Energy Harvesting in Wireless Body Area Networks

A PROJECT REPORT

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requirements for the award of the degrees

of

BACHELOR OF TECHNOLOGY

In

ELECTRICAL ENGINEERING

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CANDIDATE'S DECLARATION

We hereby declare that the project entitled **"Energy Harvesting in Wireless Body Area Networks"** submitted in partial fulfillment for the award of the degree of Bachelor of Technology in 'Electrical Engineering' completed under the supervision of **Dr. Prabhat Kumar Upadhyay, Associate Professor, Electrical Engineering,** IIT Indore is an authentic work.

Further, we declare that we have not submitted this work for the award of any other degree elsewhere.

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CERTIFICATE by BTP Supervisor

It is certified that the above statement made by the students is correct to the best of my knowledge.

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Preface

This report on **Energy Harvesting in Wireless Body Area Networks** is prepared under the guidance of Dr. Prabhat Kumar Upadhyay, Associate Professor, Discipline of Electrical Engineering, Indian Institute of Technology Indore.

Throughout this report, we have tried to give a detailed design of a wireless body area network system model and have explained the concepts to be utilized for the same. We have also tried to cover every possible aspect of the proposed design to ensure that it is energy as well as spectral efficient. We have even performed simulations in MATLAB and verified those with the help of numerical analysis of the expressions of the outage probabilities obtained from the performance analysis and have presented the results or insights gained in a well-defined manner to show that our design is technically sound and viable.

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Acknowledgements

We would like to thank our B. Tech Project supervisor **Dr. Prabhat Kumar Upadhyay** for his constant support and valuable guidance in formulation of our project's problem statement and his valuable feedback over entire duration of this project. He gave us a good opportunity to explore and work in one of the hot research topics in the modern day scenario.

We would also like to extend our thanks to **Mr. Sourabh Solanki** for his valuable guidance and his constant assistance with the problems which we faced throughout the course of this project. His useful directions helped us to get through the phases where we got perplexed.

It is their help and support, due to which we were able to complete the design, perform successful analysis and achieve significant results making the completion of this report possible.

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Abstract

In this project, the prospect of integration of cognitive radio in a wireless body area network consisting of primary and secondary networks co-existing together has been analysed. The secondary network communicates via an energy harvesting amplify-and-forward relay under the interference threshold conditions at the primary receiver. The secondary relay harvests energy from the received radio-frequency signals and then it uses this harvested energy to transmit information to the secondary destination. For our project, as we are dealing with low signal-to-noise ratio, the protocol which we have used is the time switching (TS) relay protocol because it outperforms the power splitting relay protocol in this situation. TS protocol allows the relay to switch between harvesting the energy from the received signal and transmitting the information processed in the remaining time. For the model proposed in this project, the performance of outage probability and system throughput is investigated to measure the system performance at the secondary destination for the chosen protocol. Simulation and numerical results show the effect of different system parameters on the performance of the proposed model design.

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Chapter 1

Introduction

Energy saving is one of the biggest challenges in communication networks. Limited energy reserves reduce the network lifetime and its frequent replenishment also increases the operational cost. However, the recent advancements in technology has made it possible to address the challenge posed and extensive researches are ongoing to improve energy efficiency and communication performance. This chapter illustrates the setting in which this project is applicable and also highlights the motivation for the project. The problem statement formulated regarding the application of the concepts used for the project has been described here and its implementation, performance, results, future scope, etc. will be discussed as we progress further in the chapters to follow. Towards the end of this chapter, the objectives and scope of this project will be clarified.

1.1 Background

Energy efficiency is essential for future wireless communication networks. It refers to an efficient utilization of the available energy and consequently extends the network lifetime and/or reduces the operation cost.

Battery-powered wireless communication systems suffer from short lifetime and require periodic replacement/recharging in order to maintain network connectivity. Communication systems supported by a continuous power supply such as cellular networks require a power grid infrastructure leading to large energy consumption which keeps increasing due to the growth of global wireless data traffic with no deceleration in the near future. This is due to rapid growth in the use of smart devices and applications with the emergence of continuous progress in the field of technology as a result of intensive ongoing researches. However, this increase of data traffic over a frequency band has resulted in congestion bringing in a new problem of spectrum shortage. The solution to this is either to widen the spectrum resulting in additional energy consumption or use spectrum sharing techniques.Thus, due to the limited supply of non-renewable energy resources as well as to eliminate the issue of spectrum shortage, there is a lot of interest to integrate the energy harvesting (EH) technology to power communication networks (which often comes under green radio/communications) [1, 2] with spectrum sharing techniques.

1.2 What are Wireless Body-Area Networks?

Wireless body-area networks (WBANs) are a real-time and low-power network, consisting of a set of wearable/implanted biosensor nodes, which collect/relay physiological and contextual signals profiling the human body activities [3]. The typical applications of these include health care, sports, military and remote medical treatment [4].

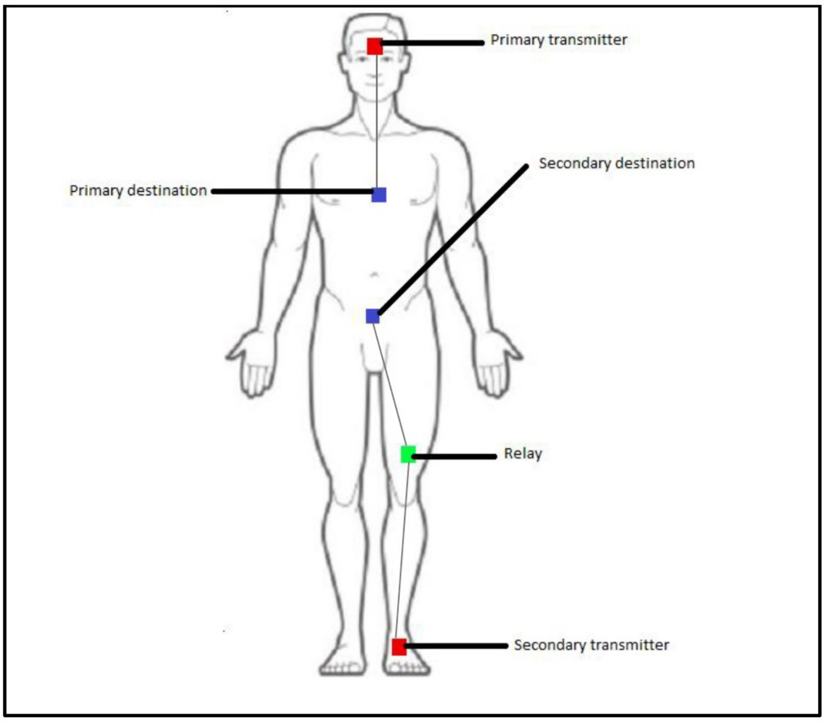

Fig. 1.1 A WBAN link depicting primary and secondary nodes

These networks usually consist of two types of nodes namely primary nodes and secondary nodes as shown in Fig.1.1. Nodes which transmit information related to our health and medical purposes have higher priorities, hence, these are called as primary nodes. The other nodes which transmit information about our movements, recording gym activities, gaming sensors, etc. have lower priorities, hence, these are called as secondary nodes. Secondary nodes can have direct links between the transmitter and receiver or have indirect links between the transmitter and receiver using relays. Line-of-sight/non-line-of-sight categorizations are not relevant and in fact is misleading for body-area communications as these are both common misconceptions.

1.3 Motivation

In the modern era, due to the stressful and hectic lifestyles, we neglect our physical and mental health. Therefore, multiple WBANs can be used to keep track of the data profiling the human body activities enabling people to monitor their health to stay fit and healthy and alert them when irregular functioning is detected.

Maintaining reliable low-power operation is particularly important for a BAN considering that their typical applications are in essential and crucial departments pertaining to health, medical and military. In such networks, battery is inconvenient to recharge/replace [5] and the outage performance is heavily influenced by the limited energy of sensors. EH is a promising technology to supply sufficient energy for sensors and also communication via relays for indirect links and long distance direct links can improve system performance. Spectrum efficiency can also be incorporated to save the bandwidth by using an underlay system where secondary nodes transmit information over the same communication channel as that of the primary nodes given that interference level at primary destination is less than some threshold. But we know that there is a trade-off between energy efficiency and spectral efficiency. Hence, to ensure that we can use energy harvesting while achieving spectral efficiency, concepts like cognitive radio and cooperative relays [6, 7] come into light to manage the interferences introduced by spectral sharing and successfully transfer information with minimalistic losses.

So the motive of our project is:

- To use energy harvesting techniques for energy efficiency and reduce the inconvenience of recharging/replacing the battery.
- To use spectral efficiency to save bandwidth.
- To manage the interferences caused due to implementation of spectral efficiency.

1.4 Concepts Employed

This section of the chapter is divided into three parts. The first part of this section gives a definition of what cognitive radio is and describes the different ways of spectrum sharing. This is followed by discussion on cooperative relay and the mode of relaying used.The last section provides explanation of energy harvesting (EH) and the corresponding protocol used in our model.

1.4.1 Cognitive Radio

The technique which enables multiple user networks to operate in a non-interfering mode over the same communication channel, i.e., frequency spectrum is called as cognitive radio. Cognitive users must have some information about the other non-cognitive users to manage the interferences caused due to spectrum sharing. As per the knowledge that is required for coexistence of the secondary users (SU) with the primary users (PU), the cognitive radio technology classifies the secondary networks into three classes namely

1. Underlay: SUs can transmit in parallel with the PU only if the interference level from SU to PU is less than some threshold. Here, threshold means the maximum level of interference that PU can tolerate without hampering its target data rate or target SNR. The PU only cares about the interference level not about the existence of SU.

2. Overlay: There is a mutual cooperation between PU and SU so the PU knows the existence of SU. SU may act as a cooperative relay for the PU and in exchange of cooperation, SU can get the opportunity to access the channel with PU.

3. Interweave: SU can access the channel only if there is no primary transmission which means that SU has to monitor the channel to know the existence of PU and the entire responsibility goes to the SU only.

As we have used an underlay system, there is a constraint on the power that can be transmitted by the secondary sources which results in reduced coverage area, limiting the distances over which a direct link is productive. To surpass this shortcoming, relays can be implemented to improve the system performance as this reduces the path loss over shorter communication ranges. Cognitive radio with relaying is recently appealing to the researchers and encouraging the prospect of using cognitive relays over wide range of networks.

1.4.2 Cooperative Relay

This technique assures increased energy efficiency and throughput for spectrum sharing relay system. Here, suppose when a SU transmits data; the PU on interpreting the interferences will convey a signal to the secondary relay and secondary destination to improve decoding at the receiver and enabling the amplify-and-forward (AF) relay being used to vary the gain accordingly [8]. Networks where users cooperate to transmit signals in spectrum sharing relay system features a new, distributed form of spatial diversity that mitigates the negative effects of signal fading and interference [9, 10].

