COMPUTER TOOLS FOR PARTICLE PHYSICS

M.Sc. Thesis

By
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DISCIPLINE OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY INDORE
MAY 2024

COMPUTER TOOLS FOR PARTICLE PHENOMENOLOGY

A THESIS

Submitted in partial fulfillment of the
requirements for the award of the degree
of
Master of Science

by
SUNNY KUMAR KESHRI

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INDIAN INSTITUTE OF TECHNOLOGY INDORE
MAY 2024

Indian Institute of Technology Indore

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled *Computer Tools for Particle Phenomenology* in the partial fulfillment of the requirements for the award of the degree of Master of Science and submitted in the Discipline of Physics, Indian Institute of Technology Indore, is an authentic record of my own work carried out during the time period from August 2023 to May 2024 under the supervision of Dr. Dipankar Das, Assistant professor, Indian Institute of Technology Indore.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

13/05/24

Signature of Supervisor
Dr. Dipankar Das
Date:

Signature of DPGC
Name:
Date: 22/5/2024

"His laboratory was his ball point pen"
$\sim$ *Julian Schwinger (1918-1994)*

INDIAN INSTITUTE OF TECHNOLOGY INDORE

Abstract

Dr. Dipankar Das
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Master Thesis

Computer Tools for Particle Phenomenology

by Sunny Kumar KESHRI

The thesis explores the utilization of "Computer Tools for Particle Phenomenology" research, focusing on the calculation of one-loop corrections and decay processes within the Standard Model (SM) of particle physics and Beyond the SM as well. Making use of various use of computational packages such as FeynArts, FeynCalc, LoopTools and FeynCalc, I have conducted a comprehensive investigation into various phenomena.

Initially, I evaluated the one-loop correction to electron anomalous magnetic moment (g-2) both analytically and using FeynCalc package in terms of Passarino-Veltman functions. I have reproduced the $H\gamma\gamma$ and $HZ\gamma$ decays in SM and BSM, including some additional new charged scalar particles by incorporating them into a new model file with the aid of FeynRules.

Finally I have evaluated the Higgs boson decay to ZZ^*, for an off-shell Z boson within the SM framework in terms of Passarino-Veltman functions.

Acknowledgements

I would like to express my special thanks and gratitude to my supervisor, Dr. Dipankar Das, for providing me the direction and opportunity to conduct research. His motivation kept me driven to learn new things. He has taught me the methodology to conduct a research. I am also grateful to my lab mates, Anugrah M. Prasad and Shreya Pandey for the stimulating discussion and talk we had in the lab. I express my gratitude to my supervisor for having those evening discussion with my lab colleagues.

I would like to thank my parents and sisters for their patience, support and preparing me for my future. I am grateful for their invaluable support.

Last but not least, I would like to thank all my friends, especially my 606 VSB hostel unit mates, without you all I would have done much better.

Contents

List of Figures

List of Abbreviations

SM	**S**tandard **M**odel
BSM	**B**eyond **S**tandard **M**odel
PV	**P**assarino **V**eltman
HEP	**H**igh **E**nergy **P**hysics
LHC	**L**arge **H**adron **C**ollider
VEV	**Vacuum** **E**xpectation **V**alue
QCD	**Q**uantum **C**hromo **Dynamics**
QED	**Q**uantum **E**lectro **D**ynamics

Dediacted to my Parents

Chapter 1

Software Packages

1.1 Introduction

In high-energy physics, the interaction of matter and its mediating forces is successfully described in consistent field theoretical models. For the evaluation of observational aspect of these theories, perturbative methods have become well-established. The quest for precision in theoretical predictions stands as a cornerstone for unraveling the mysteries of fundamental interactions in nature. A very beautiful and systematic approach to perturbation theory was provided by Richard P. Feynman., who introduced the concepts of Feynman graphs and Feynman rules. Feynman diagrams are a graphical method of representing the interactions of elementary particles. These diagrams are terms in the perturbative expansion of the matrix element for an interaction.

In general, there are an infinite number of Feynman diagrams in a full matrix element when all orders in perturbation are considered. However, in a non-trivial model, a calculation beyond the tree-level quickly becomes tedious and very susceptible to errors. This is valid for finding all Feynman graphs and amplitudes. Therefore many efforts have been put to automatize these procedures with computer packages.

A few of the Mathematica [1] based package are, FeynArts [2] that generate all the tree and loop level diagrams for a process, FeynCalc [3] which does all the tensor contractions and evaluates the Dirac matrices and the necessary integral calculations using a reduction scheme called Passarino-Veltman [4] functions to frame the final results, and FeynHelpers [5] which evaluates the scalar integrals A0, B0, C0, etc. Therefore one needs all these packages to do one-loop calculations and more. Together, these packages form a powerful toolkit for theoretical physicists engaged in high-energy physics research.

By automating tedious calculations and providing efficient algorithms for handling complex expressions, they enable researchers to focus on the conceptual aspects of their theories and explore the intricate phenomena of fundamental interactions with precision and rigor.

1.1.1 FeynArts

FeynArts is Mathematica package used for generating and visualizing Feynman diagrams and invariant amplitude. It can generate tree-level, loop-level, and counter-term diagrams, and with placeholders for one-particle-irreducible vertex functions (skeleton diagrams). The model information is generally available in two model files. The kinematical quantities like spinors or vector fields are defined in generic model file and the particle content and the actual couplings are described in classes model file. The applicability of FeynArts is virtually unlimited within perturbative quantum field theory, because the user can create their own model file using FeynRules. The installation can be done along with FeynCalc.

1.1.2 FeynCalc

FeynCalc is also a Mathematica package for symbolic evaluation of Feynman graphs and algebraic calculations in QFT and elementary particle physics. FeynCalc job is to contract all the tensors and Dirac matrices and convert the expression to a readable format. One of the features of FeynCalc is that it can be used to simplify frequently occurring tasks like Lorentz index contraction, Dirac matrix manipulation and traces, etc. Tensor and Dirac algebra manipulations and generating Feynman rules from a Lagrangian. Other works FeynCalc can do is Passarino-Veltman reduction of one-loop amplitudes to standard scalar integrals. Generation of Feynman rules from a lagrangian. Tools for non-commutative algebra, etc.

Installation

- Run the following command into a Mathematica session.
  ```
  Import["https://raw.githubusercontent.com/FeynCalc
  /feyncalc/master/install.m"];
  InstallFeynCalc[]
  ```
- It will install both FeynArts and FeynCalc.

Loading Package

- Before loading the FeynCalc package, one should always load FeynArts using the command `$LoadAddOns={"FeynArts"}`
- Then run FeynCalc using the command `<<FeynCalc``
- You can remove the startup messages using `$FeynCalcStartupMessages = False` and `$FAVerbose = 0;`

FeyArts Commands:

Some of the useful command to generate loop level diagrams are as follows:

`LoadAddOns = {"FeynArts"}`	To load the package FeynArts.
`ExcludeTopologies[]`	list of filters for excluding topologies.
`WFCorrection`	to remove self-energy diagrams.
`InsertFields[t, i1,.. -> o1,..]`	insert fields into the TopologyList.
`CreateFeynAmp[]`	translate the topologies into amplitude.
`Paint[]`	To draw the diagrams.
`ColumnXRows → {m × n}`	to draw diagrams in m × n matrix.
`Numbering→ Simple`	to give simple numbers to the diagrams.
`CreateTopologies[l,i → o]`	create topologies with l loops, i incoming and o outgoing particles.

FeynCalc Commands

`FCFAConvert[]`	generates proper FeynCalc expression.
`ScalarProduct[]`	defines the scalar product.
`Contract[]`	contracts all the tensor indices.
`FCReplace[]`	replace to a given dimension.
`TID[]`	Tensor integral decomposition.
`DiracSimplify[]`	simplifies the Dirac matrices.
`PaVeUVPart[]`	Collects the scalar integrals A_0, B_0.

and many more can be found in the FeynCalc manual.

A short example on how to use FeynArts and FeynCalc is shown below for a tree-level process where I have generated the tree level diagrams and amplitude for the process $e^- e^+ \to \mu^- \mu^+$.

Tree level scattering process, $e^- e^+ \to \mu^- \mu^+$

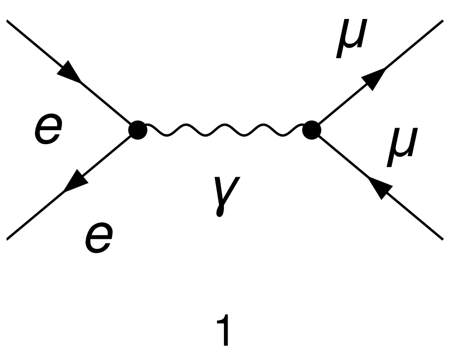
Loading packages into Mathematica session

In[ ]:=
```
$LoadAddOns = {"FeynArts"}
$FeynCalcStartupMessages = False;
<< FeynCalc`
$FAVerbose = 0;
```

Out[ ]= {FeynArts}

$e^- e^+ \to \mu^- \mu^+$ scattering diagram using FeynArts

In[ ]:=
```
diagram =
  InsertFields[CreateTopologies[0, 2 → 2], {F[2, {1}], -F[2, {1}]} → {F[2, {2}], -F[2, {2}]},
   InsertionLevel → {Classes}, Restrictions → QEDOnly, Model → "SM"];
Paint[diagram, ColumnsXRows → {2, 1}, Numbering → Simple,
  SheetHeader → None, ImageSize → {512, 256}];
```

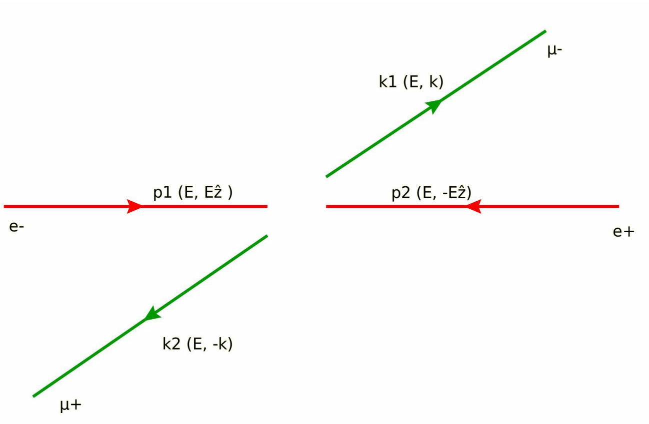


Generating amplitude using FeynCalc

In[ ]:=
```
amplitude = FCFAConvert[CreateFeynAmp[diagram], IncomingMomenta → {p1, p2},
    OutgoingMomenta → {k1, k2}, UndoChiralSplittings → True,
    ChangeDimension → 4, List → False, SMP → True, Contract → True] /.
   {SMP["m_e"] → m_e, SMP["m_mu"] → m_μ, SMP["e"] → e} // FullSimplify
```

Out[ ]=
$$-\frac{e^2 \left(\varphi(-\overline{p2}, m_e)\right).\bar{\gamma}^{Lor1}.\left(\varphi(\overline{p1}, m_e)\right)\left(\varphi(\overline{k1}, m_\mu)\right).\bar{\gamma}^{Lor1}.\left(\varphi(-\overline{k2}, m_\mu)\right)}{(\overline{k1}+\overline{k2})^2}$$

Defining Kinematics

In[ ]:=


;

In[ ]:=
```
FCClearScalarProducts[];
ScalarProduct[p1, p1] = m_e^2;
ScalarProduct[p2, p2] = m_e^2;
ScalarProduct[k1, k1] = m_μ^2;
ScalarProduct[k2, k2] = m_μ^2;
ScalarProduct[p1, k1] =  EE^2 - EE k Cosθ;
ScalarProduct[p2, k2] = EE^2 - EE k Cosθ;
ScalarProduct[p1, k2] =  EE^2 + EE k Cosθ;
ScalarProduct[p2, k1]  =  EE^2 + EE k Cosθ;
ScalarProduct[p1, p2] = 2 EE^2 - m_e^2;
ScalarProduct[k1, k2] =  2 EE^2 - m_μ^2;
```

In[ ]:=
```
squared = DiracSimplify[(FermionSpinSum[#, ExtraFactor → 1/ 2^2] &)[
    FeynAmpDenominatorExplicit[amplitude * ComplexConjugate[amplitude]]]] // FullSimplify
```

Out[ ]=
$$\frac{e^4 \left(\text{Cos}\theta^2\, k^2 + m_e^2 + \text{EE}^2 + m_\mu^2\right)}{\text{EE}^2}$$

For simplicity, taking mass of electron equal to zero.

In[ ]:=
```
squaredAmplitude  =  squared /. {m_e → 0, k →  EE^2 - m_μ^2} // FullSimplify
```

Out[ ]=
$$\frac{e^4 \left(\text{Cos}\theta^2 \left(\text{EE}^2 - m_\mu^2\right)^2 + \text{EE}^2 + m_\mu^2\right)}{\text{EE}^2}$$

In[ ]:=
```
final  =  Collect[FullSimplify[%], {e, Cos[θ]}]
```

Out[ ]=
$$\frac{e^4 \left(\text{Cos}\theta^2 \left(\text{EE}^2 - m_\mu^2\right)^2 + \text{EE}^2 + m_\mu^2\right)}{\text{EE}^2}$$

The result agrees with the corresponding expression given in Peskin and Schroeder, An Introduction to QFT, equation 5.11

1.1.4 FeynRules

FeynRules is Mathematica-based package which addresses the implementation of particle physics models that means we have to input all the model information in a Mathematica based syntax. The input is usually given in the form of a list of parameters which contains information about couplings, mixing angles, etc and particle class that contains all the particles in a model, Lagrangian, the gauge groups, etc.
It calculates the Feynman rules and outputs them in a form appropriate for various programs such as `FeynArts`, `CalcHep`, `MadGraph`, etc. Therefore one can use simulation and compare with experiments easily.

To input a model file one has to generally give the following information.

- **Model Information** which contains general information about the model implementation into the model file.
- **Indices** very often the Lagrangian describing a model file contains quantities that carry indices specifying their members. It can be specified using Indices.
- **The model parameters** that contains information about coupling constants, mixing angles and matrices, masses, etc.
- **Particle classes** that contains information about all the particles in a model.
- **Gauge group** the symmetry of the interaction of a model is described by the gauge structure which can be declared in gauge groups.
- **The Lagrangian** which is an essential requirement of a model implementation that is built out of fields of the models.

Installation

- Visit `https://feynrules.irmp.ucl.ac.be/`
- Download the latest FeynRules file, the `tar.gz` file.
- Extract it and save it in a directory and open a new Mathematica session.
- Set the directory where the FeynRules file is stored using command `SetDirectory["$Path"]`
- Run FeynRules using `<<FeynRules`‘

Useful Commands

`LoadModel["modelfile.fr"]`	to load the model file.
`FeynmanRules[]`	to calculate the Feynman rules.
`ComputeWidths[]`	to compute the decay width.

Below is an example on how to implement a model file and compute its Feynman rules for ϕ^4 theory where the Lagrangian is

Example Lagrangian

ϕ^4 theory

$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \tag{1.1}$$

where, ϕ is a complex field.

