B. TECH. PROJECT REPORT

On

Finite Element Analysis Of Crack Propagation in Spur Gear Tooth using XFEM

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DISCIPLINE OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY INDORE

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Finite Element Analysis Of Crack Propagation in Spur Gear Tooth using XFEM

A PROJECT REPORT

Submitted in partial fulfilment of the
Requirements for the award of the degrees

Of
BACHELOR OF TECHNOLOGY
In
MECHANICAL ENGINEERING

Submitted by
Gurram Sai Vilas (150003013)
Prasenjeet Jain (150003025)

Guided By
Dr. Indrasen Singh

INDIAN INSTITUTE OF TECHNOLOGY INDORE
November 2018

CANDIDATES’ DECLARATION

We hereby declare that the project entitled **“Finite Element Analysis of Crack Propagation in Spur Gear Tooth using XFEM”** submitted in partial fulfilment for the award of the degree of Bachelor of Technology in **‘Mechanical Engineering’** completed under the supervision of **Dr. Indrasen Singh, Assistant Professor, Mechanical Engineering, IIT Indore** is an authentic work.

Further, we declare that we have not submitted this work for the award of any other degree elsewhere.

Gurram Sai Vilas

Prasenjeet Jain

CERTIFICATE BY BTP GUIDE

It is certified that the above statement made by the students is correct to the best of my knowledge.

Dr. Indrasen Singh
Assistant Professor
Department of Mechanical Engineering
IIT Indore

PREFACE

This report on **"Finite Element Analysis of Crack Propagation in Spur Gear Tooth using XFEM"** is prepared under the guidance of **Dr. Indrasen Singh.**

In this project, **Continuum analysis of crack propagation in a spur gear tooth** is performed using **XFEM** in commercially available software package Abaqus. The effect of backup ratio and pressure angle on the crack path is investigated. The analysis provide useful guidelines in choosing appropriate design parameters to avoid catastrophic crack propagation towards the rim which may lead to sever damage to machine.

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Acknowledgements

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Without their support, this project would not have been possible.

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Abstract

Different types of gears are used in industries, which are operating from moderate to high speed thus it is very important to identify cracks and predict the crack paths in order to avoid catastrophic accidents.

Finite element analysis has been performed using Extended Finite Element Method in a spur gear tooth to predict the crack propagation path for a combination of different Backup Ratios and Crack position in gear tooth fillet and for different Backup Ratios and Pressure Angles. The failures are classified as safe and catastrophic failure.

A catastrophic failure is the one in which the crack propagate towards the rim, which can cause severe accidents. Thus failsafe or safe failure is a much better design.

Extended Finite Element Method is used as no re-meshing is required in this process and thus computational load is reduced. For validation purpose of XFEM, a SENT specimen is used and results are compared with the previous work in Cohesive Zone Modelling. Different mode mixities are used for this purpose. The results of XFEM matches with the previous results obtained through Cohesive Zone Modelling. The results of these numerical simulations in gear tooth will give a good criteria to predict crack propagation paths. Numerical simulations have been performed in 3D models so as to give more realistic results.

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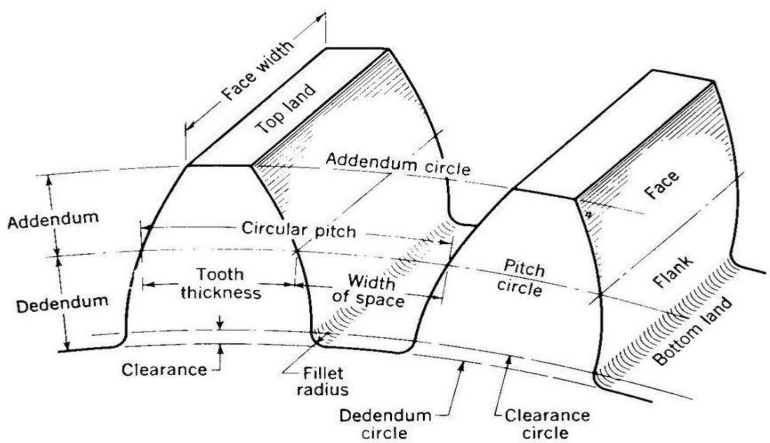
Chapter 1
Introduction

1.1 Spur gears and terminologies associated with it

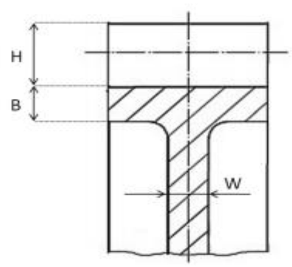
Spur gears are cogged wheels whose cogs or teeth project radially and stand parallel to the axis.



Gear Terminologies

Figure 1

Some important parameters



Important gear geometric parameters

Figure 2

Here, H is the tooth height, B is the rim thickness and W is the web thickness.

Important definitions

1. **Backup Ratio:** This geometrical parameter called backup ratio is defined as the ratio between rim thickness B and tooth height H.

   Backup Ratio (m) = B/H

2. **Pressure Angle:**

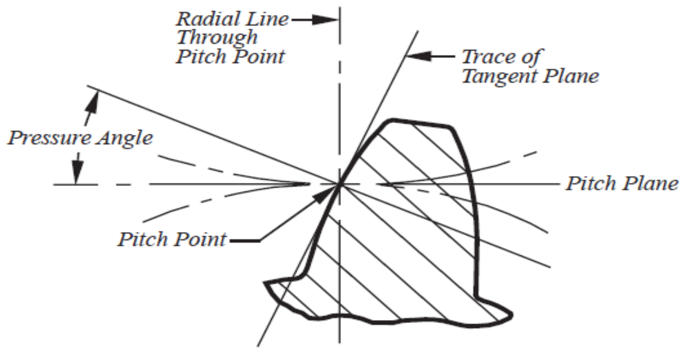


Figure 3

Pressure angle is the angle between the common normal (the line of action of Force) to the contacting teeth and the common tangent to the pitch circles of meshing gears.

1.2 Types of failures in gears

The Five common failure modes in a Spur Gear are as follows

1. **Bending fatigue:** It is a result of cycling bending stress at the tooth root, the damage process includes crack nucleation, crack propagation and final unstable fracture. It is important to note that the site of crack nucleation is generally tooth root fillet. In this work, the main focus is on this type of failure.

