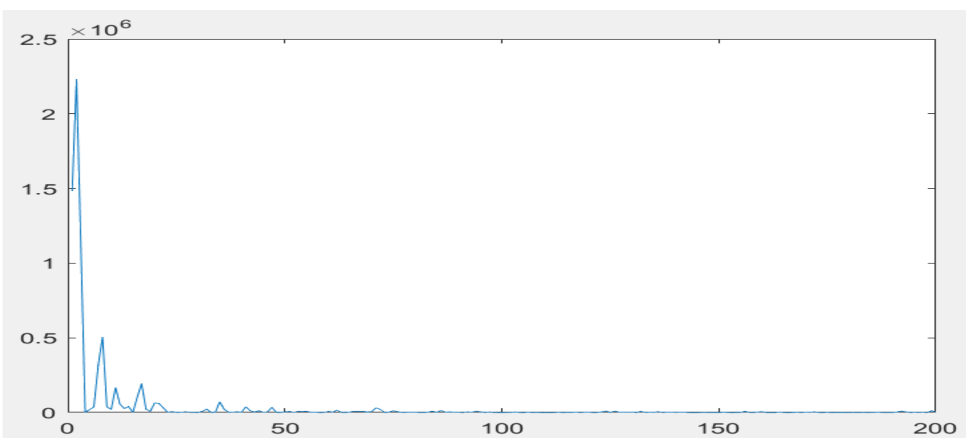

Fig 7.5c MATLAB simulated energy spectrum of EEG signal with label -1 (sad emotion)

7.6 Band-limited entropies:

Now the segmented energy spectrum is used to find band-limited entropies. Since, the number of sub bands are set to 14, the number of segments in the energy spectrum is also 14. Each segment of energy spectrum is extracted and then the entropies are computed. It was found that entropies are the best parameters to measure the uncertainty in the information of any dynamically varying signals like EEG signals. Therefore, the three entropies, Shannon-spectral entropy, Wiener entropy and log energy entropy are computed as described in section 5.2 for each of these segments separately.

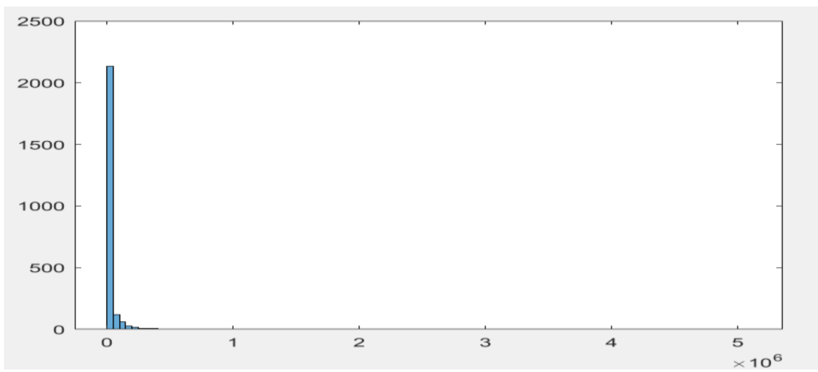

Fig 7.6.1a MATLAB simulated histogram of energy spectrum of EEG signal with label 1 (happy emotion)

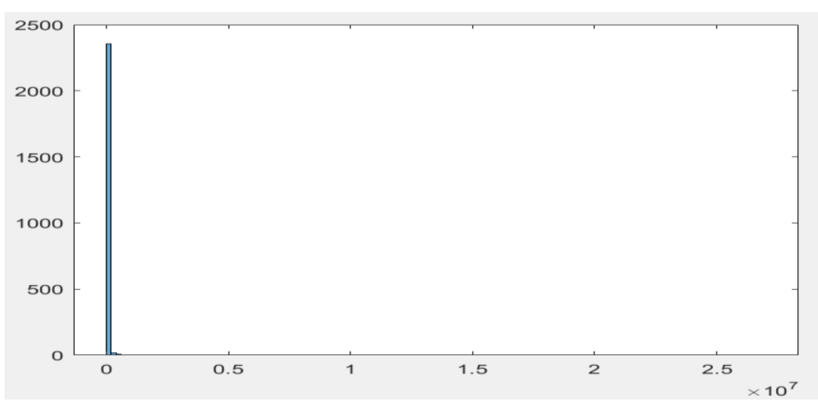

Fig 7.6.1b MATLAB simulated histogram of energy spectrum of EEG signal with label 0 (neutral emotion)

