Bulk operator reconstruction and smearing techniques in AdS/CFT

A THESIS

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By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled ***Bulk operator reconstruction and smearing techniques in AdS/CFT*** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF SCIENCE** and submitted in the **DEPARTMENT OF PHYSICS**, Indian Institute of Technology Indore, is an authentic record of my own work carried out during the time period from *August 2024* to *May 2025* under the supervision of *Dr Debajyoti Sarkar, Assitant Professor, Department of Physics, IIT Indore*.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

20/5/25

Signature of the student with date
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This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

Signature of the Supervisor
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Signature of Convener, DPGC
Date: 22-05-25

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My beloved parents and my grandmother,

for their unconditional love, unwavering support,

and constant encouragement throughout my life.

Contents

Chapter 1

Abstract

In this report, I will first examine the prescription which defines the bulk fields with boundary operators which is known as HKLL prescription. I will demonstrate when we consider free fileds in HKLL prescription, the commutator calculated using the three point function doesn't commute at spacelike separation.

By analyzing two- and three-point functions, I will show that the linear smearing transformation leads to non-local commutators in the bulk when interactions are considered. To address this non-locality, I will discuss a modification which introduces a bilocal correction term which provides a more efficient method computationally to reestablish spacelike commutativity. This bilocal correction is shown to cancel the unwanted non-local terms in the commutator, thus ensuring that the bulk operators satisfy commutativity at spacelike separation.

Chapter 2

Introduction

Two of the most foundational theories in physics are General Relativity and Quantum Field Theory. General Relativity describes the nature of gravity and the structure of the universe, while QFT describes the behavior of particles and forces at the sub-atomic levels, such as those found in particle physics. Despite their remarkable success in their respective domains, these theories are fundamentally incompatible in certain regimes, particularly when it comes to describing gravity at quantum scales—such as near black hole singularities.

For decades, theoretical physicists have sought a unified framework that can merge General Relativity and Quantum Field Theory into a single, consistent theory of quantum gravity. One of the most promising developments connecting these two theories is known as the AdS/CFT correspondence.

AdS/CFT stands for Anti-de Sitter/Conformal Field Theory correspondence. This is a powerful theoretical tool proposed by Juan Maldacena in 1997 [1], which establishes an equivalence between the gravitational theory in a higher-dimensional curved spacetime and a quantum field theory defined on the lower-dimensional boundary of that space.

An Anti-de Sitter space is a spacetime with uniform negative curvature, defined by a negative cosmological constant. This structure is significant in the correspondence because it ensures that the boundary of the space is well-defined and located at a finite 'distance' from any point within the bulk.

On the dual side is the Conformal Field Theory—a form of quantum field theory that stays unchanged under conformal transformations, which preserve angles but do not necessarily retain distances.

A key concept within the AdS/CFT framework is the HKLL prescription. This prescription provides a mathematical way to reconstruct bulk fields which live inside AdS space from boundary operators. The holographic principle is fundamentally based on the idea that all the information within a spatial volume can be encoded by operators acting on its boundary.

The AdS/CFT correspondence has changed the way we think about quantum gravity and has applications in a wide range of fields, from string theory and black hole physics to condensed matter systems and quantum information theory.

We know that local operators in the bulk can be represented by smeared operators on the boundary. In order to define these observables, it is necessary that the bulk operators commute when separated by a spacelike interval. As we will see that for an operator to be considered local in the bulk, it must commute with other operators at spacelike separations.

The large N limit considers the bulk fields as free and non-interacting which simplifies the mapping between bulk and boundary operators. We begin with the fact that for an operator to be considered local inside the bulk, it must commute with other operators at spacelike separations. This property is known as commutativity at space-like separation.

One key method to address this issue, which will be explored in this report, is bilinear smearing.

Chapter 3

Bulk boundary correspondance

3.1 HKLL prescription

This section introduces the HKLL prescription, a foundational concept for this report which forms the basis of my work. The AdS/CFT correspondence establishes a link between the fields inside the bulk with the operator on the boundary. This principle states that excitation in the bulk corresponds to an operator on the boundary [2]. The Lorentzian AdS/CFT correspondence establishes a link between the fields inside the bulk with the operator on the boundary. This principle states that excitation in the bulk corresponds to an operator on the boundary [2].

Consider a bulk field $\phi(y, x)$ that shows a normalizable fall-off at the boundary of AdS. The behavior of this bulk field is given by:

$$\phi(y, x) \sim y^{\Delta}\phi_0(x), \tag{3.1}$$

Here, y denotes the radial coordinate, tending to zero at the AdS boundary. The bulk field $\phi(y, x)$ may be expressed in terms of the boundary field $\phi_0(x)$ via an integral kernel.

$$\phi(y, x) = \int dx' K(x'|y, x)\phi_0(x'), \tag{3.2}$$

where $K(x'|y, x)$ is the smearing function that maps local bulk excitations to operators in the boundary $\phi_0(x')$. The boundary field $\phi_0(x')$ is associated with a local operator $O(x')$ in the CFT, leading to the relation:

$$\phi_0(x') \leftrightarrow O(x'). \tag{3.3}$$

Thus, the AdS/CFT correspondence suggests that local bulk fields correspond to non-local operators on the boundary:

$$\phi(y,x) \leftrightarrow \int dx' \, K(x'|y,x) O(x'). \tag{3.4}$$

Additionally, bulk-to-bulk correlation functions are equivalent to the correlation functions of the related non-local operators in the CFT:

$$\langle \phi(y_1,x_1)\phi(y_2,x_2) \rangle = \int dx'_1 dx'_2 \, K(x'_1|y_1,x_1) K(x'_2|y_2,x_2) \langle O(x'_1) O(x'_2) \rangle. \tag{3.5}$$

K is the smearing function that establishes the connection of local bulk excitations onto the boundary field theory.

3.2 Correlators in Poincare Patch

We commence by analyzing the AdS_2/CFT_1 correspondence, wherein the linear smearing transformation is applied to the two- and three-point correlation functions within the conformal field theory. This procedure yields bulk operators that fail to commute at spacelike separations. The emergence of such non-commutativity at spacelike intervals constitutes a deviation from locality, thereby indicating that the reconstructed bulk theory from the boundary CFT does not exhibit exact locality.

