CHIMERA IN MULTIPLEX NETWORKS

M.Sc. Thesis

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DISCIPLINE OF PHYSICS

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A THESIS

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of
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NAVEEN KUMAR MENDOLA

DISCIPLINE OF PHYSICS
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INDORE

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **CHIMERA IN MULTIPLEX NETWORKS** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF SCIENCE** and submitted in the **DISCIPLINE OF PHYSICS, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from **JULY 2017** to **JUNE 2019** under the supervision of **Dr. Sarika Jalan, Professor, IIT INDORE.**

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

Signature of the student with date
(NAVEEN KUMAR MENDOLA)

This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

Signature of the Supervisor (with date)
(Dr. SARIKA JALAN)

NAVEEN KUMAR MENDOLA has successfully given his M.Sc. Oral Examination held on ____________________ .

Signature of Supervisor of MSc thesis
Date:

Convener, DPGC
Date:

Signature of PSPC Member
Date:

Signature of PSPC Member
Date:

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Abstract

Delay sheds light upon plenty of different characteristics of a nonlinear system which are either nonexisting or unrevealed without it. Here we unmask one of those nonexisting characteristics of First-order Kuramoto Oscillators 'Solitary states'. Where the natural occurrence is not yet discovered in the First-order Kuramoto model, we propose a technique which can induce the Solitary states in it by applying the Delay in a multiplex network. This technique is equally useful to induce the Chimera states and is also extended to allow the switching between Solitary and Chimera states.

Keywords : Solitary States, Chimera States, Switching between Solitary state and Chimera, Delay, Delay in Multiplex network

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Chapter 1
Theory and Introduction

1. 1 Introduction

In the year 2002, a new subfield was added in the vast field of nonlinear dynamics when Dr. Yoshiki Kuramoto realized the symmetry breaking in a system of identically coupled oscillators [1]. An identical system, which seemed to be in any one the only two possible states, i.e., Synchronized or desynchronized states, he discovered a new mixed state between them. Later, Strogatz and Abrams called these states '**Chimera State**' [2]. Initially, Chimera was found in a system of '**First order Kuramoto Oscillators'** (1-KO), but by now it has been observed in various models like FitzHugh-Nagumo oscillators, Rossler's, and coupled maps.

The study of Chimera attracts the researcher not only because it is another spatiotemporal pattern to understand, but also its unexpected and probably unnoticed appearances in the places which are yet to discover. A recent study in Neuroscience doubts chimera states to be the probable cause of Epilepsy in patients. Even this single example clarifies why it is essential to build a strong theoretical background which can provide us more control over these states.

With time, people found many other chimeric patterns and named them as Virtual Chimera [4], Travelling Chimera [5], Breathing Chimera [6] and many more [7, 8, 9]. For this thesis, I would like to draw the attention to '**Solitary state**' in oscillators too, which is another Chimera-like pattern, observed in '**Second-order Kuramoto Oscillator'** (2-KO) [10].

Further in the introduction, we provide a brief discussion to Chimera state and give the necessary introduction of Solitary states.

1. 1. 1 Coherent and Incoherent States

A general system of N coupled components can show a variety of dynamical behaviors of the individuals. Each can have its own natural frequency and can be anywhere in the entire 0-2π space at any time.

If for some suitable parameter, every component is found at the same place with exactly the same frequency, the system is called to be in **'Coherent state'**. Whereas, if every node is moving with the different frequency and have completely random phase value at any time, the system is said to be in **'Incoherent state'**.