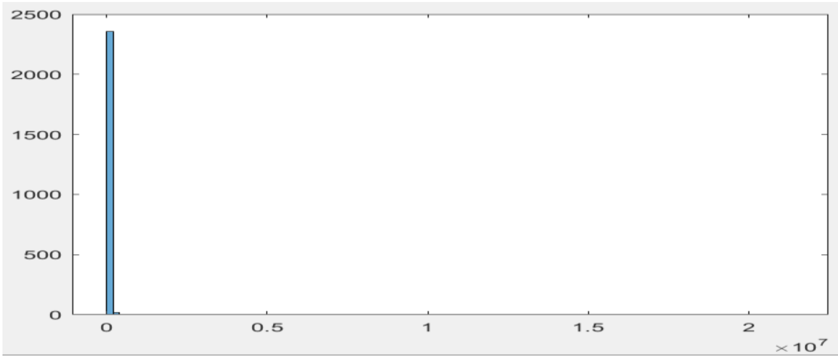

Fig 7.6.1c MATLAB simulated histogram of energy spectrum of EEG signal with label -1 (sad emotion)

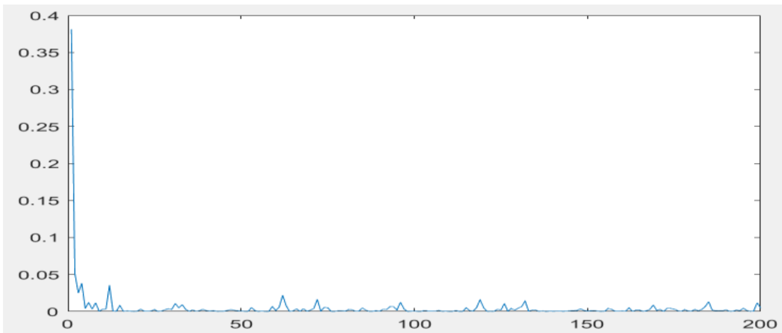

Fig 7.6.2a MATLAB simulated normalized energy distribution of EEG signal with label 1 (happy emotion)

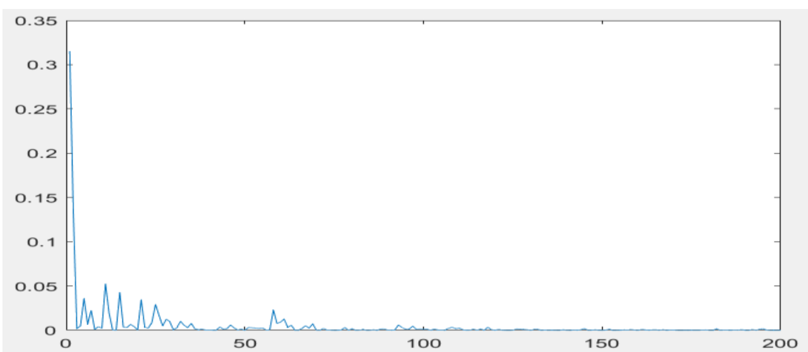

Fig 7.6.2b MATLAB simulated normalized energy distribution of EEG signal with label 0 (neutral emotion)

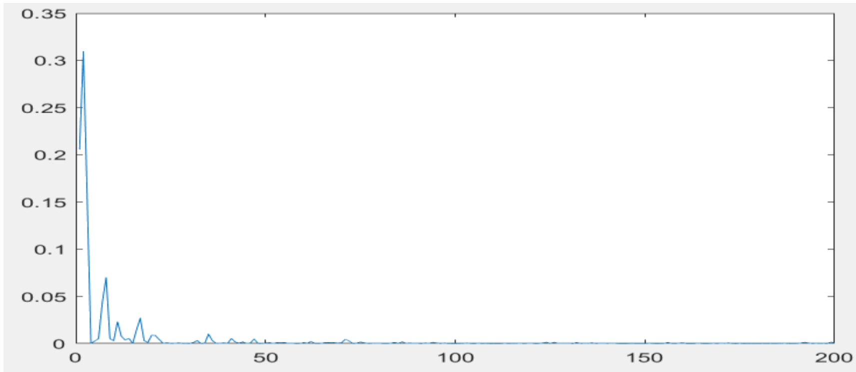

Fig 7.6.2c MATLAB simulated normalized energy distribution of EEG signal with label -1 (sad emotion)