We will first describe the bulk field correlator in a Poincare patch and then simplify the bulk two point function. We first define the Poincare metric as: [3]

$$ds^2 = \frac{R^2}{Z^2}(-dT^2 + dZ^2) \tag{3.6}$$

Let us consider a massless scalar field in the bulk and its corresponding operator on the boundary, which are connected through the following relation, where Z denotes the radial coordinate.

$$\phi(T, Z) \to Z\,O(T). \tag{3.7}$$

We begin with the bulk two-point function for a free scalar field in AdS_2:

$$\langle \phi(T, Z)\, \phi(T', Z') \rangle = \tanh^{-1}\left(\frac{1}{\sigma}\right), \tag{3.8}$$

where the invariant distance σ which is the measure of how far two points can be in AdS space is given by

$$\sigma = \frac{Z^2 + Z'^2 - (T - T')^2}{2ZZ'}. \tag{3.9}$$

To compute the bulk-boundary correlator, we will first send one of the bulk points to the boundary:

$$Z' \to 0 \quad \text{with} \quad T' \text{ fixed.}$$

If we apply this limit to the invariant distance we get,

$$\sigma \approx \frac{Z^2 - (T - T')^2}{2ZZ'}, \quad \text{so} \quad \frac{1}{\sigma} \approx \frac{2ZZ'}{Z^2 - (T - T')^2}. \tag{3.10}$$

Using the approximation $\tanh^{-1}(x) \approx x$ for small x where $x = \frac{1}{\sigma}$, the two-point function becomes

$$\langle \phi(T, Z)\, \phi(T', Z') \rangle \approx \frac{2ZZ'}{Z^2 - (T - T')^2}. \tag{3.11}$$

According to the boundary limit we get,

$$\mathcal{O}(T') = \lim_{Z' \to 0} Z'^{-\Delta} \phi(T', Z'), \tag{3.12}$$

so we find the bulk-boundary correlator by multiplying by $Z'^{-\Delta}$ and taking the limit:

$$\langle \phi(T, Z)\, \mathcal{O}(T') \rangle = \lim_{Z' \to 0} Z'^{-\Delta} \langle \phi(T, Z)\, \phi(T', Z') \rangle \tag{3.13}$$

$$= \frac{2ZZ'}{Z^2 - (T - T')^2} \cdot \lim_{Z' \to 0} Z'^{-\Delta}. \tag{3.14}$$

For the result to be finite and nonzero, we set $\Delta = 1$, which gives us:

$$\langle \phi(T, Z)\, \mathcal{O}(T') \rangle = \frac{2Z}{Z^2 - (T - T')^2}. \tag{3.15}$$

Now we take both points to the boundary:

$$Z \to 0, \quad Z' \to 0.$$

Now again we start from the approximate form of the bulk two-point function:

$$\langle \phi(T, Z)\, \phi(T', Z') \rangle \approx \frac{2ZZ'}{Z^2 + Z'^2 - (T - T')^2}. \tag{3.16}$$

Multiplying by $Z^{-\Delta}Z'^{-\Delta}$ and taking the limit gives:

$$\langle \mathcal{O}(T)\,\mathcal{O}(T')\rangle = \lim_{Z,Z'\to 0} Z^{-\Delta}Z'^{-\Delta}\cdot \frac{2ZZ'}{Z^2+Z'^2-(T-T')^2} \tag{3.17}$$

$$= \frac{2}{-(T-T')^2}\cdot \lim_{Z,Z'\to 0} Z^{1-\Delta}Z'^{1-\Delta}. \tag{3.18}$$

Again, choosing $\Delta = 1$, we find:

$$\langle \mathcal{O}(T)\,\mathcal{O}(T')\rangle = -\frac{2}{(T-T')^2}. \tag{3.19}$$

This is typically written (after rescaling constants) as:

$$\langle \mathcal{O}(T)\,\mathcal{O}(T')\rangle = -\frac{1}{(T-T')^2}. \tag{3.20}$$

We will also require the three-point correlator in the CFT. Assuming $T_1 > T_2 > T_3$, it is given by

$$\langle 0|\mathcal{O}(T_1)\mathcal{O}(T_2)\mathcal{O}(T_3)|0\rangle = -\frac{i\lambda R^2}{\pi(T_1-T_2)(T_1-T_3)(T_2-T_3)} \tag{3.21}$$

Chapter 4

Smearing techniques

4.1 Three-point correlator and bilinear smearing

4.1.1 Three-point correlator

In CFT, a three-point correlator is a quantity that defines how three different operators interact. We first start with the general form of a three-point correlator, which is given as:

$$\langle O_i(x_i)O_j(x_j)O_k(x_k)\rangle = \frac{c_{ijk}}{|x_i - x_j|^{\Delta_i+\Delta_j-\Delta_k}|x_i - x_k|^{\Delta_i+\Delta_k-\Delta_j}|x_j - x_k|^{\Delta_j+\Delta_k-\Delta_i}}. \quad (4.1)$$

- $\mathcal{O}_i(x_i)$: A local primary operator in the CFT, labeled by index i, located at position x_i.
- Δ_i: The scaling dimension of the operator $\mathcal{O}_i$.
- c_{ijk}: A structure constant — this encodes theory-specific information and is not fixed by symmetry. It is determined by the dynamics of the CFT.
- $|x_i - x_j|$: Euclidean distance between the two points x_i and x_j.

We now set the scaling dimension as $\Delta = 1$:

$$\langle 0|O(T_1)O(T_2)O(T_3)|0\rangle = \frac{C}{(T_1 - T_2)(T_1 - T_3)(T_2 - T_3)} \quad (4.2)$$

Now we need to normalize this to find the value of C:

$$C = \frac{-i\lambda R^2}{\pi} \tag{4.3}$$

$$\langle 0|O(T_1)O(T_2)O(T_3)|0\rangle = \frac{-i\lambda R^2}{\pi} \frac{1}{(T_1 - T_2)(T_1 - T_3)(T_2 - T_3)} \tag{4.4}$$

4.1.2 Linear smearing

Now we define the process of linear smearing, where the bulk field is given as the integration of the boundary operator.It's called linear smearing because the bulk field is expressed as a linear functional of the boundary operator $\mathcal{O}$. No nonlinear terms (like $\mathcal{O}^2$, etc.) are involved.