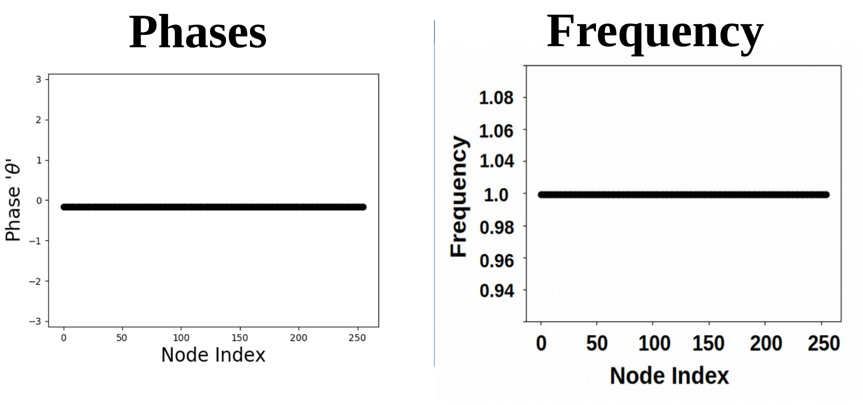


Fig 1.1(a) : Phases and frequencies of a coherent system

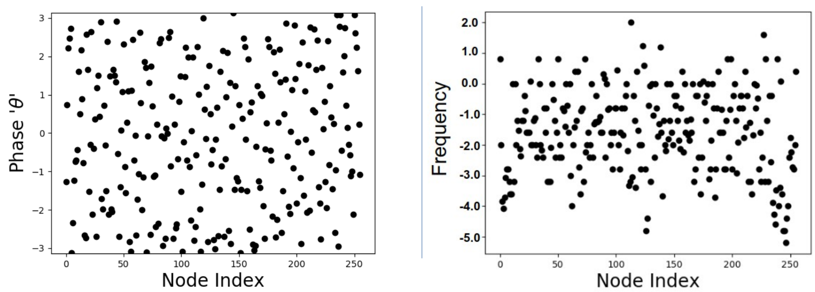


Fig 1.1(b) : Phases and frequencies of an incoherent system

Figure – 1.1 : Example of Coherent and Incoherent system

1. 1. 2 Chimera State

When a system of the identical oscillator is simulated, the dynamics do not seem much complicated, and depending on the system parameters; we expect it always to get synchronized or stay desynchronized eventually. But Kuramoto found [1] that those identical systems are not every time simple and for some specific initial condition, they can be in a mixed state, called **'Chimera State'**. Figure 1.2 is the example of this coexisting behaviour.

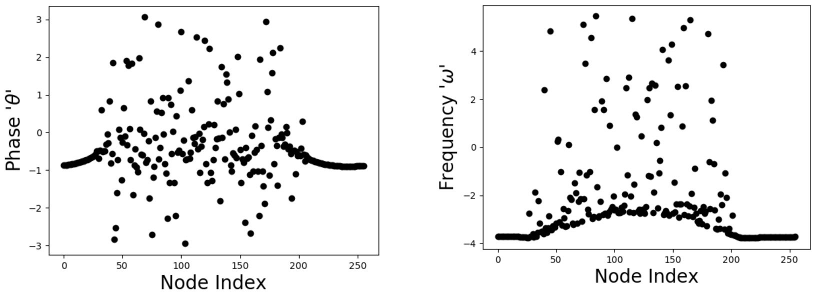


Figure 1.2 : Phases and frequencies of individual oscillators when the system is in Chimera state.

I would like to emphasis on those special initial conditions and will later discuss how important they actually are, and if they provide the necessary and sufficient conditions observe Chimera.

1. 1. 3 Solitary state

The word Solitary is taken from Latin 'Solitarius' which stands for 'alone' or 'isolated'. The overall behavior of the whole system, in this case, is similar to Chimera, but here only a few oscillators from different locations possess different frequencies and phases. One solitary unit in the system is called 1-Solitary, two units 2-Solitary and 'k' units k-Solitary [11]. **The population and the position of the isolated nodes depend on the system parameters and, especially, found for a specific combination of parameter values in the phase space.** The detailed study of such parameters and its causes is beyond the scope of this thesis and would be unnecessary for the objective of the project. but it should be noted that these states are observed only in the 2-KO and **NOT YET** observed in 1-KO. Figure 1.3 shows the example of Solitary states.