Now, these band-limited entropies are used as features both combined and separately for human emotion classification. The feature vector size obtained using the three entropies separately is 20250*168 whereas the size of the combined feature vector is 20250*336.

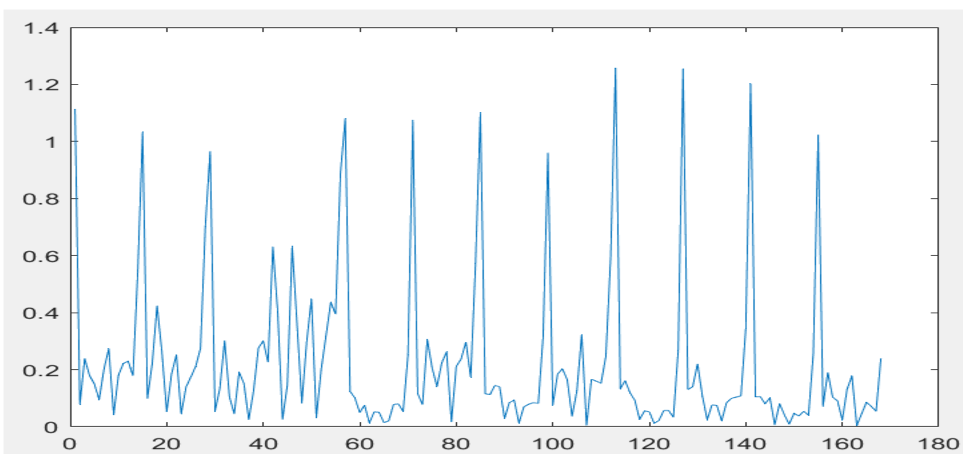

Fig 7.6.3a MATLAB simulated smoothened SHE of label 1(happy emotion)

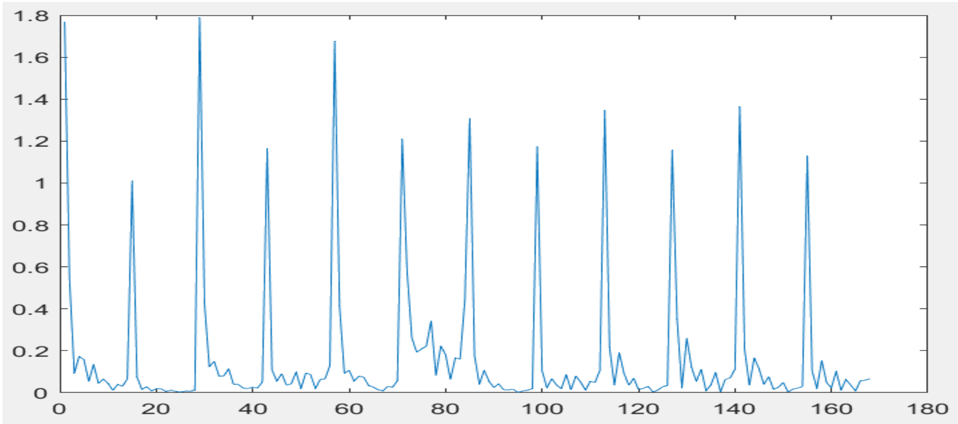

Fig 7.6.3b MATLAB simulated smoothened SHE entropy of label 0(neutral emotion)