1.4.3 Energy Harvesting

Energy harvesting is a new technology that allows the nodes to harvest energy from sources like renewable resources, mechanical vibrations and electromagnetic radiation in order to keep working. EH from renewable energy sources looks towards a fully autonomous and self-sustainable networks but it is unstable as it is weather-dependent and less efficient for applications which require critical quality-of-service (QoS). Current communication networks focus on the information content of the radio-frequency (RF) radiation and neglect the energy held by the signals but here this neglected energy is utilized for EH and this harvested energy successively powers communication devices.

The fundamental block for implementation of EH is the rectifying-antenna (rectenna) which is a diode-based circuit that converts the RF signals to DC voltage [8]. Decoding information and harvesting energy independently from the same signal at the receiver is only possible when the receiver is assumed ideal [11, 12], though it is not feasible due to practical limitations. In works [13-16], the practical RF energy harvesting mechanisms for simultaneous information and energy transfer are:

1. Time Switching (TS) - where dedicated time slots are used either for information transfer or energy harvesting.

2. Power Splitting (PS) - where one part of the received signal is used for information decoding, while the other is used for energy harvesting.

For our project, as we are dealing with low signal-to-noise ratio (SNR), the protocol which we have used in the TS relay protocol because it outperforms the PS relay protocol in this circumstances.

1.5 Objective

The objective of this project is incorporating EH technology in WBANs and spectrum sharing via cooperative cognitive radio to achieve energy as well as spectral efficiency and to ensure that information is transferred from the transmitter to the receiver optimally and without degrading the QoS.

This is carried out in our project by using a relay assisted system for an underlay having spectrum sharing communication which wirelessly harnesses energy present in the RF signals to charge the relay nodes increasing energy efficiency without degrading the communication performance.The work to be done by us involves the following things:

- Developing a relay assisted underlay communication system for EH and spectrum sharing.
- Derive the SNR expressions of the system and perform the Monte-Carlo simulations for outage probability and throughput in MATLAB.
- Mathematical analysis of the expressions obtained on further solving for outage probability with the help of the SNR expressions to reach a closed-form solution.
- If no closed-form expression for outage probability is feasible then perform numerical integrations to evaluate the values to complete the analysis using Mathematica.
- Looking for insights by examining the outage performance of the proposed system model with respect to the variations of other parameters which affect the system.

Chapter 2

Literature Review

This chapter discusses the related works in literature regarding the topic of this project and towards the end explains the reasons for choosing it.

In the recent times, due to the potential applications of WBANs in health care, sports, military and remote medical treatment, it has attracted the interest of researchers. Extensive surveys regarding WBANs have been conducted in [17-19]. The applications, structural design, involved wireless technologies and its modelling have been discussed in these surveys. Maintaining low-power operation is important for a WBAN considering the fact that it increases the human body exposure to electromagnetic radiation which can have several harmful effects. The network performance and reliability of links degrade due to low power and high attenuation caused by the path loss. In works [20-22], cooperative relaying communication technology in WBAN has been studied. In [20], it is shown that hop communication increasing energy efficiency in cases where high path losses occur. The authors in work [22] have explored the different transmission policies of relaying in WBAN and studied the outage performances of the system and also optimal distribution of power to resources to minimize the energy consumption increasing the energy efficiency using cooperative communication. Thus, we can see that as mentioned in the previous chapter that employing relays improves reliability and performance of the system.

Due to the confined nature of energy in WBANs, frequent recharging of battery is required to ensure proper functioning which is very inefficient and not feasible in many situations. EH is a promising technology to supply sufficient energy for sensors thus prolonging the lifetime of the network. EH from renewable energy sources is unstable as it is weather-dependent, thus cannot provide continuous energy supply. Therefore, it is less efficient for applications which require critical QoS [23]. To solve this problem, an alternative approach called simultaneous wireless information and power transfer (SWIPT) is applied where the nodes use the energy present in RF signals (which is often neglected) [14-16, 24]. The RF-based EH for WBANs has been discussed in research works [4, 25]. The authors in [25] have employed an EH based WBAN to maximize the throughput from the sensors to the access point and derived the optimal TS and PS ratios. In [4], the authors examined the SWIPT in a relay-based WBAN and studied an optimal strategy to obtain the maximum throughput.

The aforementioned works have solely focused on the energy efficient design of WBANs. However, due to limited availability of bandwidth and the presence of various sensor nodes in small coverage area, the problem of spectral efficiency and management of interferences introduced due to the idea of spectrum sharing is yet to be explored. Presently, very few works have looked into the concept for the integration of cognitive radio technology in WBAN [6, 7, 9, 10, 26]. However, none of them have given us an extensive analysis of outage performance for the WBAN.

From our literature review, it is worth noting the fact that no research has been conducted towards the design and analysis of energy as well as spectrum efficient model of WBAN. Therefore, the design of such WBANs is still an open area for research regarding their applications in future of smart healthcare systems. Hence, we wanted to explore, design and perform simulations and outage performance analysis of a WBAN model which integrates EH and cognitive radio techniques to achieve energy as well as spectral efficiency and to ensure that information is transferred from the transmitter to the receiver optimally without degradation of QoS.

Chapter 3

System and Channel Models

This chapter describes the system and channel models with broad explanation of the EH protocols used in the proposed model.

3.1 Model description

As shown in Fig. 3.1, in our model, we consider a primary network/user (a direct link) which consists of a primary transmitter (P_T) and a primary destination (P_D), where they communicate over a channel of certain bandwidth say B Hz. The secondary network/user (an indirect link) consists of a secondary transmitter/source (S) which communicates with a secondary destination (D) through an EH AF secondary relay (R). The SU shares the spectrum with the PU and to maintain successful decoding of information at P_D, interference management is implemented to improve QoS of the system. The interference management takes into account all kinds of interferences which are introduced by simultaneous transmissions of PUs and SUs. This introduces a constraint on the transmission power of SUs due to interference threshold (I_Q) allowed at P_D. Interference at R and D due to P_T has some impact on system performance from the SUs point of view, but here we neglect them and with help of an AF relay as R, these interferences can be reduced significantly at D.

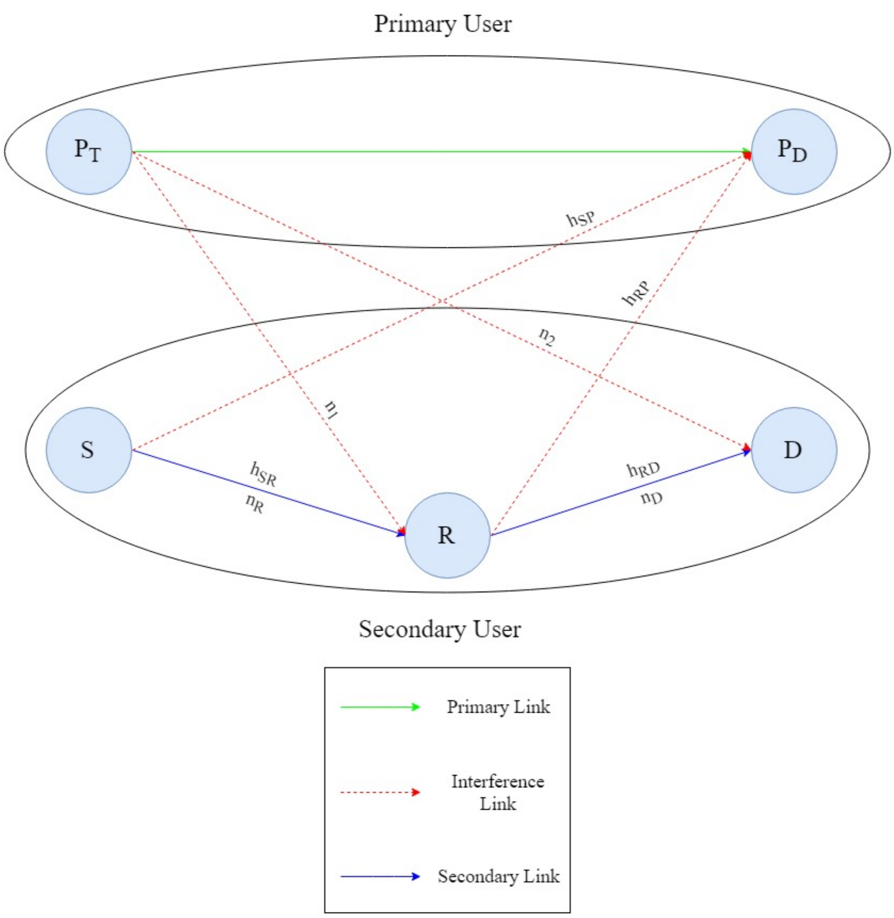

Fig. 3.1 System Model

Let h_{SR}, h_{RD}, h_{SP}, h_{RP} denote the channel coefficients of $S-R$, $R-D$, $S-P_D$ and $R-P_D$ links respectively [15, 16]. All channels are independent of each other and as the WBAN channels undergo small-scale fading in both ultra-wideband and narrowband communications they are best modelled by lognormal distributions. The instantaneous channel power gains, i.e., all $|h_K|^2$, where $k \,\epsilon\, (SR, RD, SP, RP)$ are therefore lognormally distributed random variables (RVs). All the noise is Additive White Gaussian Noise (AWGN). The channel state information (CSI) is only available at the relay node and destination node. The TS relay protocol is used here as it is better than PS when low SNR applications are considered. The TS scheme is specified in the next section of this chapter and comprehensive analysis of outage performance for the chosen scheme and the results/insights are described in chapters to follow.