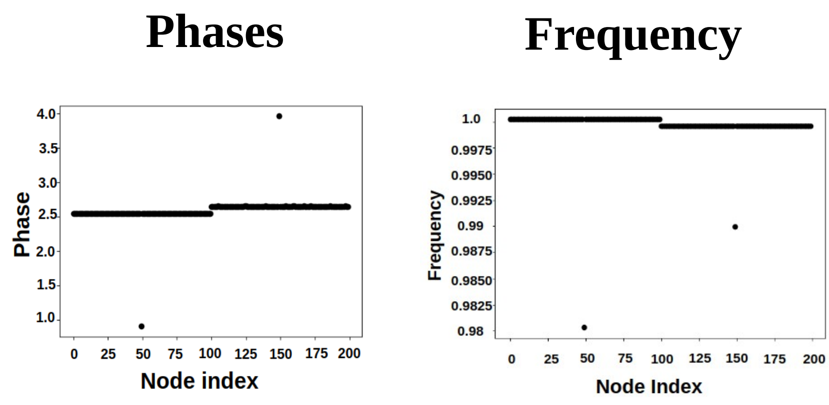


Fig 1.3 (a) : Phase and Frequency of 1-Solitary state

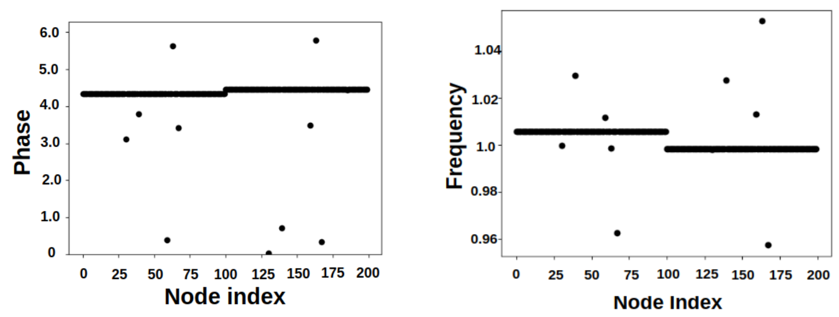


Fig 1.3 (b) : Phase and Frequency of 5-Solitary state

Figure – 1.3 : Example of solitary state

1. 2 Theory of Chimera

We will be discussing the initial conditions and the necessary parameters for Chimera in this section.

It is not very straight-forward to guess or obtain those initial conditions. According to Kuramoto, it is required to provide the system with a push start by picking a random number for each phase from a range which increases with distance as we go away from the first oscillator in a ring which is done by taking an exponential kernel

$$\dot{\theta}_i^{initial} = a * r(i) * \exp[-b(i-\lambda)^2]$$

Here, $\theta_i^{initial}$ is the initial phases of all the nodes, r(i) is the random number of i^{th} node, 'a' and 'b' are the appropriate constants, and '¸' is the mean of the distribution. Figure 1.4 shows how the system initially looks like.

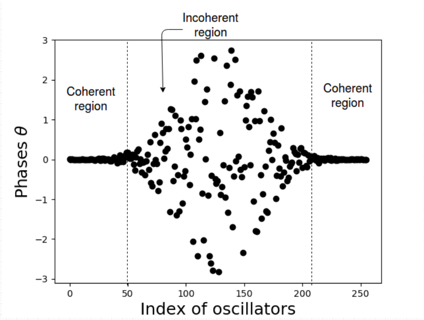


Fig 1.4 : Initial phases of the system of 256 nodes.

Evolving the system with any other initial condition without using a similar kernel leads the system to the synchronized state for enough coupling strength.

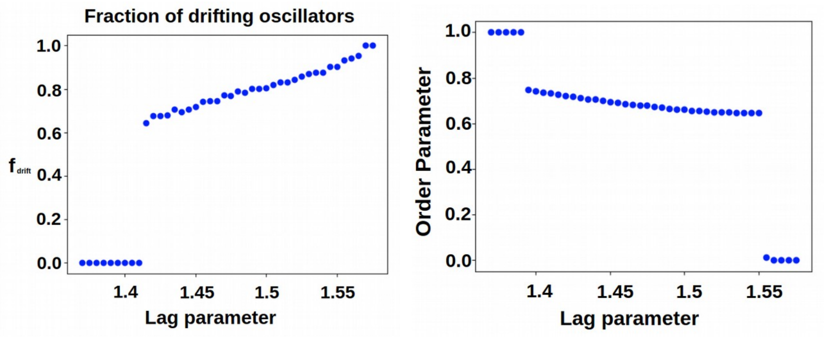
First order Kuramoto (1-KO) is given by:

$$\dot{\theta}_i = \omega + \mu \sum_{j=1}^{N} \sin(\theta_j - \theta_i - \alpha)$$

Here 'θ' is the phase, 'ω' is the natural frequency, 'μ' is the coupling strength and 'α' is the lag parameter.