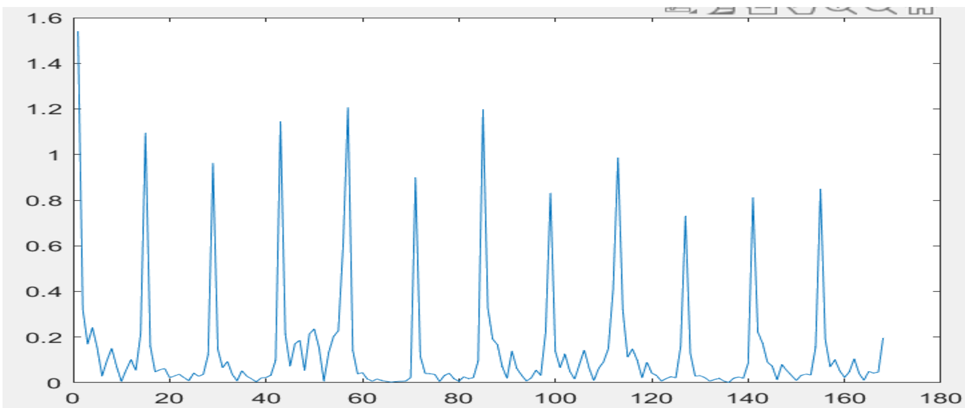

Fig 7.6.3c MATLAB simulated smoothened SHE entropy of label -1(sad emotion)

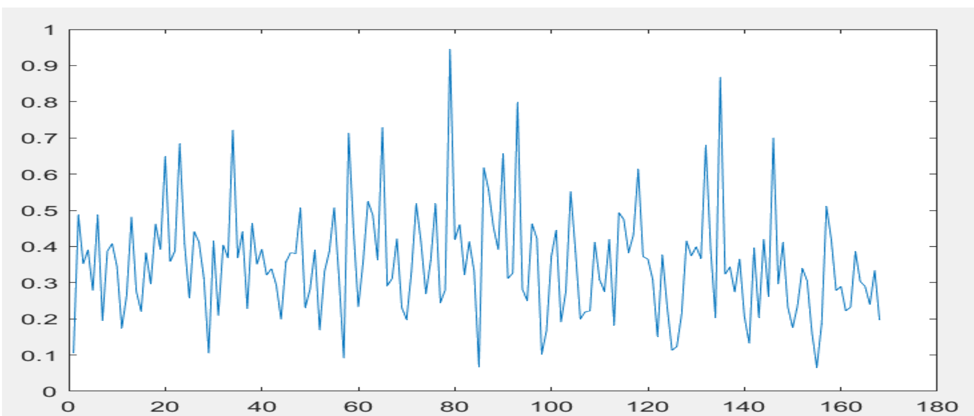

Fig 7.6.4a MATLAB simulated smoothened WE of label 1(happy emotion)

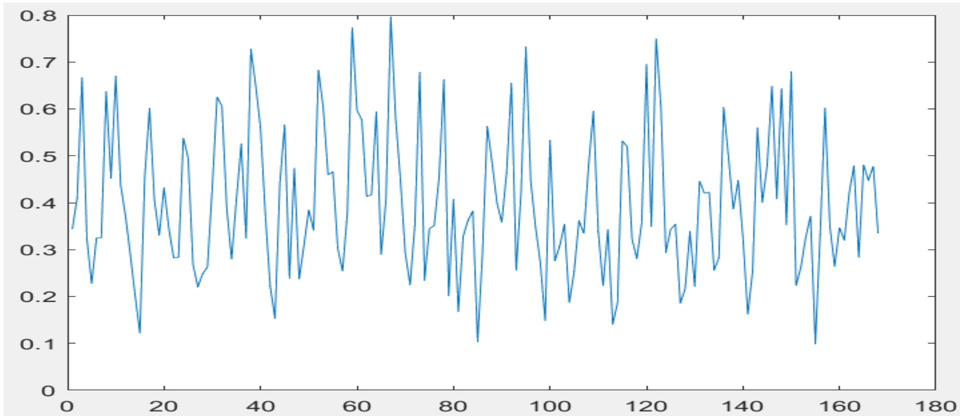

Fig 7.6.4b MATLAB simulated smoothened WE of label 0 (neutral emotion)