1.1.5 Model file

The model file for the above Lagrangian is as follows,

```
M$ModelName = "Phi_4";

M$Information = {Author->{"Sunny"}};

(*Particles*)
M$ClassesDescription = {
       S[1] == {
              ClassName   -> phi,
              SelfConjugate  -> False,
              Mass    -> {m, 100},
              Width -> 0,
       ParticleName -> "phi",
       PropagatorLabel -> "phi",
       PropagatorType -> Dashed,
       PropagatorArrow -> None,
       FullName -> "phi"
              }
};

(* Parameters *)

M$Parameters = {
       lam == {
              Parametertype  -> External
              Value    -> 0.1
       }
};

(* phi4 Lagrangian *)

L = del[phi,mu] del[phibar,mu] - m^2 phi phibar + lam (phi phibar)
   ↪ ^2;
```

ϕ^4 Lagrangian

```
In[ ]:= $FeynRulesPath = SetDirectory["/home/sunny/Documents/Software/FeynRules"];
In[ ]:= << FeynRules`;
In[ ]:= SetDirectory[$FeynRulesPath <> "/Models/SMHgg"];
```

Loading model file

```
In[ ]:= LoadModel["phi4.fr"];
   - Loading particle classes.
   - Loading parameter classes.

Model Phi_4 loaded.
```

One can check if the Lagrangian is Hermitian.

```
In[ ]:= CheckHermiticity[L]
Checking for hermiticity by calculating the Feynman rules contained in L-HC[L].
If the lagrangian is hermitian, then the number of vertices should be zero.
Starting Feynman rule calculation.
Expanding the Lagrangian...
No vertices found.
0 vertices obtained.
The lagrangian is hermitian.
Out[ ]= {}
```

```
In[ ]:= vertices = FeynmanRules[L]
Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
1 possible non-zero vertices have been found -> starting the computation:
 FeynRules`FR$FeynmanRules / 1.
1 vertex obtained.
Out[ ]= {{{{phi, 1}, {phi, 2}, {phi†, 3}, {phi†, 4}}, 4 i lam}}
```

Result: $4\,i\,\lambda$

1.1.7 LoopTools

LoopTools is used for the evaluation of scalar and tensor one-loop integrals and provides actual numerical implementation of the Passarino Veltman functions appearing in FeynCalc.

Installation

- To install LoopTools visit `https://feynarts.de/looptools/`
- Download the shell script file named `FeynInstall`
- Make it executable using `chmod 755 FeynInstall`
- Run it in a directory where you want to install it using `./FeynInstall`
- The shell script prompts you to install FeynArts and FormCalc as well. You can ignore those and proceed to install LoopTools.
- Finally include the path into Mathematica.

Using LoopTools in Mathematica

```
In[1]:= Install["LoopTools"]

Out[1]= LinkObject[LoopTools,1,1]

In[2]:= B0[1000, 50, 80]

Out[2]= -4.40593 + 2.70414 I
```

1.1.8 FeynHelpers

Finally FeynHelpers is used to evaluate the Passarino Veltman functions, B0, C0, C12, etc, which I have explained in the next section. The combination of FeynCalc and FeynHelpers allows one to obtain fully analytic results for most 1-loop amplitudes with up to 4 external legs and to rewrite many multi-loop amplitudes in terms of master loop integrals.

Installation

- Run the following command into a Mathematica session.
```
Import["https://raw.githubusercontent.com/FeynCalc/feynhelpers/master
/install.m"]
InstallFeynHelpers[]
```
- Load it before FeynCalc using `$LoadAddOns={"FeynHelpers"}`

An example on how to calculate the scalar integrals using FeynHelpers package is shown below where I have explicitly evaluated B0 and C0 scalar integrals.

The described packages represent essential tools for addressing the challenges and achieving the objective outlined in the thesis. Through the utilization of these packages, significant progress have been made in particle phenomenology calculations. Through the above packages, I have done some loop-level calculations relevant for my research.

Loading packages

In[1]:=
```
$LoadAddOns = {"FeynHelpers"};
$FeynCalcStartupMessages = False;
<< FeynCalc`;
$FAVerbose = 0;
```

Defining a scalar integral

In[7]:=
```
C0 = PaVe[0, {0, m2, m3}, {p, p, p}]
```

Out[7]= $C_0(0, \mathrm{m2}, \mathrm{m3}, p, p, p)$

Evaluating it using FeynHelpers

In[8]:=
```
PaXEvaluate[C0]
```

Out[8]=
$$\frac{1}{2\,(\mathrm{m2}-\mathrm{m3})}\left(\log\left(\frac{\sqrt{\mathrm{m2}\,(\mathrm{m2}-4\,p)}-\mathrm{m2}+2\,p}{2\,p}\right)-\log\left(\frac{\sqrt{\mathrm{m3}\,(\mathrm{m3}-4\,p)}-\mathrm{m3}+2\,p}{2\,p}\right)\right)$$
$$\left(\log\left(\frac{\sqrt{\mathrm{m2}\,(\mathrm{m2}-4\,p)}-\mathrm{m2}+2\,p}{2\,p}\right)+\log\left(\frac{\sqrt{\mathrm{m3}\,(\mathrm{m3}-4\,p)}-\mathrm{m3}+2\,p}{2\,p}\right)\right)$$

B0 scalar integral

In[12]:=
```
B0 = PaVe[0, {p1}, {p1, p1}]
```

Out[12]= $B_0(\mathrm{p1}, \mathrm{p1}, \mathrm{p1})$

In[13]:=
```
PaXEvaluate[B0]
```

Out[13]=
$$\frac{1}{\varepsilon}+\frac{1}{3}\left(3\log\left(\frac{\mu^2}{\pi\,\mathrm{p1}}\right)-\sqrt{3}\,\pi-3\,\gamma+6\right)$$

one can also find the infinite term using

In[14]:=
```
PaXEvaluateUV[B0]
```

Out[14]=
$$\frac{1}{\varepsilon_{\mathrm{UV}}}$$

1.2 Passarino-Veltman functions

Using a reduction scheme is one of the main methods for managing any general one-loop computation [6]. The Passarino Veltman [7] technique is the most widely utilized reduction scheme. FeynCalc assesses the amplitudes for a particular process using the Passarino-Veltman Reduction concept. First, let's review the fundamentals of the Passarino Veltman Reduction. Any tensorial integral can be reduced into a linear combination of scalar integrals using the Passarino-Veltman reduction technique [8]. A linear combination of a restricted subset of integrals can be used to simplify any one-loop integral. Now let us investigate a general case of a one-loop tensorial integral for any function with N points and N propagators.

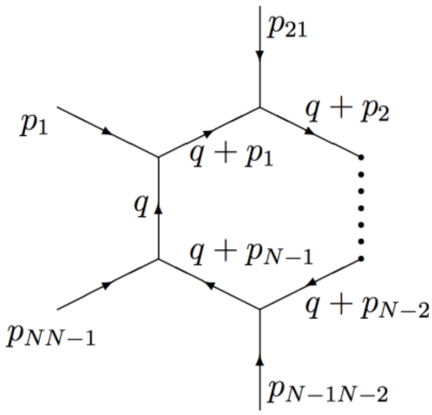


FIGURE 1.1: N-point integral (where q is the internal momentum.)

This expression can in general be written as,

$$\begin{aligned}&T^{n}_{\mu_1\ldots\mu_p}(p_1,\ldots p_{n-1},m_0,\ldots.,m_{n-1})\\&=\frac{(2\pi\mu)^{4-d}}{i\pi^2}\int\frac{N_{\mu_1\ldots.\mu_p}(k_i,m_i,p_j)}{(k_0^2-m_0^2+i\epsilon)(k_1^2-m_1^2+i\epsilon)\ldots.(k_{n-1}^2-m_{n-1}^2+i\epsilon)}\end{aligned} \tag{1.2}$$

with,

- $p_{ij} : p_i - p_j$
- $\mathcal{N}$: function dependent on internal and external momenta.
- n : number of internal lines
- k_i : ith propagator momentum
- p_j : external momentum of the jth particle
- d no. of dimensions
- m_i : ith propagator mass
- $i\epsilon$ is an infinitesimal displacement into the complex plane

- parameter μ has mass dimension and allows the dimensionality of integral.

Nomenclature

$$A^{(i)}_{(0,\mu)} = A_{0,\mu}(k_i^2; m_{i+1}^2) \tag{1.3a}$$
$$\equiv \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d q\{1, q_\mu\}}{D_i}, i = 0,1,2 \tag{1.3b}$$

$$B^{(i)}_{0,\mu,\mu\nu} = B_{0,\mu}(k_i^2; m_1^2, m_{i+1}^2) \tag{1.4a}$$
$$\equiv \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d q\{1, q_\mu, q_{\mu\nu}\}}{D_0 D_i}, i = 1,2 \tag{1.4b}$$

$$C_{0,\mu,\mu\nu} = C_{0,\mu,\mu\nu}(p_1^2, p_2^2, (p_1+p_2)^2; m_1^2, m_2^2, m_3^2) \tag{1.5a}$$
$$= \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \frac{d^d q\{1, q_\mu, q_{\mu\nu}\}}{D_0 D_1 D_2} \tag{1.5b}$$

where, $d = 4 - 2\epsilon$ $(\epsilon \to 0)$ is the integral dimension,
$D_i = (q + k_i)^2 - m_{i+1}^2$, $k_0 = 0$, $k_1 = -p_1$, $k_2 = -(p_1 + p_2)$, $i = 0,1,2$.

Chapter 2

Electron g-2

2.1 Introduction

The precision measurement of the electron $g-2$ has been a subject of profound theoretical and experimental interest due to its remarkable sensitivity to the contributions from quantum field theory and the possibility of revealing new physics beyond the Standard Model.

We embark on a comprehensive study and computational analysis aimed at calculating the electron $g-2$ using the framework of form factors. It play a pivotal role in describing the internal structure of particles and their response to electromagnetic interactions.

The goal is to provide a comprehensive insight into the calculation of electron $g-2$ using cutting-edge software like FeynArts and FeynCalc then compare it with the analytic result.

Electron's magnetic moment $g-2$ is quantified by the deviation of electron g-factor from its classical value of 2. In other words, the $g-2$ factor represents the deviation of electron's magnetic moment from the Dirac value, g = 2, due to quantum effects.

2.1.1 Form factors

Form factors are electromagnetic current matrix elements. Suppose we want to evaluate the matrix-element of electromagnetic current operator between two physical or on-shell states of a fermion with charge eQ.
we have,

$$\langle \vec{p}\,', s'|j_\mu(x)|\vec{p}, s\rangle = e^{-iq.x}\langle \vec{p}\,', s'|j_\mu(0)|\vec{p}, s\rangle \tag{2.1}$$

where,

$$q = p - p' \tag{2.2}$$

which is the transferred momentum.
The matrix element of $j_\mu(0)$ is parameterized such that,

$$\langle \vec{p}\,', s'|j_\mu(x)|\vec{p}, s\rangle = \frac{e^{-iq.x}}{\sqrt{2E_p V}\sqrt{2E_{p'} V}}\bar{u}_{s'}(\vec{p}\,')e\Gamma_\mu(p, p')u_s(\vec{p}) \tag{2.3}$$

where terms dependent on specific particle states have been dumped into Γ_μ which is also called the vertex function.

The most general parametrization consistent with gauge invariance is given by

$$\Gamma_\mu = \gamma_\mu F_1 + (iF_2 + \tilde{F}_2\gamma_5)\sigma_{\mu\nu}q^\nu + \tilde{F}_3(q_\mu \not{q} - q^2\gamma_\mu)\gamma_5 \tag{2.4}$$

where, F_s are known as "Form Factors".
It provides an insight into the internal structure and distribution of charge and other properties within the particle.
Since, QED is parity conserving, terms containing γ_5 will be zero.
hence,

$$\Gamma_\mu = \gamma_\mu F_1 + iF_2\sigma_{\mu\nu}q^\nu \tag{2.5}$$

2.1.2 Charge form factor F_1

Let us consider the situation where $\vec{p} = \vec{p}\,'$ and $s = s'$, i.e. the initial and final states of the fermions are same. Hence we have $q^2 = 0$. One now obtains,

$$\langle \vec{p}, s | j_\mu(x) | \vec{p}, s \rangle = \frac{eF_1(0)}{2E_p V}\bar{u}_s(\vec{p})\gamma_\mu u_s(\vec{p}) \tag{2.6a}$$

$$= \frac{eF_1(0)p^\mu}{E_p V} \tag{2.6b}$$

This expression is like a classical current. $F_1(q^2)$ is often called the *charge form factor*. The magnetic moment, coming from the charge form factor is usually called the *Dirac magnetic moment*, denoted as,

$$\vec{\mu}_D = \frac{2eQ}{2m}\vec{S} \tag{2.7}$$

Usually, the magnetic moment is expressed in terms of the Landé g-factor, which is defined by

$$\vec{\mu} = \frac{eq}{2m}g\vec{S} \tag{2.8}$$

where eQ is the charge and m is the mass of particle. Hence, comparing 2.7 and 2.8, we get the following contribution to the g-factor g_D = 2.

2.1.3 Anomalous magnetic moment F_2

There is another contribution to the magnetic moment coming from the form factor F_2. This is called the *anomalous magnetic moment*, denoted as,

$$\vec{\mu}_A = -eF_2(0)\vec{\sigma} \tag{2.9a}$$

$$= -2eF_2(0)\vec{S} \tag{2.9b}$$

where $\vec{S} = \frac{\vec{\sigma}}{2}$ is the spin vector for the particles.
Landé g-factor is thus obtained by summing the two contributions:

$$\vec{\mu} = \vec{\mu}_A + \vec{\mu}_D \tag{2.10a}$$

$$\frac{eQ}{2m} g\vec{S} = -2eF_2(0)\vec{S} + \frac{2eQ}{2m}\vec{S} \tag{2.10b}$$

from here,

$$g = 2 - \frac{4m}{Q} F_2(0) \tag{2.11}$$

2.2 Analytic calculation

The analytic calculation of anomalous magnetic moment of the electron (1-loop correction) is as follows.
Following represents the Feynman graph for vertex function.

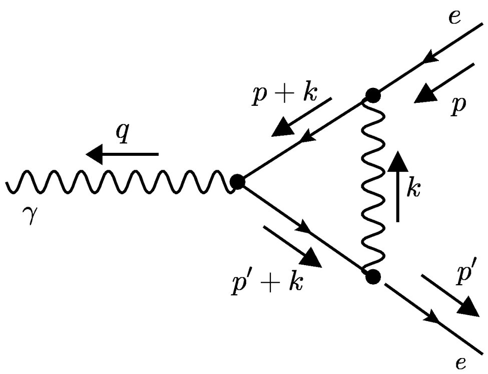


FIGURE 2.1: Vertex diagram contributing to electron g factor.