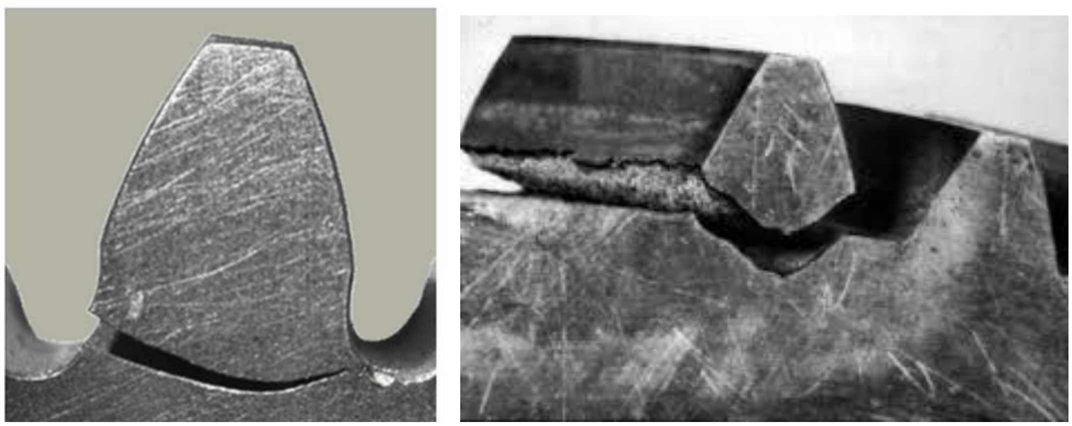
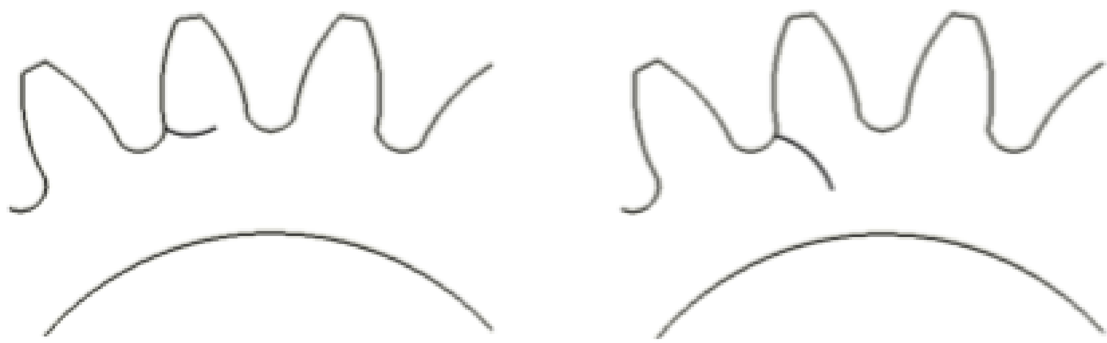


Figure 4

2. **Pitting**: Surface damage from cyclic contact stresses.
3. **Micro pitting:** Formation of small craters on the tooth surface.
4. **Scuffing:** It is a type of adhesive wear which instantly damages tooth surfaces that are in relative motion.
5. **Wear:** The Continuous abrasive process of material removal form mating gear teeth.

1.3 Concept of failsafe design

Spur gears are used in Industries on a large scale and are used at varying speeds. Thus the failure due to crack must be analyzed in spur gear and criteria must be established for the failsafe design. In the aerospace industry, there is a need for low weight gears due to which the spur gears produced are with thin rim and web. In thin rim gears, if an initial crack is located at tooth root fillet, then that crack may propagate through the tooth or crack may grow towards the rim which can cause catastrophic failure, which can affect the whole transmission and also can cause the machine to stop running. The failure in which the crack propagates towards the tooth (may cause the entire tooth to break or may be a part of it) is called as the safe failure. While the second type is the catastrophic failure.



a.) **b.)**

Possible failure modes in a thin rim gear:
a) Safe failure b) Catastrophic failure

Figure 5

1.4 Fracture Mechanics

Fracture is a degradation process which is non-uniform in time and space consisting of the initial stage of formation, the phase of propagation and coalescence of cracks. Crack is defined as the opening created when surfaces move in opposite directions. The major causes of crack propagation are mainly due to the magnitude and direction of load and its loading cycle.

In the field of mechanics, it is the fracture mechanics which deals with the study of crack propagation in materials. The three modes of fracture are as follows

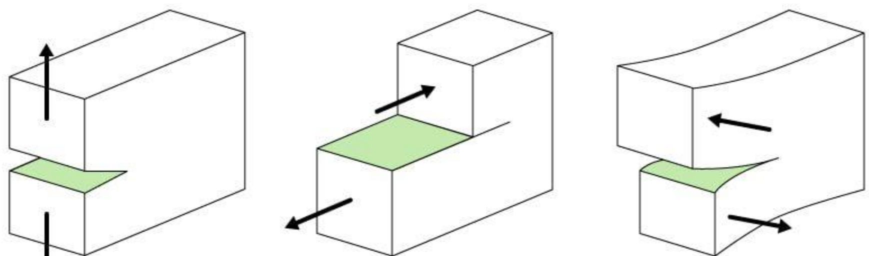


Figure 6

Mode I: Opening mode, where the tensile stress is normal to the plane of crack.

Mode II: Sliding Mode (in-plane shear)

Mode III: Tearing Mode (Out of plane shear)

Fracture mechanics approach to design

The fracture mechanics approach has 3 important variables as applied stress, flaw size, and fracture toughness.

Fracture toughness (Kc) is a property which describes the ability of a material to resist fracture and is one of the most important properties of any material for many design applications.

Comparison of fracture mechanics approach to design with the strength of materials approach

a) Strength of materials approach

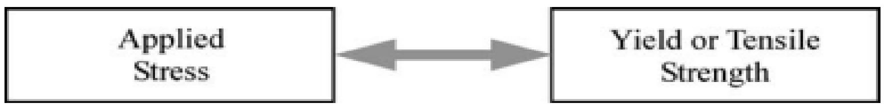


Figure 7

b) Fracture mechanics approach

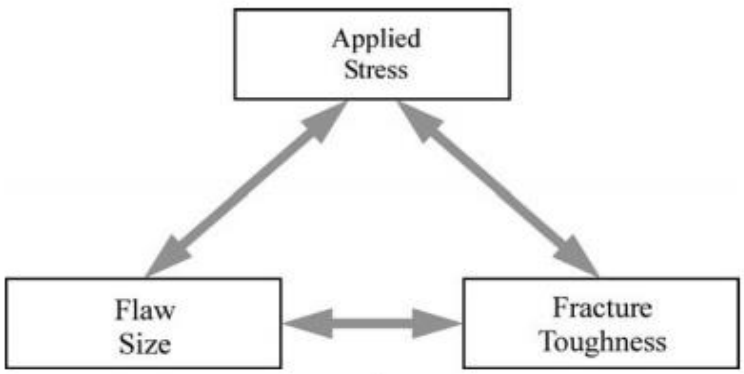


Figure 8

There are two alternative approaches to fracture analysis: the energy criterion and the stress intensity approach.

Energy criterion

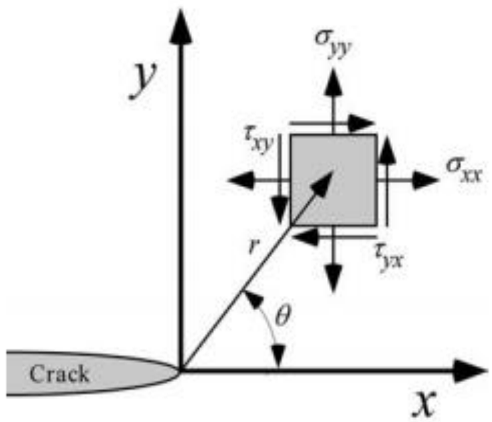
It states that fracture occurs when the energy available for crack growth is sufficient enough to overcome the surface energy, plastic work or other types of energy dissipation associated with crack propagation. Fracture energy is defined as the energy required to open unit area of the crack surface. It is a material property and does not depend on the size of the structure.