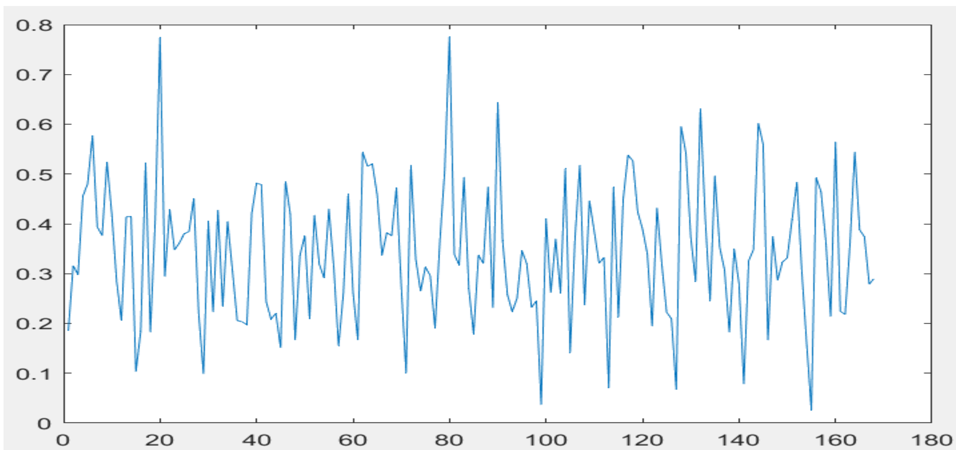

Fig 7.6.4c MATLAB simulated smoothened WE of label -1 (sad emotion)

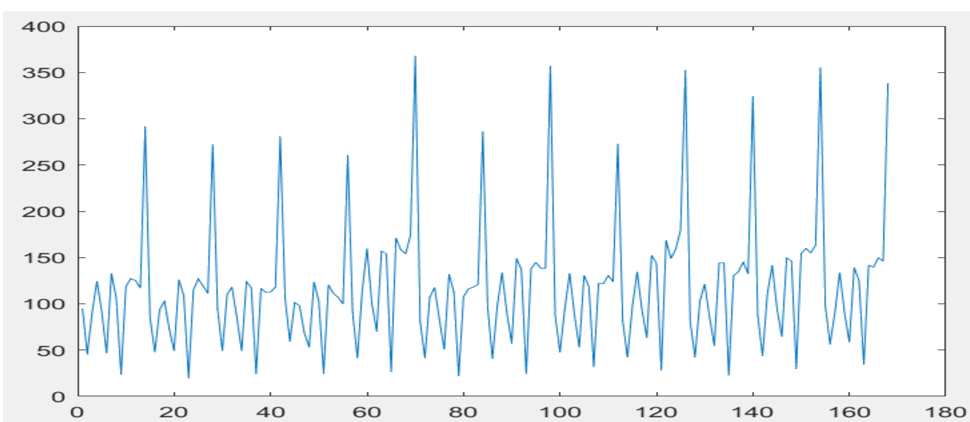

Fig 7.6.5a MATLAB simulated smoothened LEE of label 1(happy emotion)

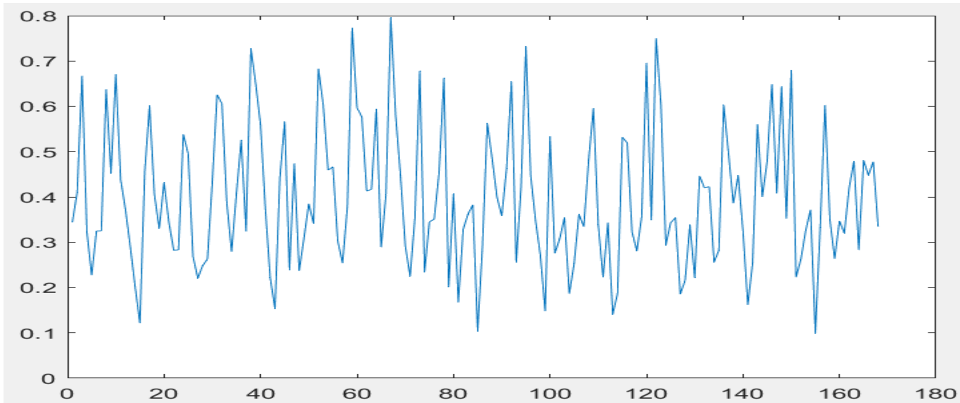

Fig 7.6.5b MATLAB simulated smoothened LEE of label 0 (neutral emotion)