Applying the Feynman rules,

$$-ie\Gamma_\mu = \int \frac{d^4k^4}{2\pi} (-ie\gamma_\lambda) \frac{(\not{p}' + \not{k} + m)}{[(\not{p}' + k)^2 - m^2]} (-ie\gamma_\mu) \frac{(\not{p} + \not{k} + m)}{[(\not{p} + k)^2 - m^2]} \tag{2.12a}$$

$$\times (-ie\gamma_\rho) \frac{(-ig^{\lambda\rho})}{k^2} \tag{2.12b}$$

$$= ie^2 \int \frac{d^k}{(2\pi)^4} \frac{\gamma_\lambda(\not{p}' + \not{k} + m)\gamma_\mu(\not{p} + \not{k} + m)\gamma^\lambda}{[[(\not{p}' + k)^2 - m^2][(\not{p} + k)^2 - m^2]k^2]} \tag{2.12c}$$

the denominator is,

$$\frac{1}{D} = \frac{1}{(\not{p}' + k)^2 - m^2][(\not{p} + k)^2 - m^2]k^2} \tag{2.13}$$

Using Feynman parametrization,

$$\frac{1}{abc} = 2\int_0^1 d\eta_1 \int_0^1 d\eta_2 \int_0^1 d\eta_3 \delta(1-\eta_1-\eta_2-\eta_3)\frac{1}{[D]^3} \tag{2.14}$$

Simplifying D,

$$D = k^2 + 2k(\eta_1 p' + \eta_2 p) \tag{2.15}$$

let,

$$k' = k + \eta_1 p' + \eta_2 p \tag{2.16a}$$
$$D = k'^2 - (\eta_1 p' + \eta_2 p)^2 \tag{2.16b}$$

eq(2.12) simplifies to

$$\begin{aligned}\Gamma_\mu =& 2ie^2 \int \frac{d^4k'}{(2\pi)^4} \int_0^1 d\eta_1 \int_0^1 d\eta_2 \int_0^1 d\eta_3 \delta(1-\eta_1-\eta_2-\eta_3) \\ &\times \frac{N_\mu(k'-\eta_1 p' - \eta_2 p)}{[k'^2 - (\eta_1 p' + \eta_2 p)^2]^3}\end{aligned} \tag{2.17}$$

Since the denominator is spherically symmetric, we have

$$\int d^4k \frac{k^\mu}{(k^2+s+i\epsilon)^n} = 0, n \geq 3 \tag{2.18}$$

hence,

$$\Gamma_\mu = 8ie^2\alpha \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\eta_1 \int_0^1 d\eta_2 \int_0^1 d\eta_3 \delta(1-\eta_1-\eta_2-\eta_3) \tag{2.19a}$$
$$\times \frac{N_\mu}{[k^2 - (\eta_1 p' + \eta_2 p)^2]^3} \tag{2.19b}$$
$$= 8\pi i\alpha \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\eta_1 \int_0^1 d\eta_2 \times \theta(1-\eta_1-\eta_2) \times \frac{N_\mu}{D^3} \tag{2.19c}$$

Simplifying eq((2.16)),

$$D = k^2 - (\eta_1 p + \eta_2 p)^2 \tag{2.20a}$$
$$= k^2 - (\eta_1+\eta_2)^2 m^2 + \eta_1\eta_2 q^2 \tag{2.20b}$$
$$q^2 = 2m^2 - 2p.p' \tag{2.20c}$$

Simplifying the numerator part,

$$N_\mu = \gamma_\lambda \left[\not{p}' + (\not{k} - \eta_1 \not{p}' - \eta_2 \not{p} + m)\gamma_\mu(\not{p} + (\not{k} - \eta_1 \not{p}_1 - \eta_2 \not{p}) + m)\right]\gamma^\lambda \tag{2.21}$$

let,

$$a_\mu = (1-\eta_1)p'_\mu - \eta_2 p_\mu \tag{2.22a}$$
$$b_\mu = (1-\eta_2)p_\mu - \eta_1 p'_\mu \tag{2.22b}$$

$$N_\mu = \gamma_\lambda \left[(\not{a} + \not{k} + m)\gamma_\mu(\not{b} + \not{k} + m) \right] \gamma^\lambda \tag{2.23}$$

Linear terms will integrate to zero and terms containing γ_μ will contribute to F_1(i.e. charged form factors) and not to F_2(i.e. anomalous magnetic moment) form factors. Thus, we are left with,

$$N_\mu = \gamma_\lambda \not{a}\gamma_\mu \not{b}\gamma^\lambda + m\gamma_\lambda(\not{a}\gamma_\mu + \gamma_m u\not{b})\gamma^\lambda \tag{2.24a}$$
$$N_\mu = -2\not{b}\gamma_\mu\not{a} + 4m(a_\mu + b_\mu) \tag{2.24b}$$

2^{nd} term can be rewritten as,

$$4m(a_\mu + b_\mu) = 4m(1-\eta_1-\eta_2)(p+p')_\mu \tag{2.25}$$

Now, the entire numerator is sandwiched between $\bar{u}$ and u spinor,
Using Gordon's Identity,

$$u(\bar{p'})\gamma^\mu u(p) = \frac{1}{2m} u(\bar{p'})[(p+p')^\mu - i\sigma^{\mu\nu}q_\nu]u(p) \tag{2.26}$$

equation (14) simplifies to,

$$4m(a_\mu + b_\mu) = 4m(1-\eta_1-\eta_2)[2m\gamma_\mu + i\sigma_{\mu\nu}q^\nu] \tag{2.27}$$

only $i\sigma_{\mu\nu}q^\nu$ will contribute to $F_2(0)$
Similarly, the first can be simplified to,(substituting b and a),

$$-2\not{b}\gamma_\mu\not{a} = -4m(1-\eta_1-\eta_2)(1-\eta_1)i\sigma_{\mu\nu}q^\nu \tag{2.28}$$

Finally, we have the numerator part,

$$N_\mu = 4m(1-\eta_1-\eta_2)(i\sigma_{\mu\nu}q^\nu)\eta_1 \tag{2.29}$$

equation (2.19) becomes,

$$\Gamma_\mu = 8\pi i\alpha \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\eta_1 \int_0^{1-\eta_1} d\eta_2 \times \frac{4m\eta_1(1-\eta_1-\eta_2)}{[k^2 - (\eta_1+\eta_2)^2 m^2 + \eta_1\eta_2 q^2]^3} \tag{2.30}$$

Taking, $q^2 \to 0$

$$\Gamma_\mu = 8\pi i\alpha \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\eta_1 \int_0^{1-\eta_1} d\eta_2 \times \frac{4m\eta_1(1-\eta_1-\eta_2)}{[k^2 - (\eta_1+\eta_2)^2 m^2]^3} \tag{2.31}$$

using the formula,

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-s)^n} = \frac{(-1)^n}{(4\pi)^{d/2}} i \frac{\Gamma(n-d/2)}{\Gamma(n)} \left(\frac{1}{s}\right)^{n-d/2} \tag{2.32}$$

we obtain,

$$F_2(0) = \frac{\alpha}{\pi m} \int_0^1 d\eta_1 \int_0^{1-\eta_1} d\eta_2 \frac{(1-\eta_1\eta_2)\eta_1}{(\eta_1+\eta_2)^2} \tag{2.33}$$

Finally, we get,

$$F_2(0) = \frac{\alpha}{4\pi m} \tag{2.34}$$

Now,

$$g = 2 - \frac{4m}{Q} F_2(0) \tag{2.35a}$$

$$g = 2 + \frac{\alpha}{\pi} > 2 \tag{2.35b}$$

This is generally quoted in the form,

$$a_e = \frac{g_s^e - 2}{2} \tag{2.36a}$$

$$= \frac{\alpha}{2\pi} \tag{2.36b}$$

$$= 0.00116 \tag{2.36c}$$

This result, first derived by Schwinger in 1948 [9], is in excellent agreement with the first measurements by Kusch and Foley (1947,1948) [10] who obtained the value,

$$a_e^{exp} = 0.00119 \pm 0.00005 \tag{2.37}$$

2.2.1 Conclusion

Till now the electron $g-2$ have been measured experimentally very precisely, offering rigorous tests for the accuracy of theoretical predictions based on QED. Any deviation between the theory and experiment signals towards new physics beyond the SM. As of now the the coefficient of analytic magnetic moment of the electron have been evaluated up to α^5 i.e. $a_e = 0.001\,159\,652\,181\,643(764)$ [11]. The QED prediction agrees with the experimentally measured value to more than 10 significant figures.

Another interesting anomalous magnetic moment is of muon, which includes three parts.

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{hadron}} \tag{2.38}$$

The theoretical results disagrees with the experiment. As of July 2017, the value disagrees with the SM, suggesting new physics beyond the SM.

Chapter 3

Reproducing the results for $H \to \gamma\gamma$ and $H \to Z\gamma$

3.1 $H \to \gamma\gamma$

3.1.1 Introduction

Once the discovery of the Higgs boson at the Large Hadron Collider (LHC) confirmed the mechanism of electroweak symmetry breaking as predicted by the Standard Model, extensive measurements were undertaken to validate its quantum properties and interactions, including its couplings. Among these, the coupling of the Higgs boson to photons holds particular interest for several reasons. Primarily, direct couplings between the Higgs boson and photons do not occur at the tree level due to the massless-ness of photons. Consequently, the $H \to \gamma\gamma$ decay mode emerges as a primary way for investigating the Higgs boson existence and properties at CERN, LHC.

3.1.2 Lagrangian

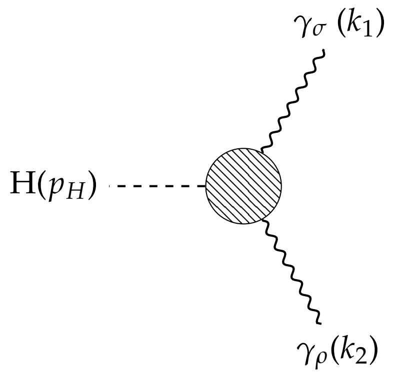
The $H \to \gamma\gamma$ one-loop corrections involves W bosons, fermions (primarily top quark) in SM [12], and charged scalar induced loops in BSM.

The effective Lagrangian for the $H \to \gamma\gamma$ interaction can be written as follows

$$\mathcal{L} = \frac{1}{4} b F_{\mu\nu} F^{\mu\nu} H \tag{3.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, is the field strength tensor.
b is a constant which contains information about the couplings, A_μ is the photon field, and H is the Higgs field.

The Feynman diagram is (with all momentum incoming)



The Feynman rules for the above lagrangian can be calculated as follows,

$$\mathcal{L} = \frac{1}{4} b[(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)]H \tag{3.2a}$$

$$= \frac{1}{4} b[\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu - \partial_\nu A_\mu \partial^\mu A^\nu + \partial_\nu A_\mu \partial^\nu A^\mu]H \tag{3.2b}$$

$$= \frac{1}{2} b[\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu]H \tag{3.2c}$$

where, I have changed the dummy indices in third and fourth term of (3.2b). Now, the photon field is defined as

$$A_\mu = \frac{1}{(2\pi)^{3/2}\sqrt{2E_p}} \int d^3p \sum_{\lambda=1}^{3} \left(\epsilon_\lambda^\mu(p) \hat{a}_{p\lambda} e^{-ipx} + \epsilon_\lambda^{\mu *}(p) \hat{a}_{p\lambda}^\dagger e^{ipx} \right), \tag{3.3}$$

where, $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operators, and since at the point of interaction two photons are being created we have,

$$\partial^\mu A^\nu = \partial^\mu g^{\nu\rho} A_\rho \tag{3.4a}$$

$$= g^{\nu\rho} \partial^\mu A_\rho \tag{3.4b}$$

$$= g^{\nu\rho} (ip^\mu) A_\rho \tag{3.4c}$$

The gauge invariant amplitude for $H \to \gamma\gamma$ is generally given by

$$M = \mathcal{M}^{\mu\nu} \epsilon_\mu(p_1) \epsilon_\nu(p_2) \tag{3.5}$$

Since, the decaying particles are same we need to multiply with the symmetry factor 2

$$\mathcal{L} = b[(\partial_\mu g_{\nu^\sigma} A_\sigma)(\partial^\mu g^{\nu\rho} A_\rho) - (\partial_\mu g_{\nu^\sigma} A_\sigma)(\partial^\nu g^{\mu\rho} A_\rho)]H \tag{3.6a}$$

$$= b[(g_\nu^\sigma (ik_{1\mu}) A_\sigma)(g^{\nu\rho}(ik_2^\mu) A_\rho) - (g_{\nu^\sigma}(ik_{1\mu}) A_\sigma)(g^{\mu\rho}(ik_2^\nu) A_\rho)]H \tag{3.6b}$$

$$= bA_\rho[-(k_1.k_2) g^{\rho\sigma} + (k_1^\sigma k_2^\rho)] A_\sigma H \tag{3.6c}$$

since, photon are massless, the kinematics are

$$p_H = k_1 + k_2, {k_1}^2 = {k_2}^2 = 0, k_1.k_2 = \frac{m_H^2}{2} \tag{3.7}$$

$$\mathcal{M}^{\rho\sigma} = ib\left[k_1^{\sigma}k_2^{\rho} - (k_1.k_2)g^{\rho\sigma}\right] \tag{3.8a}$$
$$i\mathcal{M} = ib\left[k_1^{\sigma}k_2^{\rho} - (k_1.k_2)g^{\rho\sigma}\right]\epsilon^*_{1\sigma}\epsilon^*_{2\rho} \tag{3.8b}$$

using (3.7),

$$i\mathcal{M} = i\frac{b}{2}\left[2(k_1.\epsilon^*(k_2))(k_2.\epsilon^*(k_1)) - m_H^2(\epsilon^*(k_1).\epsilon^*(k_2))\right] \tag{3.9}$$

3.1.3 Total decay rate

The total decay rate for $H \to \gamma\gamma$ is as follows
using 3.8b, $|\mathcal{M}|^2$ is,

$$|\mathcal{M}|^2 = b^2\left[k_1^{\sigma}k_2^{\rho} - (k_1.k_2)g^{\rho\sigma}\right]\left[k_1^{\eta}k_2^{\delta} - (k_1.k_2)g^{\eta\delta}\right](\epsilon_1^{*\sigma}\epsilon_2^{*\rho})(\epsilon_1^{*\delta}\epsilon_2^{*\eta}) \tag{3.10}$$

for photons,

$$\Sigma = \epsilon_\rho^r(k_1)\epsilon_\sigma^s(k_1) = -g^{\rho\sigma} \tag{3.11}$$

$$|\mathcal{M}|^2 = b^2\left[k_1^{\sigma}k_2^{\rho} - (k_1.k_2)g^{\rho\sigma}\right]\left[k_1^{\eta}k_2^{\delta} - (k_1.k_2)g^{\eta\delta}\right]g_{\rho\eta}g_{\sigma\delta} \tag{3.12a}$$
$$= b^2\left[k_1^2k_2^2 - m_H^2(k_1.k_2) + m_H^4\right] \tag{3.12b}$$
$$= \frac{b^2}{2}m_H^4 \tag{3.12c}$$

The decay rate can be calculated by integrating the phase space.

In the rest frame of reference

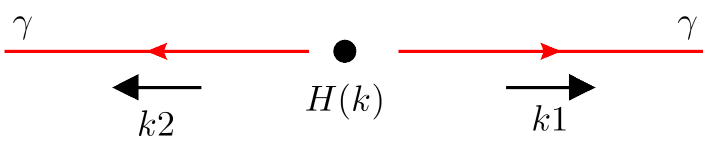


FIGURE 3.1: Decay kinematics for $H \to \gamma\,\gamma$

where $k_1 = (E = 0, \vec{p}\hat{z})$, $(k = k_1 + k_2)$, $k_2 = (E' = 0, \vec{p'}\hat{z})$

$$\Gamma = \frac{1}{2E}\Pi_f \int \frac{d^3p_f}{(2\pi^3)2E_f}(2\pi^4)\delta^4(p_i - \Sigma p_f)|\mathcal{M}_{\text{fi}}|^2 \tag{3.13}$$

let,

$$\rho = \int \frac{d^3p}{(2\pi)^3 2E}\int \frac{d^3p'}{(2\pi^3)2E'}(2\pi^4)\delta^4(p_i - \Sigma p_f) \tag{3.14}$$

$$\delta^4(k-p-p') = \delta(k^0-p^0-p'^0)\delta^3(\vec{0}-\vec{p}-\vec{p}') \tag{3.15}$$

the δ^3 implies, $p = p'$

$$\rho = \int \frac{d^3p}{4EE'}\frac{1}{(2\pi)^2}\delta(k^0-p^0-p'^0) \tag{3.16}$$

such that,

$$k_0 = M, p_0 = p'_0 = 0, E = E' \tag{3.17}$$

Let, $p_0 = p'_0 = E$ for simplicity and later take E $\to 0$

$$\rho = \frac{1}{(2\pi)^2}\int \frac{d^3p}{4E^2}\delta(M-2E) \tag{3.18a}$$

$$= \frac{1}{(2\pi)^2}\int \frac{d^3p}{4E^2}\frac{1}{2}\delta\left(E-\frac{M}{2}\right) \tag{3.18b}$$

let

$$pdp = EdE \tag{3.19}$$

$$\begin{aligned} \rho &= \frac{1}{\pi}\int \frac{dpp^2}{4E^2}\frac{1}{2}\delta\left(E-\frac{M}{2}\right) \\ \rho &= \frac{1}{8\pi}\int \frac{E^2dE}{E^2}\delta\left(E-\frac{M}{2}\right) \end{aligned} \tag{3.20}$$

$$\rho = \frac{1}{8\pi} \tag{3.21}$$

The phase space factor has to be multiplied by $\frac{1}{n!}$ if there are n identical particles in the final state.

$$\Gamma = \frac{1}{64\pi m_H}b^2 m_H^4 \tag{3.22}$$

The decay rate, since m$\to 0$ i.e. photons are massless,

$$\Gamma(H\to\gamma\gamma) = \frac{1}{64\pi}b^2 m_H^3 \tag{3.23}$$

Now, we have to find the value of b which I have calculated using FeynCalc where

$$b = b_W + b_t + b_s\,, \tag{3.24}$$

where b_W refers to contributions from W-loop, b_t fermionic contributions and b_s scalar-loop contributions.

