Exploring Chimera in Networks

M.Sc. Thesis

by

Pramodini Mallik

Discipline of Physics

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Submitted in partial fulfillment of the
requirements for the award of the degree
of
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by

Pramodini Mallik

Discipline of Physics

Indian Institute of Technology Indore

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Dedicated to my Family and Friends.

INDIAN INSTITUTE OF TECHNOLOGY INDORE

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**Exploring Chimera in Networks"** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF SCIENCE** and submitted in the **DISCIPLINE OF PHYSICS, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July 2018 to June 2019 under the supervision of Dr. Sarika Jalan, Professor, Indian Institute of Technology Indore.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

Pramodini Mallik

This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

M.Sc. Thesis Supervisor
Dr. Sarika Jalan

Pramodini Mallik has successfully given her M.Sc. Oral Examination held on ______________

M.Sc. Thesis Supervisor
Date:

Convener, DPGC
Date:

PSPC Member 1.
Date:

PSPC Member 2.
Date:

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Discipline of Physics

Indian Institute of Technology Indore

Abstract

The coexistence nature of both coherent and incoherent state in non-locally coupled identical oscillators in a structurally symmetric network is termed as chimera. Study of chimera has become very popular in the past decades due to its peculiar behavior. There has been numerous method adopted to observe chimera. Here we investigate on Kuramoto model and the logistic map model. We study the impact of multiplexing a repulsively coupled second layer on the positively coupled first layer. We report that a repulsively coupled layer enhance the appearance of chimera in the positively coupled layer. We study the logistic map model for both single and multiplex network, ignoring the effect of the delay. Furthermore adding delay to the nodes of a single layer network, we investigate how does this affect the dynamics of the system. We also study the role of homogeneous and heterogeneous delay to produce chimera state.

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List of abbreviations

sIPR	Spatial Inverse Participation Ratio
ICR	Incoherent Region
CR	Coherent Region
SR	Synchronous Region

Chapter 1

Introduction

We have various problems related to complex systems around us, and such systems are studied and solved by constructing networks. The dynamical behavior of a network is either observed to be in a synchronized or desynchronized state (i.e., each node either behaves in the same manner like the all other nodes, or they are independent of each other). A different phenomenon, unlike the synchronized and desynchronized state, was observed by Kuramoto and Battogtokh in 2002 for a model of identical but non-locally coupled oscillators in a ring network [1]. Later in 2006, it was defined as chimera state by Abrams and Strogatz [2]. Chimera state is referred to as a mathematical hybrid state in which coherent and incoherent dynamics coexist in non-locally coupled oscillators in a structurally symmetric network [3].

In Greek mythology, the fire-breathing hybrid creature of Lycia is known as Chimera. Chimera occurs in a variety of physical, chemical, biological, neuronal, ecological, technological, or socio-economic systems [4]. The coexisting behavior of coherent and incoherent state at the same time fascinates and motivates the researchers to find out the factors that are responsible for the emergence of chimera. Chimera has observed in many network systems. Recent literature has indicated a strong connection between the occurrence of chimera and various responses of neurons in brain networks. For example, the chimera state has been related to the uni-hemispheric sleep in mammals where half of the brain remains asleep, while the other half remains active [2].

1.1 Network

To better understand a complex system, we need to apprehend how its components interact with each other. It consists of different types of networks. Nodes in a network are connected through edges. These nodes are connected to each other through links. N represents the number of nodes, and We often call N as the size of the network. To distinguish the nodes, we denote them with i = 1, 2, ..., N. L represents the number of links, which gives us a basic idea about the total number of interactions exists between the nodes [5]. The network depicted in figure 1.1 depicting has N=7 and L=8.

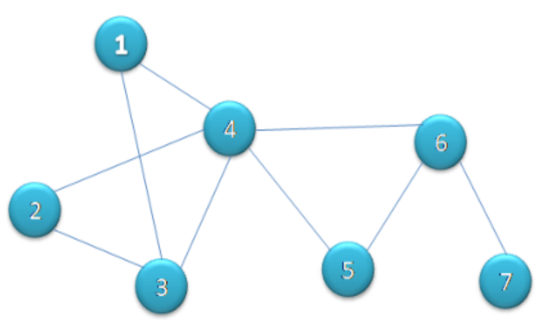


Figure 1.1: Schematic diagram of a network consisting of 7 nodes.

For example, two persons in a social network can interact academically or socially or both, two communication hubs may be connected via rail or road or air or by all of them. Real world complex system from various field ranging from science and engineering to sociology or economics and model them as networks which will help us to make some understanding of the underlying interaction patterns [3]. There are different types of network, but here we used a regular network to study its behavior, and how does the state of the system evolve with time? Some of the important properties of a network are listed below.