Stress-Intensity Approach

The stress intensity factor (K) is used in fracture mechanics to predict the stress state near the tip of a crack caused by load or residual stresses. Kc is called as critical stress intensity factor which is an alternate measure of fracture toughness.

The figure below shows an element near the crack tip of an elastic material along with the in-plane stresses on the element. Each stress component is proportional to K1, thus entire stress distribution at the tip of the crack can be calculated with help of the equations shown. This stress intensity factor tells the stress conditions at the crack tip.

Failure occurs when K1=K1c.



$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

Figure 9

Stresses near the tip of the crack in an elastic material.

The following relationship holds between the fracture energy and Stress intensity factor

$$\mathbf{G = (K_1)^2 / E}$$

The similar relation holds for K_{1c} and G_c. Thus, the energy and stress intensity approaches are equivalent to linear elastic materials.

1.5 Different methods for analyzing crack propagation and Finite element modelling of crack growth

FEA has proven to be an important tool in the field of computational fracture mechanics. It has been used to study various properties of crack growth such as crack growth resistance, the stability of crack growth etc. Some of the important methods used in analyzing crack propagation are

I. Cohesive Zone Modeling

In the Cohesive zone modelling (CZM) approach, the fracture is assumed to occur by continuous separation of the crack surfaces ahead of the crack tip. The crack grows when the crack separation reaches a critical value, δ_c. The cohesive zone model is working according to the traction separation law.

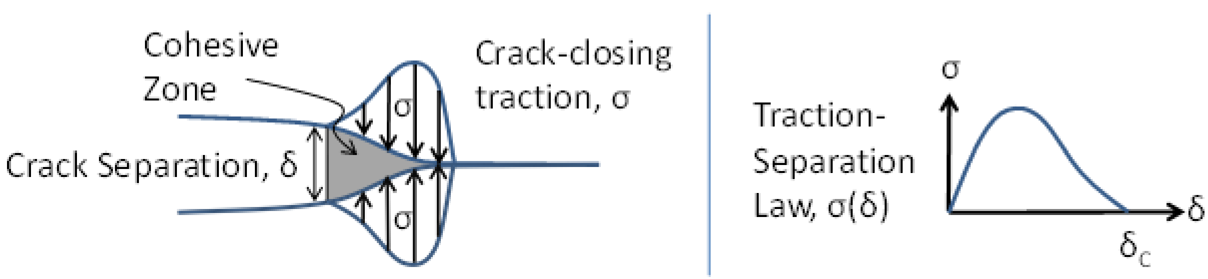


Figure 10

II. Phase field Modelling

In this approach, a phase parameter which represents damage is coupled with the elasticity equations. In CZM, one needs to track the crack interface as it grows which greatly increases the computational effort. By using the phase field model, it is not required to track the crack interface as it grows.

III. Extended Finite Element Method (XFEM)

FEM is important for solutions to problems related to crack propagation analysis. But the finite element analysis method has a disadvantage in the crack propagation analysis. The mesh must be consistent with the discontinuity (Abdelrahman 2011) that is the mesh must be aligned with the domain boundaries. Due to this re-meshing, the computational load increases and also it consumes time and computer memory capacity. In a study by Eriki et al. (2012) on the crack propagation in spur gear using the Finite Element Method, 'DELETE And FILL' method for re-meshing. This process was mind-numbing and tiresome. Thus, XFEM served as a substantial tool for analysis of crack propagation. XFEM allows the crack to pass through an element, thus XFEM is independent of crack as it can be modelled or fixed mesh and mesh refinement is avoided. This reduces a lot of computational burden, also prevent singularities of crack.

In this study, the focus is on the XFEM method and crack modelling is done in Abaqus. The crack propagation paths are visualized using the 'STATUSXFEM'.

1.6 Motivation and Objective

Motivation

It is important to avoid catastrophic failures as it can cause a machine to stop running, can affect the whole transmission system and may even cause many severe accidents. Thus it is very important to have vigorous design criteria in order to avoid catastrophic failures.

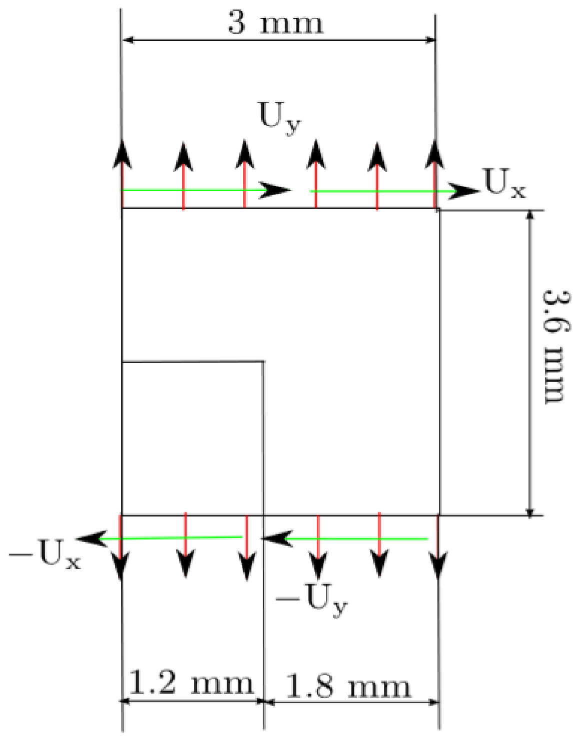
Objective

1. To study the crack propagation path for different Backup Ratio and Initial Crack Position
2. To study the crack propagation path for different Backup Ratio and Pressure Angle
3. To give a design criteria for failsafe failure

Chapter 2
Validation of XFEM (Benchmark)

This chapter represents some benchmark examples to examine the performance of Extended Finite Element Method.

R. Karthikeyan (MSc Thesis, 2014 IISc Bangalore) has performed mixed mode fracture analysis using Cohesive Element Method on a SENT specimen of the following geometry.



Single Edge Notch Tension Plate

Figure 11

Mode mixity m is defined as:

$$m = \frac{2}{\pi} \tan^{-1} R\sqrt{\frac{1-2\nu}{2(1-\nu)}}$$

Here,

$$R = \frac{U_x}{U_y}$$

and ν is the Poisson's ratio.

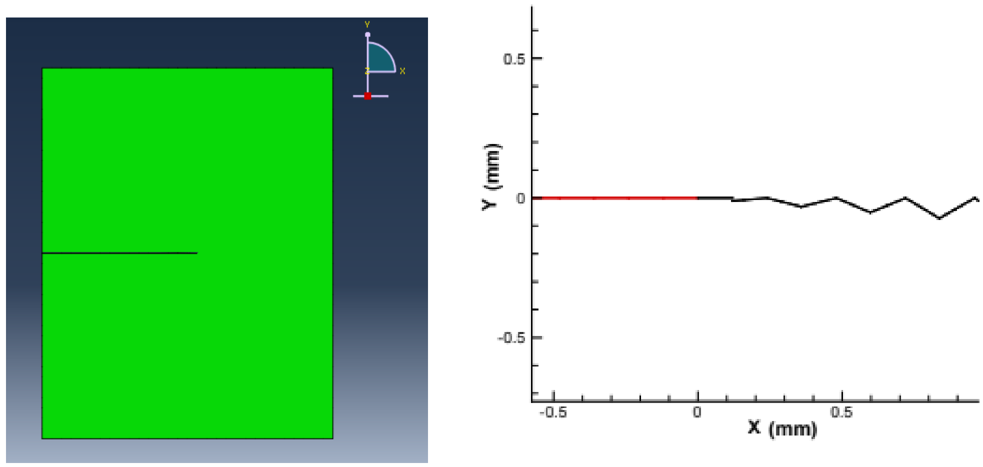
Four values of m = 0, 0.49, 0.86 and 1 are considered in this report.