The known results for $H\gamma\gamma$ is [13]

$$i\mathcal{M} = \frac{\alpha g}{4\pi m_W}\left|F_W(\tau_W) + \frac{4}{3}F_t(\tau_f) + \kappa F_s(\tau_s)\right| \tag{3.25}$$

and

$$\mathcal{M}^2 = \frac{\alpha g}{4\pi m_W}\left|F_W(\tau_W) + \frac{4}{3}F_t(\tau_f) + \kappa F_s(\tau_s)\right|^2 \tag{3.26}$$

$$\tau_W = \frac{4m_W^2}{m_H^2}, \tau_f = \frac{4m_f^2}{m_H^2}, \tau_s = \frac{4m_s^2}{m_h^2}, \kappa = 1 \tag{3.27}$$

and some known functions [13]

$$\mathcal{F}_W(\tau_W) = 2 + 3\tau_W + 3\tau_W(2-\tau_W)f(\tau_W) \tag{3.28}$$

$$\mathcal{F}_f(\tau_f) = -2\tau_f[1 + (1-\tau_f)f(\tau_f)] \tag{3.29}$$

$$\mathcal{F}_s(\tau_s) = -\tau_s[1 - \tau_s f(\tau_s)] \tag{3.30}$$

where,

$$f(\tau) = \begin{cases} \arcsin^2\left(\sqrt{\frac{1}{\tau}}\right), & \text{for } \tau \geq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2, & \text{for } \tau < 1 \end{cases} \tag{3.31}$$

The standard expression for the diphoton decay width is given by [14]:

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 g^2}{2^{10}\pi^3}\frac{m_h^3}{M_W^2}\left|\mathcal{F}_W + \frac{4}{3}\mathcal{F}_t + \kappa_i\mathcal{F}_s\right|^2 \tag{3.32}$$

The above equations 3.8a and (3.23) can be verified using FeynRules.

3.1.4 Model file

The model file for the above Lagrangian is as follows,

```
M$ModelName = "H␣gamma␣gamma␣Lagrangian";

M$Information = {Author -> {"Sunny"}};
(********** Classes ***********)
M$ClassesDescription = {
    V[1] == {
        ClassName -> A,
        SelfConjugate -> True,
        Mass -> 0,
        Width -> 0,
        ParticleName -> "a",
        PropagatorLabel -> "a",
        PropagatorType -> W,
        PropagatorArrow -> None,
        FullName -> "Photon"
        },
    S[1] == {
        ClassName -> H,
        SelfConjugate -> True,
        Mass -> {MH,125},
        Width -> {WH,0.00407},
        PropagatorLabel -> "H",
        PropagatorType -> D,
        PropagatorArrow -> None,
        ParticleName -> "H",
        FullName -> "H"
        }
};
(*************** Parameters *******************)
M$Parameters = {
    b == {
        ParameterType -> External,
        Value -> 1
        }
};
(************** Lagrangian ********************)

L = 1/4 b FS[A, mu, nu] FS[A, mu, nu] H
```

H-> $\gamma\gamma$ Lagrangian

In[1]:=
```
$FeynRulesPath = SetDirectory["/home/sunny/Documents/Software/FeynRules"]
```
Out[1]=
```
/home/sunny/Documents/Software/FeynRules
```

In[2]:=
```
<< FeynRules`
```

In[3]:=
```
SetDirectory[$FeynRulesPath <> "/Models/SMHgg"]
```
Out[3]=
```
/home/sunny/Documents/Software/FeynRules/Models/SMHgg
```

In[4]:=
```
LoadModel["SMHgg.fr"]
```
```
 - Loading particle classes.
 - Loading parameter classes.

Model H gamma gamma Lagrangian loaded.
```

In[5]:=
```
vertices = FeynmanRules[L]
```
```
Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
1 possible non-zero vertices have been found -> starting the computation: 1 / 1.
1 vertex obtained.
```
Out[5]=

$\{\{\{\{A, 1\}, \{A, 2\}, \{H, 3\}\}, i\, b\, p_1^{\mu_2}\, p_2^{\mu_1} - i\, b\, \eta_{\mu_1,\mu_2}\, p_1 . p_2\}\}$

The result agrees with eq (6).

3.1.6 Results for $H \to \gamma\gamma$

The contribution to decay of $H \to \gamma\gamma$ consists of top loop, W-loop and charged scalars. Using FeynCalc one can evaluate the $\Gamma(H \to \gamma\gamma)$. The results for the given decay are,

W-loop contribution

The W-loop contributing diagrams at one-loop level are

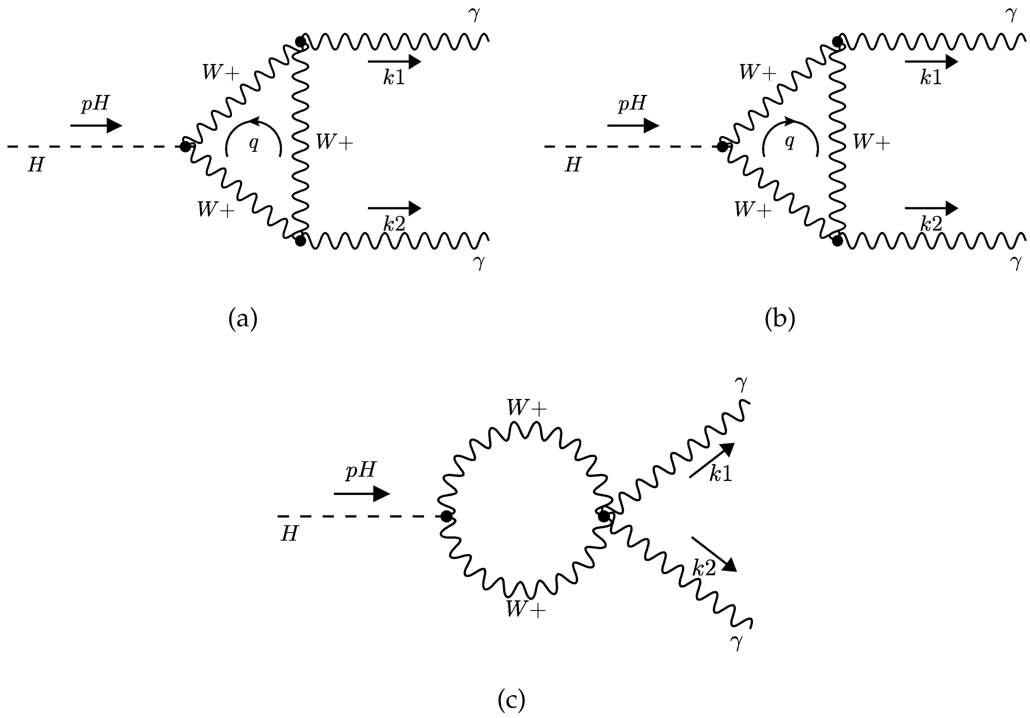


FIGURE 3.2: W-loop contributing diagrams

The amplitude obtained using FeynCalc package for the case of $H \to \gamma\gamma$ in terms of PV functions reads,

$$\begin{aligned} i\mathcal{M} = {} & \frac{ie^3}{16\pi^2 m_H^2 m_W \sin(\theta_W)} \left(m_H^2 \left(6m_W^2 \mathrm{C}_0 \left(0,0,m_H^2,m_W^2,m_W^2,m_W^2\right) - 1 \right) \right. \\ & - 6\left(2m_W^4 \mathrm{C}_0 \left(0,0,m_H^2,m_W^2,m_W^2,m_W^2\right) + m_W^2 \right) \\ & \left. \times \left(2\left(\mathrm{k1}\cdot\varepsilon^*(\mathrm{k2})\right)\left(\mathrm{k2}\cdot\varepsilon^*(\mathrm{k1})\right) - m_H^2 \left(\varepsilon^*(\mathrm{k1})\cdot\varepsilon^*(\mathrm{k2})\right) \right) \right) \end{aligned} \tag{3.33a}$$

Using FeynHelpers, one can evaluate the PV function and quote the results in terms of $\tau_W = \frac{4m_W^2}{m_H^2}$ as

$$
\begin{aligned}
i\mathcal{M} = &- \frac{ie^3}{128\pi^2 m_W \sin(\theta_W)} \\
&\times \left(12\tau + \frac{3(\tau-2)\tau\left(\left(\tau + 2\sqrt{\frac{1}{\tau}-1}\sqrt{\tau}-2\right)\log^2\right)}{\tau} + 8\right) \\
&\times \left(2\left(\mathrm{k1}\cdot\varepsilon^*(\mathrm{k2})\right)\left(\mathrm{k2}\cdot\varepsilon^*(\mathrm{k1})\right) - m_H^2\left(\varepsilon^*(\mathrm{k1})\cdot\varepsilon^*(\mathrm{k2})\right)\right)
\end{aligned}
\tag{3.34}
$$

Comparing to 3.9 equation,

$$
\begin{aligned}
\frac{b_W}{2} = &- \frac{ie^3}{128\pi^2 m_W \sin(\theta_W)} \\
&\times \left(12\tau + \frac{3(\tau-2)\tau\left(\left(\tau + 2\sqrt{\frac{1}{\tau}-1}\sqrt{\tau}-2\right)\log^2\right)}{\tau} + 8\right)
\end{aligned}
\tag{3.35}
$$

3.35 is equivalent to 3.25 (W-loop contribution).

Fermion-loop contributions

The fermionic one-loop diagrams are

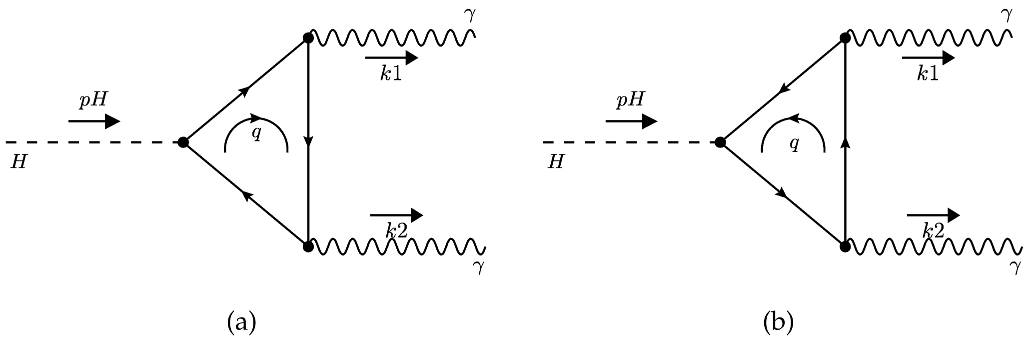


FIGURE 3.3: Fermionic diagrams

The fermionic contribution reads,

$$
\begin{aligned}
i\mathcal{M} = &\frac{ie^3 m_t^2}{6\pi^2 m_H^2 m_W (\sin(\theta_W))} \\
&\times \left((m_H^2 - 4m_t^2)C_0\left(0,0,m_H^2,m_t^2,m_t^2,m_t^2\right) - 2\right. \\
&\times \left.\left(2(\mathrm{k1}\cdot\varepsilon^*(\mathrm{k2}))(\mathrm{k2}\cdot\varepsilon^*(\mathrm{k1})) - m_H^2(\varepsilon^*(\mathrm{k1})\cdot\varepsilon^*(\mathrm{k2}))\right)\right)
\end{aligned}
\tag{3.36}
$$

$$
\begin{aligned}
i\mathcal{M} = &- \frac{ie^3\tau}{48\pi^2 m_W \sin(\theta_W)} \\
&\times \left(4 + \frac{(\tau-1)\left((\tau + 2i\sqrt{\tau-1} - 2)\log^2\right)}{\tau}\right) \\
&\quad \times \left(2(\mathrm{k1}\cdot\varepsilon^*(\mathrm{k2}))(\mathrm{k2}\cdot\varepsilon^*(\mathrm{k1})) - m_H^2(\varepsilon^*(\mathrm{k1})\cdot\varepsilon^*(\mathrm{k2}))\right)
\end{aligned}
\tag{3.37}
$$

Comparing to 3.9,

$$
\begin{aligned}
\frac{b_t}{2} = &- \frac{ie^3\tau}{48\pi^2 m_W \sin(\theta_W)} \\
&\times \left(4 + \frac{(\tau-1)\left((\tau + 2i\sqrt{\tau-1} - 2)\log^2\right)}{\tau}\right)
\end{aligned}
\tag{3.38}
$$

One can show that 3.38 is equivalent to 3.25 (top-loop contribution) by plotting both the equations as a function of τ and plotting their ratios.

Scalar-loop contribution

The scalar contributing diagrams are

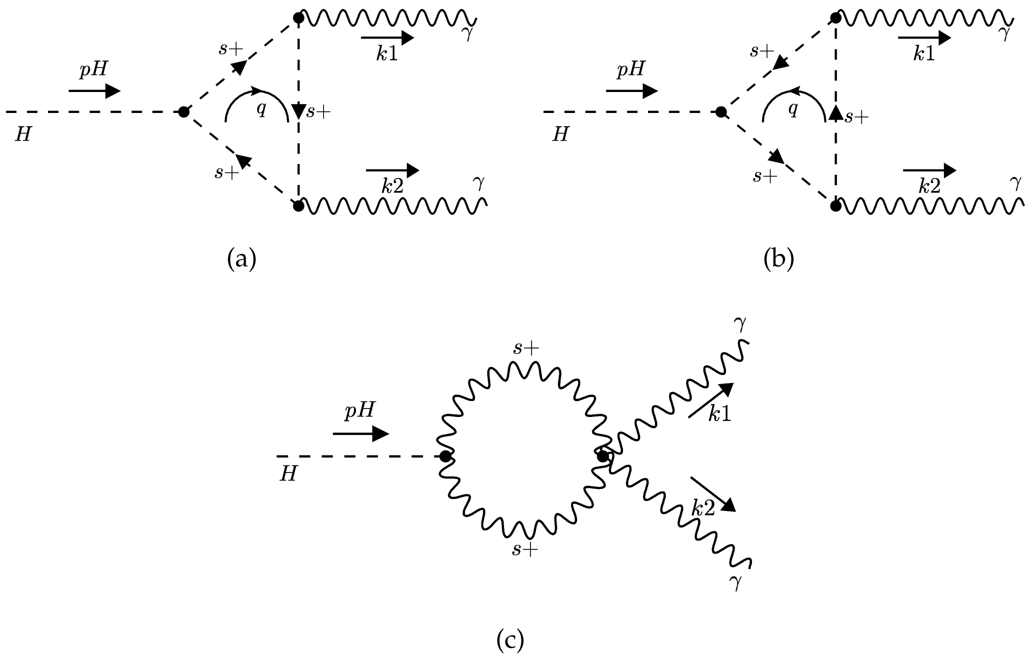


FIGURE 3.4: Scalar contributing diagrams

The amplitude for this process is

$$
\begin{aligned}
i\mathcal{M} = &-\frac{i\alpha^{3/2}m_s^2}{\sqrt{\pi}m_H^2 m_W(\sin(\theta_W))} \\
&\times \left(2m_s^2 \mathrm{C}_0\left(0,0,m_H^2,m_s^2,m_s^2,m_s^2\right)+1\right. \\
&\times \left.\left(m_H^2(\varepsilon^*(\mathrm{k1})\cdot\varepsilon^*(\mathrm{k2})) - 2(\mathrm{k1}\cdot\varepsilon^*(\mathrm{k2}))(\mathrm{k2}\cdot\varepsilon^*(\mathrm{k1}))\right)\right)
\end{aligned}
\tag{3.39}
$$

or

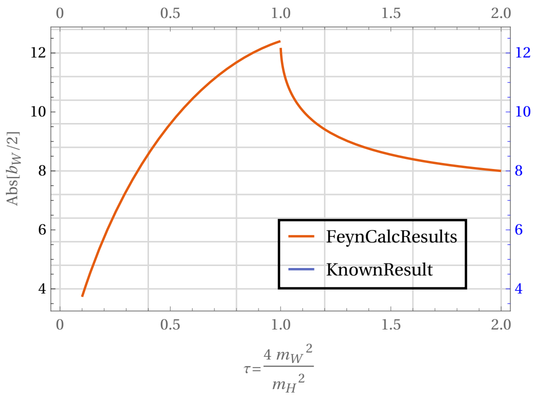
$$
\begin{aligned}
i\mathcal{M} = &-\frac{i\alpha^{3/2}m_s^2}{\sqrt{\pi}m_H^4 m_W \sin(\theta_W)} \\
&\times \left(m_H^2 + \frac{m_s^2 \log^2\left(-\frac{Im_H\sqrt{4m_s^2-m_H^2}+m_H^2-2m_s^2}{2m_s^2}\right)}{2m_s^2} \right) \\
&\times \left(m_H^2(\varepsilon^*(\mathrm{k1})\cdot\varepsilon^*(\mathrm{k2})) - 2(\mathrm{k1}\cdot\varepsilon^*(\mathrm{k2}))(\mathrm{k2}\cdot\varepsilon^*(\mathrm{k1}))\right)
\end{aligned}
\tag{3.40}
$$