1.1.1 Degree

The total number of connection of a node has to all the other nodes in a network is said to be the degree of a network. Degree of the i^{th} node is denoted by k_i.

1.1.2 Average degree

Average Degree is denoted by $\langle k \rangle$ and is defined for this network is

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i \tag{1.1}$$

1.2 Adjacency matrix

The architecture of a network can be understood by analyzing an adjacency matrix. A network consisting of N nodes represented by a N×N matrix containing elements either 0 or 1. The elements A_{ij} of the adjacency matrix A are defined as

$$A_{\text{ij}} = \begin{cases} 1 \ \text{ if } \ \text{i} \sim \text{j i.e. } i^{th} \text{ and } j^{th} \text{ node are connected.} \\ 0 \ \text{ otherwise} \end{cases}$$

The diagonal entries of the adjacency matrix are zero represents no self-connection in the network.

1.3 Theoretical Model

In the previous section, we have mentioned there are a plethora of systems which shows different behavior such as synchronization, desynchronization, or chimera. To understand the mechanism behind such phenomenon that draws the attention of many researchers we need to construct a theoretical model which must contain all the parameters that are involved in a system and hence will be able to give us the accurate and necessary results we expect to observe. Here we used two different types of mathematical models.

1.3.1 Kuramoto Model

To solve such one of real-world phenomenon related to oscillators, Kuramoto model came into existence. A Japanese physicist, Yoshiki Kuramoto, proposed this model, which later used to solve various problems. Synchronization is well defined and studied using this model [1].

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{2N} A_{ij}(\sin(\theta_j - \theta_i + \alpha)) \tag{1.2}$$

Where θ_i and ω_i are the phase and natural frequency of the i^{th} oscillator respectively. Here i varies from 1,2,...,2N. λ is coupling strength. α_i is a constant phase lag parameter.

Equation(1.2) is a first order differential equation (θ_i is differentiated with respect to time). Phase (θ_i) could be easily solved and analyzed by using the Runge-Kutta method, and as a result, we would be able to predict the dynamics of the system [6].

1.3.1.1 Order parameter(R):

Another important parameter used in the Kuramoto model is order parameter, denoted by R. Order parameter is used to determine the state of a system and help us to predict whether the state is in a coherent or in an incoherent state. Defined as:

$$R(t) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(t)}\right| \tag{1.3}$$

We calculate order parameter for the transient time i.e.

$$R = \frac{1}{T}\sum_{t=0}^{T} R(t) \tag{1.4}$$

The magnitude of R lies between 0 and 1. R=1 denotes the system is in a completely coherent state, and 0 denotes the system is completely in an incoherent state [7].

1.3.2 Logistic Map Model

The dynamics of a system is determined using the second model which is defined as:

$$z_{t+1}(i) = f(z_t(i)) + \frac{\varepsilon}{\sum_{j=1} A_{ij}} \sum_{j=1}^{mN} A_{ij} \times [f(z_t(j)) - f(z_t(i))] \tag{1.5}$$

Where ε is the coupling strength, m is the number of layers in the network, i and j are the indices of the oscillators. N is the total number of nodes in a layer. The denominator term $\sum_{j=1}^{N} A_{ij}$ in the above equation is the average degree of i^{th} node i.e. with how many nodes the i^{th} node is connected with. A is the adjacency matrix which gives us information about node connection in a network. The function f(z) that repeatedly appears in the above model is the logistic map, and defined as [3].

$$f(z) = \mu z(1 - z) \tag{1.6}$$

The logistic map is an iterative second order polynomial used in this model. This map gives the value of z for the next time step provided that we know the previous value of z. With the help of this model, we were able to determine the dynamics of any node at any time in terms of $z_t(i)$.

Chapter 2

Inhibition induces Chimera

Since its discovery, a numerous approach has been taken to study chimera state. We are going to discuss various method followed by us in this chapter, to study chimera and its peculiar behavior. In this chapter we are going to observe the impact of multiplexing a repulsively coupled layer with a positively coupled layer. Furthermore, this approach of constructing a framework of network is acceptable. Here, we consider a multiplex network consisting of two identical layers but nature of coupling is different. Specifically, we consider attractive and repulsive couplings nature of a layer. Such type of attractive and repulsive coupling nature has observed in many real world complex system. For example, the brain has inhibitory and excitatory neurons representing repulsive and attractive couplings, respectively. Studies on the impact of inhibition on the emergence of the chimera state in the multiplex framework will be useful for a better understanding of such complex systems in different conditions [6].