Since these conditions are necessary but not sufficient for Chimera states, there is one more parameter which has a vital role in its appearance, the 'phase lag α'. This is the only parameter which gives us at least a little control over chimera states. By tuning it, we can increase or decrease the width of the incoherent region. But even this parameter has its limitations and works only in its specific range. Being below or above that range also leads the system to the synchronized state.

Though figure 1.5 shows that range but this thesis presents the method for which the study of the role of lag isn't important and therefore, is not given here. The only thing, which is of our concern, is to notice that only this range of lag is the second necessary condition for Chimera.



(a) **(b)**

Fig 1.5 : (a) Shows the number of drifting oscillators and **(b)** the corresponding order parameter of the system.

The fraction of drifting oscillator is just the inverse of the number of oscillators which are drifting. Thus, it takes the value 1 when all are drifting and ‘0’ when no one is. Similarly, (b) is the measure of the order in the system, shows almost the opposite thing than (a). It takes the minimum value for all drifting oscillators and ‘1’ for the completely synchronized system.

Chapter 2
Methods and Techniques

2.1 Theoretical modeling

We are using a ring of standard first-order Kuramoto oscillators connected to another identical ring with only one to one connection between the mirror nodes. The schematic diagram can be imagined like figure 2.1.

The dynamics of each ring is given by :

$$\begin{aligned}\dot{\theta} &= \omega + \mu_1 \sum_{j=1}^{N} \sin(\theta_j^1 - \theta_i^1) + \mu_{inter} \sin(\theta_i^2 - \theta_i^1) \\ \dot{\theta} &= \omega + \mu_2 \sum_{j=1}^{N} \sin(\theta_j^2 - \theta_i^2) + \mu_{inter} \sin(\theta_i^1 - \theta_i^2)\end{aligned} \qquad \textbf{(2.1)}$$

Here,

θi is the phase of each oscillator in the 1st or the 2nd layer accordingly, N is the number of oscillators in each layer, ω is the (equal) frequency, μ_1 and μ_2 are the coupling strengths between the oscillators in the first and second layer respectively, and μ_{inter} is the coupling strength of the inter-layer connection. We choose the conditions for which the whole system synchronizes.

This article uses 'delay' as a tool to get some of the synchronized oscillators to drift with a different frequency than the rests. A similar study was done for coupled chaotic maps by introducing delay in some of the nodes, making them behave differently. Here, we choose to have the delay in the inter layer connection keeping each layer as close to the previous model (eq. 2.1) as possible. The governing equation changes slightly and is given by:

$$\dot{\theta} = \omega + \mu_1 \sum_{j=1}^{N} \sin(\theta_j^1 - \theta_i^1) + \mu_{inter} \sin(\theta_i^2 (t - \tau_i) - \theta_i^1 (t))$$

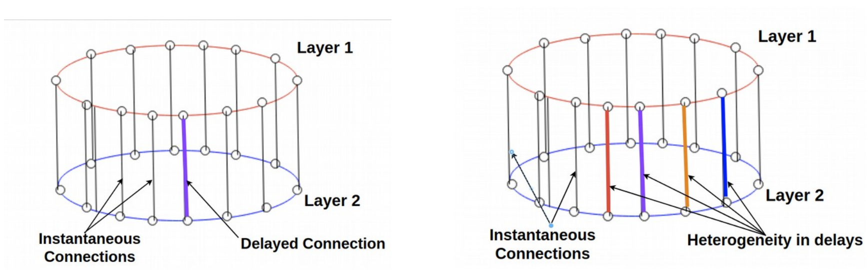
$$\dot{\theta} = \omega + \mu_2 \sum_{j=1}^{N} \sin(\theta_j^2 - \theta_i^2) + \mu_{inter} \sin(\theta_i^1 (t - \tau_i) - \theta_i^2 (t)) \qquad \textbf{(2.2)}$$

Where τ_i is the appropriate time delay in each inter-layer connection. We expect that as soon as we apply delay in any of the links, the corresponding nodes should respond and behave differently, making it a Solitary node.

