B.Tech Project Report

On

Nonlinear Bending Analysis of Stiffened Ruled Spherical Shell Surfaces

by

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Under the guidance of

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Nonlinear Bending Analysis of Stiffened Ruled Spherical Shell Surfaces

A PROJECT REPORT

Submitted in partial fulfillment of the requirements for the award of the degrees

of

BACHELOR OF TECHNOLOGY
in

CIVIL ENGINEERING

Submitted by:
Deshdeepak Singh

Guided by:
Dr. Kaustav Bakshi
Assistant Professor

INDIAN INSTITUTE OF TECHNOLOGY INDORE

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CANDIDATE'S DECLARATION

We hereby declare that the project entitled "Nonlinear Finite Element Analysis of Singly Curved Ruled Surfaces" submitted in partial fulfillment for the award of the degree of Bachelor of Technology in 'Civil Engineering' completed under the supervision of Dr.Kaustav Bakshi, Assistant Professor, Department of Civil Engineering, IIT Indore is an authentic work. Further, we declare that we have not submitted this work for the award of any other degree elsewhere.

Deshdeepak Singh

Deshdeepak Singh

Signature and name of the student(s) with date

CERTIFICATE by BTP Guide(s)

It is certified that the above statement made by the students is correct to the best of my knowledge.

Signature of BTP Guide with the date and designation

Preface

This report on “**Nonlinear Finite element Analysis of singly curved ruled Surfaces**” is prepared under the guidance of Dr. Kaustav Bakshi. during this report, we've studied static bending and free vibration of laminated composite one by one incurvate spherical shells victimization the geometrically nonlinear approach. A finite component model is developed and valid by examination the results with those printed within the literature. difficult boundary conditions are accustomed replicate the state of affairs that the shell could face in industrial condition.

The study is aimed to support active civil engineers whereas planning roofs and sheds in buildings/industries following Indian codes of apply. I even have tried to the most effective of my talents and information to elucidate the content in an exceedingly lucid manner. I even have additionally another 3-D models and figures to form it a lot of illustrative.

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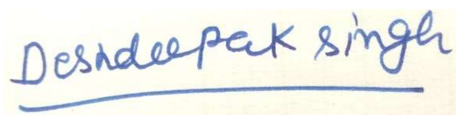
Dr. Kaustav Bakshi for his kind support, expertise, and valuable guidance. He provided a perfect environment for critical thinking and research insight and was always available for discussions, doubt clearance, and guidance at every part of the project. He has constantly motivated us to take the project to its very culmination.

I would also like to acknowledge my friends, family members, and colleagues for their direct and indirect support without which this report would not have been possible.



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Abstract

In this report ,a nonlinear finite element solutions of bending responses of functionally graded spherical panels are presented. the material properties of functionally graded material are graded in thickness direction in line with a power-law distribution of volume fractions. A general nonlinear mathematical shell model has been developed based on higher order shear deformation theory. The model is discretised using finite element steps and the governing equations are obtained through variational principle. The nonlinear responses area unit evaluated through an on the spot unvarying methodology. The model is validated by comparing the responses with the vailable published literatures. The efficacy of gift model has also been established by demonstrating a simulation based nonlinear model developed in ABAQUS environment. the effects of power-law indices, support conditions and different geometrical parameters on bending behaviour of functionally graded shells are obtained and discussed in detail.

Table of Contents

List of Figures

List of Tables

CHAPTER 1

Introduction

A thin shell is outlined as a shell with a thickness that is little compared to its alternative dimensions and within which deformations aren't giant compared to thickness. A primary difference between a shell structure and a plate structure is that, within the unaccented state, the shell structure has curvature as critical the plates structure that is flat Membrane action in a very shell is primarily caused by in-plane forces (Plane stress), but there is also secondary forces ensuing from flexural deformations. wherever a flat plate acts the same as a beam with bending and shear stresses, shells square measure analogous to a cable that resists hundreds through tensile stresses. the perfect skinny shell should be capable of developing each tension and compression.

In this report we are going to show regarding the past studies conducted by numerous researchers on shell structures and an in-depth finite component modeling for a laminated composite plate subjected to static load whereas future study involves extending the model to incorporate geometric nonlinearity.

CHAPTER 2

Literature Review

1. **J.N. Reddy (1984)** presented an extension of the Sanders shell theory for doubly curved shells to a shear deformation theory of laminated shells. The theory accounts for transverse shear strains and rotation about the normal to the shell mid-surface. Exact solutions of the equations were presented for simply supported, doubly curved, cross-ply laminated shells under sinusoidal, uniformly distributed, and concentrated point load at the center. Fundamental frequencies of cross-ply laminated shells were also presented.
2. **Alwar and Narasimha (1991)** proposed an analytical solution based on the Chebyshev-Galerkin spectral method. The effects of various geometric and material parameters on the nonlinear axisymmetric response of laminated annular shells subjected to uniform external pressure have been studied. It was also observed that softening nonlinearity of these shells increases with an increase in the ratio of radius to heights but decreases with an increase in orthotropy ratio and number of layers.
3. **Gautham and Ganesa (1992)** analyzed axisymmetric and non-symmetric vibrations of spherical shells using the thick shell theory and semi-analytical

method to reduce the dimension of the problem. It can be observed that influence of boundary condition was grater in the shallow shells.