Comparing to 3.9

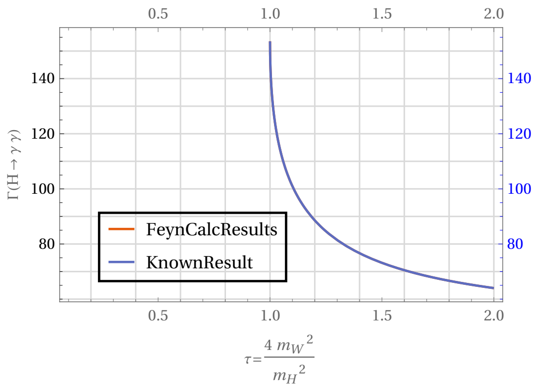
$$\frac{b_s}{2} = \frac{i\alpha^{3/2} m_s^2}{\sqrt{\pi} m_H^4 m_W \sin(\theta_W)} \times \left(m_H^2 + \frac{m_s^2 \log^2\left(-\frac{I m_H \sqrt{4m_s^2 - m_H^2} + m_H^2 - 2m_s^2}{2m_s^2}\right)}{2m_s^2} \right) \tag{3.41}$$

3.41 is equivalent to scalar part of 3.25.

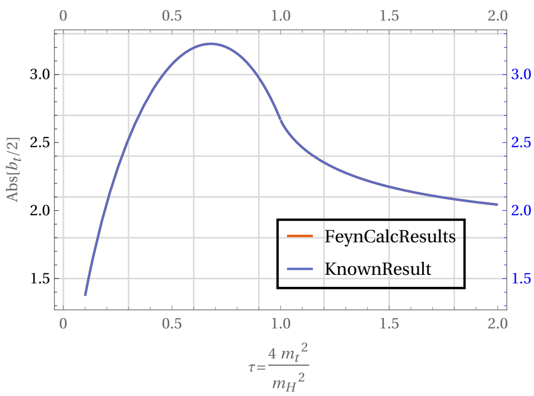
The above amplitudes and its decay rates are plotted against τ.



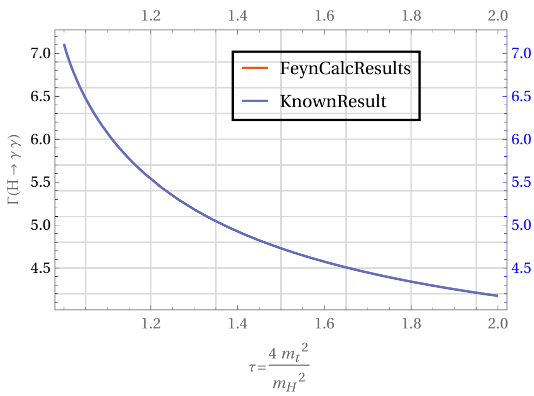
(a) b/2 vs $\tau = \frac{4m_W^2}{m_H^2}$ for W-loop



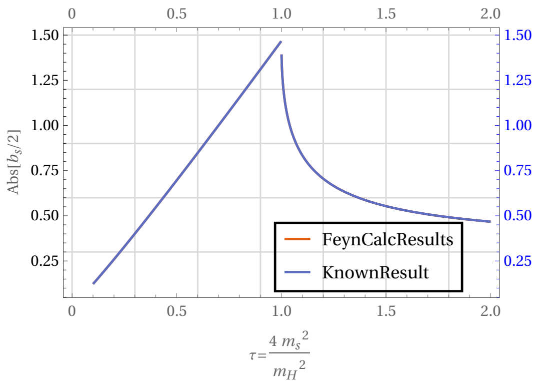
(b) $\Gamma(H \to \gamma\gamma)$ via W-loop



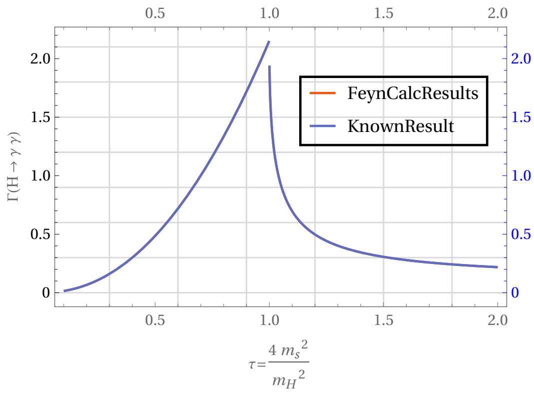
(c) b/2 vs $\tau = \frac{4m_t^2}{m_H^2}$ for top-loop



(d) $\Gamma(H \to \gamma\gamma)$ via top-loop



(e) b/2 vs $\tau = \frac{4m_s^2}{m_H^2}$ for scalar-loop



(f) $\Gamma(H \to \gamma\gamma)$ via scalar-loop

FIGURE 3.5: Results for $H \to \gamma\gamma$

In the above plots, the FeynCalc results matches with the known results i.e. they overlap.
One can find the large τ behavior for the above.
In the limit $\tau \to 0$, we have $F_s \to -\frac{1}{3}$, $F_t \to -\frac{4}{3}$, and $F_W \to 7$ which I have verified using FeynCalc.

3.2 $H \to Z\gamma$

3.2.1 Introduction

Another interesting channel of Higgs boson decay is Higgs decay to a Z boson and a photon. Recently, ATLAS and CMS collaboration jointly announced the first evidence of this rare Higgs boson decay channel [15]. It greatly deviates from the SM prediction. Same as $H\gamma\gamma$, there is no tree level coupling in $HZ\gamma$.

3.2.2 Lagrangian

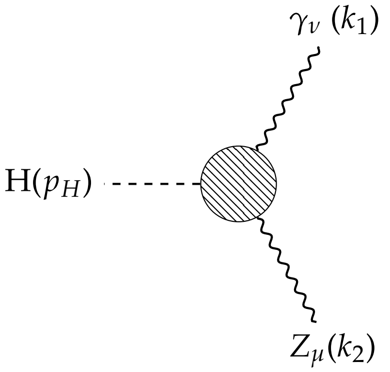
Same as $H \to \gamma\gamma$, for the case of $H \to Z\gamma$, the one loop corrections involves W bosons, fermions in SM, and charged scalars in BSM.
The effective lagrangian is

$$\mathcal{L} = \frac{1}{2} b Z_{\mu\nu} F^{\mu\nu} H \tag{3.42}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \;\; Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu,$

H is the Higgs field, b refers to form factor and, Z_μ is the Z-boson field.

The Feynman rule is,



Here, the kinematics are,

$$k_2{}^2 = m_Z^2, k_2{}^2 = 0, k_1.k_2 = \frac{m_H^2 - m_Z^2}{2} \tag{3.43}$$

The amplitude for the case of $H \to Z\gamma$ can be written as

$$\begin{aligned} \mathcal{M}(H \to Z\gamma) &\equiv \mathcal{M}\left(Z_\mu(k_2), \gamma_\nu(k_2), h(p_H)\right) \varepsilon_1^{\mu *}(k_1) \varepsilon_2^{\nu *}(k_2) \\ &\equiv \mathcal{M}_{\mu\nu} \varepsilon_1^{\mu *} \varepsilon_2^{\nu *}, \end{aligned} \tag{3.44}$$

where ε_1^μ and ε_2^ν are the polarization vectors of the photon and the Z boson γ, respectively. The decay amplitude is generally written in the following form [16]:

$$\mathcal{M}_{\mu\nu} \equiv F_{00}\, g_{\mu\nu} + \sum_{i,j=1}^{2} F_{ij} k_{i\mu} k_{j\nu} + F_5 \times i\epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta, \tag{3.45}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor.

Expanding 3.45,

$$\begin{aligned}\mathcal{M}_{\mu\nu} &= F_{00}\, g_{\mu\nu} + F_{11}\, k_{1\mu}k_{1\nu} + F_{12}\, k_{1\mu}k_{2\nu} + F_{21}\, k_{2\mu}k_{1\nu} + F_{22}\, k_{2\mu}k_{2\nu} \\ &\quad + F_5 \times i\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta}\end{aligned} \tag{3.46}$$

Using the equality, $\varepsilon_2^{\nu *}k_{1\nu} = 0$ for the external photon implies that $F_{12,22}$ do not contribute to the total amplitude.

$$\begin{aligned}\mathcal{M}_{\mu\nu} &= F_{00}\, g_{\mu\nu} + F_{11}\, k_{1\mu}k_{1\nu} + F_{21}\, k_{2\mu}k_{1\nu} \\ &\quad + F_5 \times i\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta}\end{aligned} \tag{3.47}$$

Also using Ward Identity, $k_1^{\nu}\mathcal{M}_{\mu\nu} = 0$, results in $F_{11} = 0$ and

$$F_{00} = -(k_1.k_2)F_{21} = \frac{(m_Z^2 - m_H^2)}{2}F_{21}. \tag{3.48}$$

Hence the amplitude is

$$\begin{aligned}\mathcal{M}(H \to Z\gamma) &= \mathcal{M}_{\mu\nu}\varepsilon_1^{\mu *}\varepsilon_2^{\nu *}, \\ \mathcal{M}_{\mu\nu} &= F_{21}\left[-(k_2.k_1)g_{\mu\nu} + k_{2\mu}k_{1\nu}\right] + F_5 \times i\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta}.\end{aligned} \tag{3.49}$$

This implies that we need to only evaluate F_{21} and F_5, where F_5 arises from chiral fermion loops. We need to only evaluate the terms that are proportional to $F_{21}k_{2\mu}k_{1\nu}$.

$$F_{21} = F_W + F_t + F_s \tag{3.50}$$

The known results for $HZ\gamma$ is [13]

$$i\mathcal{M} = \frac{\alpha g}{4\pi m_W}\left|\mathcal{A}_W + \mathcal{A}_t + \kappa_s\mathcal{A}_s\right| \tag{3.51}$$

therefore,

$$\begin{aligned}F_W &= \frac{\alpha g}{4\pi m_W}|\mathcal{A}_W| \\ F_t &= \frac{\alpha g}{4\pi m_W}|\mathcal{A}_t| \\ F_s &= \frac{\alpha g}{4\pi m_W}|\mathcal{A}_s|\end{aligned} \tag{3.52}$$

where

$$\begin{aligned}
\mathcal{A}_W &= \cot\theta_w \Bigg[4(\tan^2\theta_w - 3) I_2(\tau_W, \eta_W) \\
&\quad + \left\{ \left(5 + \frac{2}{\tau_W}\right) - \left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_w \right\} I_1(\tau_W, \eta_W)\Bigg], \qquad (3.53a) \\
\mathcal{A}_t &= \frac{4\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w\right)}{\sin\theta_w \cos\theta_w} \Big[I_2(\tau_t, \eta_t) - I_1(\tau_t, \eta_t)\Big], \qquad (3.53b) \\
\mathcal{A}_s &= \frac{(2\sin^2\theta_w - 1)}{\sin\theta_w \cos\theta_w} I_1(\tau_s, \eta_s). \qquad (3.53c)
\end{aligned}$$

The functions I_1 and I_2 are defined as,

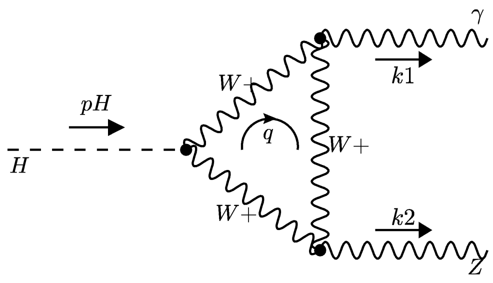
$$\begin{aligned}
I_1(\tau, \eta) &= \frac{\tau\eta}{2(\tau - \eta)} + \frac{\tau^2\eta^2}{2(\tau - \eta)^2}\Big[f(\tau) - f(\eta)\Big] \qquad (3.54a) \\
&\quad + \frac{\tau^2\eta}{(\tau - \eta)^2}\Big[g(\tau) - g(\eta)\Big], \\
I_2(\tau, \eta) &= -\frac{\tau\eta}{2(\tau - \eta)}\Big[f(\tau) - f(\eta)\Big], \qquad (3.54b)
\end{aligned}$$

where $f(\tau)$ has the same definition as eq and

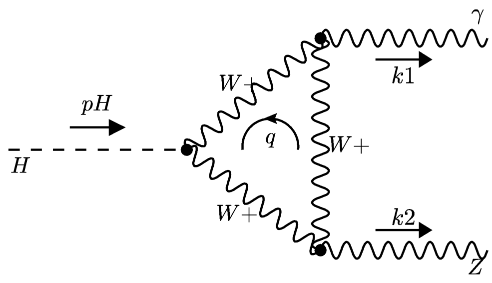
$$g(x) = \sqrt{x - 1}\sin^{-1}\left(\sqrt{1/x}\right). \qquad (3.55)$$

3.2.3 Results for $H \to Z\gamma$

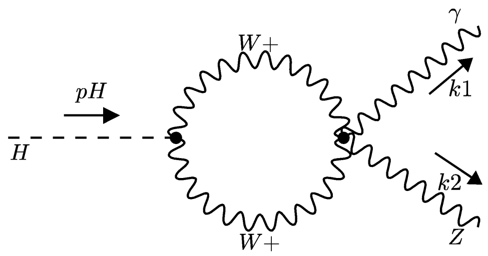
W-loop contribution



(a)



(b)



(c)

For $HZ\gamma$ case, the W-loop contribution reads in terms of PV functions

$$
\begin{aligned}
i\mathcal{M} = {} & \frac{i\mathrm{e}^3}{16\pi^2 m_W \cos(\theta_W)(m_H^2 - m_Z^2)^2 \sin^2(\theta_W)} \quad \times k_1.\varepsilon^*(k_2) k_2.\varepsilon^*(k_1) \\
& \quad \times \Bigg(m_Z^2 B_0(m_H^2, m_W^2, m_W^2) \times \left(\cos^2(\theta_W) m_H^2 + 10 m_W^2 - m_H^2 + 2 m_W^2 \sin^2(\theta_W) \right) \\
& - m_Z^2 B_0(m_Z^2, m_W^2, m_W^2) \times \left(\cos^2(\theta_W) m_H^2 + 10 m_W^2 - m_H^2 + 2 m_W^2 \sin^2(\theta_W) \right) \\
& + 2 m_W^2 m_H^2 - m_Z^2 C_0(0, m_H^2, m_Z^2, m_W^2, m_W^2, m_W^2) \\
& \times \left(\cos^2(\theta_W) - 5 m_H^2 + 10 m_W^2 + 6 m_Z^2 + \sin^2(\theta_W) m_H^2 - 2(m_W^2 + m_Z^2) \right) \\
& + m_H^2 - m_Z^2 \left(\cos^2(\theta_W) m_H^2 + 10 m_W^2 - m_H^2 + 2 m_W^2 \sin^2(\theta_W) \right) \Bigg)
\end{aligned}
\tag{3.56}
$$