Though the delay plays the most important role here, it's not the only player in the game. Actually, the inter and intra-layer coupling strength also have crucial parts in the overall dynamics. Where the delay initializes the frequency difference, the combination of inter and intra-layer coupling strength take part in amplifying it. Thus, It needs comparatively more amount of inter-layer coupling strength to see the visible and legitimate frequency difference.

Hopefully, this method will enable us to induce Solitary states in a synchronized system of 1-KO where these states are not discovered yet. Similarly, we will be able to induce Chimera optionally too. Also, **this method will allow us the switching between the Solitary state and Chimera**, which is going to be the main objective of this thesis.

The schematic diagram of the network after applying delay is shown in figure 2.1. Different color, shown in 2.1(b) shows the different delay value in the interlayer connection.



(a) **(b)**

Figure 2.1 : Schematic diagram of the multiplex network when only 1 (2.1a) and 2 or more connections (2.1b) are delayed with different values

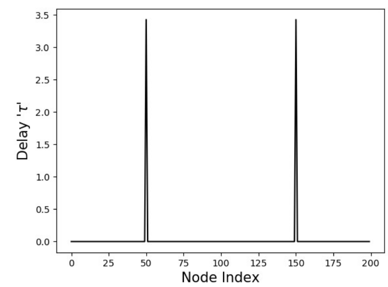
For k-Solitary state, we will randomly choose k connections in any of the networks and will randomly distribute some delay values on those positions only. As a result, the nodes corresponding to those delayed connections will pop out of the synchronized chunk and drift with the different frequency than the rests. By applying the same heterogeneous delay values in the adjacent connection instead of in the random position will switch the k-solitary state with chimera state.

Chapter 3
Results and conclusion

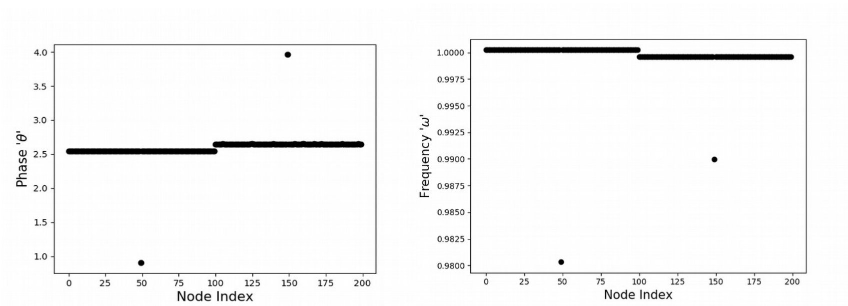
3.1 Results

As discussed, to get a synchronized system, we take $\mu_1 = \mu_2 = 0.5$ and $\mu_{intra} = 4.0$. Figure 1.1(a) shows how the phases, as well as the frequencies, become equal for these parameters without introducing delay. Since these parameters show enough frequency difference, we are going to use them for further investigation.

We start by taking a random delay value at the 50th position. The phases and the frequency difference is quite visible in figure (3.1).



(a)



(b)

Fig 3.1 : (a) Delay value only at 50th position. (b) shows how the synchronized system has evolved to 1-Solitary state.

Similarly, random positions and the delay values were chosen to obtain 2-Solitary, 5-Solitary, and 10-Solitary states. All these states are shown in Fig 3.2 with their corresponding delays.