4. **Chakravorty et. al. (1994)** studied the free vibration behavior of doubly curved, point-supported laminated composite shells. The fundamental frequency was observed to increase with increase in the number of layers for both antwasymmetric and symmetric stacking sequences.
5. **Chakravorty, Bandyopadhyay and Sinha (1994)** conducted a finite element analyswas to study the free vibration behavior of doubly curved, point-supported laminated composite shells. The fundamental frequency was observed to increase with increase in the number of layers for both antisymmetric and symmetric stacking sequences.
6. **Okasha el-Nady and Negm (2004)** presented a simple method for the solution of arbitrarily laminated composite spherical shells by expanding displacement functions in Chebyshev series. The method was used to solve a variety of spherical shell problems with different fiber orientations and boundary conditions.
7. **Okasha el-Nady and Negm (2005)** presented a simple method for the free vibration analysis of cross-ply laminated composite spherical shells by expanding the displacement functions in Chebyshev series The method was made highly automated by casting the equations in a standardized matrix form, and then systematically converting them to a standard eigenvalue problem .The method was used to obtain the natural frequencies and mode shapes of isotropic as well as laminated shell.
8. **Arciniega, Reddy (2006)** have presented a finite element computational model for the nonlinear analysis of shell structures and considered a consistent shell formulation for the nonlinear analysis of multilayered composites and functionally graded shells. A simple tensor-based displacement finite element model was developed and a family of Lagrangian elements with high-order interpolation polynomials was employed. The first-order shell theory with seven parameters was derived with exact nonlinear deformations and under the framework of the Lagrangian description.
9. **Lee, Reddy and Rostam-Abadi (2006)** did nonlinear finite element analysis of laminated composite shell structures with smart material laminae. Third-order shear deformation theory using Sanders nonlinear shell kinematics was chosen for the laminated composite shell formulations. A number of parametric studies were carried out to understand the damping characteristics of laminated composites with embedded smart material layers. It was observed that the spherical shell has the biggest deflection suppression and smallest maximum deflection.
10. **Umut Topal (2006)** studied about mode-frequency analyses of laminated spherical shell using a finite element model, based on first-order shear deformation theory and it was observed that when width to thickness ratio, material anisotropy and angle of fiber orientation increase then the non-dimensional fundamental frequency increased. The fundamental frequencies in the case of angle-ply

laminate were higher than those in the case of both cross-ply laminate for the effect of the material anisotropy.

11. **Panda and Sing (2009)** studied the buckling and post-buckling behaviors of a laminated composite spherical shallow shell panel embedded with shape memory alloy (SMA) fibers under a thermal environment. It was observed that the consideration of full geometric non-linearity effect in the stiffness matrix. The non-linear stiffness matrices was the reason for the divergence trend noted in a few cases and it can be observe that the geometric non-linearity was predominant on the material non-linearity for laminated structures.
12. **Lee and Chung (2009**) developed a finite element formulation based on the Sanders higher order theory to study the free vibration of laminated composite shell panels with delamination around quadrilateral cutouts. The authors showed that the effect of interactions between the radius–length ratio and various other parameters.
13. **Umut Topal (2012)** studied the frequency optimization of symmetrically laminated angle-ply spherical shells. It was observed that the shell aspect ratio increases, the maximum fundamental frequency deiminases because of a decrease in the shell stiffness. The maximum fundamental frequency decreases as the curvature ratio increases and it can be observed that the curvature ratio has no effect on the optimum fiber orientations. The maximum and minimum fundamental frequencies occur for CCCC and SSSS (where C – Clamped and S
– Simply supported) boundary conditions, respectively.
14. Lal et. al. (2012) The sensitivity of failure index (FI) changes with the layup sequences, number of layers,plate side to thickness ratio, plate aspect ratio, boundary conditions, amplitude ratios, types of loading, types of failure theory and modulus ratio. Among the system properties studied, the elastic modulus in transverse direction and lateral loading have dominant effect on the COV of FI when compared to other system properties subjected UDL or sinusoidal loading using Tsai–Wu and Hoff-man's failure. Between the Tsai–Wu and Hoffman's theory of failures considered Hoffman's failure theory is more sensitive as compare to Tsai–Wu failure theory.

15. Bakshi and Chakravorty (2013) concluded to provide accurate non linear first ply failure load of composite conoidal shells the proposed code is capable. and we also observe that cross ply laminations are relatively better options to fabricate the conoidal shell surface than angle ply ones.

16. **Panda and Singh (2013)** studied linear static and free vibration behaviors of laminated composite square spherical shell panel using finite element method. The model had discretized using eight- noded, six degrees of freedom shell element. It was observed that thick laminate suffers with higher transverse bending and normal stresses as compared to thin laminates. The non-dimensional natural frequency

increases with increase in number of layers and was higher for angle ply than the cross-ply laminates.