This integration typically provides the bulk field at leading order. To incorporate interaction effects or higher-order corrections, modifications to this linear approach, such as bilinear smearing, are necessary.

$$\phi^{(0)}(T, Z) = \frac{1}{2} \int_{T-Z}^{T+Z} dT_1 O(T_1) \tag{4.5}$$

Substituting this relation into the three-point operator correlator function and replacing one operator with the bulk field, we get the following integration:

$$= -\frac{i\lambda R^2}{\pi} \int_{T-Z}^{T+Z} \frac{1}{(T_1 - T_2)(T_1 - T_3)(T_2 - T_3)} dT_1 \tag{4.6}$$

$$= -\frac{i\lambda R^2}{\pi(T_2 - T_3)} \int_{T-Z}^{T+Z} \left(\frac{1}{(T_1 - T_2)(T_1 - T_3)} \right) dT_1 \tag{4.7}$$

$$\frac{1}{(T_1 - T_2)(T_1 - T_3)} = \frac{A}{T_1 - T_2} + \frac{B}{T_1 - T_3} \tag{4.8}$$

$$1 = A(T_2 - T_3), \quad A = \frac{1}{T_2 - T_3} \tag{4.9}$$

$$1 = B(T_3 - T_2), \quad B = \frac{-1}{T_2 - T_3} \tag{4.10}$$

Thus, the fraction becomes:

$$\frac{1}{(T_1 - T_2)(T_1 - T_3)} = \frac{1}{(T_2 - T_3)}\left(\frac{1}{T_1 - T_2} - \frac{1}{T_1 - T_3}\right) \tag{4.11}$$

Thus,

$$= -\frac{i\lambda R^2}{\pi(T_2 - T_3)^2}\int_{T-Z}^{T+Z}\left(\frac{1}{T_1 - T_2} - \frac{1}{T_1 - T_3}\right) dT_1 \tag{4.12}$$

$$I = \frac{-i\lambda R^2}{2\pi(T_2 - T_3)^2} \ln\left(\frac{T + Z - T_2}{T - Z - T_2}\right) - \ln\left(\frac{T + Z - T_3}{T - Z - T_3}\right) \tag{4.13}$$

Thus, the final solution of the three-point correlator is given as:

$$I = \frac{-i\lambda R^2}{2\pi(T_2 - T_3)^2} \ln\left|\frac{(T + Z - T_3)(T - Z - T_3)}{(T + Z - T_3)(T - Z - T_2)}\right| \tag{4.14}$$

If we repeat the same calculation for $\langle 0|O(T_2)\phi^{(0)}(T, Z)O(T_3)|0\rangle$,

$$= \frac{-i\lambda R^2}{2\pi(T_2 - T_3)^2} \log\left(\frac{(T + Z - T_3)(T_2 - T + Z)}{(T_2 - T - Z)(T - Z - T_3)}\right) \tag{4.15}$$

Here we replace the second operator with the bulk field to obtain the following result.

Taking the difference to find the commutator, we get:

$$\langle 0| \left[\phi^{(0)}(T, T_2), O(T_2)\right] O(T_3) \big| \, 0\rangle = \langle 0|\phi^{(0)}(T, T_2)O(T_2)O(T_3)|0\rangle - \langle 0|O(T_2)\phi^{(0)}(T, T_2)O(T_3)|0\rangle. \tag{4.16}$$

$$= \frac{-i\lambda R^2}{\pi}\frac{1}{(T_2 - T_3)^2}\left[\log\left(\frac{(T + Z - T_3)(T - Z - T_2)}{(T + Z - T_2)(T - Z - T_3)}\right) - \log\left(\frac{(T + Z - T_3)(T_2 - T + Z)}{(T_2 - T - Z)(T - Z - T_3)}\right)\right] \tag{4.17}$$

$$= \frac{-\lambda R^2}{\pi} \frac{1}{(T_2 - T_3)^2} \log \left(\frac{(T + Z - T_3)(T_2 - T + Z)}{(T + Z - T_2)(T - Z - T_3)} \right) \tag{4.18}$$

Thus, we can see that the commutator is not vanishing for a three-point correlator, meaning that the commutators do not satisfy commutativity at spacelike separation. This holds specifically for the three-point function, and a way to solve this is by bilinear smearing.

4.1.3 Bilinear smearing

Bilinear smearing is a way to determine the bulk field up to $\frac{1}{N}$ corrections. This technique is used to construct well-defined bulk operators by integrating bilinear combinations of boundary operators with a smearing function.Bilinear smearing refers to a generalization of the linear smearing technique used in AdS/CFT bulk reconstruction, where the bulk field is reconstructed as a bilinear (i.e., second-order) functional of boundary operators, rather than a linear one.

Now we define the bilocal operator which takes into account the correction term as:

$$\phi^{(1)}(T_1, Z) = \frac{\lambda R^2}{8} \int_0^Z \frac{dZ'}{Z'^2} \int_{T-Z-Z'}^{T-Z'} dT' \int d\bar{T}_1 \, d\bar{T}_2 \, : \mathcal{O}(\bar{T}_1)\mathcal{O}(\bar{T}_2) : \tag{4.19}$$

Now substituting this bilocal operator inside the three-point operator function, we get the following equation:

$$\begin{aligned} \langle 0|\phi^{(1)}(T_1, Z)\mathcal{O}(T_2)\mathcal{O}(T_3)|0\rangle = \frac{\lambda R^2}{8} \int_0^Z \frac{dZ'}{Z'^2} \int_{T-Z-Z'}^{T-Z'} dT' \\ \times \int d\bar{T}_1 \, d\bar{T}_2 \, \langle 0| : \mathcal{O}(\bar{T}_1)\mathcal{O}(\bar{T}_2)\mathcal{O}(T_2)\mathcal{O}(T_3) : |0\rangle. \end{aligned} \tag{4.20}$$

From the HKLL prescription, we know that: [2]

$$\phi^{(1)}(T, Z) = \int dT' \, dZ' \, K(T, Z \mid T', Z') \, O(T') \tag{4.21}$$