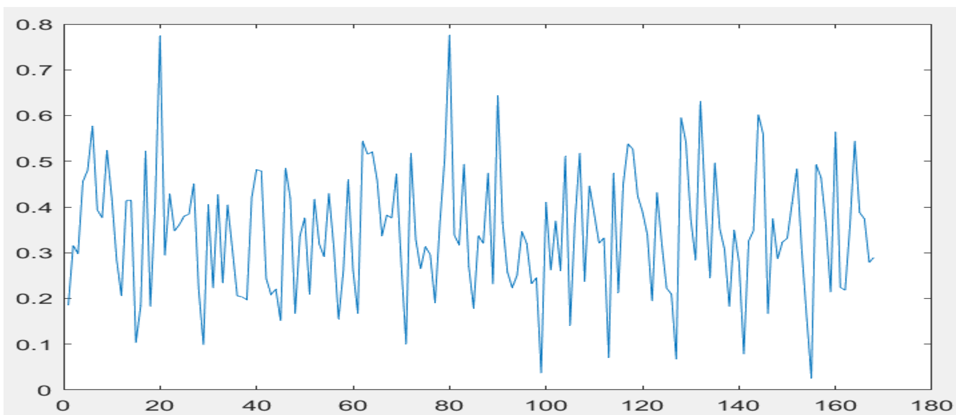

Fig 7.6.5c MATLAB simulated smoothened LEE of label -1(sad emotion)

7.7 Classification:

Once the feature matrices are obtained, several machine learning classifiers whose specifications are mentioned in section 6.1 and 6.2 are used for classification. The dataset is generated by stimulating the subjects and then three emotions namely, happy, sad and neutral are recorded. The emotions are labelled as 1, -1 and 0 for happy, sad and neutral emotions respectively. Once the classification is performed, accuracy is computed in order to analyze the performance of the classifiers.

CHAPTER 8

RESULTS AND DISCUSSIONS

In order to evaluate the performance of the proposed methodology, SEED EEG dataset that is described in section 7.2 is used. The preprocessed EEG data is segmented into small epochs each of one second. An epoch contains 200 samples. The last 30 seconds i.e., a total of 6000 samples for each channel are considered for analysis. The required number of boundaries are set to six to obtain uniform number of features. The dimension of the feature matrices obtained is 20250*168. The dimension of the joint feature matrix obtained is 20250*336. These feature matrices are classified using SVM, KNN classifiers.

The following are the accuracy values obtained after feeding the features to classifiers. Classification is performed using the classifier learner application available on MATLAB. It is observed that Wiener entropy doesn't perform well with band-limited entropies. The best accuracy obtained is 44.7% with quadratic SVM. Shannon spectral entropy and log energy entropy perform well for human emotion classification.

	SHE	**LEE**	**Combination of SHE and LEE**
Fine KNN	92.3%	91.5%	92.5%
Subspace KNN	90%	90.8%	91.2%
Weighted KNN	88.3%	90.3%	87.3%
Quadratic SVM	86.5%	86.8%	86.3%

Table 8.1. Accuracy values of the classifiers

CHAPTER 9

CONCLUSION

In this work, a novel human emotion detection is explored using Fourier-Bessel domain based band-limited entropies. The preprocessed SEED EEG dataset is used for evaluating the proposed method. 12 channels out of 62 channels were considered for analysis. The Fourier-Bessel coefficients obtained are stored. Boundaries are computed form the multi-variate Fourier-Bessel series expansion. Energy spectrum is computed from the stored FB coefficients and the boundaries obtained. The non-linear band-limited entropies computed are used as features. Three different entropies, $SHE,\ WIE$ and LOE are computed for every sub band of the energy spectrum. The computational complexity is reduced to a large extent by computing entropies for each sub band when compared to the full-scale entropy used in the previous study. The proposed method achieved the classification accuracy values comparable to the existing state of art methods especially with the combined feature of SHE and LOE. Therefore this method proves to be effective for a three class human emotion classification.

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