17. **Panda and Singh (2013)** worked on thermal post buckling of doubly curved composite spherical shell panel using nonlinear finite element method. It can be observed that the buckling was very sensitive to the support condition and the type of lamination scheme, and the post buckling strength increases with increase in the curvature ratio, the thickness ratio, the amplitude ratio, and the number of layers.

18. **Khaire, Ambhore and Jagtap (2014)** investigated nonlinear free vibration response of functionally graded (FGMs) spherical shell. For this study a nine noded Lagrange element having 63 DOFs per element was used for discretizing the laminate. An HSDT model was used, and 4 × 4 mesh was used. It was observed that the higher order shear deformation theory can provide accurate results for natural frequencies of FGM spherical shell.
19. **Das, Singla, Srivastava (2016)** studied three-dimensional finite element method (FEM) based on thermo-mechanical stress analysis of a laminated fiber reinforced polymer (FRP) composite spherical shell structure subjected to elevated thermal filed. It can be observed that due to curvature effect of the shell structure the stress concentration was not only limited to free edges rather may exist inside the shell boundaries hence care should be taken to select appropriate mesh to capture stress concentrations.
20. **Ghosh and Chakravorty (2017)** observed that the finite element code can be accepted as a successful tool to explore the first and ultimate ply failure aspects of composite hypar shells. Solutions obtained for the benchmark problem using the present method indicated this fact. The lamina wise failure investigation and using that information to evolve tailored laminates was utilized as design guidelines to fabricate stiff shell surfaces for a given material consumption
21. **Ghosh and Chakravorty (2017)** compared the first ply and ultimate ply failure load values for different boundary conditions and found that clamped shells having the maximum number of support constraints yield the highest load values.
22. **Behera, Garg, Patro and Sharma (2018)** presented free vibration responses of laminated composite spherical shell panels (SSLCP) without and with central cutouts (square, circular and rectangular) using finite elements. It was observed that among simply supported and clamped boundary conditions, clamped boundary condition was the more desirable for SSLCP with or without cutouts.
23. **Mohammad Zannon (2018)** applied third-order shear deformation theory to analytically derive the frequency characteristics of thick spherical laminated composite shells. The theory for thick laminated shells was applied to solve simply supported shells with cross-ply laminates. It was observed that the natural frequency of the shell increase if the radius to span ratio increase.
24. **Ghahfarokhi and Rahimi (2018)** worked on vibration correlation technique (VCT) which presented a very good correlation for grid-stiffened composite

cylindrical shells when the maximum load adopted in the VCT was higher than 68% of the experimental buckling load.

25. **Eva Kormanikova (2018)** presented mode-frequency analysis of laminated spherical shell using a finite element model. The model was based on first order shear deformation theory. It was observed that the frequencies in the case of angle-ply laminate were higher than in the same for cross-ply laminate. The frequencies in the case of symmetric layup were the same than in the case of antisymmetric layup for both types of laminates. This topic can be extended for different finite element formulations, boundary conditions, aspect ratios and number of layers.
26. **Chaubey, Kumar and Chakrabarti (2019)** studied the shear buckling of laminated spherical shells with multiple cutouts in the present mathematical model a transverse shear stresses have been varied parabolically across the thickness. It was observed that with an increase in cutout size, dimensionless buckling load decreases and it also observed that with increase in eccentricity (e) dimensionless shear buckling load increases for SSSS, CCCC, CCSS and CCFF (Where C – Clamped, S – Simply supported and F – free) boundary conditions.
27. **Zolt an Juhasz and Andras Szekr enyes (2019)** proposed an efficient analytical solution technique for delaminated doubly curved shells. It was shown that the results obtained with the use of the improved version of the Sanders shell theory were very close to the FE results. The through-the-width delamination has been modelled with the use of the systems of exact kinematic conditions, which results in a very compact model as there was no need for contacts between the delaminated top and bottom regions.
28. **Bing Hu, Cong Gao, Hang Zhang,Haichao Li ,Fuzhen Pang and Jicai Lang (2020)** used Ritz method to investigate the vibration characteristics of isotropic moderately thick annular spherical shell with general boundary conditions. The Ritz method has fast convergence and delightful accuracy through the comparative study.
29. **Mohammad Zannon,Abdullah Abu-Rqayiq,Ammar Al-bdour (2020)** conducted free vibration analyses of FG spherical shells. A third-order shear deformation theory that allows extensibility along the thickness direction was executed and the influence was studied. It was observed that the fundamental frequency decreases as the ratio R/a increases, clamped FG shell panels presented higher frequency values than the simply supported ones and the fundamental frequency of FG spherical shell panels decreases as the exponent p in power-law increases.
30. **Ahmadi et. al. (2020)** observed that softening behavior of the frequency–response curve was inversely proportional to linear elastic foundation and also observed that increasing the volume fraction power leads to increase the response amplitude.
31. **Shamloofard, Hosseinzadeh and Movahhedy (2020**) presented a new shell super element for finite element analysis of spherical shell structures. This super element had first-order shear deformation theory. The presented super element can analyze partial spherical sectors with and without apex and complete spherical shells

properly and it was observed that the super element predicts the behavior of spherical shells under local loads and boundary conditions.