Material properties used for simulation:

Max. Principle Stress	=	100 MPa
Fracture Energy	=	0.05 mJ/mm2
Young’s Modulus	=	3240 MPa
Poisson’s Ratio	=	0.35

Comparison of results obtained by XFEM method with CZM

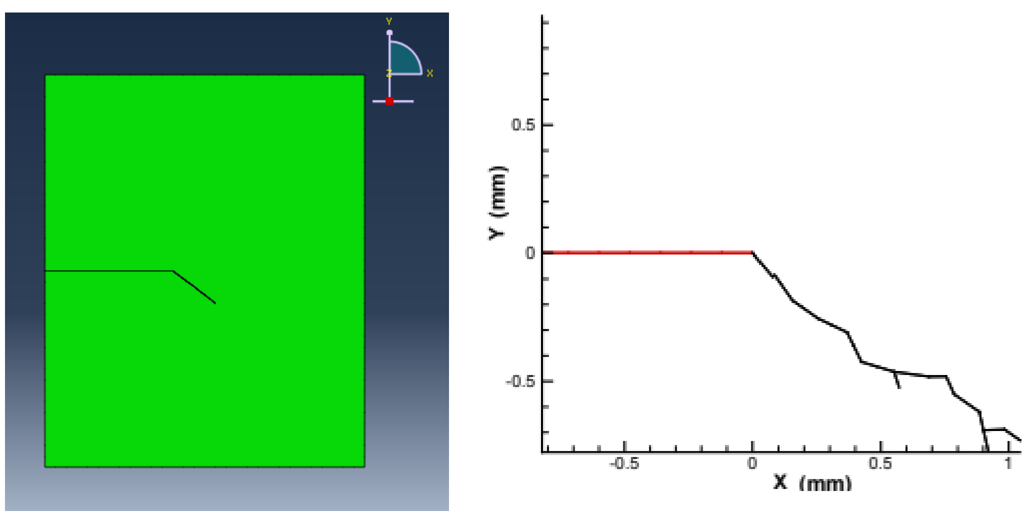
1. Mode Mixity (m)=0



Angle of crack: **XFEM = 0°** **CZM = 0°**

Figure 12

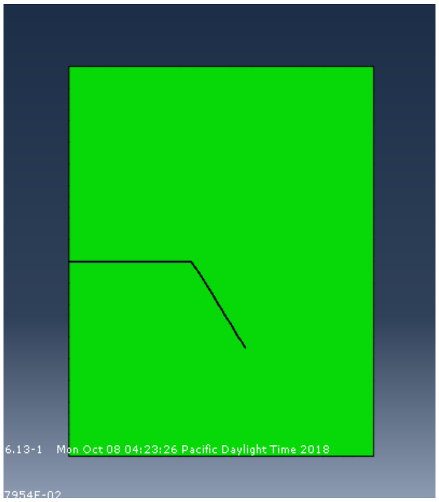
2. Mode Mixity (m)=0.49

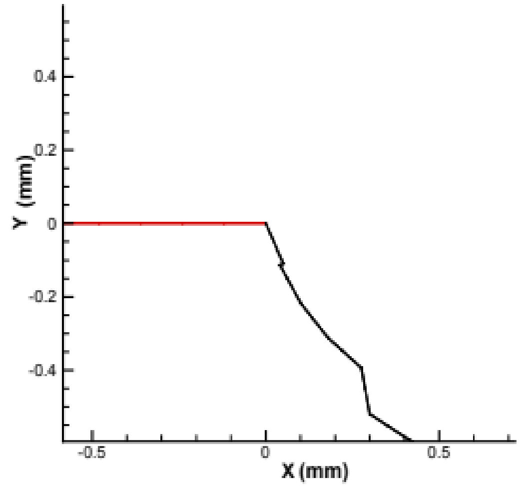


Angle of crack : XFEM = 33 degrees **CZM = 33.7 degrees**

Figure 13

3. Mode Mixity (m)=0.86

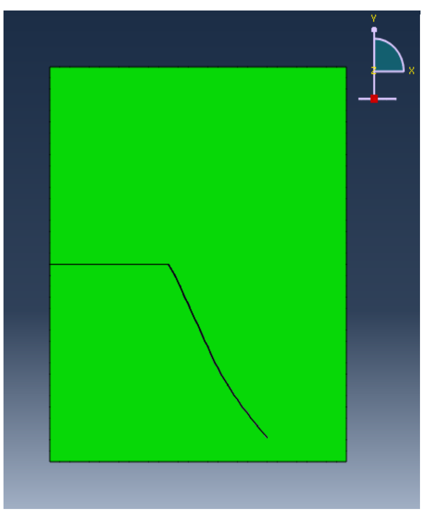


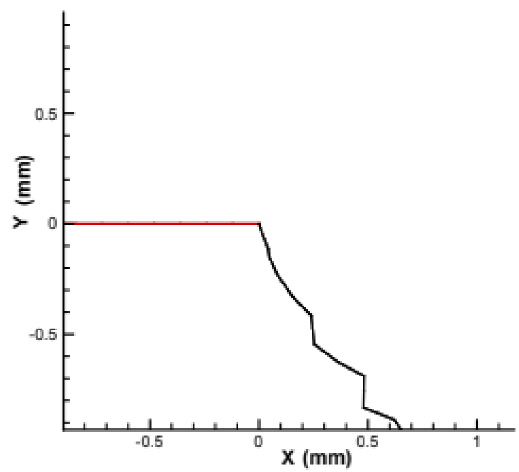


Angle of crack : XFEM = 52.2 degrees **CZM =53 degrees**

Figure 14

4.Mode Mixity (m)=1





Angle of crack : XFEM = 57.1 degrees **CZM =55 degrees**

Figure 15

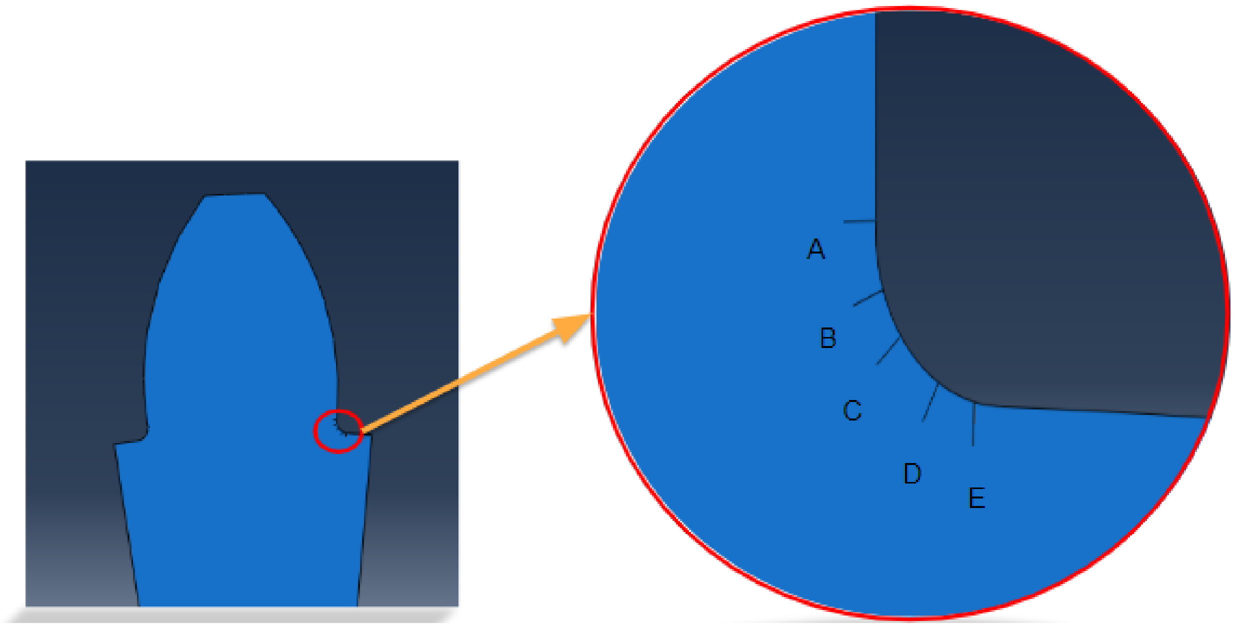
The results from obtained by XFEM are comparable and matches with results by CZM. Thus validation of XFEM through 2D crack modelling is successfully done.

Chapter 3

FEA of Crack Propagation in Spur Gear Tooth using XFEM

3.1) 3D Crack Modeling in a Spur gear tooth

A small initial crack is considered at 5 different positions at the Spur Gear Tooth Foot (fillet), which are marked as A, B, C, D and E as displayed in Fig. 16. . The load is applied on the tooth along the Pitch Circle considering Pressure Angle of 20°.