3.2 Energy Harvesting Protocol (Time Switching)

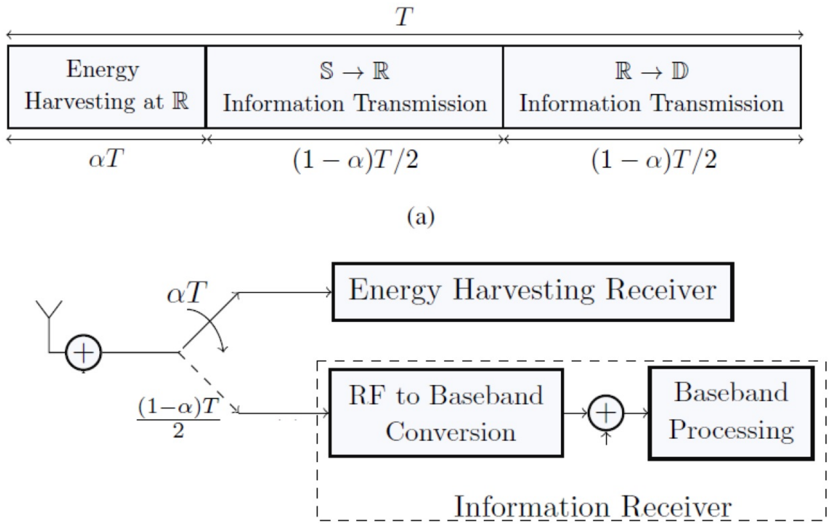

Fig. 3.2 Time Switching Protocol

In this project, we have opted for TS protocol due to its enhanced performance for low power networks and for its easiness to harvest energy from RF signals received at the secondary relay (R). In this protocol, the transmission block time is denoted as T and $\alpha \epsilon\ (0, 1)$ denotes the time switching parameter. As shown in Fig. 3.2, the relay node R harvests energy for αT duration from the RF signals received from the secondary transmitter (S) and the remaining block time of $(1 - \alpha)\, T$ duration is used for transmission of information to secondary destination (D). This remaining block of time is divided into 2 equal parts as the relays employ half-duplex scheme, where $(1 - \alpha)\, T/2$ time is used for receiving information from S and the other half is used for transmitting information to D. Hence, if S transmits with power P_S and the information x_S (with unit energy), the received signal at R is given as

$$y_R = \sqrt{P_S} x_S h_{SR} + n_R + n_1, \tag{3.1}$$

where h_{SR} is the channel coefficient between S and R, n_R is the channel AWGN and n_1 is the interference from the primary transmitter modelled as AWGN. The harvested energy at R is given by

$$E_H = \eta \alpha T P_S |h_{SR}|^2, \tag{3.2}$$

where $\eta \, \epsilon \, (0, 1)$ is the energy conversion efficiency which depends on the architecture of the nodes. The relay uses this harvested energy to transmit the information received from the S to D in the second half of the block of time left after the harvesting time i.e. in $(1 - \alpha) \, T/2$ time. Thus the transmitted power of the secondary relay without taking into account the power constraint on P_S due to the interference threshold I_Q is given by

$$P_R = \frac{2\eta \alpha P_S |h_{SR}|^2}{1 - \alpha}. \tag{3.3}$$

Taking into consideration the spectrum sharing scenario, the interference constraints/threshold (I_Q) at P_D limit the maximum transmitted powers of S and R. The interference threshold at P_D can be modelled with the help of its outage probability. Hence, the transmit powers of the secondary transmitter and the secondary relay must be restricted so that the received energy at P_D due to S and R should be below the given threshold (I_Q). Therefore, we have

$$P_S |h_{SP}|^2 \leq I_Q, \tag{3.4}$$

$$P_R |h_{RP}|^2 \leq I_Q. \tag{3.5}$$

Using equations (3.3), (3.4), (3.5), the power constraint on P_S due to the tolerable interference levels at P_D is given as

$$P_S \leq \min \left(\frac{I_Q}{|h_{SP}|^2}, \frac{I_Q \, (1 - \alpha)}{2\eta \alpha |h_{RP}|^2 |h_{SR}|^2} \right). \tag{3.6}$$

Chapter 4

Performance Analysis

This chapter provides a comprehensive analysis of outage performance and system throughput for the proposed model described in the previous chapter.

4.1 Calculation of SNR

Signal-to-noise ratio (SNR) measures the power of a desired signal to that of the background noise present in it. It is often expressed in decibels. If SNR is higher than 1 (0 dB), then it indicates that there is more signal power present than noise.

In this section, we derive the expression for SNR using the described model, the interference constraints specified and the channel coefficients modelled in the previous chapter.

From equations (3.4) and (3.5), we have

$$P_S|h_{SP}|^2 \leq I_Q \qquad \Rightarrow P_S \leq \frac{I_Q}{|h_{SP}|^2}. \tag{4.1}$$

$$P_R|h_{RP}|^2 \leq I_Q \qquad \Rightarrow P_R \leq \frac{I_Q}{|h_{RP}|^2}. \tag{4.2}$$

Signal received at R and D can be written as

$$\left.\begin{aligned} y_R &= \sqrt{P_S} x_S h_{SR} + n_R + n_1 \\ N_1 &= n_R + n_1 \end{aligned}\right\} \Rightarrow y_R = \sqrt{P_S} x_S h_{SR} + N_1, \tag{4.3}$$

$$\left.\begin{aligned} y_D &= H y_R h_{RD} + n_D + n_2 \\ N_2 &= n_D + n_2 \end{aligned}\right\} \Rightarrow y_D = H y_R h_{RD} + N_2, \tag{4.4}$$

$$H = \sqrt{\frac{P_R}{P_S|h_{SR}|^2 + {\sigma_1}^2}}. \tag{4.5}$$

By substituting the values of y_R and H from equations (4.3) and (4.5) respectively in equation (4.4), we get

$$y_D = H\sqrt{P_S}x_S h_{SR} h_{RD} + HN_1 h_{RD} + N_2. \tag{4.6}$$

Therefore, from equation (4.6) we get SNR_D as

$$SNR_D = \frac{H^2 P_S |h_{SR}|^2 |h_{RD}|^2}{H^2 |h_{RD}|^2 \sigma_1^2 + \sigma_2^2}. \tag{4.7}$$

Also

$$P_R = \frac{\eta\alpha T P_S |h_{SR}|^2}{\left(\frac{(1-\alpha)T}{2}\right)} \Rightarrow P_R = \frac{2\eta\alpha P_S |h_{SR}|^2}{1-\alpha}. \tag{4.8}$$

From now on, let

$$|h_{SR}|^2 = X \qquad |h_{RD}|^2 = Y \qquad |h_{SP}|^2 = Z \qquad |h_{RP}|^2 = W.$$

Therefore, on substituting the values of H and P_R from equations (4.5) and (4.8) respectively in equation (4.7), we get

$$SNR_D = \frac{2\eta\alpha P_S^2 X^2 Y}{2\eta\alpha P_S XY \sigma_2^2 + \sigma_2^2 (1-\alpha)(P_S X + \sigma_1^2)}. \tag{4.9}$$

Also, from equation (4.2) and (4.8), we derive another inequality on P_S as follows

$$P_S \leq \frac{I_Q (1-\alpha)}{2\eta\alpha W X}. \tag{4.10}$$

From equations (4.1) and (4.10), we get a final condition on P_S as follows

$$P_S \leq \min\left(\frac{I_Q}{Z}, \frac{I_Q (1-\alpha)}{2\eta\alpha W X}\right). \tag{4.11}$$

If $\frac{I_Q}{Z} \leq \frac{I_Q(1-\alpha)}{2\eta\alpha WX}$, we substitute $P_S = \frac{I_Q}{Z}$ in equation (4.9) and get

$$SNR_D = \frac{2\eta\alpha I_Q^2 X^2 Y}{2\eta\alpha I_Q XYZ\sigma_1^2 + \sigma_2^2(1-\alpha)(I_Q XZ + \sigma_1^2 Z^2)}. \tag{4.12}$$

Else, we substitute $P_S = \frac{I_Q(1-\alpha)}{2\eta\alpha WX}$ in equation (4.9) and get

$$SNR_D = \frac{I_Q^2(1-\alpha)Y}{2\eta\alpha WYI_Q\sigma_1^2 + \sigma_2^2[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2]}. \tag{4.13}$$

4.2 Calculation of Outage Probability

Outage probability is defined as the probability that information transfer rate of a communication network is less than the required threshold information rate. It can also be defined as the probability that SNR of a system is less than a required specified threshold.

Mathematically, we can express it as

$$P_{out} = P[SNR_D \leq \delta], \tag{4.14}$$

where δ is the threshold SNR.

$$\begin{aligned} P_{out} = \;& P\left[SNR_D \leq \delta,\, \tfrac{I_Q}{Z} \leq \tfrac{I_Q(1-\alpha)}{2\eta\alpha WX}\right] \Big\} \Rightarrow A \\ +& \\ & P\left[SNR_D \leq \delta,\, \tfrac{I_Q}{Z} > \tfrac{I_Q(1-\alpha)}{2\eta\alpha WX}\right] \Big\} \Rightarrow B. \end{aligned} \tag{4.15}$$

This is because $P(A \cap B) = 0$ (since A and B are mutually exclusive events).

4.2.1 Solving for event A

Here, we solve for both the conditions mentioned in event A i.e. $SNR_D \leq \delta$ and $\frac{I_Q}{Z} \leq \frac{I_Q(1-\alpha)}{2\eta\alpha WX}$.

$SNR_D \leq \delta \Rightarrow$

$$\frac{2\eta\alpha I_Q^2 X^2 Y}{2\eta\alpha I_Q XYZ\sigma_1^2 + \sigma_2^2 (1-\alpha)(I_Q XZ + \sigma_1^2 Z^2)} \leq \delta.$$
$$2\eta\alpha I_Q^2 X^2 Y \leq \delta \left[2\eta\alpha I_Q XYZ\sigma_1^2 + \sigma_2^2 (1-\alpha)\left(I_Q XZ + \sigma_1^2 Z^2\right)\right].$$

$$Y\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q XZ\sigma_1^2\right] \leq \sigma_2^2 \delta (1-\alpha)\left(I_Q XZ + \sigma_1^2 Z^2\right). \tag{4.16}$$

Now if the coefficient on Y is greater than 0, then we can write

$$Y \leq \frac{\sigma_2^2 \delta (1-\alpha)(I_Q XZ + \sigma_1^2 Z^2)}{\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q XZ\sigma_1^2\right]}. \tag{4.17}$$

The constraint for making the coefficient of Y greater than 0 is as follows

$$2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q XZ\sigma_1^2 > 0.$$
$$\frac{Z}{X} < \frac{I_Q}{\sigma_1^2 \delta}. \tag{4.18}$$

Now let $\frac{Z}{X} = G$ [Transformation of Random Variables].