32. **Amabili and Reddy (2020)** introduced a rigorous higher-order polynomial along the thickness coordinate to develop a theory for doubly curved, laminated composite shells. The developed third-order thickness and shear deformation theory had a single additional parameter to describe the thickness deformation with respect to the well-known Reddy's shear deformation theory. The developed theory keeps geometric nonlinear terms in all the kinematic parameters in the formulation. That enabled its application to problems with large displacements and large strains.
33. **Bakshi et. al. (2020)** presented a finite element code predict the linear and nonlinear transverse displacement of the shell. Among the shell options considered the cross-ply laminations of clamped ones turned out to be the best option for the practicing civil engineers to achieve the minimum displacement within a given cost of fabrication. The $0^0/90^0/0^0$ shell showed the best performance considering all the shell options.
34. **Bakshi et. al. (2020)** observed that if the practicing engineers adopt only one stiffener for the shell roof, then that should be oriented along the arch direction (*y*-stiffener) to achieve the minimum transverse displacements. The nonlinear approach must be implemented for all other stiffener arrangements considered by the authors.
35. **Ahmadi, Bayat and Duc (2021)** presented a semi-analytical procedure for investigating the non-linear primary resonant of an imperfect stiffened FG doubly curved shallow shells exposed to thermal conditions and harmonic excitation. Material properties of shells were gradually changed along the thickness direction of the FGM shallow shells, and it was observed that the frequency response curve with or without initial imperfection and stiffener, increases the amplitude of response.
36. **Zhi-Min Li, Tao Liu, Pizhong Qiao (2021)** studied a new shell of arbitrary curvatures and arbitrary fiber stacking sequences for large amplitude vibration of shear deformable laminated doubly curved shells. The governing equations were derived based on an extended higher order shear deformation shell theory, including von Kármán type of kinematic nonlinearity and stiffness couplings. It was a two-step perturbation technique combining with the Galerkin method was employed to determine the linear and nonlinear vibration frequencies and forced responses.
37. **Bakshi (2021**) studied the non-dimensional fundamental frequencies and mode shapes of laminated composite singly curved stiffened shells for varying boundary condition, lamination and stiffener properties like orientations, eccentricities, numbers and depth. The conclusion was the nonlinear approach was essential for both shell and stiffener for correct predictions of natural frequencies and mode

shapes. The relatively simpler linear approach was recommended for shells having single x – stiffener only.

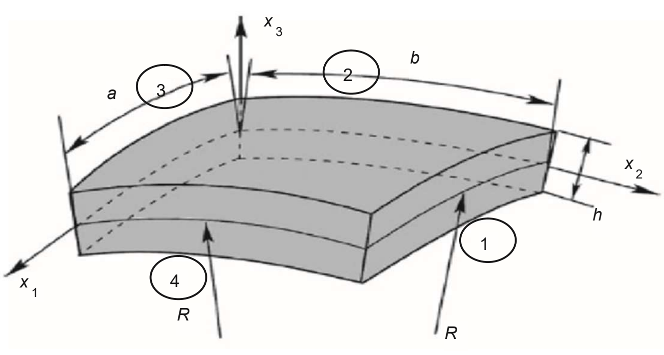
38. **Chatterjee, Ghosh and Chakravorty(2021)** concluded that the uniformly distributed FPF pressures of cross ply shells were significantly more than those of angle ply shells and among cross ply shells again the antisymmetric lamination turns out to be the best choice. They also concluded that CFFC shell can be preferred over CFCF shell options.
39. **Bakshi (2021)** studied the non-dimensional fundamental frequencies and mode shapes of laminated composite singly curved stiffened shells for varying boundary condition, lamination and stiffener properties like orientations, eccentricities, numbers and depth. it was observed that the first mode of vibration governs the dynamic performance of 450/-450/-450/450 clamped shell. The higher modes must be considered for the 450/-450/-450 simply supported shell.

CHAPTER -3

Mathematical Formulation and FE Modeling

3.1 Governing Equation

We consider a spherical shell of length a, width b, thickness h, and mean radius R. An orthogonal curvilinear coordinate system (x_1, x_2, and x_3) is considered to represent the geometry and deformation of the spherical shell when x_1 and x_2 axes are located in the midplane of the shell and considering the first-order shear deformation theory



Spherical shell with uniformly distributed pressure (q)

The total potential energy (π) of the shell is defined as the summation of total strain energy (U) stored in the volume and the external work done on the surface (W).

$$\pi = \underbrace{\frac{1}{2}\int \{\varepsilon\}^{T}\{\sigma\}dv}_{U} + \underbrace{-\int\int \{d\}^{T}\{q\}dA}_{W}$$

Where, ε is the strain matrix, σ is the stress matrix, d is the deformation vector and q is the external load vector applied on the shell. $\{q\} = \{0 \quad 0 \quad q_z \quad 0 \quad 0\}^T$, where q_z is the pressure applied on the shell. The governing equation for the shell is derived based on the principle of minimum potential energy. Thus to minimize with respect to $\{d\}$, it has to satisfy the following condition:

$$\frac{d\pi}{d\{d\}} = \psi; \begin{cases} \psi = 0, geometrically\ linear\ analysis \\ \psi = 0. geometrically\ nonlinear\ analysis \end{cases}$$

The expression Ψ for an initial estimate of the displacements, which indicates the existence of residual unbalanced forces, and geometrically nonlinear analysis, becomes necessary for minimizing the potential energy using a Newton-Raphson iterative approach. At the end of the iterations, the shell follows the equilibrium condition, i.e. $\Psi = 0$.