We start with the correlator:

$$\langle 0|\phi^{(1)}(T_1, Z)O(T_2)O(T_3)|0\rangle \tag{4.22}$$

Substituting, we get:

$$\langle 0|\phi^{(1)}(T_1, Z)O(T_2)O(T_3)|0\rangle = \left\langle 0 \middle| \left(\int dT'\, dZ'\, K(T_1, Z \mid T', Z')\, O(T') \right) O(T_2)O(T_3) \middle| 0 \right\rangle \tag{4.23}$$

$$= \int dT'\, dZ'\, K(T_1, Z \mid T', Z')\, \langle 0|O(T')O(T_2)O(T_3)|0\rangle. \tag{4.24}$$

The three point function is given as,

$$\langle 0|\mathcal{O}(T_1)\mathcal{O}(T_2)\mathcal{O}(T_3)|0\rangle = -\frac{i\lambda R^2}{\pi(T_1 - T_2)(T_1 - T_3)(T_2 - T_3)} \tag{4.25}$$

Now, substituting equation (2.25) into the above equation, we get the following relation:

$$\langle 0|\phi^{(1)}(T_1, Z)O(T_2)O(T_3)|0\rangle = \int dT'\, dZ'\, K(T_1, Z \mid T', Z')\, \langle 0|O(T')O(T_2)O(T_3)|0\rangle \tag{4.26}$$

$$\approx \int dT'\, dZ'\, K(T_1, Z \mid T', Z') \frac{i\lambda R^2}{\pi(T_1 - T_2)(T_1 - T_3)(T_2 - T_3)}, \tag{4.27}$$

We now substitute the value of the smearing function from [4] as:

$$K(T_1, Z \mid T', Z') = \frac{\lambda R^2}{\pi} \frac{Z}{(T + Z' - T_2)(T' + Z' - T_3)(T' - Z' - T_2)(T' - Z' - T_3)} \tag{4.28}$$

Putting this value, we get the following integral:

$$\begin{aligned} \langle 0|\phi^{(1)}(T_1, Z)O(T_2)O(T_3)|0\rangle \approx \int dT'\, dZ'\, & \frac{\lambda R^2}{\pi} \frac{Z}{(T' + Z' - T_2)(T' + Z' - T_3)} \\ & \times \frac{1}{(T' - Z' - T_2)(T' - Z' - T_3)} \\ & \langle 0|\mathcal{O}(T_1)\mathcal{O}(T_2)\mathcal{O}(T_3)|0\rangle. \end{aligned} \tag{4.29}$$

$$\langle 0|\phi^{(1)}(T_1, Z)O(T_2)O(T_3)|0\rangle = \frac{i\lambda R^2}{\pi(T_1 - T_2)(T_1 - T_3)(T_2 - T_3)} \int_0^Z \frac{dZ'}{Z'} \int_{T+Z-Z'}^{T+Z'} dT' \times \frac{1}{(T' + Z' - T_2)(T' + Z' - T_3)(T' - Z' - T_2)(T' - Z' - T_3)} \tag{4.30}$$

We can calculate similarly for:

$$\langle 0|O(T_2)\phi^{(1)}(T, Z)O(T_3)|0\rangle \tag{4.31}$$

Taking the commutator relations of both of these quantities, we get the following integration:

$$\langle 0|i[\phi^{(1)}(T, Z), O(T_2)]O(T_3)|0\rangle = -\frac{2\lambda R^2}{\pi} \frac{1}{(T_2 - T_3)^2} \left[\int_0^{\frac{(T+Z-T_3)}{2}} \frac{dZ'}{2Z'(2Z' + T_2 - T_3)} + \int_0^{\frac{(T_2-T+Z)}{2}} \frac{dZ'}{2Z'(2Z' + T_3 - T_2)} \right] \tag{4.32}$$

$$I = \int_0^{\frac{(T+Z-T_3)}{2}} \frac{dZ'}{2Z'(2Z' + T_2 - T_3)} \tag{4.33}$$

$$\frac{1}{2Z'(2Z' + T_2 - T_3)} = \frac{C}{2Z'} + \frac{D}{2Z' + T_2 - T_3} \tag{4.34}$$

$$1 = C(2Z' + T_2 - T_3) + D(2Z') \tag{4.35}$$

Setting $Z' = 0$, we obtain:

$$C = \frac{1}{T_2 - T_3} \tag{4.36}$$

Setting:

$$Z' = \frac{T_3 - T_2}{2} \tag{4.37}$$

$$1 = D(T_3 - T_2) \tag{4.38}$$

$$D = \frac{1}{T_3 - T_2} \tag{4.39}$$

Substituting above values we get,

$$I_{1w} = \int_0^{\frac{T+Z-T_3}{2}} \left(\frac{1}{(T_2 - T_3)2Z'} - \frac{1}{(T_2 - T_3)(2Z' + T_2 - T_3)} \right) dZ' \tag{4.40}$$

$$I_1 = \int_0^{\frac{T+Z-T_2}{2}} \frac{1}{(T_2 - T_3)2Z'} dZ' \tag{4.41}$$

$$= \frac{1}{(T_2 - T_3)2} \ln Z' \Big|_0^{\frac{T+Z-T_2}{2}} \tag{4.42}$$

$$= \frac{1}{2(T_2 - T_3)} \ln \left(\frac{T + Z - T_2}{2} \right) \tag{4.43}$$

$$I_2 = \int_0^{\frac{T+Z-T_2}{2}} \frac{1}{(T_2 - T_3)(2Z' + T_2 - T_3)} dZ' \tag{4.44}$$

$$= \frac{1}{2(T_2 - T_3)} \ln(2Z' + T_2 - T_3) \Big|_0^{\frac{T+Z-T_2}{2}} \tag{4.45}$$

$$= \frac{1}{2(T_2 - T_3)} \ln \left(\frac{T + Z - T_3}{T_2 - T_3} \right) \tag{4.46}$$

Combining:

$$I_{1w} = \frac{1}{2(T_2 - T_3)} \ln \left| \frac{T + Z - T_2}{2} \right| - \frac{1}{2(T_2 - T_3)} \ln \left| \frac{T + Z - T_3}{T_2 - T_3} \right| \tag{4.47}$$

$$= \frac{1}{2(T_2 - T_3)} \ln \left| \frac{(T + Z - T_2)(T_2 - T_3)}{2(T + Z - T_3)} \right| \tag{4.48}$$

The second integral is the same with just a change in limits.