Crack positions

Figure 16

3.2) Table 1: Gear Parameters

Module	7.4
No. of Teeth	28
Pitch Circle Diameter	206.4mm
Pressure Angle	20°
Face Width	8mm

3.3) Table 2: Material properties used for simulation

Max. Principle Stress	472 MPa
Fracture Energy	21.125 mJ/mm^2
Young's Modulus	200 MPa
Poisson's Ratio	0.3

3.4) Boundary Conditions

All the nodes on hub, rim and web are constrained to move along X, Y and directions, while a constant force of 2000 N is applied along the pitch circle (refer Fig. 17).

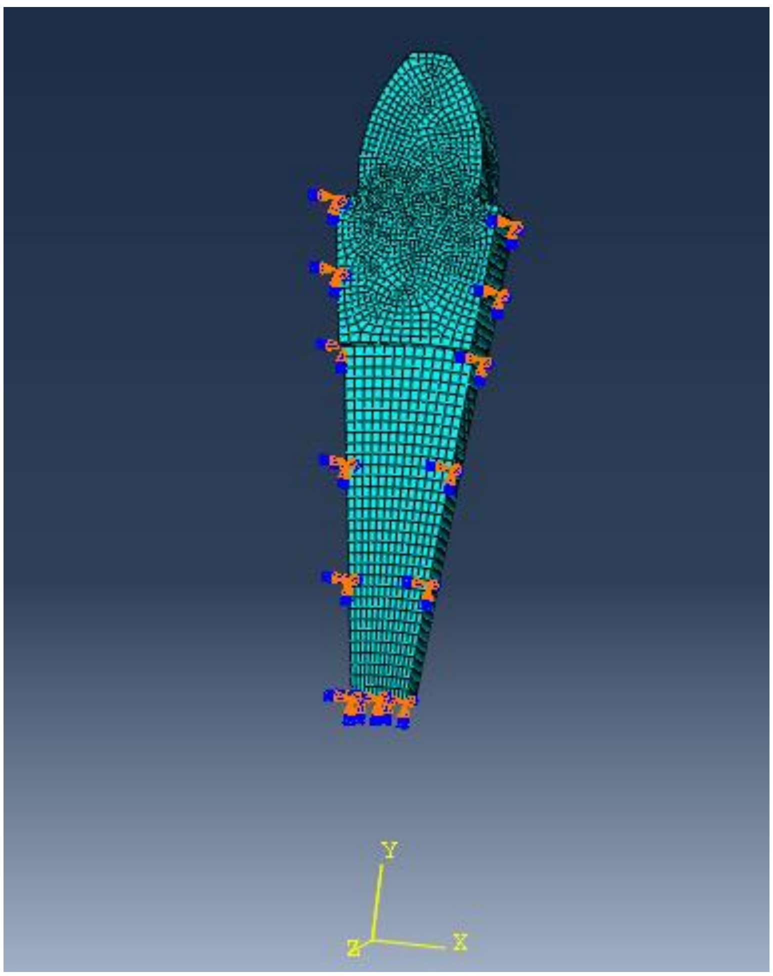


Figure 17

3.5) Simulation Results

In these simulations, Maximum Principal Stress damage criterion is employed to initiate damage, and energy criterion is used for damage evolution. Here crack path is determined for initial flaw being at five different positions, as shown in Fig. 16, and for every case four values of m_B = 0.3, 0.4, 0.5 and 1 are considered.

. Also, the analysis is performed for position A for all the backup ratios for different pressure angles to understand the effect of pressure angle on the crack path.

3.5.1) Variation of backup ratios for different crack positions

Analysis is done for various Backup Ratios (m_B = 0.3, 0.4, 0.5, 1) and various Initial Crack Positions (A, B, C, D & E), keeping the pressure angle constant (θ =20°)