3.2 Laminate constitutive relation

The constitutive relation for the laminated composite is given as

$$\{\sigma\} = [D]\{\varepsilon\}$$

Where $[D]$ is adopted from Chakravorty et al. (1995) and $\{\sigma\}$ is the laminate stress resultant vector given as

$$as\ \{\sigma\} = \int_{-h/2}^{h/2} \{\sigma_x\ \sigma_y\ \tau_{xy}\ \sigma_x.z\ \sigma_y.z\ \tau_{xy}.z\ \tau_{xz}\ \tau_{yz}\}' dz$$

3.3 Finite Element Formulation

An eight noded doubly curved thin shell is used with five degrees of freedom per node. The displacement field for each element is given as $\{d\} = \{u\ v\ w\ \alpha\ \beta\}^T$, where u, v, and w are the translational deformation of shell along x, y and z axis, and α, β are the rotations of the mid surface along y and x direction, respectively. The nodal values for $\{d\}$ are estimated by using the shape functions N as

$$\{d\} = \sum_{i=1}^{8} [N_i]\{d_i\}$$

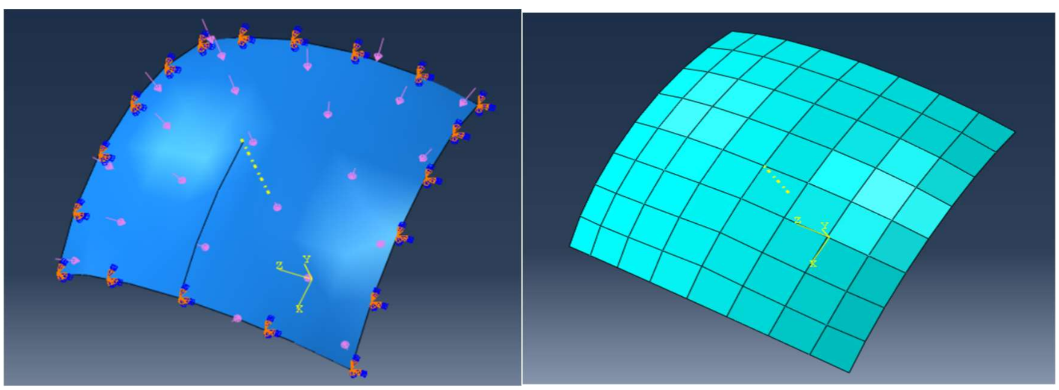
The mid surface strain vector is expressed as

$$\{\varepsilon\} = \{\varepsilon\}^L + \{\varepsilon\}^{NL}$$

Where $\{\varepsilon\}^{L}$ and $\{\varepsilon\}^{NL}$are linear and nonlinear strains at the mid-surface of the shell adopted from Sander's nonlinear strain displacement relations and Von Karman type geometric nonlinearity. The obtained nonlinear equilibrium formulation is solved using the Newton-Raphson iterative method. For this, an initial guess for the shell displacements is used to calculate the residual forces. After certain iterations, an improved solution is obtained using the Taylor series expansion.

2.4 ABAQUS Modeling

The data information for the FE modeling of the spherical shell is done using ABAQUS FE package. Dimensions of the spherical shell, material properties of the composite laminate and boundary conditions are given as input to the application.



(a) BCs and loading on shell (b) Meshing details of the shell

Figure 2: A typical representation of the FE model on ABAQUS application

0.1 Material properties are as follows and kept same for all the laminates unless specified. 0.1 N/m^2 is given as loading on the top of the shell surface. Width and length of the plan area is taken as 1000 mm.

Material constants		
E11	142.5	GPa
E22	9.79	GPa
E33	9.79	GPa
G12=G13	4.72	GPa
G23	1.192	GPa
ν12=ν13	0.27	
ν23	0.25	

CHAPTER-4
Numerical Problems and Results

4.1 Validation-

To validate the finite element model, the static bending and free vibration values for the spherical shell is compared with those given in Reddy (1984). Simply supported boundary conditions are chosen with static displacement and fundamental frequencies being computed via FE simulations. The material and geometric properties are given as the footnote in the tables. The values are mentioned in nondimensional form, which for transverse displacement and fundamental frequency is given as follows. The results are presented in table 1 and 2, respectively.

$$\bar{w} = \frac{wE_{22}h^3}{qa^4}$$

$$\bar{\omega} = \omega a^2 \sqrt{\frac{\rho}{E_{22}h^2}}$$

From tables 1 and 2, on comparing with the published values we can depict that the values are in good agreement.