$$I = \int_0^{\frac{T_2 - T + Z}{2}} \frac{dZ'}{2Z'(2Z' + T_3 - T_2)} \tag{4.49}$$

$$I_{2w} = \int_0^{\frac{T_2 - T + Z}{2}} \frac{dZ'}{2(T_2 - T_3)Z'} + \int_0^{\frac{T_2 - T + Z}{2}} \frac{dZ'}{(T_3 - T_2)(2Z' + T_3 - T_2)} \tag{4.50}$$

$$I_2 w = I_1 + I_2 \tag{4.51}$$

$$I_1 = \frac{1}{2(T_2 - T_3)} \ln |Z'| \Big|_0^{\frac{T_2 - T + Z}{2}} \tag{4.52}$$

$$= \frac{1}{2(T_2 - T_3)} \ln \left| \frac{T_2 - T + Z}{2} \right| \tag{4.53}$$

$$I_2 = \int_0^{\frac{T_2 - T + Z}{2}} \frac{dZ'}{2(T_2 - T_3)(2Z' + T_3 - T_2)} \tag{4.54}$$

$$= \frac{1}{2(T_2 - T_3)} \ln |2Z' + T_3 - T_3| \Big|_0^{\frac{T_2 - T + Z}{2}} \tag{4.55}$$

$$= \frac{1}{2(T_2 - T_3)} \ln [2T_2 - T + Z - T_3] - \ln(T_2 - T_3) \tag{4.56}$$

$$= \frac{1}{2(T_2 - T_3)} \ln \left[\frac{2T_2 - T + Z - T_3}{T_2 - T_3} \right] \tag{4.57}$$

$$I = I_{1w} - I_{2w} \tag{4.58}$$

$$= \frac{\lambda R^2}{\pi} \frac{1}{(T_2 - T_3)^2} \log\left(\frac{(T + Z - T_3)(T_2 - T + Z)}{(T + Z - T_2)(T - T_3 - Z)}\right) \tag{4.59}$$

As we can see, using the bilocal operator in the commutator, we get the following term, which exactly cancels the extra term we calculated in equation (2.18). Thus, adding this correction term satisfies the commutator relation.

4.2 Two point correlator and linear smearing function

A two point correlator measures the correlation between two operators at different point in spacetime. A two point function is defined as:

$$\langle O_i(x) O_j(0)\rangle = \frac{\delta_{ij}}{(X^2 - T^2)^{\Delta_i}}. \tag{4.60}$$

We can expand the two point function in the following way,

$$O_i(X,T)O_j(0) = \frac{\delta_{ij}}{(X^2 - T^2)^{\Delta_i}} + \sum_k \frac{c_{ijk}}{(X^2 - T^2)^{(\Delta_i + \Delta_j - \Delta_k)/2}} O_k(0) + \dots \tag{4.61}$$

For simplicity, we consider a dimension-two operator, where the expansion simplifies to:

$$O(X,T)O(0) = \frac{1}{(X^2 - T^2)^2} + \frac{1}{N}\frac{1}{(X^2 - T^2)} O(0) + \dots \tag{4.62}$$

The first term simply represents the bulk-boundary correlator, while the second term contributes additional corrections.

I will now define linear smearing in AdS/CFT used to express a bulk field in terms of its corresponding boundary operator via an integral transformation.

The integral is given by:

$$\phi^{(0)}(T,Z) = \frac{1}{2}\int_{T-Z}^{T+Z} dT_1\, O(T_1) \tag{4.63}$$

Now, using the linear smearing function, we can show the bulk-boundary correlator in terms of integration as:

$$\langle \phi^{(0)}(Z,X,T) O(0,0)\rangle = \frac{1}{N} O(0) \int \frac{dY' dT'}{(X + iY')^2 - (T + T')^2} \tag{4.64}$$

Now, we make the following substitution:

$$Y' = r\sin\theta, \quad T' = r\cos\theta, \quad \alpha = e^{i\theta}, \tag{4.65}$$

$$0 \le r \le Z, \quad 0 \le \theta < 2\pi \tag{4.66}$$

$$dY' dT' = r\, dr\, d\theta \tag{4.67}$$

$$T' = r\cos\theta, \quad Y' = r\sin\theta \tag{4.68}$$

The denominator can be written as:

$$(X + iY')^2 - (T + T')^2 = \big[(X + iY') - (T + T')\big]\big[(X + iY') + (T + T')\big] \tag{4.69}$$

$$(X + iY') - (T + T') = \big[X + ir\sin\theta\big] - \big[T + r\cos\theta\big] \tag{4.70}$$

$$= (X - T) + ir\sin\theta - r\cos\theta \tag{4.71}$$

$$= (X - T) - r(\cos\theta - i\sin\theta) \tag{4.72}$$

$$= (X - T) - r\alpha^{-1} \tag{4.73}$$

$$(X + iY') + (T + T') = \big[X + ir\sin\theta\big] + \big[T + r\cos\theta\big] \tag{4.74}$$

$$= (X + T) + r\alpha \tag{4.75}$$

$$= \big[(X - T) - r\alpha^{-1}\big]\big[(X + T) + r\alpha\big] \tag{4.76}$$

$$= -\alpha^{-1}\big[(X+T)+r\alpha\big]\big[r+\alpha(T-X)\big] \tag{4.77}$$

$$(X+iY')^2-(T+T')^2 = -\alpha^{-1}\big[(T+X)+r\alpha\big]\big[r+\alpha(T-X)\big] \tag{4.78}$$

$$(X+iY')^2-(T+T')^2 = (T+X+r\alpha)(r+\alpha(T-X)) \tag{4.79}$$

Substituting the above relation back into the integral, we get:

$$I = \int_0^Z r\,dr \int \frac{d\alpha}{(T+X+r\alpha)(r+\alpha(T-X))} \tag{4.80}$$