Crack Propagation Path for Backup Ratio = 0.3

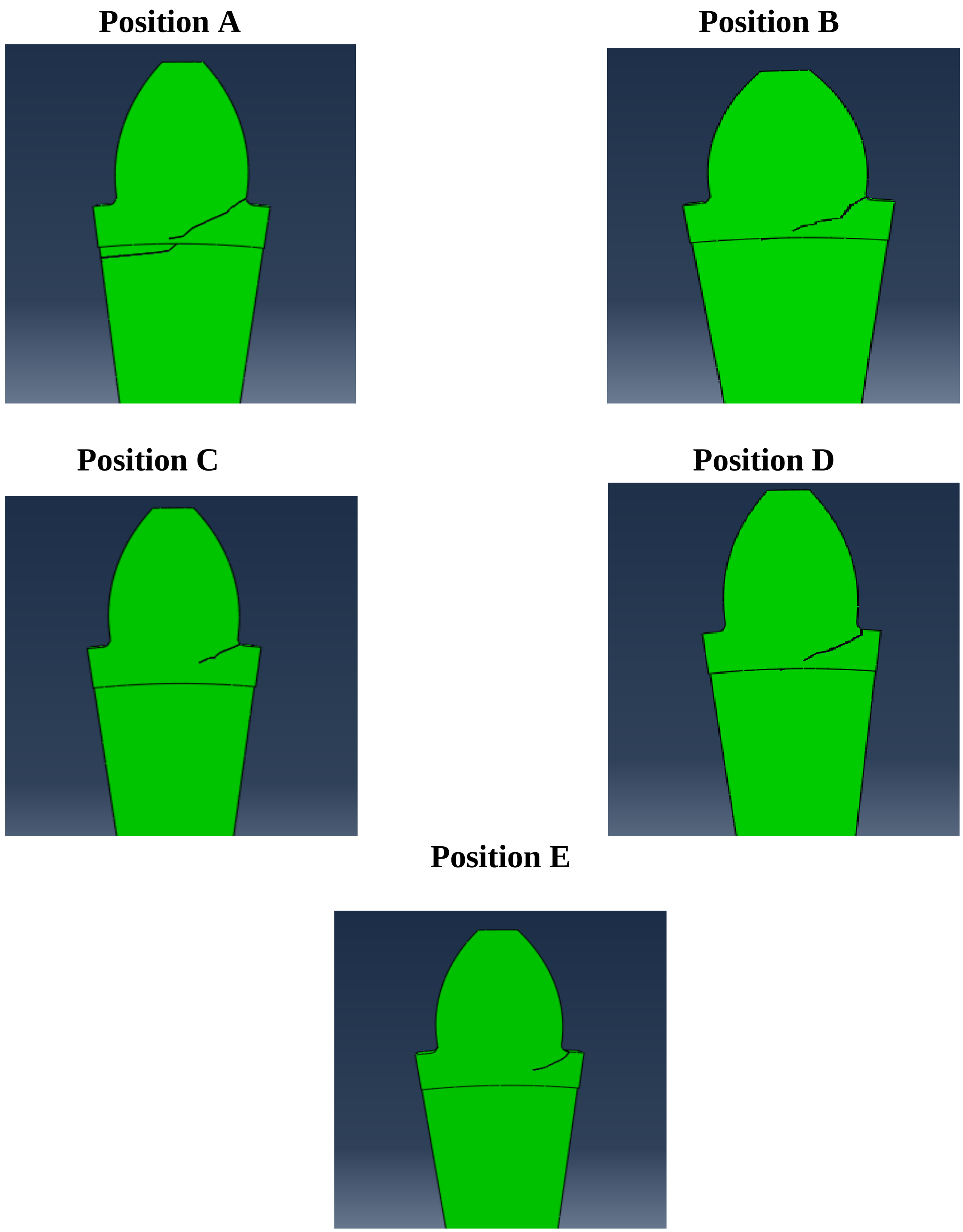
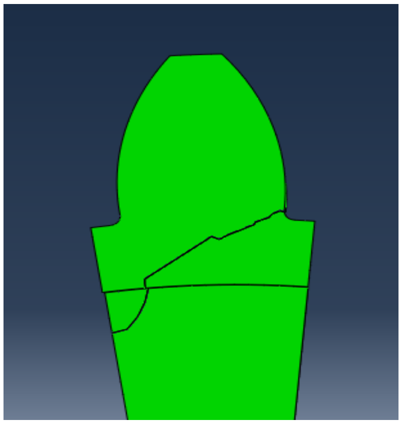


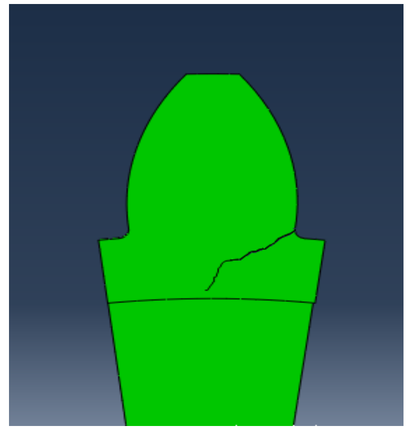
Figure 18

Crack Propagation Path for Backup Ratio = 0.4

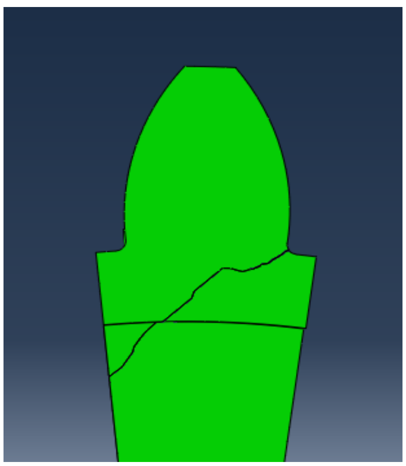
Position A



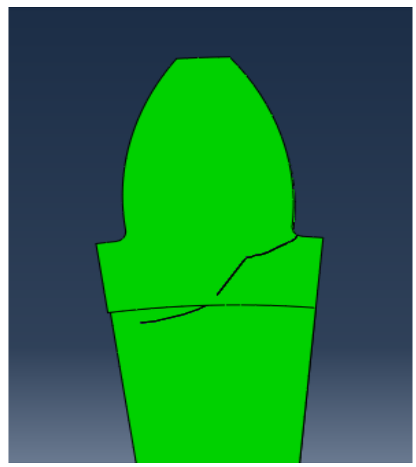
Position B



Position C



Position D



Position E

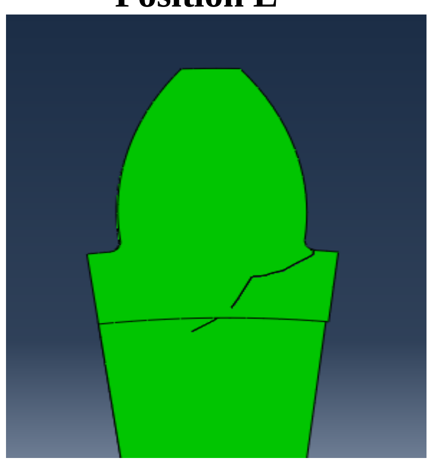
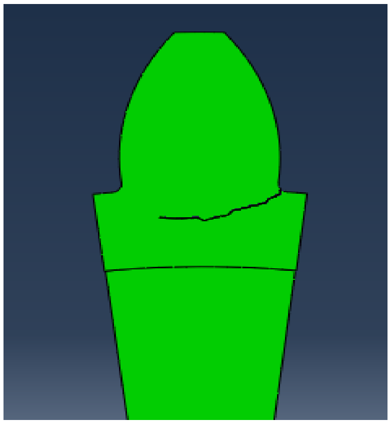


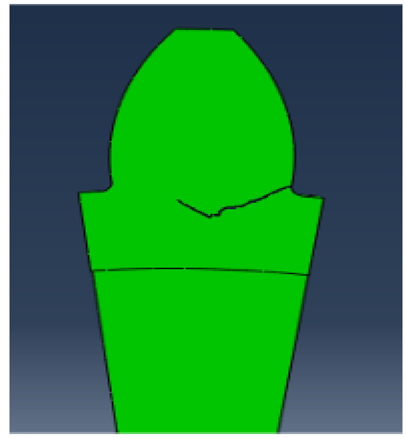
Figure 19

Crack Propagation Path for Backup Ratio=0.5

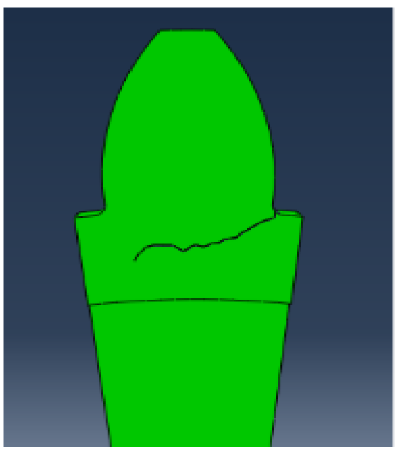
Position A:



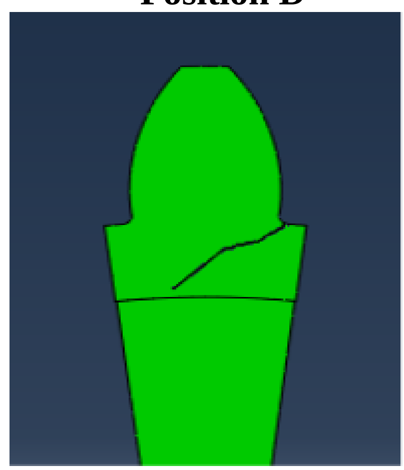
Position B



Position C



Position D



Position E

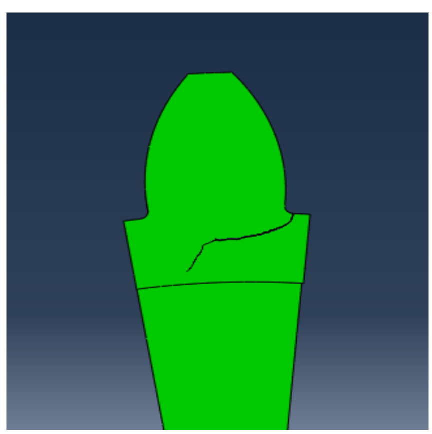
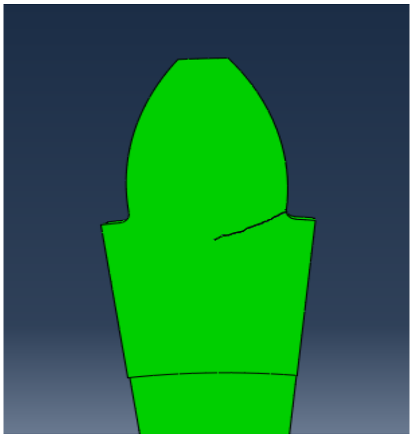


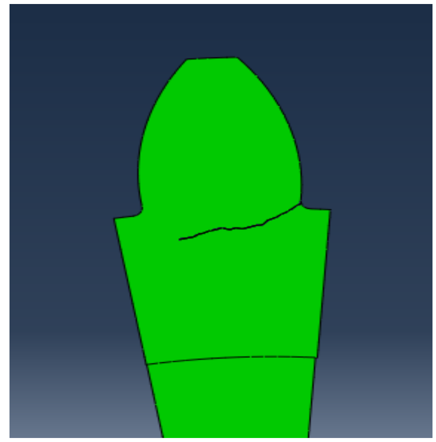
Figure 20

Crack Propagation Path for Backup Ratio = 1

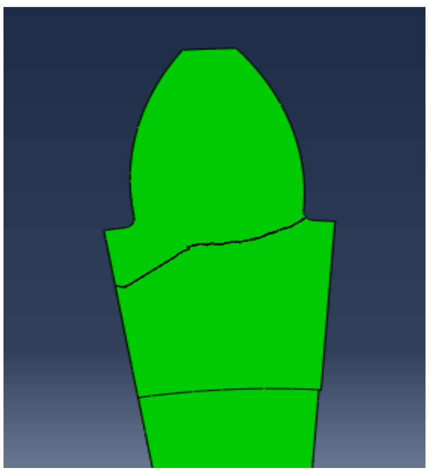
Position A



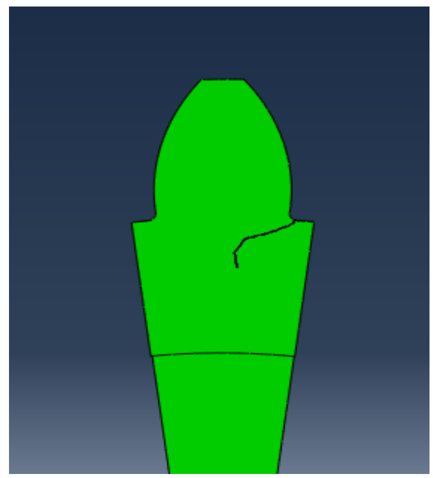
Position B



Position C



Position D



Position E

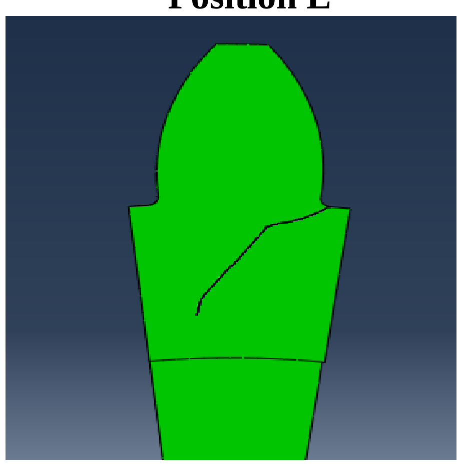
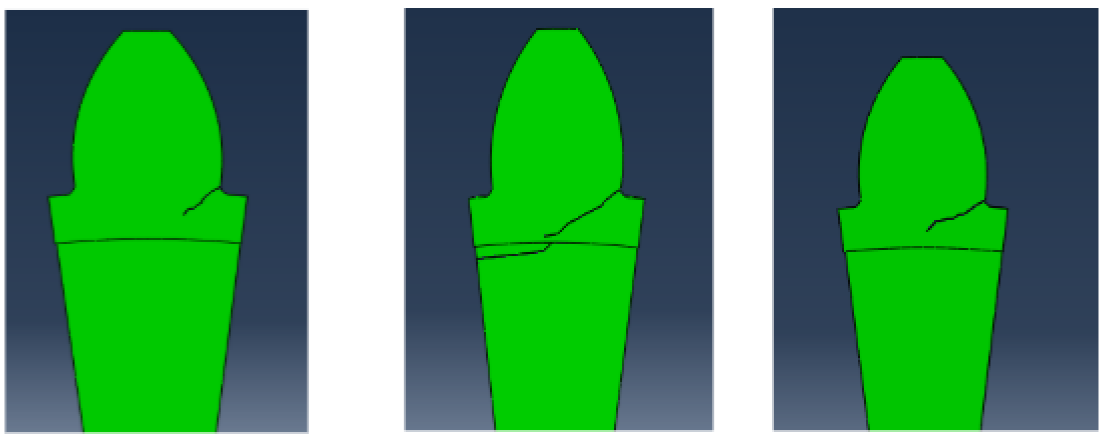


Figure 21

3.5.2) Effect of Pressure angle on Crack Path

Analysis has been done for position A in the fillet of spur gear tooth for different backup ratios and pressure angle= 15 degrees, 20 degrees, 30 degrees.