Comparing to 3.50,

$$
\begin{aligned}
F_W = & \frac{ie^3}{16\pi^2 m_W \cos(\theta_W)(m_H^2 - m_Z^2)^2 \sin^2(\theta_W)} \\
& \times \Bigg(m_Z^2 B_0(m_H^2, m_W^2, m_W^2) \times \Big(\cos^2(\theta_W) m_H^2 + 10 m_W^2 - m_H^2 + 2 m_W^2 \sin^2(\theta_W) \Big) \\
& - m_Z^2 B_0(m_Z^2, m_W^2, m_W^2) \times \Big(\cos^2(\theta_W) m_H^2 + 10 m_W^2 - m_H^2 + 2 m_W^2 \sin^2(\theta_W) \Big) \\
& + 2 m_W^2 m_H^2 - m_Z^2 C_0(0, m_H^2, m_Z^2, m_W^2, m_W^2, m_W^2) \\
& \times \Big(\cos^2(\theta_W) - 5 m_H^2 + 10 m_W^2 + 6 m_Z^2 + \sin^2(\theta_W) m_H^2 - 2(m_W^2 + m_Z^2) \Big) \\
& + m_H^2 - m_Z^2 \Big(\cos^2(\theta_W) m_H^2 + 10 m_W^2 - m_H^2 + 2 m_W^2 \sin^2(\theta_W) \Big) \Bigg)
\end{aligned}
\tag{3.57}
$$

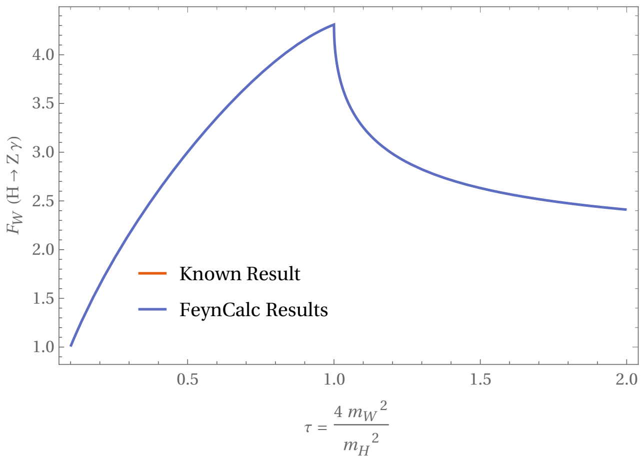


FIGURE 3.6: Plot of form factor F_W w.r.t $\tau_W = \frac{4m_W^2}{m_H^2}$

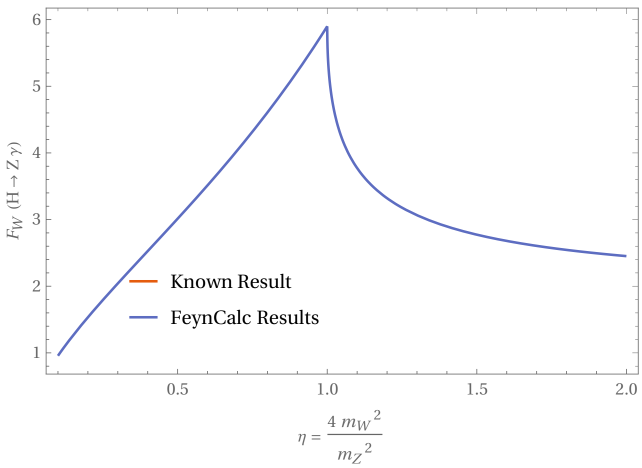
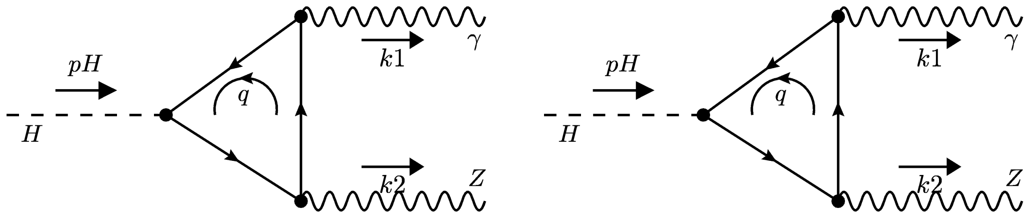


FIGURE 3.7: Plot of form factor F_W w.r.t $\eta_W = \frac{4m_W^2}{m_Z^2}$

Fermionic contribution



(a) (b)

the top-loop contribution reads in terms of PV functions

$$
\begin{aligned}
i\mathcal{M} = {} & \frac{ie^3 m_t^2 \left(3 - 8\left(\sin(\theta_W)\right)^2\right) \left(\mathrm{k1}.\varepsilon^*(\mathrm{k2})\,\mathrm{k2}.\varepsilon^*(\mathrm{k1})\right) m_Z^2\, \mathrm{B}_0\left(m_H^2, m_t^2, m_t^2\right)}{48\pi^2 m_W \left(\cos(\theta_W)\right) \left(m_H^2 - m_Z^2\right)^2 \left(\sin(\theta_W)\right)^2} \\
& - 2m_Z^2\, \mathrm{B}_0\left(m_Z^2, m_t^2, m_t^2\right) \\
& - \left(m_H^2 - m_Z^2\right)\left(m_H^2 - 4m_t^2 - m_Z^2\right) \mathrm{C}_0\left(0, m_H^2, m_Z^2, m_t^2, m_t^2, m_t^2\right) \\
& + 2m_H^2 - 2m_Z^2
\end{aligned}
\tag{3.58}
$$

Comparing to 3.50

$$
\begin{aligned}
F_t = {} & \frac{ie^3 m_t^2 \left(3 - 8\left(\sin(\theta_W)\right)^2\right) m_Z^2\, \mathrm{B}_0\left(m_H^2, m_t^2, m_t^2\right)}{48\pi^2 m_W \left(\cos(\theta_W)\right) \left(m_H^2 - m_Z^2\right)^2 \left(\sin(\theta_W)\right)^2} \\
& - 2m_Z^2\, \mathrm{B}_0\left(m_Z^2, m_t^2, m_t^2\right) \\
& - \left(m_H^2 - m_Z^2\right)\left(m_H^2 - 4m_t^2 - m_Z^2\right) \mathrm{C}_0\left(0, m_H^2, m_Z^2, m_t^2, m_t^2, m_t^2\right) \\
& + 2m_H^2 - 2m_Z^2
\end{aligned}
\tag{3.59}
$$

Plotting F_t w.r.t of τ and η

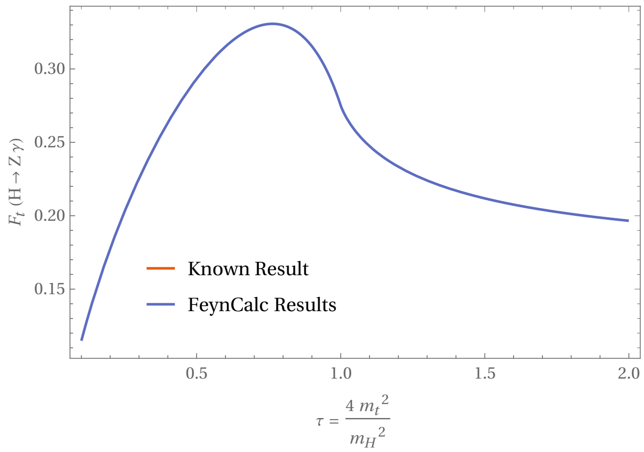


FIGURE 3.8: Plot of form factor F_t w.r.t $\tau_t = \frac{4m_t^2}{m_H^2}$

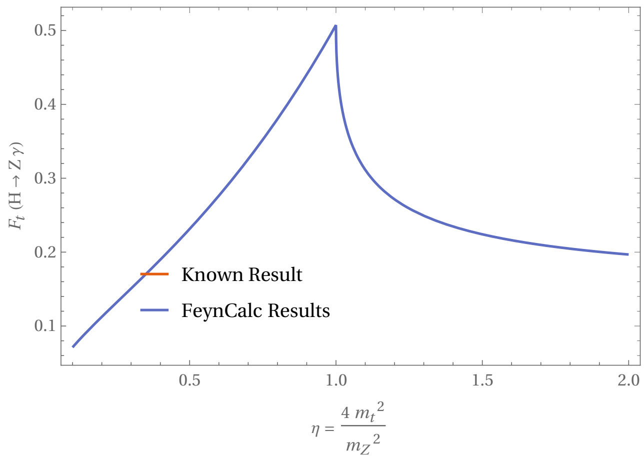
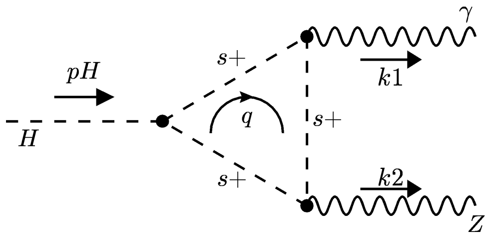
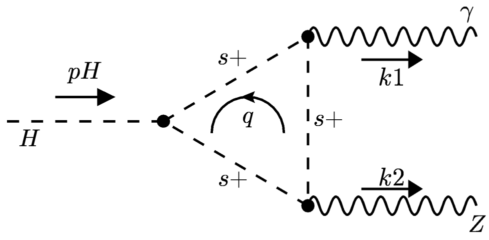


FIGURE 3.9: Plot of form factor F_t w.r.t $\eta_t = \frac{4m_t^2}{m_Z^2}$

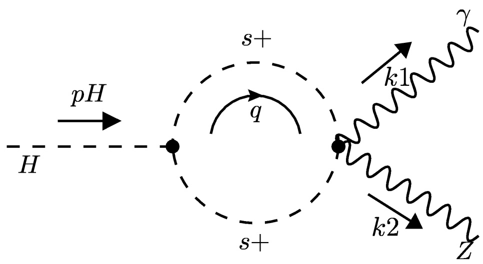
Scalar contribution



(a)



(b)



(c)

The amplitude in terms of PV functions read

$$i\mathcal{M} = -\frac{ie^2 g\left(2sw^2-1\right)m_s^2\left(k1.\varepsilon^*(k2)\right)\left(k2.\varepsilon^*(k1)\right)}{8\pi^2\,\mathrm{cw}\,\mathrm{sw}\,m_W\left(m_H^2-m_Z^2\right)^2} \times\Big(m_Z^2\left(\mathrm{B}_0\left(m_H^2,m_s^2,m_s^2\right)-\mathrm{B}_0\left(m_Z^2,m_s^2,m_s^2\right)-2m_s^2\mathrm{C}_0\left(0,m_H^2,m_Z^2,m_s^2,m_s^2,m_s^2\right)-1\right) + m_H^2\left(2m_s^2\mathrm{C}_0\left(0,m_H^2,m_Z^2,m_s^2,m_s^2,m_s^2\right)+1\right)\Big) \tag{3.60}$$

Comparing to 3.50

$$F_s = -\frac{ie^2 g\left(2sw^2-1\right)m_s^2}{8\pi^2\,\mathrm{cw}\,\mathrm{sw}\,m_W\left(m_H^2-m_Z^2\right)^2} \times\Big(m_Z^2\left(\mathrm{B}_0\left(m_H^2,m_s^2,m_s^2\right)-\mathrm{B}_0\left(m_Z^2,m_s^2,m_s^2\right)-2m_s^2\mathrm{C}_0\left(0,m_H^2,m_Z^2,m_s^2,m_s^2,m_s^2\right)-1\right) + m_H^2\left(2m_s^2\mathrm{C}_0\left(0,m_H^2,m_Z^2,m_s^2,m_s^2,m_s^2\right)+1\right)\Big) \tag{3.61}$$

Plotting F_s w.r.t to τ and η

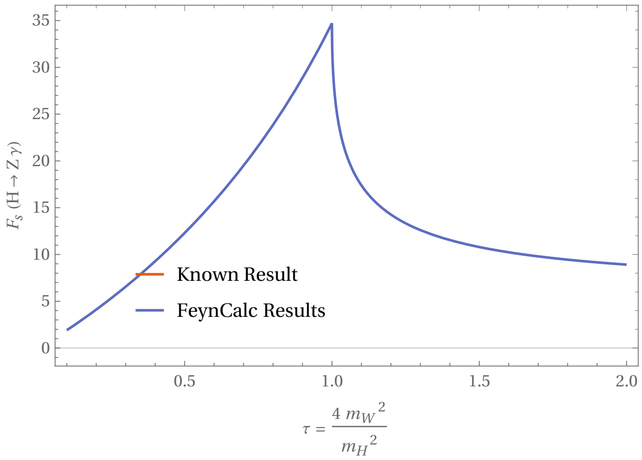


FIGURE 3.10: Plot of form factor F_s w.r.t $\tau_s = \frac{4m_s^2}{m_H^2}$

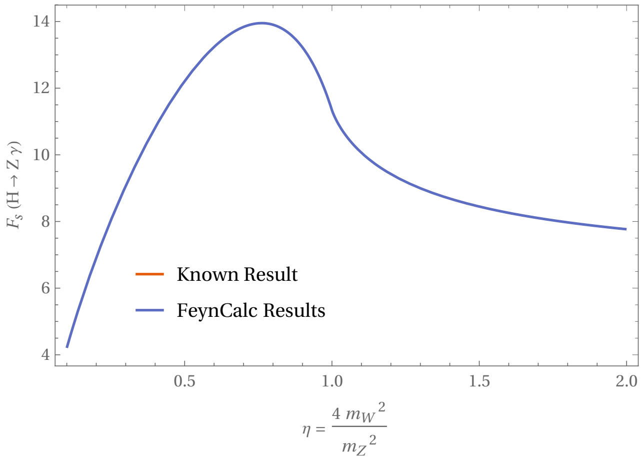


FIGURE 3.11: Plot of form factor F_s w.r.t $\eta_s = \frac{4m_s^2}{m_Z^2}$

The results for $H \to Z\gamma$ matches exactly with known results.

Chapter 4

$H \to ZZ^*$ decay

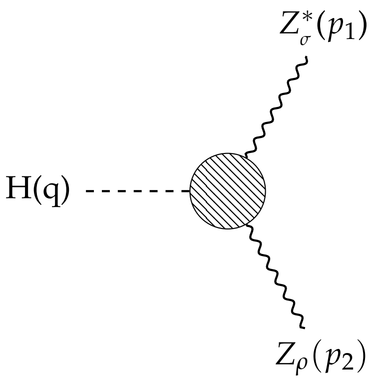
4.1 Introduction

Till now, the data presented by LHC has confirmed the properties of Higgs particle are consistent with SM predictions [17]. Although $H\gamma\gamma$ and $HZ\gamma$ couplings have been measured, there are many couplings to Higgs that are yet to be measured, including the light fermions couplings, self couplings, etc. Out of these is the HZZ^* (for an off-shell Z-boson) coupling that has several phenomenological implications which is studied through HZ production.

Here we study the $H \to ZZ^*$ decay where one of the Z-boson is off-shell. Off-shell couplings, due to optical theorem can develop an imaginary part and has been of great interest in recent years [18]. The HZZ^* one-loop corrections involves W-loop, fermionic and HZ-loop contributions in SM.