Therefore, equation (4.17) becomes

$$Y \leq \frac{\sigma_2^2 \delta (1-\alpha)(I_Q G + \sigma_1^2 G^2)}{2\eta\alpha I_Q (I_Q - G\delta\sigma_1^2)} \qquad \left[\text{iff} \quad G < \frac{I_Q}{\sigma_1^2 \delta}\right]. \tag{4.19}$$

Now we'll define the PDF and CDF of G

$$F_G(g) = \iint\limits_{\frac{z}{x} \le g} f_{z,x}(z,x)\, dz\, dx. \tag{\#4.1}$$

$$F_G(g) = \iint\limits_{\frac{z}{x} \le g} f_z(z)\, f_x(x)\, dz\, dx \qquad \left[\begin{array}{l} f_{z,x}(z,x) = f_z(z)\, f_x(x) \\ \text{because z and x are independent Random Variables} \end{array}\right].$$

$$F_G(g) = \int\limits_0^{\infty} f_x(x) \left[\int\limits_0^{gx} f_z(z)\, dz\right] dx.$$

$$F_G(g) = \int\limits_0^{\infty} f_X(x)\, F_Z(gx)\, dx. \tag{\#4.2}$$

$$f_G(g) = \int\limits_0^{\infty} x f_X(x)\, f_Z(gx)\, dx. \tag{\#4.3}$$

$$\begin{aligned} &\therefore\ P\left[SNR_D \le \delta,\ \frac{I_Q}{Z} \le \frac{I_Q(1-\alpha)}{2\eta\alpha W X}\right] \\ &= P\left[Y\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q X Z \sigma_1^2\right] \le \sigma_2^2 \delta (1-\alpha)\left(I_Q X Z + \sigma_1^2 Z^2\right),\ G \ge \frac{2\eta\alpha W}{1-\alpha}\right] \\ &= P\left[Y \le h(G),\ W \le v(G),\ G < \frac{I_Q}{\sigma_1^2 \delta}\right] + P\left[W \le v(G),\ G \ge \frac{I_Q}{\sigma_1^2\delta}\right], \end{aligned} \tag{4.20}$$

where

$$h(G) = \frac{\sigma_2^2 \delta (1-\alpha)(I_Q G + \sigma_1^2 G^2)}{2\eta\alpha I_Q (I_Q - G\delta\sigma_1^2)} \qquad \text{and} \qquad v(G) = \frac{G(1-\alpha)}{2\eta\alpha}.$$

This is due to the fact that if $G \geq \dfrac{I_Q}{\sigma_1^2\delta}$, equation (4.16) will always be true as a negative number is always smaller than a positive number and hence we don't need to take its probability into consideration as it is always 1.

$$\therefore\ P\left[SNR_D \leq \delta,\ \frac{I_Q}{Z} \leq \frac{I_Q\left(1-\alpha\right)}{2\eta\alpha WX}\right]$$
$$= \int_0^{\frac{I_Q}{\sigma_1^2\delta}} F_{Y|G}\left(h\left(g\right)\right) F_{W|G}\left(v\left(g\right)\right) f_G\left(g\right)\, dg + \int_{\frac{I_Q}{\sigma_1^2\delta}}^{\infty} F_{W|G}\left(v\left(g\right)\right) f_G\left(g\right)\, dg$$

$$= \int_0^{\frac{I_Q}{\sigma_1^2\delta}} \int_0^{\infty} \frac{1}{8\pi\sigma_X\sigma_Z} \left[1 + erf\left[\frac{log\left(\frac{\sigma_2^2\delta(1-\alpha)\left(I_Q g+\sigma_1^2 g^2\right)}{2\eta\alpha I_Q\left(I_Q - g\delta\sigma_1^2\right)}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right] \times$$
$$\left[1 + erf\left[\frac{log\left(\frac{g(1-\alpha)}{2\eta\alpha}\right) - \mu_W}{\sqrt{2}\sigma_W}\right]\right] \left[exp\left[-\left(\frac{log\left(x\right) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \times$$
$$\left[\frac{1}{gx} exp\left[-\left(\frac{log\left(gx\right) - \mu_Z}{\sqrt{2}\sigma_Z}\right)^2\right]\right]\, dx\, dg$$
$$+ \int_{\frac{I_Q}{\sigma_1^2\delta}}^{\infty} \int_0^{\infty} \frac{1}{4\pi\sigma_X\sigma_Z} \left[1 + erf\left[\frac{log\left(\frac{g(1-\alpha)}{2\eta\alpha}\right) - \mu_W}{\sqrt{2}\sigma_W}\right]\right] \times$$
$$\left[exp\left[-\left(\frac{log\left(x\right) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \left[\frac{1}{gx} exp\left[-\left(\frac{log\left(gx\right) - \mu_Z}{\sqrt{2}\sigma_Z}\right)^2\right]\right]\, dx\, dg. \quad (*4.1)$$

4.2.2 Solving for event B

Here, we solve for both the conditions mentioned in event B i.e. $SNR_D \leq \delta$ and $\frac{I_Q}{Z} > \frac{I_Q(1-\alpha)}{2\eta\alpha WX}$.
$SNR_D \leq \delta \Rightarrow$

$$\frac{I_Q^2(1-\alpha)Y}{2\eta\alpha WYI_Q\sigma_1^2 + \sigma_2^2\left[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2\right]} \leq \delta.$$
$$I_Q^2(1-\alpha)Y \leq \delta\left[2\eta\alpha WYI_Q\sigma_1^2 + \sigma_2^2\left[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2\right]\right].$$

$$YI_Q\left[I_Q(1-\alpha) - 2\eta\alpha W\sigma_1^2\delta\right] \leq \delta\sigma_2^2\left[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2\right]. \tag{4.21}$$

Now if the coefficient on Y is greater than 0, then we can write

$$Y \leq \frac{\delta\sigma_2^2\left[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2\right]}{I_Q\left[I_Q(1-\alpha) - 2\eta\alpha W\sigma_1^2\delta\right]}. \tag{4.22}$$

The constraint for making the coefficient of Y greater than 0 is as follows

$$I_Q\left[I_Q(1-\alpha) - 2\eta\alpha W\sigma_1^2\delta\right] > 0.$$
$$W < \frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}. \tag{4.23}$$

$$\begin{aligned}
&\therefore\ P\left[SNR_D \leq \delta,\ \frac{I_Q}{Z} > \frac{I_Q(1-\alpha)}{2\eta\alpha WX}\right] \\
&= P\left[YI_Q\left[I_Q(1-\alpha) - 2\eta\alpha W\sigma_1^2\delta\right] \leq \delta\sigma_2^2\left[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2\right],\ G < \frac{2\eta\alpha W}{1-\alpha}\right] \\
&= P\left[Y \leq l(W),\ G < s(W),\ W < \frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}\right] + P\left[G < s(W),\ W \geq \frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}\right],
\end{aligned} \tag{4.24}$$

where

$$l(W) = \frac{\delta\sigma_2^2\left[I_Q(1-\alpha)W + 2\eta\alpha W^2\sigma_1^2\right]}{I_Q\left[I_Q(1-\alpha) - 2\eta\alpha W\sigma_1^2\delta\right]} \text{and} \qquad s(W) = \frac{2\eta\alpha W}{1-\alpha}.$$

This is due to the fact that if $W \geq \dfrac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}$, equation (4.21) will always be true as a negative number is always smaller than a positive number and hence we don't need to take its probability into consideration as it is always 1.

$$\therefore\ P\left[SNR_D \leq \delta,\ \frac{I_Q}{Z} > \frac{I_Q(1-\alpha)}{2\eta\alpha WX}\right]$$

$$= \int_0^{\frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}} F_{Y|W}(l(w))\, F_{G|W}(s(w))\, f_W(w)\ dw + \int_{\frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}}^{\infty} F_{G|W}(s(w))\, f_W(w)\ dw$$

$$\begin{aligned}
= &\int_0^{\frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}} \int_0^{\infty} \frac{1}{8\pi\sigma_X\sigma_W} \left[1 + erf\left[\frac{log\left(\frac{\delta\sigma_2^2\left[I_Q(1-\alpha)w + 2\eta\alpha w^2\sigma_1^2\right]}{I_Q\left[I_Q(1-\alpha) - 2\eta\alpha w\sigma_1^2\delta\right]}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right] \times \\
&\left[1 + erf\left[\frac{log\left(\frac{2x\eta\alpha w}{1-\alpha}\right) - \mu_Z}{\sqrt{2}\sigma_Z}\right]\right] \left[\frac{1}{x} exp\left[-\left(\frac{log(x) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \times \\
&\left[\frac{1}{w} exp\left[-\left(\frac{log(w) - \mu_W}{\sqrt{2}\sigma_W}\right)^2\right]\right]\ dx\, dw \\
+ &\int_{\frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\delta}}^{\infty} \int_0^{\infty} \frac{1}{4\pi\sigma_X\sigma_W} \left[1 + erf\left[\frac{log\left(\frac{2x\eta\alpha w}{1-\alpha}\right) - \mu_Z}{\sqrt{2}\sigma_Z}\right]\right] \times \\
&\left[\frac{1}{x} exp\left[-\left(\frac{log(x) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \left[\frac{1}{w} exp\left[-\left(\frac{log(w) - \mu_W}{\sqrt{2}\sigma_W}\right)^2\right]\right]\ dx\, dw.
\end{aligned} \tag{*4.2}$$

Also

$$\delta = 2^{\frac{2r_{th}}{1-\alpha}} - 1. \tag{4.25}$$

By equations (*4.1), (*4.2) and (4.25) we get

$$P_{out} = \int_0^{\frac{I_Q}{\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}} \int_0^{\infty} \frac{1}{8\pi\sigma_X\sigma_Z} \left[1 + erf\left[\frac{log\left(\frac{\sigma_2^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)(1-\alpha)(I_Q g+\sigma_1^2 g^2)}{2\eta\alpha I_Q\left(I_Q - g\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\sigma_1^2\right)}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right] \times$$
$$\left[1 + erf\left[\frac{log\left(\frac{g(1-\alpha)}{2\eta\alpha}\right) - \mu_W}{\sqrt{2}\sigma_W}\right]\right] \left[exp\left[-\left(\frac{log\,(x) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \times$$
$$\left[\frac{1}{gx} exp\left[-\left(\frac{log\,(gx) - \mu_Z}{\sqrt{2}\sigma_Z}\right)^2\right]\right] dx\, dg$$
$$+ \int_{\frac{I_Q}{\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}}^{\infty} \int_0^{\infty} \frac{1}{4\pi\sigma_X\sigma_Z} \left[1 + erf\left[\frac{log\left(\frac{g(1-\alpha)}{2\eta\alpha}\right) - \mu_W}{\sqrt{2}\sigma_W}\right]\right] \times$$
$$\left[exp\left[-\left(\frac{log\,(x) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \left[\frac{1}{gx} exp\left[-\left(\frac{log\,(gx) - \mu_Z}{\sqrt{2}\sigma_Z}\right)^2\right]\right] dx\, dg$$
$$+ \int_0^{\frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}} \int_0^{\infty} \frac{1}{8\pi\sigma_X\sigma_W} \left[1 + erf\left[\frac{log\left(\frac{\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\sigma_2^2\left[I_Q(1-\alpha)w + 2\eta\alpha w^2\sigma_1^2\right]}{I_Q\left[I_Q(1-\alpha) - 2\eta\alpha w\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\right]}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right] \times$$
$$\left[1 + erf\left[\frac{log\left(\frac{2x\eta\alpha w}{1-\alpha}\right) - \mu_Z}{\sqrt{2}\sigma_Z}\right]\right] \left[\frac{1}{x} exp\left[-\left(\frac{log\,(x) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \times$$
$$\left[\frac{1}{w} exp\left[-\left(\frac{log\,(w) - \mu_W}{\sqrt{2}\sigma_W}\right)^2\right]\right] dx\, dw$$
$$+ \int_{\frac{I_Q(1-\alpha)}{2\eta\alpha\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}}^{\infty} \int_0^{\infty} \frac{1}{4\pi\sigma_X\sigma_W} \left[1 + erf\left[\frac{log\left(\frac{2x\eta\alpha w}{1-\alpha}\right) - \mu_Z}{\sqrt{2}\sigma_Z}\right]\right] \times$$
$$\left[\frac{1}{x} exp\left[-\left(\frac{log\,(x) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] \left[\frac{1}{w} exp\left[-\left(\frac{log\,(w) - \mu_W}{\sqrt{2}\sigma_W}\right)^2\right]\right] dx\, dw. \tag{*4.3}$$

As we can see that the expression for outage probability comes out to be quite complex in for the proposed model and can only be solved by numerical integration models. Thus, to simplify the expression of outage probability we use a approximated model as explained in the next chapter.