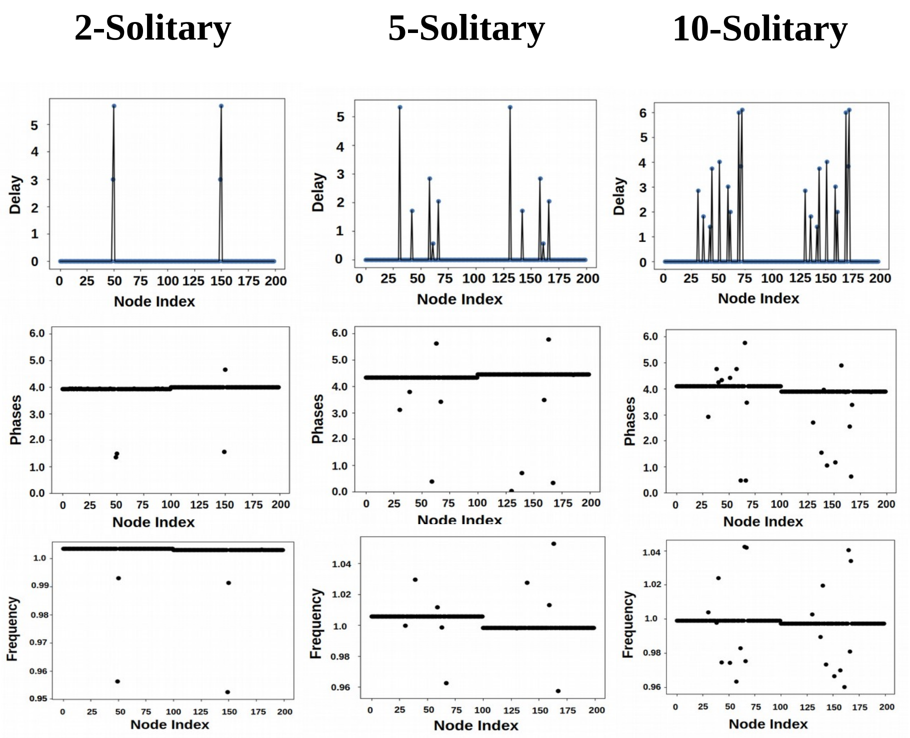


Fig 3.2 : Delay, Phases and Frequency of 2, 5 and 10-Solitary states.

Changing the positions of the delayed nodes makes possible the switching between the Solitary and Chimera state. In the next figure (3.3), we’ve shown how this simple change can give us the chimera state without using any initial condition or tuning any parameter. In fact, this state is obtained by taking random initial conditions and by not at all considering the lag parameter in the equation.

We put random values of delays in 20 of the connections at random positions first and later put them on the adjacent nodes, switching 20-Solitary to a Chimera state.

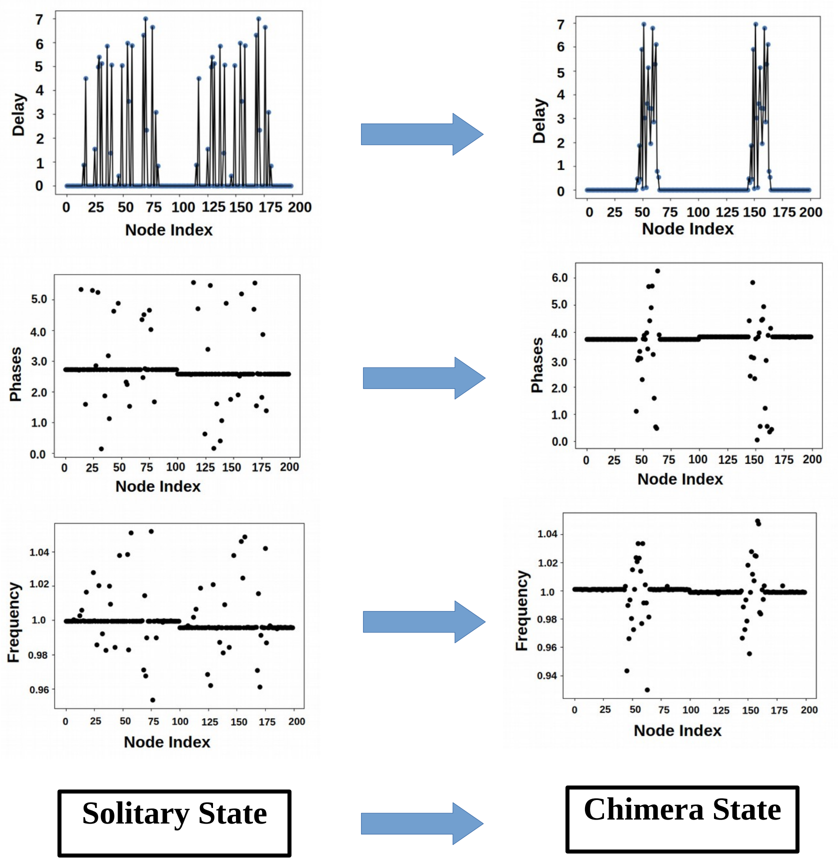


Figure 3.3 : Switching between Solitary and Chimera state

3.2 Discussion

Since, other than delay, these states are dependent on inter and intra-layer coupling strength, it seems necessary to see the coupling strength combination for which these states are achievable. Moreover, It is essential to know how the number of those combinations increase or decrease with increasing delay value.