The two poles can be interpreted as:

$$T+X+r\alpha = 0 \quad \Rightarrow \quad \alpha = -\frac{T+X}{r} \tag{4.81}$$

$$r+\alpha(T-X) = 0 \quad \Rightarrow \quad \alpha = \frac{r}{X-T} \tag{4.82}$$

Using the standard method to calculate the residue, we get:

$$\text{Res}_{\alpha=\alpha_2} = \frac{1}{T+X+r\alpha}\left[\frac{d}{d\alpha}\Big(r+\alpha(T-X)\Big)\right]^{-1} \tag{4.83}$$

$$= \frac{1}{(T+X)(T-X)-r^2} \tag{4.84}$$

$$(T+X)(T-X) = T^2-X^2 = -(X^2-T^2) \tag{4.85}$$

$$\Delta = X^2-T^2 \tag{4.86}$$

$$I = 2\pi i \left(\frac{-1}{r^2 + \Delta} \right) \tag{4.87}$$

For $r > T + X$, we can calculate the residue to be:

$$\alpha = -\frac{T + X}{r} \tag{4.88}$$

$$\mathrm{Res}_{\alpha=\alpha_1} = \frac{1}{r + \alpha(T - X)} \left[\frac{d}{d\alpha} \Big(T + X + \alpha r \Big) \right]^{-1} \Bigg|_{\alpha = -\frac{T+X}{r}} \tag{4.89}$$

$$= \frac{1}{\Big[r^2 - (T - X)(T + X) \Big]} \tag{4.90}$$

$$= \frac{1}{r^2 + \Delta} \tag{4.91}$$

$$(T + X)(T - X) = -(X^2 - T^2) \tag{4.92}$$

Finally, substituting these results into the integration, we get:

$$I = 2\pi i \left[-\int_0^Z \frac{r\,dr}{r^2 + \Delta} + \int_{-Z}^0 \frac{r\,dr}{r^2 + \Delta} \right] \tag{4.93}$$

$$\int \frac{r\,dr}{r^2 + \Delta} = \frac{1}{2} \ln(r^2 + \Delta) + \mathrm{const} \tag{4.94}$$

Let $r^2 + \Delta = t$, then:

$$2r\,dr = dt \tag{4.95}$$

$$r\,dr = \frac{dt}{2} \tag{4.96}$$

$$\frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t \tag{4.97}$$

$$= \frac{1}{2}\ln(r^2 + \Delta) + \text{const} \tag{4.98}$$

Putting the limits, we get:

$$\begin{aligned}
&= \frac{1}{2}\ln\left[\frac{1}{Z^2 + \Delta}\right] - \ln\left[\frac{1}{\Delta}\right] \\
&= \frac{1}{2}\ln\left[\frac{\Delta}{Z^2 + \Delta}\right] \\
I &= \frac{1}{2}\ln\left(r^2 + \Delta\right)\Big|_{-Z}^{0} \\
&= \frac{1}{2}(\ln\Delta - \ln(Z^2 + \Delta)) \\
&= \frac{1}{2}\ln\left[\frac{\Delta}{Z^2 + \Delta}\right] \\
I &= 2\pi i \ln\left[\frac{\Delta}{Z^2 + \Delta}\right] \\
&= \frac{O(0)}{\pi N} \cdot 2\pi i \ln\left[\frac{X^2 - T^2}{X^2 + Z^2 - T^2}\right] \\
&= \frac{2O(0)}{N}\ln\left[\frac{X^2 - T^2}{X^2 + Z^2 - T^2}\right]
\end{aligned} \tag{4.99}$$

As we can see here, if we consider $\frac{1}{N}$ corrections, then the two point function is nonzero.

Chapter 5

Smearing techniques in AdS_2

5.1 Correlators in AdS_2

In the AdS/CFT correspondence, AdS_2 plays a unique role due to its simplified structure and relevance in near-horizon geometries of extremal black holes. Correlators in AdS_2 are particularly important for understanding the dynamics of conformal quantum mechanics on the boundary.

In the free theory, bulk-to-boundary and bulk-to-bulk two-point correlators can be computed using Green's functions and smearing functions. The two-point function between boundary operators typically takes the form:

$$\langle \mathcal{O}(T_1)\, \mathcal{O}(\bar{T}_2) \rangle \propto \frac{1}{|T_1 - \bar{T}_2|^{\Delta}} \tag{5.1}$$

where Δ is the scaling dimension of the operator. This form reflects the conformal symmetry of the boundary theory. In our analysis, we take $\Delta = 1$.

However, in interacting theories, naive smearing leads to violations of *spacelike commutativity*, which in turn challenges bulk causality. To restore this, one must go beyond linear order—introducing corrections such as *bilinear smearing terms* that cancel interaction-induced deviations. These refined constructions ensure that correlators respect bulk locality and maintain the causal structure expected from quantum field theory in curved spacetime.

Taking the linear smearing of equation (3.1) we get the following operator relation,

$$\langle \phi^{(0)} \mathcal{O}(\bar{T}_2) \rangle \mathcal{O}(0) = \frac{\mathcal{O}(0)}{N} \int_{T-Z}^{T+Z} \frac{1}{(T_1 - \bar{T}_2)} \, dT_1 \tag{5.2}$$

Changing the integration limits we get the following integral,

$$\langle \phi^{(0)} \mathcal{O}(\bar{T}_2) \rangle \mathcal{O}(0) = -\frac{\mathcal{O}(0)}{N} \int_{T+Z}^{T-Z} \frac{1}{(T_1 - \bar{T}_2)} \, dT_1 \tag{5.3}$$

$$\int_{T+Z}^{T-Z} \frac{1}{T_1 - \bar{T}_2} \, dT_1 \tag{5.4}$$

$$\int \frac{1}{T_1 - \bar{T}_2} \, dT_1 = \ln |T_1 - \bar{T}_2| + C \tag{5.5}$$

$$\int_{T+Z}^{T-Z} \frac{1}{T_1 - \bar{T}_2} \, dT_1 = \ln |T - Z - \bar{T}_2| - \ln |T + Z - \bar{T}_2| \tag{5.6}$$

$$= \ln \left| \frac{T - Z - \bar{T}_2}{T + Z - \bar{T}_2} \right| \tag{5.7}$$

This can be expressed as:

$$\ln |T - Z - \bar{T}_2| - \ln |T + Z - \bar{T}_2| \tag{5.8}$$

We know that:

$$\int_{T+Z}^{T-Z} \frac{1}{T_1 - \bar{T}_2} \, dT_1 = \ln |T - Z - \bar{T}_2| - \ln |T + Z - \bar{T}_2| \tag{5.9}$$