4.3 System Throughput

Throughput is usually measured in bits per second (bit/s or bps) and sometimes in data packets per second (p/s or pps) or data packets per time slot. The system throughput or aggregate throughput is the sum of the data rates that are delivered to all terminals in a network.

The throughput of the proposed system is defined as follows [27]

$$S_T = \left(\frac{1-\alpha}{2}\right)\left[\left(1 - P_{out}\right) r_{th}\right]. \tag{4.26}$$

Chapter 5

Approximate Model

Here we discuss about the proposed approximate model for simplification of the expressions for outage probability and system throughput without deviating much from the results obtained from the actual model.

5.1 Model Description

The actual model is studied under average interference-constraints [28] where we try to approximate the instantaneous channel power gains for the channels $S - P_D$ and $R - P_D$ by their mean (or average) values, i.e., we replace the random variables $|h_{SP}|^2$ and $|h_{RP}|^2$ by their expectations in order to achieve simplification of the desired expressions.
Therefore, we replace the random variables W and Z by $E[W]$ and $E[Z]$ respectively in the equations derived in chapter 4 and solve for SNR, P_{out} and S_T of the approximated model.

5.1.1 Calculation of SNR

On replacing W and Z by $E[W]$ and $E[Z]$ respectively, in equation (4.11) we get

$$P_S \leq \min\left(\frac{I_Q}{E[Z]}, \frac{I_Q(1-\alpha)}{2\eta\alpha E[W]X}\right).$$

$$\therefore P_S = \begin{cases} \frac{I_Q}{E[Z]}, & \text{if } X \leq \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]} \\ \frac{I_Q(1-\alpha)}{2\eta\alpha E[W]X}, & \text{if } X > \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]} \end{cases}. \tag{5.1}$$

Therefore, by equations (4.9) & (5.1) we get the expression of SNR_D as follows

$$SNR_D = \begin{cases} \frac{2\eta\alpha I_Q^2 X^2 Y}{2\eta\alpha I_Q XYE[Z]\sigma_1^2 + \sigma_2^2(1-\alpha)\left(I_Q XE[Z] + \sigma_1^2 E[Z]^2\right)}, & \text{if } X \leq \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]} \\ \frac{I_Q^2(1-\alpha)Y}{2\eta\alpha E[W]YI_Q\sigma_1^2 + \sigma_2^2\left[I_Q(1-\alpha)E[W] + 2\eta\alpha E[W]^2\sigma_1^2\right]}, & \text{if } X > \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]} \end{cases}. \tag{5.2}$$

5.2 Calculation of Outage Probability

By equations (5.1) & (4.14) we get

$$\begin{aligned} P_{out} = \; & P\left[SNR_D \leq \delta,\, X \leq \tfrac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right] \Big\} \Rightarrow A \\ + & \\ & P\left[SNR_D \leq \delta,\, X > \tfrac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right] \Big\} \Rightarrow B. \end{aligned} \tag{5.3}$$

This is because $P(A \cap B) = 0$ (since A and B are mutually exclusive events).

5.2.1 Solving for event A

On replacing Z by $E[Z]$ in equation (4.16) we get

$$Y\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q XE[Z]\sigma_1^2\right] \leq \sigma_2^2\delta(1-\alpha)\left(I_Q XE[Z] + \sigma_1^2 E[Z]^2\right). \tag{5.4}$$

Now if the coefficient on Y is greater than 0, then we can write

$$Y \leq \frac{\sigma_2^2\delta(1-\alpha)\left(I_Q XE[Z] + \sigma_1^2 E[Z]^2\right)}{\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q XE[Z]\sigma_1^2\right]}. \tag{5.5}$$

The constraint for making the coefficient of Y greater than 0 is as follows

$$\begin{aligned} 2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q XE[Z]\sigma_1^2 > 0. \\ X > \frac{E[Z]\sigma_1^2\delta}{I_Q}. \end{aligned} \tag{5.6}$$

$$\begin{aligned}
&\therefore\ P\left[SNR_D \leq \delta,\ X \leq \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right] \\
&= P\left[Y\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q X E\,[Z]\,\sigma_1^2\right] \leq \sigma_2^2\delta\,(1-\alpha)\left(I_Q X E\,[Z] + \sigma_1^2 E\,[Z]^2\right),\right. \\
&\left. X \leq \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right] \\
&= P\left[X \leq \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]},\ X \leq \frac{E\,[Z]\,\sigma_1^2\delta}{I_Q}\right] \\
&+ P\left[Y \leq b\,(X)\,,\ X \leq \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]},\ X > \frac{E\,[Z]\,\sigma_1^2\delta}{I_Q}\right], \qquad (5.7)
\end{aligned}$$

where

$$b\,(X) = \frac{\sigma_2^2\delta\,(1-\alpha)\left(I_Q X E\,[Z] + \sigma_1^2 E\,[Z]^2\right)}{\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q X E\,[Z]\,\sigma_1^2\right]}.$$

This is due to the fact that if $X \leq \frac{E[Z]\sigma_1^2\delta}{I_Q}$, equation (5.4) will always be true as a negative number is always smaller than a positive number and hence we don't need to take its probability into consideration as it is always 1.

$$\begin{aligned}
&\therefore\ P\left[SNR_D \leq \delta,\ X \leq \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right] \\
&= \int_0^{\frac{E[Z]\sigma_1^2\delta}{I_Q}} f_X\,(x)\,u\left[x - \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right]\,dx \\
&+ \int_{\frac{E[Z]\sigma_1^2\delta}{I_Q}}^{\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}} F_{Y|X}\,(b\,(x))\,f_X\,(x)\,u\left[x - \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right]\,dx.
\end{aligned}$$

$$\therefore\ P\left[SNR_D \leq \delta,\, X \leq \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right]$$

$$= F_X\left(\min\left(\frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]},\, \frac{E\,[Z]\,\sigma_1^2\delta}{I_Q}\right)\right)$$

$$+ \int_{\frac{E[Z]\sigma_1^2\delta}{I_Q}}^{\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}} F_{Y|X}\left(b\left(x\right)\right) f_X\left(x\right) u\left[x - \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right]\, dx$$

$$= \frac{1}{2}\left[1 + erf\left[\frac{log\left(\min\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]},\, \frac{E[Z]\sigma_1^2\delta}{I_Q}\right)\right) - \mu_X}{\sqrt{2}\sigma_X}\right]\right]$$

$$+ \int_{\frac{E[Z]\sigma_1^2\delta}{I_Q}}^{\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}} \frac{1}{2\sqrt{2\pi}}\left[1 + erf\left[\frac{log\left(\frac{\sigma_2^2\delta(1-\alpha)\left(I_Q X E[Z] + \sigma_1^2 E[Z]^2\right)}{\left[2\eta\alpha I_Q^2 X^2 - 2\eta\delta\alpha I_Q X E[Z]\sigma_1^2\right]}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right] \times$$

$$\left[\frac{1}{x\sigma_X} exp\left[-\left(\frac{log\left(x\right) - \mu_X}{\sqrt{2}\sigma_X}\right)^2\right]\right] u\left[x - \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right]\, dx. \qquad (*5.1)$$

5.2.2 Solving for event B

On replacing W by $E\,[W]$ in equation (4.21) we get

$$Y I_Q\left[I_Q\left(1-\alpha\right) - 2\eta\alpha E\,[W]\,\sigma_1^2\delta\right] \leq \delta\sigma_2^2\left[I_Q\left(1-\alpha\right) E\,[W] + 2\eta\alpha E\,[W]^2\sigma_1^2\right]. \qquad (5.8)$$

Now if the coefficient on Y is greater than 0, then we can write

$$Y \leq \frac{\delta\sigma_2^2\left[I_Q\left(1-\alpha\right) E\,[W] + 2\eta\alpha E\,[W]^2\sigma_1^2\right]}{I_Q\left[I_Q\left(1-\alpha\right) - 2\eta\alpha E\,[W]\,\sigma_1^2\delta\right]}. \qquad (5.9)$$

On substituting the value of δ from equation (4.25) in the above equation, we get

$$Y \leq \frac{\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)\sigma_2^2\left[I_Q\left(1-\alpha\right)E\left[W\right] + 2\eta\alpha E\left[W\right]^2\sigma_1^2\right]}{I_Q\left[I_Q\left(1-\alpha\right) - 2\eta\alpha E\left[W\right]\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)\right]}. \tag{5.10}$$

The constraint for making the coefficient of Y greater than 0 is as follows

$$I_Q\left[I_Q\left(1-\alpha\right) - 2\eta\alpha E\left[W\right]\sigma_1^2\delta\right] > 0.$$

$$\alpha < \frac{I_Q}{I_Q + 2\eta\delta\sigma_1^2 E\left[W\right]}. \tag{5.11}$$

On substituting the value of δ from equation (4.25) in the above equation, we get

$$\alpha < \frac{I_Q}{I_Q + 2\eta\sigma_1^2 E\left[W\right].\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)}. \tag{5.12}$$

$$P\left[SNR_D \leq \delta,\, X > \frac{\left(1-\alpha\right)E\left[Z\right]}{2\eta\alpha E\left[W\right]}\right] = \begin{cases} P\left[Y \leq r\left(\alpha\right),\, X > \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right], & \text{if } \alpha < s\left(\alpha\right) \\ P\left[X > \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right], & \text{if } \alpha \geq s\left(\alpha\right) \end{cases}, \tag{5.13}$$

where $r\left(\alpha\right) = \dfrac{\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)\sigma_2^2\left[I_Q\left(1-\alpha\right)E\left[W\right] + 2\eta\alpha E\left[W\right]^2\sigma_1^2\right]}{I_Q\left[I_Q\left(1-\alpha\right) - 2\eta\alpha E\left[W\right]\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)\right]}$,

and $s\left(\alpha\right) = \dfrac{I_Q}{I_Q + 2\eta\sigma_1^2 E\left[W\right]\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)}$.