Table 1: Fundamental frequencies (ϖ) in radian/sec of laminated composite shell

Lamination	$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/90^0/0^0$
J.N Reddy [1]	9.687	15.183	15.184
Present FEM (2×2)	4.23	4.296	4.294
(4×4)	8.58	8.67	8.66
(6×6)	8.88	12.96	12.93
(8×8)	8.88	13.94	13.33

a/b=1,*a/h*=100,*E11*=25*E22*,*G12*=*G13*=0.5*E22*,*G23*=0.2*E22*,*v*=0.25,*E22*=106*N/cm2*,*R/a* =1030

Table 2: Maximum downward deformation in ‘mm’ of laminated composite shell

Lamination	$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/90^0/0^0$
J.N Reddy [1]	16.980	6.697	6.833
Present FEM (2×2)	16.54	6.728	7.304
(4×4)	16.58	6.720	7.302
(6×6)	16.59	6.719	7.301
(8×8)	16.59	6.718	7.301

a/b=1,*a/h*=100,*E11*=25*E22*,*G12*=*G13*=0.5*E22*,*G23*=0.2*E22*,*v*=0.25,*E22*=106*N/cm2*,*R/a* =1030

4.2 Nonlinear response with different boundary conditions and lamination angle For the spherical shell

Nonlinear deflections and fundamental frequencies are computed for the spherical shell with varying radii as given in Table 3 and 4, respectively. The results are in non-dimensional form. Here, a and b are kept constant and radii is varying as per its ratio with a. The results highly vary with different boundary conditions and lamination angle. However, as we increase the ratio, the shell takes a plate configuration and deflections are significantly high.

Table 3: Non-dimensional deflections ($\hat{w} \times 10^5$) of laminated composite spherical shell

Boundary conditions	Lamination	R_{yy}/a = 0.75	R_{yy}/a = 1.0	R_{yy}/a = 1.25	R_{yy}/a = 1.5	R_{yy}/a = 3.0	R_{yy}/a = 5.0	R_{yy}/a = 10.0
CCCC	0/90	0.3702	0. 6621	1.051	1.513	5.132	17.08	70.83
	0/90/0	0.3451	0.5967	0.9297	1.396	6.185	15.8.0	49.37
	0/90/0/90	0.3526	0.6297	0.9568	1.365	5.826	18.14	59.69
	0/90/90/0	0.3521	0.6201	0.9736	1.345	5.604	16.48	53.78
	45/-45	0.3305	0.6290	1.025	1.502	5.403	15.70	70.78
	45/-45/45	0.3926	0.7166	1.105	1.543	5.556	17.93	63.23
	45/-45/45/-45	0.3851	0.7130	1.091	1.518	5.590	17.99	61.19
	45/-45/-45/45	0.3998	0.7369	1.116	1.541	5.642	18.23	61.76
Boundary conditions	Lamination	R_{yy}/a = 0.75	R_{yy}/a = 1.0	R_{yy}/a = 1.25	R_{yy}/a = 1.5	R_{yy}/a = 3.0	R_{yy}/a = 5.0	R_{yy}/a = 10.0
SSSS	0/90	0.5383	0.9186	1.389	1.922	6.422	19.78	84.08
	0/90/0	0.3727	0.6466	0.9773	1.357	5.177	15.32	58.01

	0/90/0/90	0.3646	0.6101	0.9126	1.285	5.001	15.48	67.46
	0/90/90/0	0.3375	0.5770	0.8709	1.211	4.937	14.33	61.92
	45/-45	0.4924	0.7414	1.091	1.596	6.281	15.97	73.09
	45/-45/45	0.3294	0.5850	0.9340	1.358	5.170	14.53	62.43
	45/-45/45/-45	0.3413	0.6024	0.9684	1.403	5.017	15.07	63.15
	45/-45/-45/45	0.3396	0.5807	0.9267	1.343	4.903	14.73	63.15
CSCS	0/90	0.3838	0.5911	1.088	1.553	5.314	17.54	69.37
	0/90/0	0.3862	0.6712	0.9858	1.335	5.858	16.03	46.68
	0/90/0/90	0.3254	0.5924	0.9543	1.316	5.275	16.63	58.97
	0/90/90/0	0.3517	0.6070	0.8941	1.214	5.450	14.88	49.48
	45/-45	0.4611	0.7102	1.032	1.507	5.819	15.54	71.13
	45/-45/45	0.3718	0.6572	1.025	1.458	5.511	16.11	62.95
	45/-45/45/-45	0.3645	0.6558	1.030	1.463	5.340	16.54	62.40
	45/-45/-45/45	0.3759	0.6511	1.009	1.424	5.338	16.52	62.68
SCSC	0/90	0.5086	0.8848	1.376	1.876	6.266	19.25	84.33
	0/90/0	0.3240	0.5810	0.9463	1.341	5.487	15.20	63.13
	0/90/0/90	0.3885	0.6531	0.9807	1.324	5.350	16.85	67.31
	0/90/90/0	0.3264	0.5945	0.9617	1.371	5.002	16.12	66.71
	45/-45	0.4329	0.7128	1.193	1.524	5.801	15.57	71.14
	45/-45/45	0.3413	0.6375	1.335	1.442	5.520	16.11	62.96
	45/-45/45/-45	0.3499	0.6551	0.9724	1.457	5.344	16.54	62.40
	45/-45/-45/45	0.3519	0.6520	0.9867	1.432	5.344	16.52	62.68