For this matter, we plot the phase diagram for inter and intra-layer coupling strengths with increasing delay for the 1-Solitary state. Since the frequency difference is the only legitimate parameter, we check the corresponding frequency difference in the system of 8 nodes.

In figure (3.4), we see how the obtainable region can be reached sooner if we increase the delay. Below the middle line is the region where the system is synchronized, and we do not have any frequency difference among the nodes.

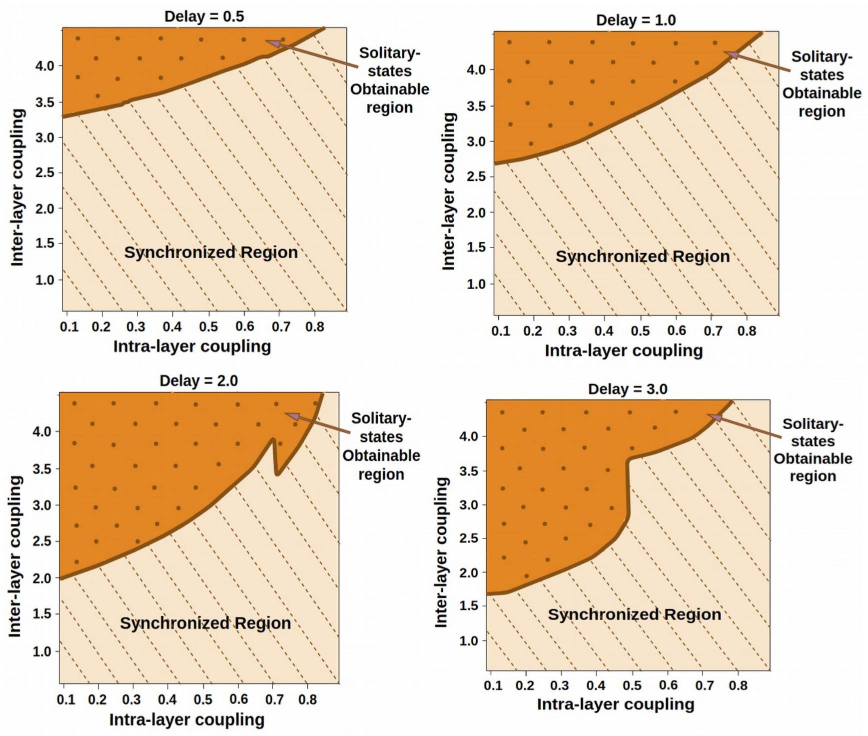


Fig 3.4 : Phase spaces with different delay

Though the value in Y and X axis are correct, these are just the schematic diagrams of the two regions. The real plots are [A], [B], [C] and [D] in the appendix. Below 0.1 intra-layer coupling is the region when the system is not synchronized and is of no interest to us. One can see from these diagrams that the states are obtainable only for the combinations of low intra-layer and high inter-layer coupling strength.

3.3 Conclusion

I want like to conclude the discussion by bringing in light, the progress we've made so far.

i. Chimera states could not be achieved without using special initial conditions and the lag parameter which we can now.

ii. The natural appearance of Solitary state in 1-KO is not yet discovered. Still, we can induce it using this method.

iii. Even when we apply the appropriate conditions for Chimera, it was completely unknown where in the network Chimera will appear. So is the case for Solitary states. The number and position of solitary nodes were parameter dependent, which now is completely on our control.

iv. Most importantly, by simple tuning, this method allows the switching between the two chimera-like states.