This is due to the fact that if $\alpha \geq s\left(\alpha\right)$, equation (5.8) will always be true as a negative number is always smaller than a positive number and hence we don't need to take its probability into consideration as it is always 1.

$$\therefore\ P\left[SNR_D \leq \delta,\, X > \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right] = \begin{cases} F_Y\left(r\left(\alpha\right)\right)\left[1 - F_X\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right)\right], & \text{if } \alpha < s\left(\alpha\right) \\ 1 - F_X\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right), & \text{if } \alpha \geq s\left(\alpha\right) \end{cases}. \tag{5.14}$$

$$\begin{aligned} \therefore P&\left[SNR_D \leq \delta\, X > \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right] \\ &= F_Y\left(r\left(\alpha\right)\right)\left[1 - F_X\left(\frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right)\right]\left[1 - u\left[\alpha - s\left(\alpha\right)\right]\right] \\ &+ \left[1 - F_X\left(\frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right)\right] u\left[\alpha - s\left(\alpha\right)\right]. \end{aligned} \tag{5.15}$$

$$\begin{aligned} &P\left[SNR_D \leq \delta\, X > \frac{(1-\alpha)\,E\,[Z]}{2\eta\alpha E\,[W]}\right] \\ &= \left[\frac{1}{4}\left[1 + erf\left[\frac{log\left(\frac{\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\sigma_2^2\left[I_Q(1-\alpha)E[W]+2\eta\alpha E[W]^2\sigma_1^2\right]}{I_Q\left[I_Q(1-\alpha)-2\eta\alpha E[W]\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\right]}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right]\right] \times \\ &\left[1 - erf\left[\frac{log\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right) - \mu_X}{\sqrt{2}\sigma_X}\right]\right]\left[1 - u\left[\alpha - \frac{I_Q}{I_Q + 2\eta\sigma_1^2 E\,[W]\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)}\right]\right]\Bigg]\Bigg] \\ &+\frac{1}{2}\left[1 - erf\left[\frac{log\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right) - \mu_X}{\sqrt{2}\sigma_X}\right]\right] u\left[\alpha - \frac{I_Q}{I_Q + 2\eta\sigma_1^2 E\,[W]\left(2^{\frac{2r_{th}}{1-\alpha}} - 1\right)}\right]. \end{aligned} \tag{*5.2}$$

By equations (*5.1), (*5.2) and (4.25) we get

$$P_{out} = \frac{1}{2}\left[1 + erf\left[\frac{log\left(\min\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}, \frac{E[Z]\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}{I_Q}\right)\right) - \mu_X}{\sqrt{2}\sigma_X}\right]\right]$$
$$+ \int_{\frac{E[Z]\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}{I_Q}}^{\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}} \frac{1}{2}\left[\left[1 + erf\left[\frac{log\left(\frac{\sigma_2^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)(1-\alpha)\left(I_Q X E[Z]+\sigma_1^2 E[Z]^2\right)}{\left[2\eta\alpha I_Q^2 X^2 - 2\eta\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\alpha I_Q X E[Z]\sigma_1^2\right]}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right]\right] \times$$
$$u\left[x - \frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right]\Bigg] \, dx$$
$$+ \left[\frac{1}{4}\left[1 + erf\left[\frac{log\left(\frac{\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\sigma_2^2\left[I_Q(1-\alpha)E[W]+2\eta\alpha E[W]^2\sigma_1^2\right]}{I_Q\left[I_Q(1-\alpha)-2\eta\alpha E[W]\sigma_1^2\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)\right]}\right) - \mu_Y}{\sqrt{2}\sigma_Y}\right]\right]\right. \times$$
$$\left[1 - erf\left[\frac{log\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right) - \mu_X}{\sqrt{2}\sigma_X}\right]\right]\left.\left[1 - u\left[\alpha - \frac{I_Q}{I_Q + 2\eta\sigma_1^2 E[W]\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}\right]\right]\right]$$
$$+\frac{1}{2}\left[1 - erf\left[\frac{log\left(\frac{(1-\alpha)E[Z]}{2\eta\alpha E[W]}\right) - \mu_X}{\sqrt{2}\sigma_X}\right]\right] u\left[\alpha - \frac{I_Q}{I_Q + 2\eta\sigma_1^2 E[W]\left(2^{\frac{2r_{th}}{1-\alpha}}-1\right)}\right]. \tag{*5.3}$$

5.3 System Throughput

The throughput of the approximated system is defined as follows

$$S_T = \left(\frac{1-\alpha}{2}\right)\left[(1 - P_{out})\, r_{th}\right]. \tag{5.16}$$

Chapter 6

Results and Inferences

Here, we discuss the results or insights obtained from the analysis of the outage performance with the help of its expression obtained in the previous chapter and its simulation in MATLAB of the model described in Chapter 3 in the form of graphs. It is also evident from all the graphs included in this chapter that the simulated and analytical values of outage probabilities match in all cases.

6.1 For varying time switching parameter (α)

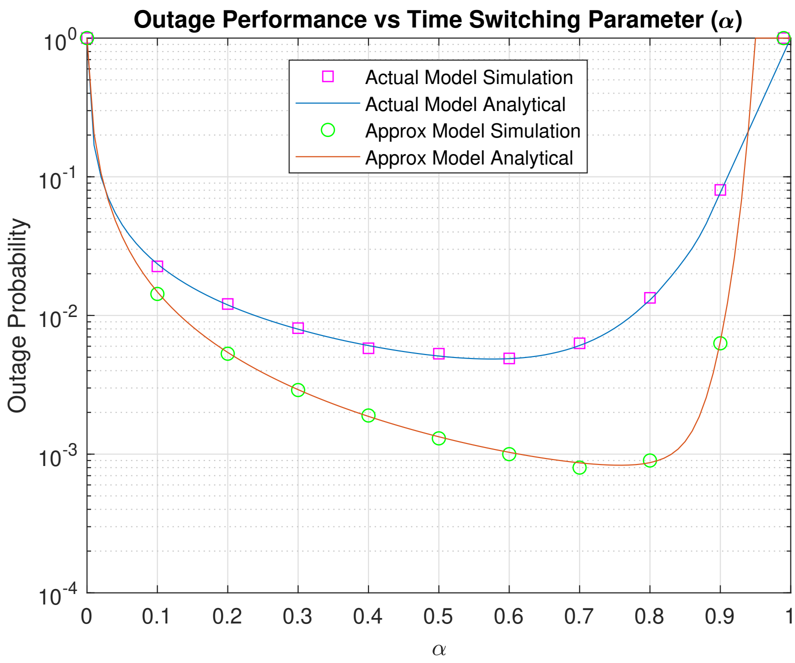

Fig. 6.1 P_{out} vs α

In Fig. 6.1, the simulated and analytical outage probabilities of both the actual and approx. models have been observed for varying values of $\alpha \epsilon (0,\ 1)$ where other parameters are constant ($\eta = 0.5$, $r_{th} = 0.01$ bits/s and $I_Q = 10$ W). We can see that as the value of α increases the outage probability decreases and after a point, it starts to increase for both the models. Therefore we get an optimal value of α for which system performance is maximum.

6.2 For varying interference threshold (I_Q)

Here, the performances of actual and approx. models have been compared and also observed individually for varying values of $I_Q \epsilon (10, 50)$. We can see from the graphs obtained that as the value of I_Q increases, the outage probability keeps decreasing for both the models. This implies that system performance increases as I_Q increases.

6.2.1 Comparing actual and approx. model

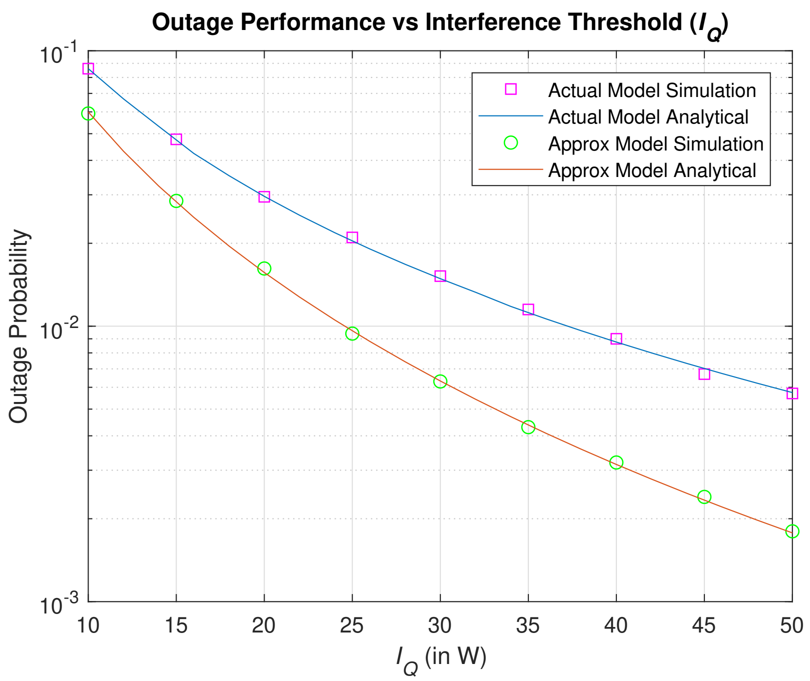

Fig. 6.2 P_{out} vs I_Q

In Fig. 6.2, the simulated and analytical outage probabilities of both the actual and approx. models have been observed for varying values of $I_Q \epsilon (10, 50)$ where other parameters are constant ($\eta = 0.5$, $\alpha = 0.5$ and $r_{th} = 0.1$ bits/s).