Table 4- Non-dimensional fundamental frequency (ϖ) of laminated composite spherical shell

Boundary conditions	Lamination	R_{yy}/a = 0.75	R_{yy}/a = 1.0	R_{yy}/a = 1.25	R_{yy}/a = 1.5	R_{yy}/a = 3.0	R_{yy}/a = 5.0	R_{yy}/a = 10.0
CCCC	0/90	73.64	59.21	49.76	43.35	28.02	23.25	15.02
	0/90/0	75.21	62.20	53.97	46.15	32.19	26.55	17.28
	0/90/0/90	75.15	61.89	52.35	45.63	31.52	28.46	16.83
	0/90/90/0	75.52	62.57	55.23	48.76	35.56	27.23	17.28
	45/-45	65.97	55.42	47.63	43.19	29.97	23.49	14.38
	45/-45/45	72.01	62.02	50.72	50.77	32.57	24.13	15.96
	45/-45/45/-45	71.81	61.70	49.19	49.12	38.04	27.08	16.21
	45/-45/-45/45	72.38	62.39	50.16	51.46	37.95	26.89	16.15
SSSS	0/90	72.63	57.93	48.07	41.22	24.06	18.09	12.18

	0/90/0	73.83	59.75	51.46	43.25	25.79	19.01	13.89
	0/90/0/90	74.04	59.75	52.94	43.92	26.05	19.85	14.95
	0/90/90/0	74.14	60.07	53.72	44.05	28.33	22.43	13.88
	45/-45	64.03	53.66	45.43	39.52	29.84	19.66	11.52
	45/-45/45	69.80	58.18	49.85	43.18	30.89	20.35	12.45
	45/-45/45/-45	69.81	58.12	49.89	43.23	33.23	24.06	15.26
	45/-45/-45/45	70.18	58.49	50.61	45.30	34.16	24.44	15.08
Boundary conditions	Lamination	R_{yy}/a = 0.75	R_{yy}/a = 1.0	R_{yy}/a = 1.25	R_{yy}/a = 1.5	R_{yy}/a = 3.0	R_{yy}/a = 5.0	R_{yy}/a = 10.0
CSCS	0/90	73.51	59.12	49.69	43.10	27.08	21.61	14.14
	0/90/0	58.18	62.14	51.32	45.65	29.88	23.92	14.99
	0/90/0/90	75.02	61.19	52.21	47.27	31.85	23.62	15.46
	0/90/90/0	75.46	62.32	53.87	48.19	34.30	24.32	16.02
	45/-45	64.97	54.60	46.68	41.09	25.45	21.68	14.01
	45/-45/45	72.31	60.25	49.16	44.32	28.53	23.46	15.18
	45/-45/45/-45	70.94	60.06	52.84	47.94	35.68	26.01	15.65
	45/-45/-45/45	71.50	60.57	53.34	48.44	35.88	25.62	15.58
SCSC	0/90	72.76	57.93	48.13	41.40	25.19	20.11	13.14
	0/90/0	73.89	59.85	49.65	42.86	27.17	23.54	15.19
	0/90/0/90	74.27	60.32	51.30	44.32	29.56	24.16	15.32
	0/90/90/0	74.20	60.25	51.18	44.19	29.42	24.50	15.33
	45/-45	65.16	54.60	46.62	41.15	27.89	24.98	13.99
	45/-45/45	70.87	60.26	52.89	46.12	32.53	25.65	15.44
	45/-45/45/-45	70.81	60.07	53.01	47.56	34.03	25.99	15.70
	45/-45/-45/45	71.25	60.63	53.39	48.50	35.88	26.64	15.61

4.3 Nonlinear response with different boundary conditions and a/h ratio for the spherical shell

Next we vary a/h ratio for the same boundary conditions as indicated above. The results are given in tabular form in Table 5 and 6.

Table 5: Non-dimensional deflections ($\hat{w} \times 10^5$) of laminated composite spherical shell, a/b = 1.0, a = 1.0 m

Boundary conditions	a/h ratio	R_{yy}= 0.75 m	R_{yy} = 1.0 m	R_{yy} = 1.25 m	R_{yy}/= 1.5 m	R_{yy}= 3.0 m	R_{yy}= 5.0 m	R_{yy}= 10.0 m
0/90/0 CCCC	80	0.5273	0.9076	1.471	2.2187	9.3671	27.3304	66.0545
	100	0.3451	0.5967	0.9297	1.396	6.1851	15.8	49.37
	120	0.2469	0.4223	0.6501	0.9461	4.3101	11.4871	37.8990
0/90/0 SSSS	80	0.5658	0.9714	1.4468	1.9882	8.4023	23.6522	86.2614
	100	0.3727	0.6466	0.9773	1.3574	5.1770	15.32	58.01
	120	0.2624	0.4585	0.7013	0.9832	3.4351	10.5786	40.5321
0/90/0 CSCS	80	0.5771	0.9796	1.4185	1.9883	9.1972	23.5155	62.2654
	100	0.3862	0.6712	0.9858	1.3355	5.8582	16.03	46.68
	120	0.2757	0.9896	0.7262	0.9930	3.9687	11.464	18.4304
0/90/0 SCSC	80	0.5105	0.9119	1.4355	2.0624	8.5566	24.238	95.6442
	100	0.3240	0.5810	0.9245	1.341	5.4873	15.2	63.13
	120	0.2373	0.3987	0.6394	0.9340	3.7910	10.5728	43.7844