Let us define:

$$A = T - Z - \bar{T}_2, \quad B = T + Z - \bar{T}_2$$

Then:

$$\ln |A| - \ln |B| \approx (1 + A) - (1 + B) = A - B = -2Z \tag{5.10}$$

Thus, the integral approximately evaluates to:

$$\int_{T+Z}^{T-Z} \frac{1}{T_1 - \bar{T}_2} \, dT_1 \approx -AZ \tag{5.11}$$

Similarly we can calculate another correlator which will be,

$$\langle 0|\phi^{(0)}(T_1, Z)\, \mathcal{O}(T_2)|0\rangle\, \mathcal{O}(0) = AZ \tag{5.12}$$

Thus the commutator will be given as,

$$\begin{aligned} \langle 0|[\mathcal{O}(T_2),\, \phi^{(0)}(T_1, Z)]|0\rangle\, \mathcal{O}(0) &= \langle 0|\mathcal{O}(T_2)\, \phi^{(0)}(T_1, Z)|0\rangle\, \mathcal{O}(0) \\ &\quad - \langle 0|\phi^{(0)}(T_1, Z)\, \mathcal{O}(T_2)|0\rangle\, \mathcal{O}(0) \\ &= -2AZ \end{aligned} \tag{5.13}$$

As we can see the integration is giving a non zero quantity which we need to cancel out using methods discussed above. The method I would be using to cancel this extra term will be to use bilinear smearing.

Further, we know that the total bulk field can be expressed as the sum of the bulk field at zeroth order in N and a correction term of order $\frac{1}{N}$:

$$\phi^{com} = \phi^{(0)} + \phi^{(1)} \tag{5.14}$$

As we know the integral for bilinear smearing is given by:

$$\phi^{(1)}(T, Z) = \frac{\lambda R^2}{8} \int_0^Z \frac{dZ'}{Z'^2} \int_{T+Z-Z'}^{T'+Z'-Z} dT' \int_{T'-Z'}^{T'-Z'} dT_1\, dT_2 \;:\mathcal{O}(T_1)\mathcal{O}(T_2): \tag{5.15}$$

Now taking the correalator of this field with an operator at *T*=0 gives us the following integration.

$$\phi^{(1)}(T,Z) = \frac{\lambda R^2}{8}\int_0^Z \frac{dZ'}{Z'^2}\int_{T+Z-Z'}^{T'+Z'-Z} dT' \int_{T'-Z'}^{T'-Z'} dT_1\, dT_2\; :\mathcal{O}(T_1)\mathcal{O}(T_2): O(0) \tag{5.16}$$

Now we know from linear smearing that,

$$\phi^{(0)}(T,Z) = \int_{T-Z}^{T+Z} dT_1\, \mathcal{O}(T_1) \tag{5.17}$$

Applying this in the above equation and using the bulk field correlator (3.8) we get the following integral,

$$\langle \phi^{(1)}(T,Z)\mathcal{O}(0)\rangle = \frac{\lambda R^2}{8}\int_0^Z \frac{dZ'}{Z'^2}\int_{T-Z+Z'}^{T+Z-Z'} dT'\, \tanh^{-1}\left(\frac{Z^2+Z'^2-(T-T')^2}{2ZZ'}\right) \tag{5.18}$$

To solve this we will start with the following integral:

$$\int_{T-Z+Z'}^{T+Z-Z'} dT'\, \tanh^{-1}\left(\frac{Z^2+Z'^2-(T-T')^2}{2ZZ'}\right) \tag{5.19}$$

Let us use the substitution:

$$u = T - T' \quad \Rightarrow \quad dT' = -du \tag{5.20}$$

Now, updating the limits of integration:

When $T' = T - Z + Z'$, then $u = Z - Z'$

When $T' = T + Z - Z'$, then $u = -Z + Z'$

Rewriting the integral with the new variable and flipping the limits due to the negative differential:

$$\int_{-Z+Z'}^{Z-Z'} du\, \tanh^{-1}\left(\frac{Z^2+Z'^2-u^2}{2ZZ'}\right) \tag{5.21}$$

Since the integrand is even in u (it only depends on u^2), we can simplify:

$$2\int_0^{Z-Z'} du\, \tanh^{-1}\left(\frac{Z^2+Z'^2-u^2}{2ZZ'}\right) \tag{5.22}$$

Define parameters:

$$A = Z^2 + Z'^2, \quad B = 2ZZ' \tag{5.23}$$

Series expansion of $\tanh^{-1}(x)$ is given as:

$$\tanh^{-1}(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \tag{5.24}$$

Apply this with $x = \frac{A-u^2}{B}$, and expand the denominator:

$$\left(A - u^2\right)^{-(2n+1)} = A^{-(2n+1)} \sum_{k=0}^{\infty} \binom{2n+k}{k} \left(\frac{u^2}{A}\right)^k \tag{5.25}$$

Leading order approximation:

$$\tanh^{-1}\left(\frac{A-u^2}{B}\right) \approx \frac{A-u^2}{B} + \frac{1}{3}\left(\frac{A-u^2}{B}\right)^3 + \dots \tag{5.26}$$

Now,

$$\frac{1}{A-u^2} \approx \frac{1}{A}\left(1 + \frac{u^2}{A} + \frac{u^4}{A^2} + \dots\right) \tag{5.27}$$

So the first-order approximation becomes:

$$\tanh^{-1}\left(\frac{A-u^2}{B}\right) \approx \frac{A}{B}\left(1 - \frac{u^2}{A}\right) \tag{5.28}$$

$$I \approx 2\int_0^{Z-Z'} \frac{A}{B}\left(1 - \frac{u^2}{A}\right) du \tag{5.29}$$

$$= \frac{2A}{B}\int_0^{Z-Z'} \left(1 - \frac{u^2}{A}\right) du \tag{5.30}$$

$$= \frac{2A}{B}\left[u - \frac{u^3}{3A}\right]_0^{Z-Z'} \tag{5.31}$$

$$= \frac{2A}{B}\left(Z - Z' - \frac{(Z-Z')^3}{3A}\right) \tag{5.32}$$

Substitute back $A = Z^2 + Z'^2, \quad B = 2ZZ'$:

$$I \approx \frac{4ZZ'}{Z^2 + Z'^2} \left(Z - Z' - \frac{(Z - Z')^3}{3(Z^2 + Z'^2)} \right) \tag{5.33}$$