6.2.2 For actual model

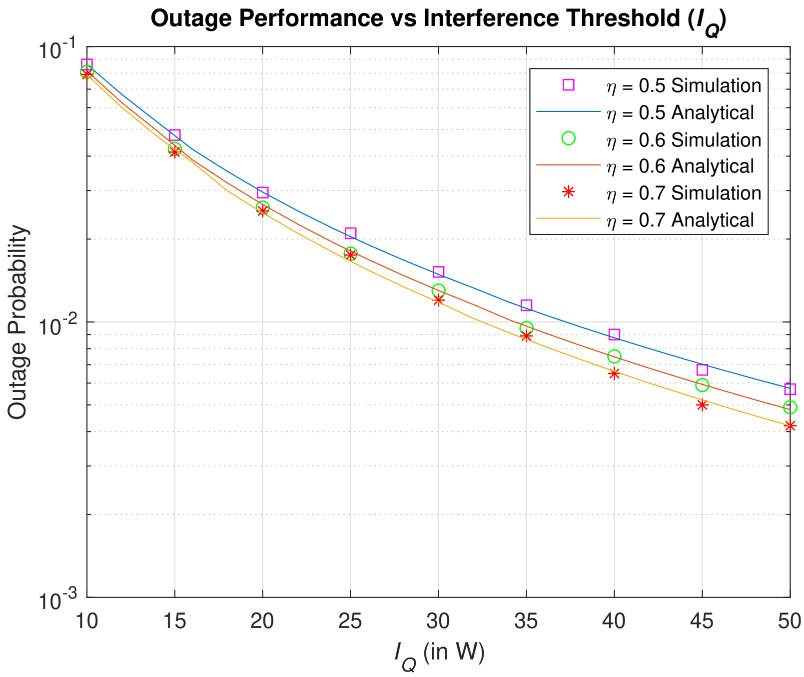

Fig. 6.3 P_{out} (actual) vs I_Q

In Fig. 6.3, the simulated and analytical outage probabilities of the actual model have been observed for different value of $\eta \, \epsilon \, \{0.5, \, 0.6, \, 0.7\}$ and varying values of $I_Q \, \epsilon \, (10, \, 50)$ where other parameters are constant (α = 0.5 and r_{th} = 0.1 bits/s). It is also evident from the graph that as value of η is increased the graph moves downward showing an improvement in system performance.

6.2.3 For approximate model

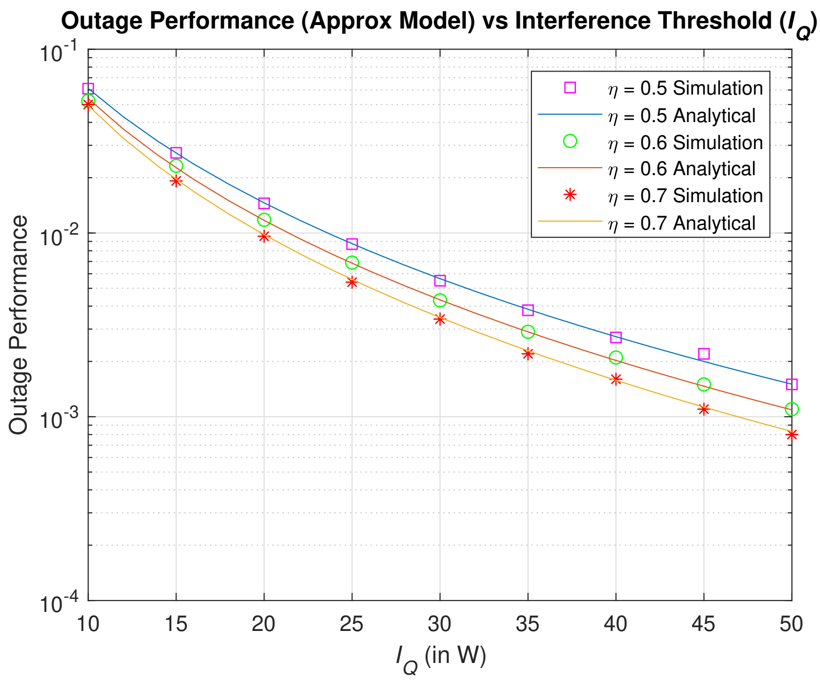

Fig. 6.4 P_{out} (approx) vs I_Q

In Fig. 6.4, the simulated and analytical outage probabilities of the approx. model have been observed for different value of $\eta \, \epsilon \, \{0.5, \, 0.6, \, 0.7\}$ and varying values of $I_Q \, \epsilon \, (10, \, 50)$ where other parameters are constant (α = 0.6 and r_{th} = 0.1 bits/s). It is also evident from the graph that as value of η is increased the graph moves downward showing an improvement in system performance.

6.3 For varying energy conversion efficiency (η)

Here, the performances of actual and approx. models have been compared and also observed individually for varying values of $\eta \, \epsilon \, (0.3, \, 0.7)$. We can see from the graphs obtained that as the value of η increases, the outage probability keeps decreasing for both the models. This implies that system performance increases as η increases.

6.3.1 Comparing actual and approx. model

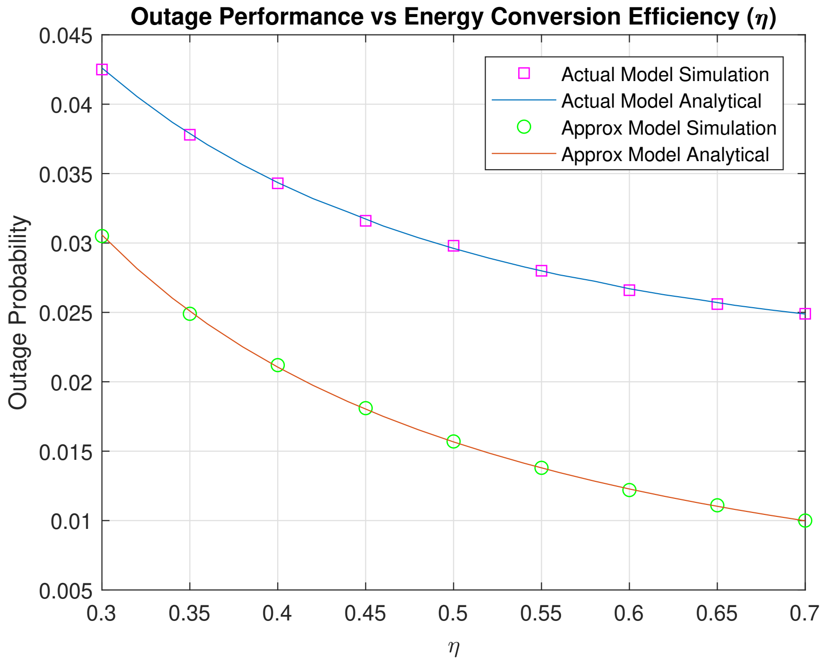

Fig. 6.5 P_{out} vs η

In Fig. 6.5, the simulated and analytical outage probabilities of both the actual and approx. models have been observed for varying values of $\eta \epsilon (0.3, 0.7)$ where other parameters are constant (I_Q = 20 W, α = 0.5 and r_{th} = 0.1 bits/s).

6.3.2 For actual model

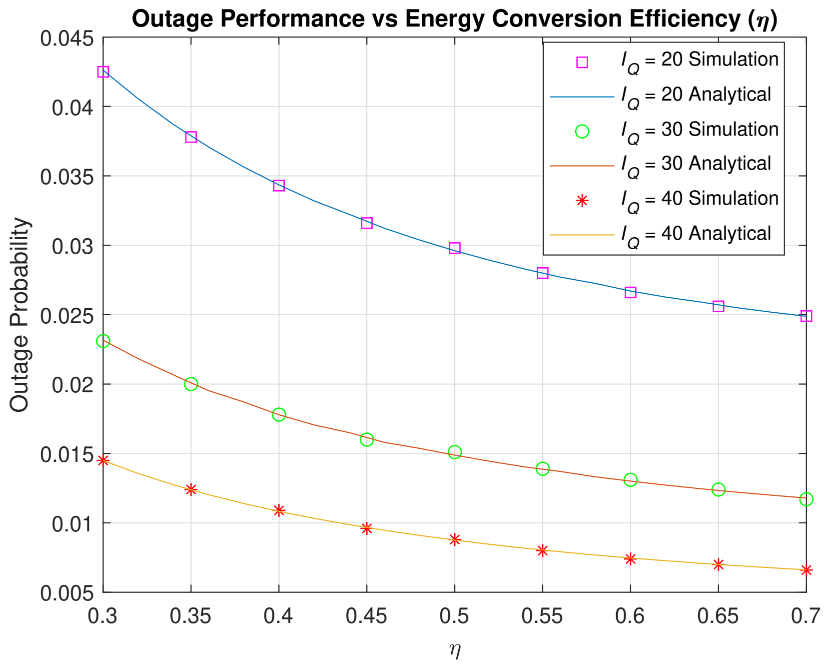

Fig. 6.6 P_{out} (actual) vs η

In Fig. 6.6, the simulated and analytical outage probabilities of the actual model have been observed for different value of $I_Q \, \epsilon \, \{20,\ 30,\ 40\}$ and varying values of $\eta \, \epsilon \, (0.3,\ 0.7)$ where other parameters are constant (α = 0.5 and r_{th} = 0.1 bits/s). It is also evident from the graph that as value of I_Q is increased the graph moves downward showing an improvement in system performance.

6.3.3 For approximate model

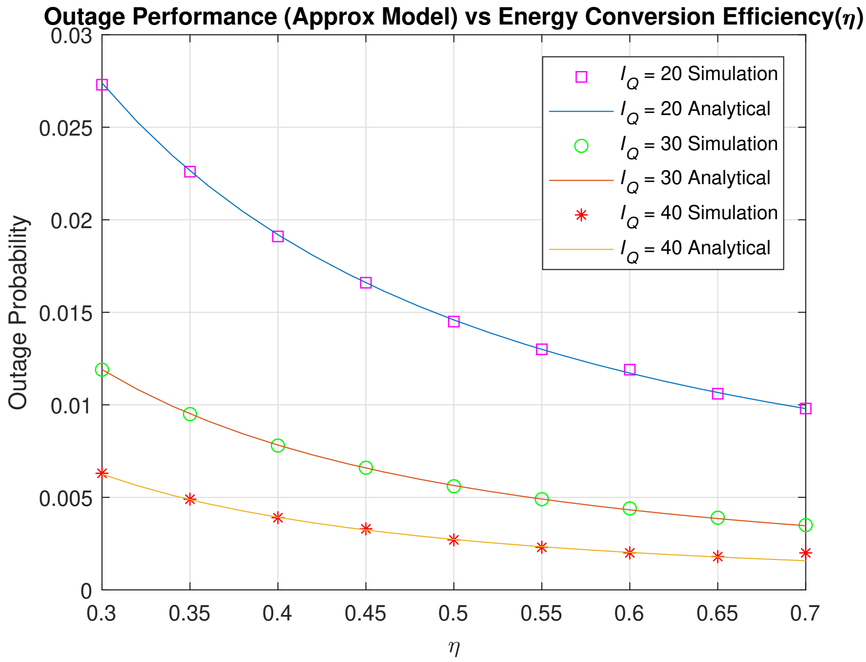

Fig. 6.7 P_{out} (approx) vs η

In Fig. 6.7, the simulated and analytical outage probabilities of the approx. model have been observed for different value of $I_Q \epsilon \{20, 30, 40\}$ and varying values of $\eta \epsilon (0.3, 0.7)$ where other parameters are constant ($\alpha = 0.6$ and $r_{th} = 0.1$ bits/s). It is also evident from the graph that as value of I_Q is increased the graph moves downward showing an improvement in system performance.