Table 6: Non-dimensional fundamental frequencies (ϖ) of laminated composite spherical shell, a/b = 1.0, a = 1.0 m

Boundary conditions	a/h ratio	R_{yy}= 0.75 m	R_{yy} = 1.0 m	R_{yy} = 1.25 m	R_{yy}/= 1.5 m	R_{yy}= 3.0 m	R_{yy}= 5.0 m	R_{yy}= 10.0 m
0/90/0 CCCC	80	194.02	51.77	45.94	42.02	31.74	22.18	15.23
	100	237.81	62.27	53.97	48.51	36.43	25.76	17.29
	120	278.97	72.76	62.20	55.17	39.09	29.52	19.08
0/90/0 SSSS	80	189.19	48.81	41.67	36.75	25.32	20.01	11.96
	100	195.84	59.78	50.34	43.86	28.24	23.29	13.89
	120	277.46	70.80	59.11	42.59	31.29	27.76	14.23

0/90/0 CSCS	80	194.02	51.67	57.12	41.67	31.11	21.11	14.64
	100	237.69	62.14	53.85	48.25	35.89	24.88	16.15
	120	281.24	72.68	51.77	54.97	38.60	28.80	17.78
0/90/0 SCSC	80	189.29	48.96	52.46	37.15	26.14	21.11	12.63
	100	233.73	59.88	50.55	44.17	28.97	24.25	15.18
	120	278.22	70.79	49.45	51.27	31.97	25.64	17.34

4.4 Nonlinear response with different boundary conditions and a/h ratio for the spherical shell

Next we vary a/b ratio for the same boundary conditions as indicated above. The results are given in tabular form in Table 7 and 8.

Table 7: Non-dimensional deflections ($\hat{w} \times 10^5$) of laminated composite spherical shell (a/h = 100)

Boundary conditions	a/b ratio	R_{yy}= 0.75 m	R_{yy} = 1.0 m	R_{yy} = 1.25 m	R_{yy}/= 1.5 m	R_{yy}= 3.0 m	R_{yy}= 5.0 m	R_{yy}= 10.0 m
0/90/0 CCCC	0.5	0.6807	1.1567	1.8001	4.0861	49.0492	18.23E-04	9.425E-04
	1.0	0.3451	0.5967	0.9297	1.396	6.1851	15.8	49.37
	2.0	8.4468	12.2496	16.9473	34.0194	35.4154	39.4735	60.0492
0/90/0 SSSS	0.5	0.9543	1.3194	1.5426	1.7634	13.9467	22.6486	50.0101
	1.0	0.3727	0.6466	0.9773	1.3574	5.1770	15.32	58.01
	2.0	11.1460	24.5193	30.2174	39.4203	40.493	6.43E-04	10.243E-03
0/90/0 CSCS	0.5	1.0132	1.4164	1.9467	2.2197	16.820	6.18E-04	2.642E-04
	1.0	0.3862	0.6712	0.9858	1.3355	5.8582	16.03	46.68
	2.0	0.1103	0.1729	0.2890	0.3094	1.1093	2.6435	17.6272
0/90/0 SCSC	0.5	0.6426	0.4014	2.4865	4.4253	45.6197	23.6185	68.9473
	1.0	0.3240	0.5810	0.9245	1.341	5.4873	15.2	63.13
	2.0	7.9258	24.3196	29.8429	32.1846	0.8644	1.5497	4.2843

Table 8: Non-dimensional fundamental frequencies (ϖ) of laminated composite spherical shell

Boundary conditions	a/b ratio	R_{yy}= 0.75 m	R_{yy} = 1.0 m	R_{yy} = 1.25 m	R_{yy}/= 1.5 m	R_{yy}= 3.0 m	R_{yy}= 5.0 m	R_{yy}= 10.0 m
0/90/0 CCCC	0.5	254.184	39.422	27.459	25.163	12.784	8.596	12.502
	1.0	237.81	62.270	53.97	48.51	36.43	25.76	17.29
	2.0	246.617	151.227	124.365	130.095	99.568	64.167	35.426
0/90/0 SSSS	0.5	213.597	34.526	28.753	80.159	40.456	20.312	4.167
	1.0	195.84	59.72	50.34	43.86	28.24	23.29	13.89
	2.0	259.35	63.301	53.357	45.852	38.429	51.496	86.426
0/90/0 CSCS	0.5	287.159	35.842	28.453	32.256	19.547	10.129	13.92
	1.0	237.69	62.14	53.85	48.25	35.89	24.88	16.15
	2.0	298.365	111.852	107.745	104.54	95.914	67.997	30.589
0/90/0 SCSC	0.5	280.365	35.928	25.856	22.698	11.036	7.889	5.248
	1.0	233.73	59.88	50.55	44.17	28.970	24.25	15.18
	2.0	279.159	155.946	115.756	122.800	77.542	49.835	29.426

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