We consider the integral:

$$\int \frac{Z}{Z'} \cdot \frac{Z^2 + Z'^2}{Z - Z'} \left[1 - \frac{(Z - Z')^2}{3(Z^2 + Z'^2)} \right] dZ' \tag{5.34}$$

Let

$$u = Z - Z' \Rightarrow Z' = Z - u, \quad dZ' = -du \tag{5.35}$$

Substituting into the integral:

$$Z' = Z - u, \quad \frac{1}{Z'} = \frac{1}{Z - u} \tag{5.36}$$

$$Z^2 + Z'^2 = Z^2 + (Z - u)^2 = Z^2 + Z^2 - 2Zu + u^2 = 2Z^2 - 2Zu + u^2 \tag{5.37}$$

Now the integrand becomes:

$$-\int \frac{Z}{Z - u} \cdot \frac{2Z^2 - 2Zu + u^2}{u} \left[1 - \frac{u^2}{3(2Z^2 - 2Zu + u^2)} \right] du \tag{5.38}$$

Expand each term in powers of $\frac{u}{Z}$:

$$\frac{1}{Z - u} \approx \frac{1}{Z} \left(1 + \frac{u}{Z} + \frac{u^2}{Z^2} + \cdots \right) \tag{5.39}$$

$$2Z^2 - 2Zu + u^2 \approx 2Z^2 \left(1 - \frac{u}{Z} + \frac{u^2}{2Z^2} \right) \tag{5.40}$$

Thus the entire expression simplifies to:

$$\frac{2Z^2 - 2Zu + u^2}{u} \approx 2Z - 2u + \frac{u^2}{Z} \tag{5.41}$$

Combining:

$$\frac{Z}{Z-u} \cdot \frac{2Z^2 - 2Zu + u^2}{u} \left[1 - \frac{u^2}{3(2Z^2 - 2Zu + u^2)}\right] \approx \frac{2Z}{u}\left(1 + \frac{u}{Z}\right)\left(1 - \frac{u^2}{6Z^2}\right) \quad (5.42)$$

Multiplying out and keeping terms up to $\mathcal{O}(u^2)$:

$$\approx \frac{2Z}{u} + 2 + \frac{u}{3Z} \quad (5.43)$$

Now integrate:

$$-\int \left(\frac{2Z}{u} + 2 + \frac{u}{3Z}\right) du = -\left(2Z \ln|u| + 2u + \frac{u^2}{6Z}\right) + C \quad (5.44)$$

Substitute back $u = Z - Z'$:

$$\int \frac{Z}{Z'} \cdot \frac{Z^2 + Z'^2}{Z - Z'} \left[1 + \frac{(Z-Z')^2}{3(Z^2 + Z'^2)}\right] dZ' \approx -\left(2Z \ln|Z - Z'| + 2(Z - Z') + \frac{(Z-Z')^2}{6Z}\right) + C \quad (5.45)$$

We compute the definite integral from $Z' = 0$ to $Z' = Z$ using the approximation:

$$\int_0^Z \frac{Z}{Z'} \cdot \frac{Z^2 + Z'^2}{Z - Z'} \left[1 + \frac{(Z-Z')^2}{3(Z^2 + Z'^2)}\right] dZ' \approx -\left[\frac{(Z-Z')^2}{4Z} + \frac{(Z-Z')^3}{6Z^2}\right]_0^Z \quad (5.46)$$

Lower Limit $Z' = 0$:

When $Z' = 0$, we have:

$$Z - Z' = Z \quad (5.47)$$

Substitute into the expression:

$$\frac{(Z-Z')^2}{4Z} + \frac{(Z-Z')^3}{6Z^2} = \frac{Z^2}{4Z} + \frac{Z^3}{6Z^2} = \frac{Z^2}{4Z} + \frac{Z}{6} = \frac{Z}{4} + \frac{Z}{6} = \frac{5Z}{12} \quad (5.48)$$

Upper Limit $Z' = Z$:

When $Z' = Z$, we have:

$$Z - Z' = 0 \tag{5.49}$$

So the entire expression is zero:

$$\frac{(Z - Z')^2}{4Z} + \frac{(Z - Z')^3}{6Z^2} = 0 \tag{5.50}$$

Therefore, the result of the definite integral is:

$$\int_0^Z \frac{Z}{Z'} \cdot \frac{Z^2 + Z'^2}{Z - Z'} \left[1 + \frac{(Z - Z')^2}{3(Z^2 + Z'^2)}\right] dZ' \approx \frac{5Z}{12} = AZ \tag{5.51}$$

As we can see there is a similar Z term which comes from bilinear smearing which exactly cancels the extra Z which arises from the two point function.

Now, performing a similar calculation, we find:

$$\langle 0|\mathcal{O}(T_2)\, \phi^{(1)}(T_1, Z)|0\rangle = -AZ$$

Thus now if we commute the commutation relation we get,

$$\begin{aligned} \langle 0|[\mathcal{O}(T_2),\, \phi^{(1)}(T_1, Z)]|0\rangle\, \mathcal{O}(0) &= \langle 0|\mathcal{O}(T_2)\, \phi^{(1)}(T_1, Z)|0\rangle\, \mathcal{O}(0) \\ &\quad - \langle 0|\phi^{(1)}(T_1, Z)\, \mathcal{O}(T_2)|0\rangle\, \mathcal{O}(0) \\ &= 2AZ \end{aligned} \tag{5.52}$$

which exactly cancels the extra term to the leading order.

Chapter 6

Conclusion

As we have seen, maintaining spacelike commutativity is crucial because it ensures the preservation of **causality**—a fundamental principle in both quantum field theory and holography. If two operators do not commute when located at spacelike-separated points (i.e., outside each other's light cones), then the outcome of one measurement can influence the other, violating the causality constraint.

To address this issue, we explored techniques such as *bilinear smearing*, which effectively cancel out the unwanted interaction terms that lead to non-commutativity. While this was initially demonstrated in the context of AdS_3, we showed that a similar strategy applies to AdS_2 as well. In particular, we examined a specific a specific commutator that must vanish to preserve spacelike commutativity. By applying the bilinear smearing correction, we successfully canceled the additional terms, restoring the expected causal structure.

Although the bilinear smearing was able to cancel the additional term, it did so only to first order, and that too under several approximations. Due to the complex nature of the integral involved, we were unable to solve it beyond the first-order approximation. A more exact calculation can be pursued in future work.

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