6.4 System Throughput (S_T) vs varying interference threshold (I_Q)

Here, the system throughputs of actual and approx. models have been compared and also observed individually for varying values of $I_Q \epsilon (10, 50)$. We can see from the graphs obtained that as the value of I_Q increases, the system throughput keeps increasing for both the models. This implies that system performance increases as I_Q increases.

6.4.1 Comparing actual and approximate model

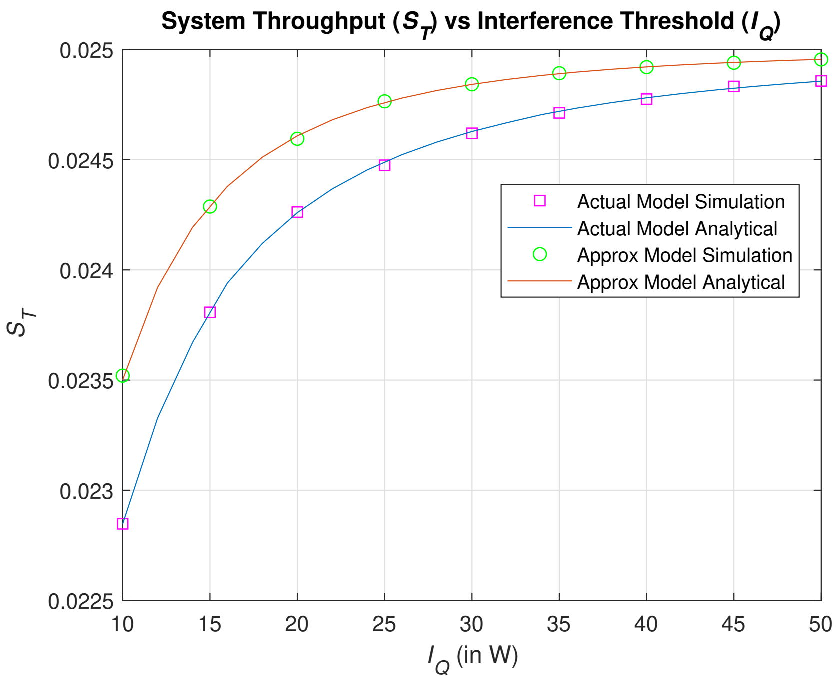

Fig. 6.8 S_T vs I_Q

In Fig. 6.8, the simulated and analytical system throughputs of both the actual and approx. models have been observed for varying values of $I_Q \,\epsilon\, (10,\, 50)$ where other parameters are constant (η = 0.5, α = 0.5 and r_{th} = 0.1 bits/s).

6.4.2 For actual model

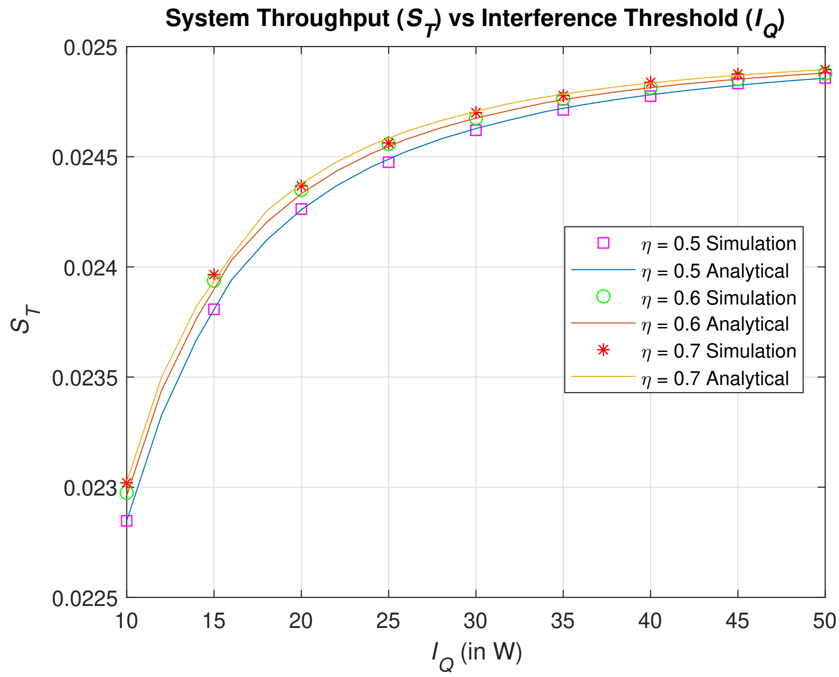

Fig. 6.9 S_T (actual) vs I_Q

In Fig. 6.9, the simulated and analytical system throughputs of the actual model have been observed for different value of $\eta \, \epsilon \, \{0.5,\ 0.6,\ 0.7\}$ and varying values of $I_Q \, \epsilon \, (10,\ 50)$ where other parameters are constant (α = 0.5 and r_{th} = 0.1 bits/s). It is also evident from the graph that as value of η is increased the graph moves upward showing an improvement in system performance.

6.4.3 For approximate model

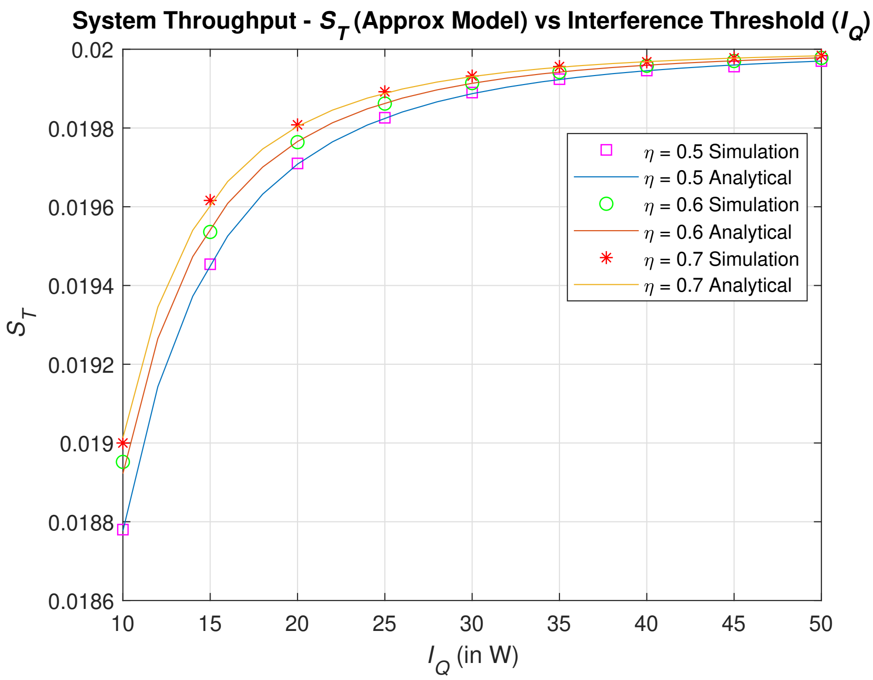

Fig. 6.10 S_T (approx) vs I_Q

In Fig. 6.10, the simulated and analytical system throughputs of the approx. model have been observed for different value of $\eta \, \epsilon \, \{0.5, \, 0.6, \, 0.7\}$ and varying values of $I_Q \, \epsilon \, (10, \, 50)$ where other parameters are constant (α = 0.6 and r_{th} = 0.1 bits/s). It is also evident from the graph that as value of η is increased the graph moves upward showing an improvement in system performance.

Chapter 7

Conclusion and Future Scope

With the help of this project, we have analysed the prospect of integration of cooperative cognitive radio in a WBAN system consisting of primary and secondary networks, co-existing together, where the secondary network communicates via an EH AF relay under the interference threshold conditions at the primary receiver. The TS protocol allows the relay to switch between the harvesting energy from the received signal and the transmission of information processed in the remaining time.

For the model proposed in chapter, the performance of outage probability and system throughput is investigated for TS protocol at the secondary destination. The graphs obtained provide significant insights into the effect of different system parameters on the performance. The results exemplified that higher values of the energy conversion efficiency and interference threshold improve the performance of our proposed network model and also obtained the optimal time switching parameter to achieve minimum outage probability. Moreover, the simulation results and numerical analysis are provided to verify that our proposed model design is tenable.

The networks integrating energy as well as spectrum efficiency form the base of development of green and sustainable networks and will help in deployment of energy saving fifth-generation (5G) communication systems on a larger scale rather than dealing only with body area networks. The design of WBAN models related to our project is still an open area for research. Their applications down the road will be in futuristic smart healthcare systems.

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Appendix A

Important Mathematical Concepts

A description of some of the important mathematical concepts used in the project.

A.1 Lognormal random variable

In probability theory, a log-normal (or lognormal) distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable X is log-normally distributed, then $Y = ln\left(X\right)$ has a normal distribution. Likewise, if Y has a normal distribution, then the exponential function of Y, $X = exp\left(Y\right)$, has a log-normal distribution. A random variable which is log-normally distributed takes only positive real values.
If X is a log-normally distributed random variable and let μ and $sigma$ be the mean and standard deviation of it's natural logarithm respectively.
CDF of X is defined as

$$F_X\left(x\right) = \frac{1}{2}\left[1 + erf\left(\frac{log\left(x\right)) - \mu}{\sqrt{2}\sigma}\right)\right]. \tag{A.1}$$

PDF of X is defined as

$$f_X\left(x\right) = \frac{1}{x\sigma\sqrt{2\pi}}exp\left(-\left(\frac{log\left(x\right)) - \mu}{\sqrt{2}\sigma}\right)^2\right). \tag{A.2}$$

Expectation of X is defined as

$$E\left[X\right] = exp\left(\mu + \frac{\sigma^2}{2}\right). \tag{A.3}$$

A.2 Unit-step function

It is a discontinuous function which takes value 1 for all non negative arguments and 0 for all negative arguments. It is denoted by u and can be expressed as

$$u[z] = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0 \end{cases} . \tag{A.4}$$