4.1.1 Feynman rules

The notation introduced throughout the rest of this work is



The Lagrangian for HZZ^* decay can be written as

$$\mathcal{L} = g\frac{m_Z}{c_W} Z^\mu Z_\mu H + \frac{1}{4} b\, Z_{\mu\nu} Z^{\mu\nu} H + \frac{1}{4}\tilde{b}\, Z_{\mu\nu}\tilde{Z}^{\mu\nu} H\,, \tag{4.1}$$

where

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu\,, \tilde{Z}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} Z^{\alpha\beta} \tag{4.2}$$

The first term in 4.1 represents the tree level contributions whereas the second term will give the same contribution as in the case of $H\gamma\gamma$. The third term can be evaluated as

$$\mathcal{L} = \frac{1}{4}\tilde{b}\, HZ_{\mu\nu}\tilde{Z}^{\mu\nu} \tag{4.3a}$$
$$= \frac{1}{4}\tilde{b}\left[(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\left(\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}Z_{\alpha\beta}\right)\right]H \tag{4.3b}$$
$$= \frac{1}{8}\tilde{b}\,\epsilon^{\mu\nu\alpha\beta}\left[(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial_\alpha Z_\beta - \partial_\beta Z_\alpha)\right]H \tag{4.3c}$$
$$= \frac{1}{4}\tilde{b}\,\epsilon^{\mu\nu\alpha\beta}\left[(\partial_\mu Z_\nu\,\partial_\alpha Z_\beta) - (\partial_\mu Z_\nu\,\partial_\beta Z_\alpha)\right]H \tag{4.3d}$$

where in 4.3c, I have interchanged μ and ν index in third and fourth term. Multiplying with symmetry factor 2, we get

$$\mathcal{M}^{\rho\sigma} = \frac{1}{2}\tilde{b}\,\epsilon^{\mu\nu\alpha\beta}\left[-p_{1\mu}p_{2\alpha}\,g^\rho_\nu g^\sigma_\beta + p_{1\mu}p_{2\beta}\,g^\rho_\nu g^\sigma_\alpha\right] \tag{4.4a}$$
$$= \frac{1}{2}\tilde{b}\left[-p_{1\mu}p_{2\alpha}\epsilon^{\mu\rho\alpha\sigma} + p_{1\mu}p_{2\beta}\epsilon^{\mu\rho\sigma\beta}\right] \tag{4.4b}$$
$$= \frac{1}{2}\tilde{b}\left[-p_{1\mu}p_{2\alpha}\epsilon^{\mu\rho\alpha\sigma} + p_{1\mu}p_{2\alpha}\epsilon^{\mu\rho\sigma\alpha}\right] \tag{4.4c}$$
$$= \frac{1}{2}\tilde{b}\left[-p_{1\mu}p_{2\alpha}\epsilon^{\mu\rho\alpha\sigma} - p_{1\mu}p_{2\alpha}\epsilon^{\mu\rho\alpha\sigma}\right] \tag{4.4d}$$
$$= -\,\tilde{b}\,p_{1\mu}p_{2\alpha}\,\epsilon^{\mu\rho\alpha\sigma} \tag{4.4e}$$

where in 4.4c, I have interchanged β with α in the second term.
Therefore

$$i\mathcal{M} = \left[ib\,(p_1^\rho p_2^\sigma - g^{\rho\sigma}(p_1.p_2)) - i\tilde{b}\,p_{1\mu}p_{2\alpha}\,\epsilon^{\mu\rho\alpha\sigma}\right]\epsilon^*_{1\rho}\epsilon^*_{2\sigma} \tag{4.5}$$

Unlike $H\gamma\gamma$ and $HZ\gamma$ case, HZZ^* has a tree level contribution as well. The term proportional to $g^{\rho\sigma}$ will have infinities which

Thus, we need to extract the terms proportional to $(p_1.\epsilon_2)(p_2.\epsilon_1)$, so that we can extract the form factors b and $\tilde{b}$ where,

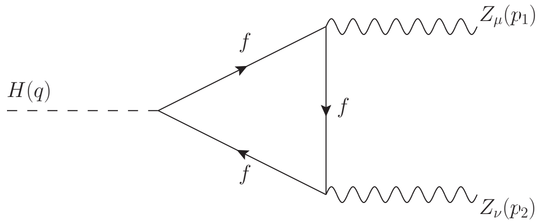
$$b = b_W + b_{HZ} + (b+\tilde{b})_f \tag{4.6}$$

b_W is W-loop contribution, and $(b+\tilde{b})_f$ and b_{HZ} are fermionic and HZ-loop contributions respectively.

I have used FeynArts to generate the one-loop level diagrams and the amplitudes have been evaluated using FeynCalc in terms of PV functions. Finally I have used LoopTools to plot these form-factors (real and imaginary part) w.r.t to p_1 i.e. as a function of off-shell Z-boson transfer momentum.

4.1.2 Implementation in FeynCalc

Fermionic contribution



(a)

FIGURE 4.1: Fermionic diagram

The amplitude in terms of PV functions for the fermionic part is

$$
\begin{aligned}
i\mathcal{M} = \frac{i\mathrm{e}^3 m_f^2}{576\pi^2\mathrm{cw}^2\mathrm{sw}^3 m_W} &\times \Bigg(\Big(\mathrm{p2}.\varepsilon^*(\mathrm{p1})\Big)\Big(\mathrm{p1}.\varepsilon^*(\mathrm{p2})\Big) \\
&\times \Big[8\left(4\mathrm{sw}^2-3\right)\mathrm{sw}^2 \mathrm{C}_0\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_f^2, m_f^2, m_f^2\right) \\
&-9\left(\mathrm{C}_1\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_f^2, m_f^2, m_f^2\right) \quad + \mathrm{C}_1\left(m_H^2, m_Z^2, \mathrm{p1}^2, m_f^2, m_f^2, m_f^2\right)\right) \\
&-4\left(32\mathrm{sw}^4-24\mathrm{sw}^2+9\right)\mathrm{C}_{12}\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_f^2, m_f^2, m_f^2\right)\Big) \\
&+3i\left(8\mathrm{sw}^2-3\right)\bar{\varepsilon}^{\overline{\mathrm{p1}}\overline{\mathrm{p2}}\bar{\varepsilon}^*(\mathrm{p1})\bar{\varepsilon}^*(\mathrm{p2})} \quad \times \Big(\mathrm{C}_1\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_f^2, m_f^2, m_f^2\right) \\
&-\mathrm{C}_1\left(m_H^2, m_Z^2, \mathrm{p1}^2, m_f^2, m_f^2, m_f^2\right)\Big)\Bigg)
\end{aligned}
\tag{4.7}
$$

I have plotted the results for fermionic contribution using LoopTools package. g_f w.r.t p_1, both it's real and imaginary part.

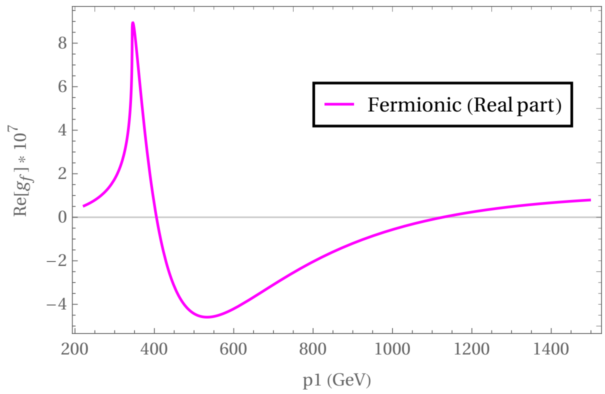


FIGURE 4.2: Plot of g_f (real part) w.r.t p_1 i.e. off-shell Z-boson transfer momentum.

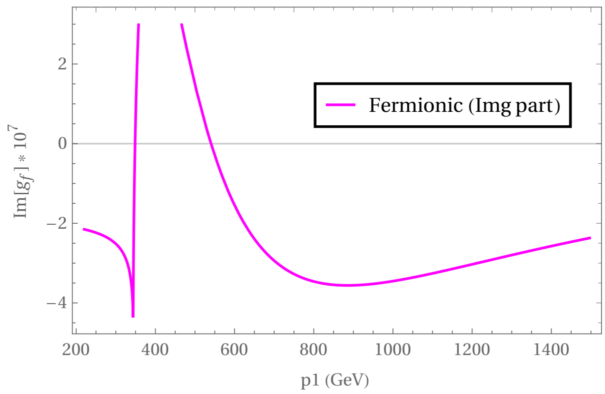


FIGURE 4.3: Plot of g_f (Imaginary part) w.r.t p_1 i.e. off-shell Z-boson transfer momentum.

W-loop contributing diagrams

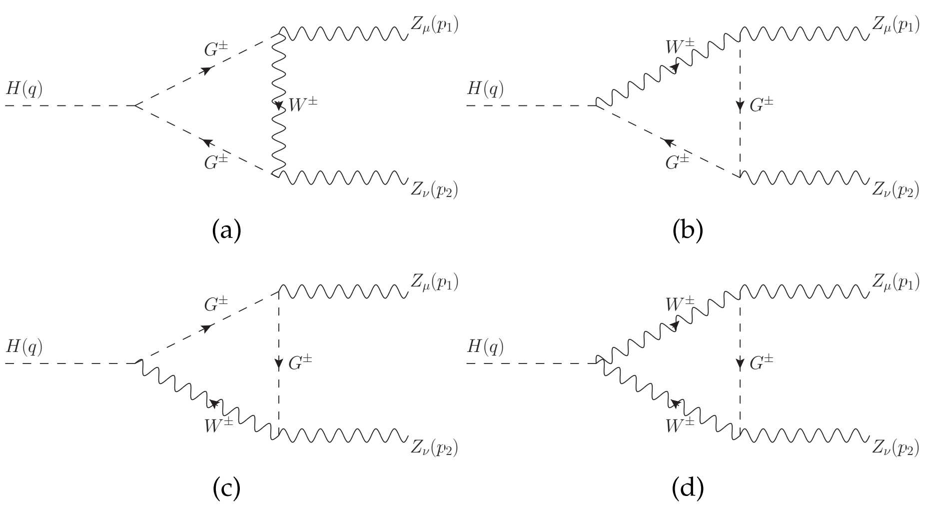


FIGURE 4.4: W-loop contributing diagrams

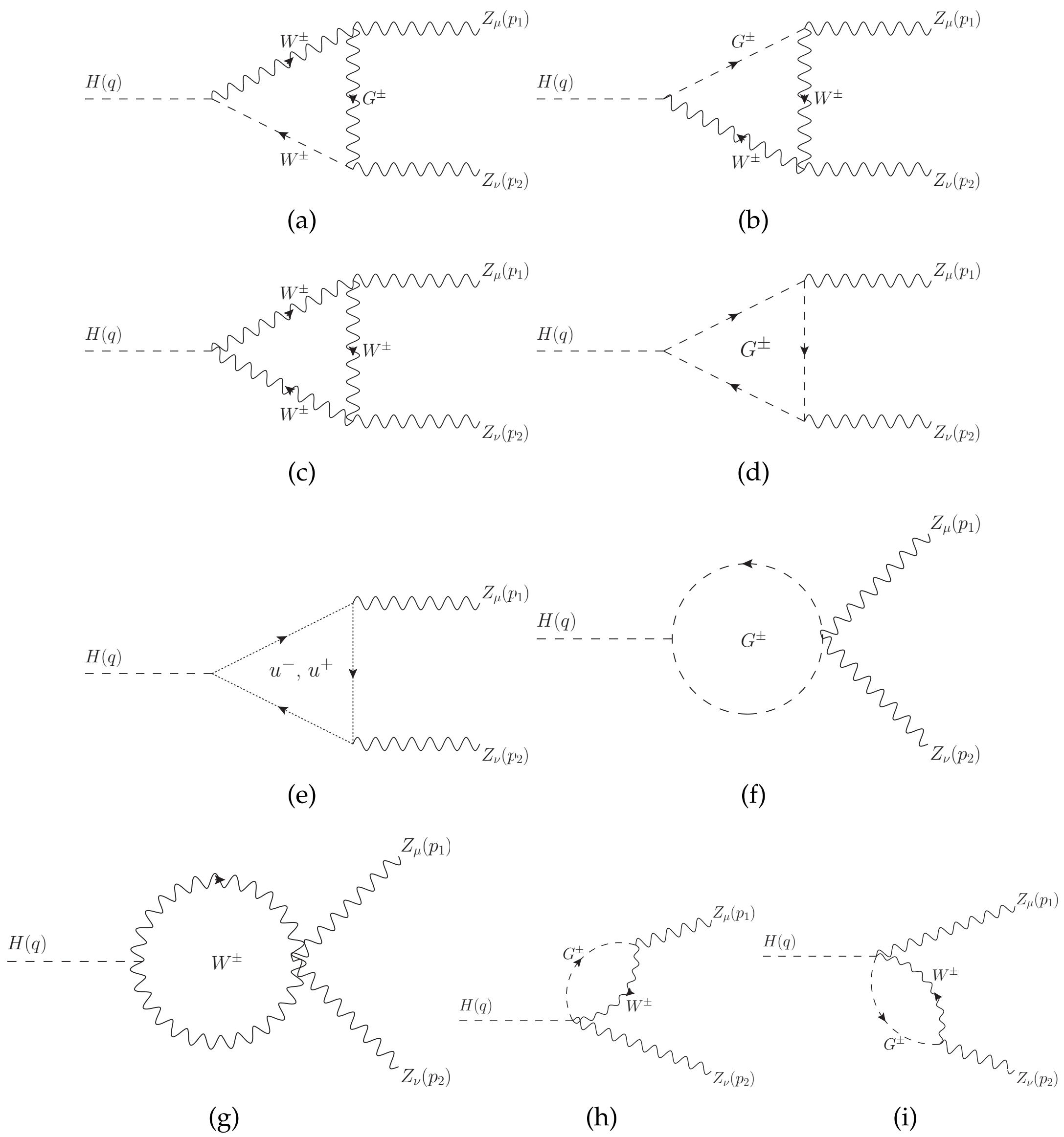
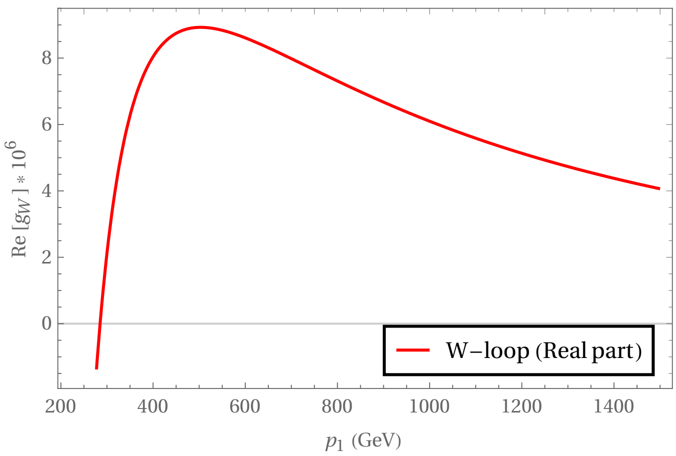


FIGURE 4.5: W-loop contributing diagrams

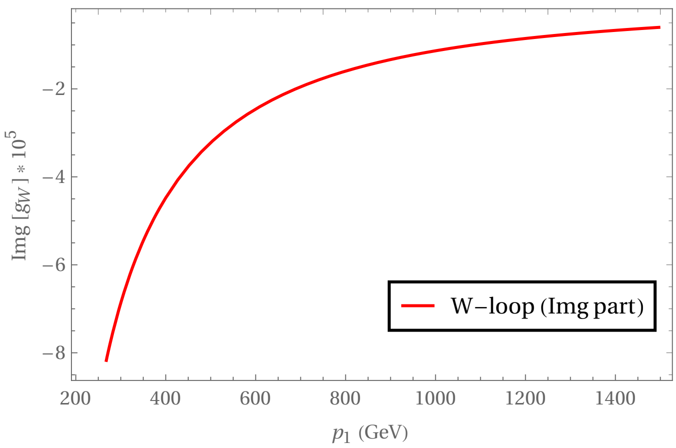
The amplitude in this case is,

$$
\begin{aligned}
i\mathcal{M} = & -\frac{ie^3\left(\mathrm{p1}.\varepsilon^*(\mathrm{p2})\right)\left(\mathrm{p2}.\varepsilon^*(\mathrm{p1})\right)}{16\pi^2\mathrm{cw}^2\mathrm{sw}^3 m_W} \\
& \times \left(2m_W^2\left(4\mathrm{cw}^2\left(\mathrm{sw}^2-\mathrm{cw}^2\right)\mathrm{C}_0\left(\mathrm{p1}^2,m_H^2,m_Z^2,m_W^2,m_W^2,m_W^2\right)\right.\right. \\
& + \left(\mathrm{cw}^2+\mathrm{sw}^2\right)^2\left(\mathrm{C}_1\left(\mathrm{p1}^2,m_H^2,m_Z^2,m_W^2,m_W^2,m_W^2\right)\right. \\
& \left.\left. + \mathrm{C}_1\left(m_H^2,m_Z^2,\mathrm{p1}^2,m_W^2,m_W^2,m_W^2\right)\right)\right) \\
& + \left(\left(\mathrm{cw}^2-\mathrm{sw}^2\right)^2 m_H^2 + 2\left(9\mathrm{cw}^4-2\mathrm{cw}^2\mathrm{sw}^2+\mathrm{sw}^4\right)m_W^2\right) \\
& \left. \times \mathrm{C}_{12}\left(\mathrm{p1}^2,m_H^2,m_Z^2,m_W^2,m_W^2,m_W^2\right)\right)
\end{aligned}
\tag{4.8}
$$

Same as fermion part, I have plotted the results for W-loop using LoopTools package. g_w w.r.t p_1.