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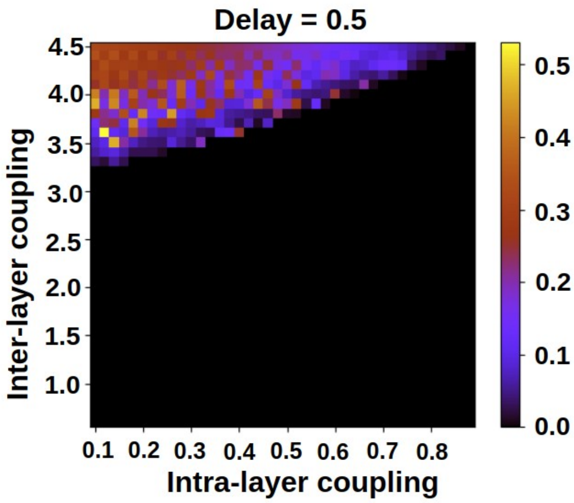
Appendix :-

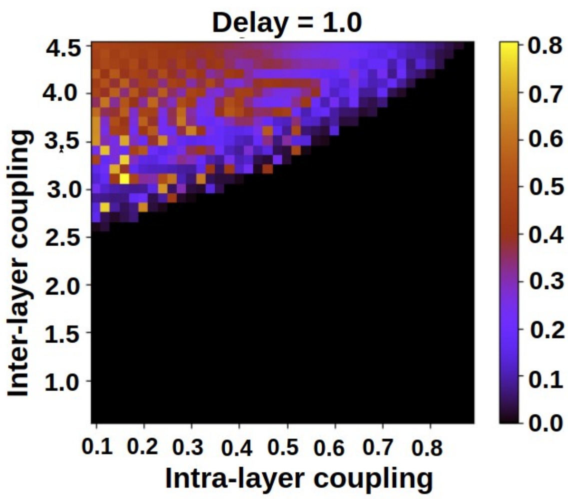
Please note that each pixel in the figure represents the frequency difference of the solitary node and the synchronized chunk. The value of those differences in the desired location can be read from the colorbar next to each figure.

Its clear that as we increase the intra-layer coupling strength, the difference in the frequency decreases no matter what delay value we take. On those locations, all the nodes are so tightly sync due to the intra-layer coupling that the inter-layer coupling is unable to amplify the effect of the delay.

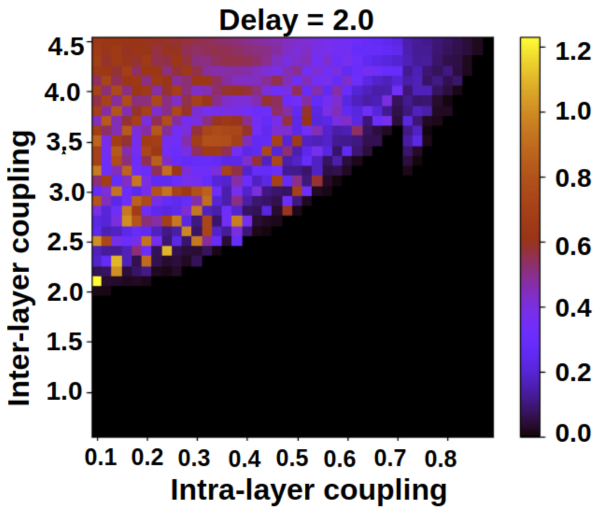
[A] .



[B] .

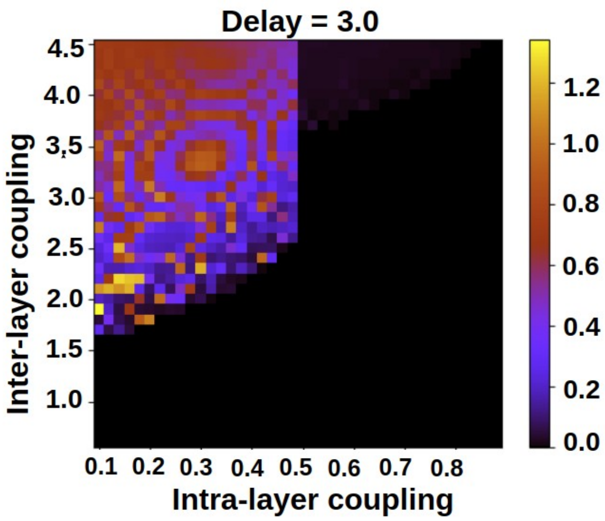


[C] .



Delay = 2.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
Inter-layer coupling
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
Intra-layer coupling
1.2
1.0
0.8
0.6
0.4
0.2
0.0

[D] .



Delay = 3.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
Inter-layer coupling
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
Intra-layer coupling
1.2
1.0
0.8
0.6
0.4
0.2
0.0