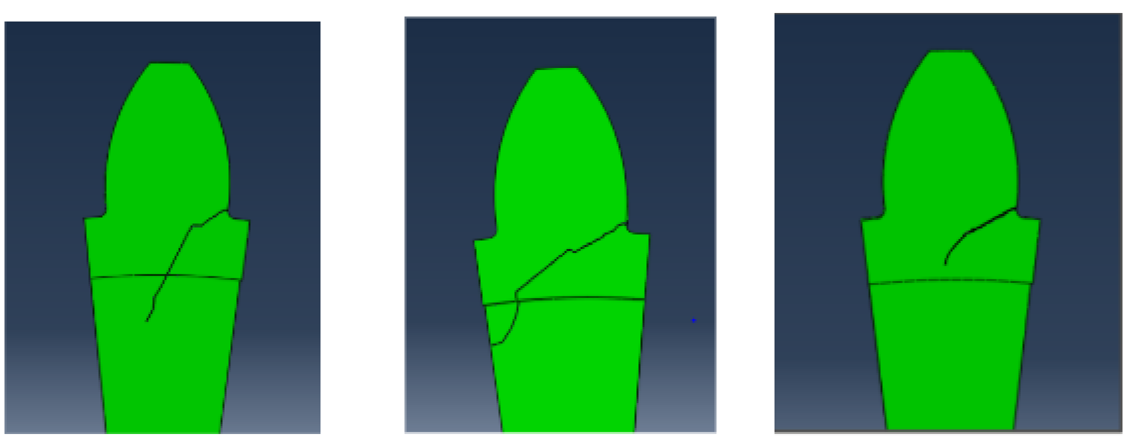
For Backup Ratio = 0.3



ϴ=30° **ϴ =20°** **ϴ=15°**

Figure 22

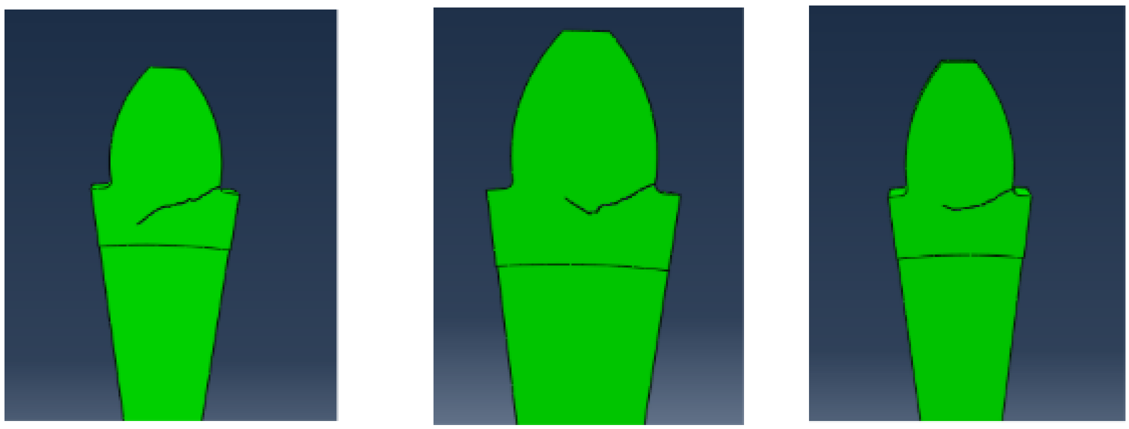
For Backup Ratio = 0.4



ϴ =30° **ϴ =20°** **ϴ =15°**

Figure 23

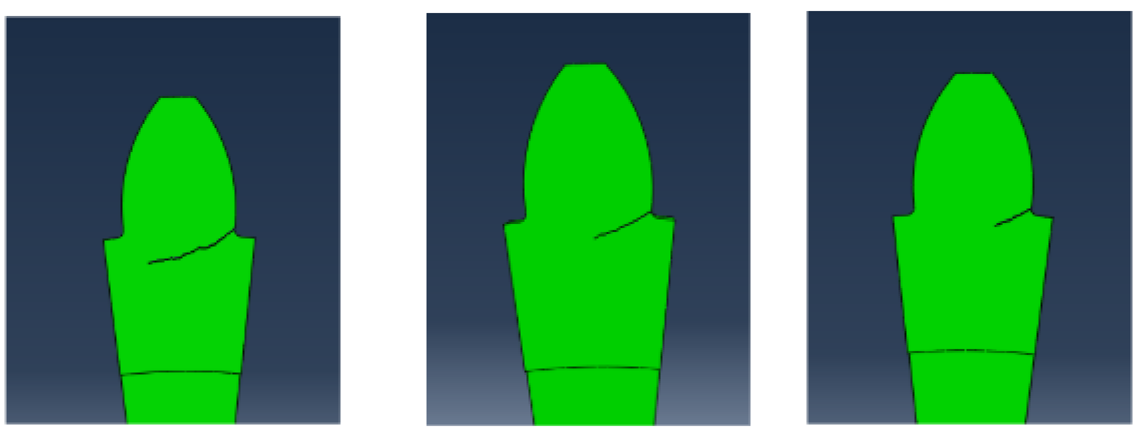
For Backup Ratio = 0.5



ϴ =30° ϴ =20° ϴ =15°

Figure 24

For Backup Ratio = 1



ϴ =30° ϴ =20° ϴ =15°

Figure 25

Chapter 4
Discussion and Conclusion

Table 3: Failure modes for various positions of initial crack and backup ratio.

Position	A	B	C	D	E
Backup Ratio					
1	Safe	Safe	Safe	Catastrophic	Catastrophic
0.5	Safe	Safe	Catastrophic	Catastrophic	Catastrophic
0.4	Catastrophic	Catastrophic	Catastrophic	Catastrophic	Catastrophic
0.3	Catastrophic	Catastrophic	Catastrophic	Catastrophic	Catastrophic

- It can be clearly seen that the crack propagation path depends not only on the backup ratio but also on the initial crack position.
- It can be seen that for lower backup ratios (<=0.5), i.e., for very thin rim spur gears, the failure is catastrophic independent of the initial crack position.
- It can be observed that as backup ratio increases to 0.5, the two positions A and B give failsafe failure, while for backup ratio 1, the three positions A, B and C are giving failsafe failure. Thus it can be concluded that as the backup ratio increases from 0.5, the possibility of safe failure increases.

Table 4: Failure modes for different values of Pressure angles and Backup Ratio

Pressure Angle / Backup Ratio	30°	20°	15°
0.3	Catastrophic	Catastrophic	Catastrophic
0.4	Catastrophic	Catastrophic	Catastrophic
0.5	Catastrophic	Safe	Safe
1	Safe	Safe	Safe

- It can be observed that the crack propagation path depends on the pressure angle.
- Thus for a given pressure angle the first criteria of variation of the backup ratio can be used as design criteria to predict the crack propagation path.
- It is important to note that both the criteria are important in terms of design criteria

Thus, Crack propagation paths due to gear tooth bending fatigue have been analyzed for a number of backup ratio values and pressure angles, to establish design criteria to prevent catastrophic failure modes. Thus the main aim was to predict the design criteria so that catastrophic failures can be avoided, and by understanding the behavior of crack propagation it is possible to design proper safety measures and maintenance plans.

Future scope

1. Future work may be focused on better defining the safe region limits also considering different gear configurations as follows
    - Web Ratios
    - Face Widths
    - Tooth Thickness
    - Tooth profile (Tooth Crowning)

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