(a) Plot of g_W (real part) w.r.t p_1 i.e. off-shell Z-boson transfer momentum.



(b) Plot of g_W (Imaginary part) w.r.t p_1 i.e. off-shell Z-boson transfer momentum.

HZ contributing diagrams

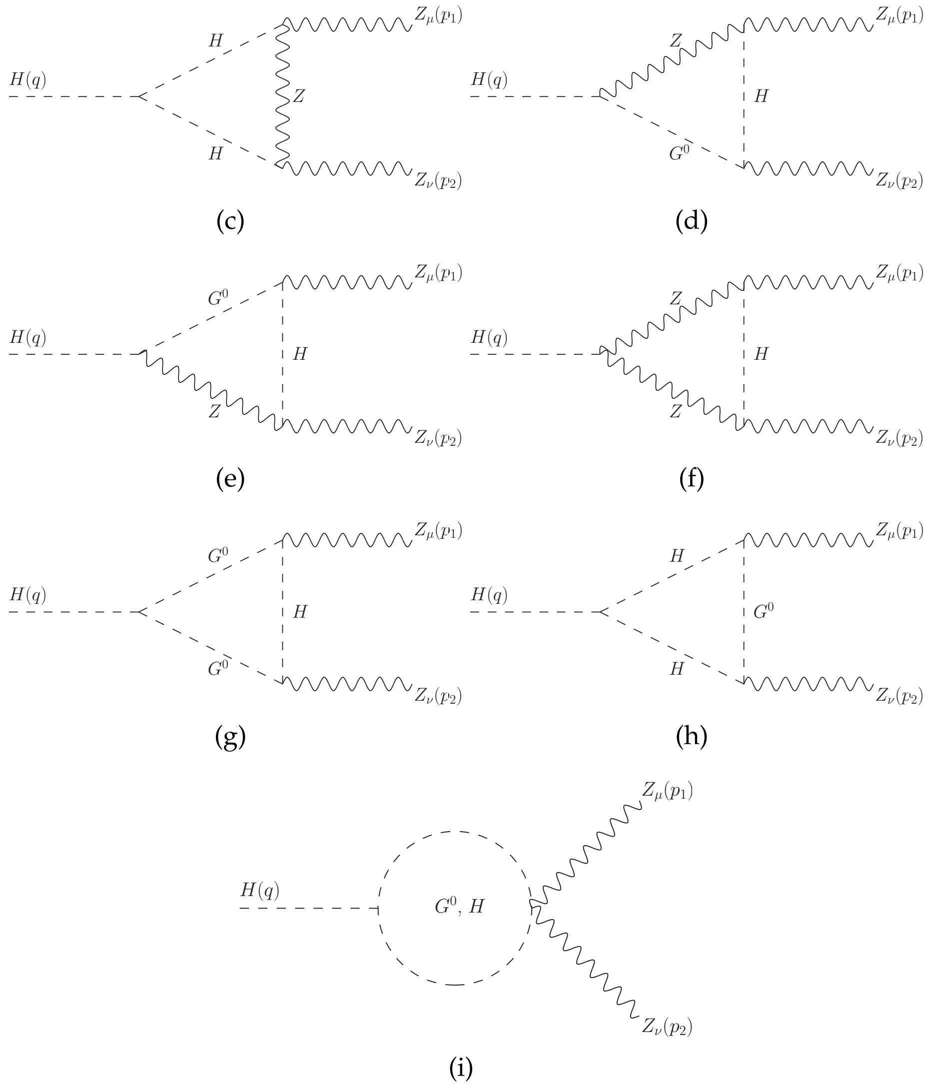
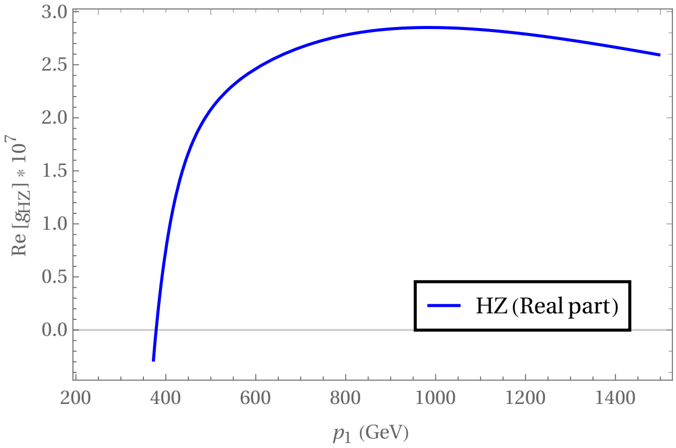


FIGURE 4.6: HZ contributing diagrams

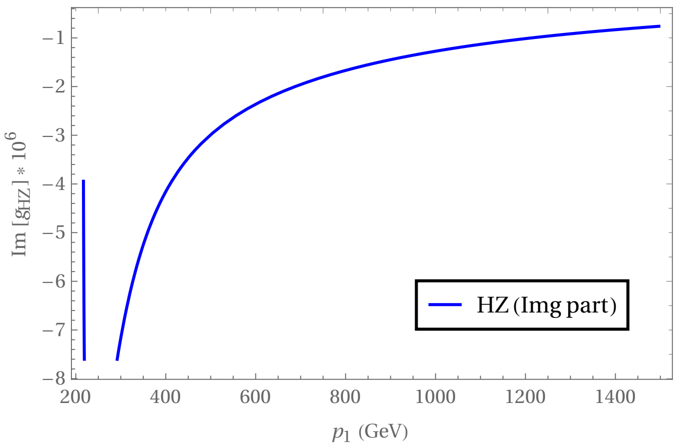
The amplitude for this case is,

$$
\begin{aligned}
i\mathcal{M} = &-\frac{i\mathrm{e}^3}{32\pi^2\mathrm{cw}^4\mathrm{sw}^3 m_W} \times \Bigg(\Big(\mathrm{p1}.\varepsilon^*(\mathrm{p2})\Big)\Big(\mathrm{p2}.\varepsilon^*(\mathrm{p1})\Big) \\
&\times \Bigg(2m_W^2\Big(\mathrm{C}_1\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_H^2, m_Z^2, m_Z^2\right) + \mathrm{C}_1\left(m_H^2, m_Z^2, \mathrm{p1}^2, m_Z^2, m_Z^2, m_H^2\right)\Big) \\
&+ \left(\mathrm{cw}^2 m_H^2 + 2m_W^2\right)\mathrm{C}_{12}\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_H^2, m_Z^2, m_Z^2\right) \\
&+ 3\mathrm{cw}^2 m_H^2 \mathrm{C}_{12}\left(\mathrm{p1}^2, m_H^2, m_Z^2, m_Z^2, m_H^2, m_H^2\right)\Bigg)\Bigg)
\end{aligned} \tag{4.9}
$$

The results for HZ contribution in the decay HZZ^* is,



(a) Plot of g_{HZ} (real part) w.r.t p_1 i.e. off-shell Z-boson transfer momentum.



(b) Plot of g_{HZ} (Imaginary part) w.r.t p_1 i.e. off-shell Z-boson transfer momentum.

One can evaluate the decay rate for the tree level Γ_{tree} part of 4.1 and the decay rate for full lagrangian Γ_{All}. For simplicity, let's assume that $\tilde{b} = 0$.

$$\Gamma_{\text{tree}} = \frac{e^2\sqrt{1-\frac{m_Z^2}{m_H^2}}\left(2m_Z^2\left(5\text{p1}^2-m_H^2\right)+\left(\text{p1}^2-m_H^2\right)^2+m_Z^4\right)}{32\pi\,\text{cw}^2\text{p1}^2\text{sw}^2 m_H} \tag{4.10a}$$

$$\begin{aligned}\Gamma_{All} = \frac{e^2\sqrt{1-\frac{m_Z^2}{m_H^2}}}{256\pi\,\text{cw}^2\text{p1}^2\text{sw}^2 m_H m_W^2}\Big(&b^2\text{cw}^2\text{p1}^2\left(-2m_H^2\left(m_Z^2+\text{p1}^2\right)+m_H^4\right.\\ &\left.+4\text{p1}^2m_Z^2+m_Z^4+\text{p1}^4\right)+24b\,cwp1^2m_Wm_Z\left(-m_H^2+m_Z^2+\text{p1}^2\right)\\ &+8m_W^2\left(2m_Z^2\left(5\text{p1}^2-m_H^2\right)+\left(\text{p1}^2-m_H^2\right)^2+m_Z^4\right)\Big)\end{aligned} \tag{4.10b}$$

The decay rate Γ_{All} comes out to be proportional to b, and the ratio,

$$\frac{\Gamma_{All}}{\Gamma_{tree}} \propto f(b) \tag{4.11}$$

and

$$1 - \frac{\Gamma_{All}}{\Gamma_{tree}} \propto f(b) \tag{4.12}$$

One can plot this ratio as a function of b to analyze the behavior of this ratio. At b=0, this ratio is exactly equal to 1. One can compare this result with experiments. We can always evaluate b using FeynCalc and FeynHelpers. If we have an experimental data for the decay, we can compare the results with this ratio. If it deviates from the experimental data, we can extend the model to analyze its behavior.

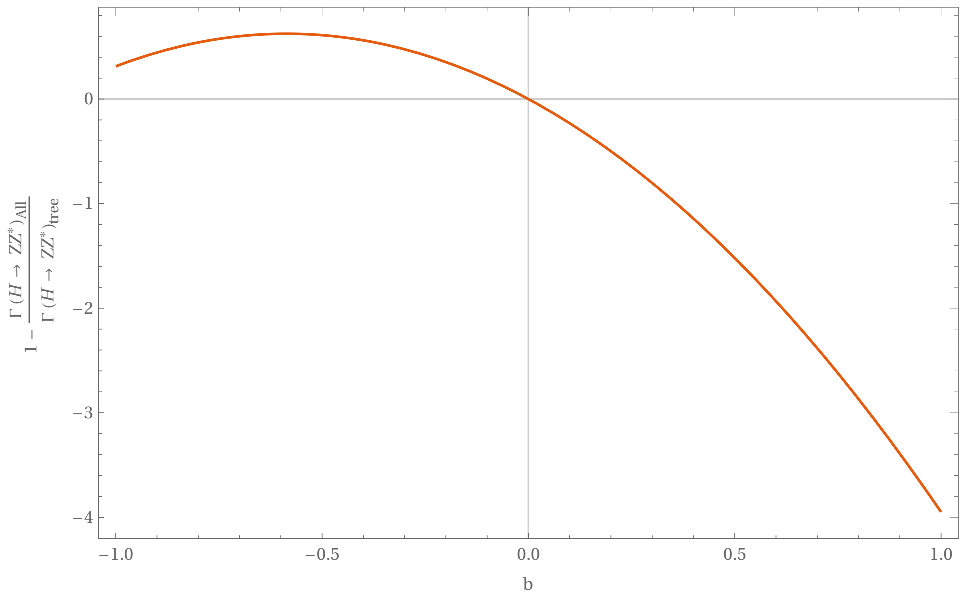


FIGURE 4.7: $\left(1 - \frac{\Gamma_{All}}{\Gamma_{tree}}\right)$ vs form-factor b.

4.2 Conclusion

In conclusion, this thesis have embarked on an investigation of precision calculations within and beyond the SM with the use of various computational packages such as, FeynArts, FeynCalc, LoopTools and FeynRules. We have studied electron anomalous magnetic moment (g-2) as a first example on how to use these tools. We looked at the one-loop calculations of $H\gamma\gamma$ and $HZ\gamma$ in SM and beyond making use of Passarino Veltman functions and making use of FeynRules to generate new model file to do calculations beyond.

By generating model files and conducting calculations beyond SM processes, we delved into realms of new particle decays, exotic interactions, and deviations from SM predictions. We evaluated the decay rate as well and matched it with the known results. Finally, we studied the decay HZZ^*, for an off-shell Z-boson at one-loop level. The interplay between theory and experiment, guided by theoretical predictions, continue to drive progress in our quest to explore the mysteries of the universe.

Appendix A

Appendix

Some important formulas for one loop calculations.

$$\int \frac{d^4}{(k^2+s+i\epsilon)n} = i\pi^2 \frac{\Gamma(n-2)}{\Gamma(n)} \frac{1}{s^{n-2}}, \qquad \text{for } \ n\geq 3 \tag{A.1a}$$

$$\int d^4k \frac{k^\mu}{(k^2+s+i\epsilon)n} = 0, \qquad \text{for } \ n\geq 3 \tag{A.1b}$$

$$\int d^4k \frac{k^\mu k^\nu}{(k^2+s+i\epsilon)^n} = i\pi^2 \frac{\Gamma(n-3)}{2\Gamma(n)} \frac{g^{\mu\nu}}{s^{n-3}}, \qquad \text{for } \ n\geq 4 \tag{A.1c}$$

$$\int d^Dk \frac{k^\mu}{(k^2+s+i\epsilon)^n} = 0 \tag{A.1d}$$

$$\int \frac{d^D}{(k^2+s+i\epsilon)^n} = i\pi^{D/2} \frac{\Gamma(n-\frac{D}{2}}{\Gamma(n)} \frac{1}{s^{n-D/2}} \tag{A.1e}$$

$$g^{\mu\nu} = D \tag{A.2a}$$

$$\gamma^\mu \gamma_\mu = D\mathcal{I} \tag{A.2b}$$

$$\gamma_\mu \gamma^\nu \gamma^\mu = -(D-2)\gamma^\nu \tag{A.2c}$$

$$\gamma_\mu \gamma^\alpha \gamma^\beta \gamma^\mu = (D-4)\gamma^\alpha \gamma^\beta + 4g^{\alpha\beta} \tag{A.2d}$$

$$Tr(\text{odd no. of } \gamma \text{ matrices }) = 0 \tag{A.3a}$$

$$Tr(\gamma^\alpha \gamma^\beta) = f(D) g^{\alpha\beta} \tag{A.3b}$$

$$Tr(\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta) = f(D) \left[g^{\alpha\beta} g^{\gamma\delta} - g^{\gamma\alpha} g^{\beta\gamma} + g^{\alpha\delta} g^{\beta\gamma} \right] \tag{A.3c}$$

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