2.1 Network Architecture

We considered a multiplex network comprising two layers, where each layer is a regular ring network consisting of 100 non-local identical oscillators coupled to each other by some coupling strength λ, and it varies from 0 to 5. In a layer, a node is connected to it's "p" number of nearest neighboring nodes in both the direction. The mirror nodes of the two layers are connected to each other. Hence the degree of each node in a layer is equal to 2p+1.

We connect the nodes of the second layer in the multiplex network in two different ways:

(1). Attractively coupled

(2). Repulsively coupled

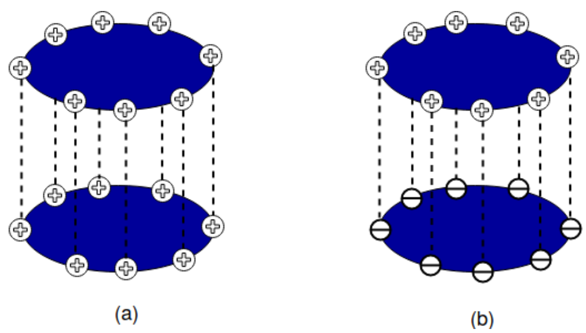


Figure 2.1: Schematic diagram depicting the two different types of coupling architecture followed to observe chimera state.

The mathematical representation of a network is obtained by constructing an adjacency matrix in C++, which shows the connectivity of each node in a network. Adjacency matrix for this multiplex network is

$$\mathrm{A}= \begin{pmatrix} A^1 & I \\ I & A^2 \end{pmatrix}$$

Here A^1 and A^2 is the adjacency matrix for the first and second layer respectively. I is the identity matrix representing the connectivity between the mirror nodes.

A_{ij} are the elements of the adjacency matrix(A^2) for the second layer, where the nodes are repulsively coupled and defined by

$$A_{ij} = \begin{cases} \text{-1} \quad \text{if} \quad \text{i} \sim \text{j i.e. } i^{th} \text{ and } j^{th} \text{ node are connected.} \\ 0 \quad \text{otherwise} \end{cases}$$

2.2 Method:

The Kuramoto model mentioned in chapter 1 was solved in C++ using 4th order Runge–Kutta method. To understand the evolution of oscillators with time, we run the code for a sufficiently long time. Here we simulated the code up to 15000 times. The phase of each oscillator for the last 5000 transient time is taken to study how these oscillators are interdependent in a network. Using these phase values, we were able to estimate the order parameter, which in turn could define the state of the system. We also computed laplacian distance measure $|D|$ defined by [6]

$$|D| = \nabla_i^2\theta = |(\theta_{i+1}(t) - \theta_i(t)) - (\theta_i(t) - \theta_{i-1}(t))| \tag{2.1}$$

2.2.1 Initial condition:

To observe chimera it is very necessary to consider some specific initial conditions. We take the initial phases of the oscillator from a uniform random distribution denoted by r(i) which is multiplied by a Gaussian distribution as given below.

$$\theta_i(t = 0) = 6r(i)\exp[-30(\frac{i}{N} - \frac{1}{2})] \tag{2.2}$$

θ_i values resulted from the above equation lies between -π to π.

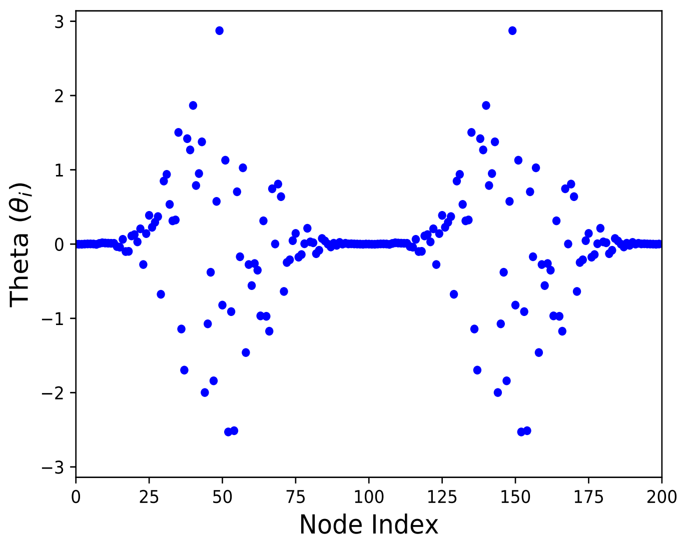


Figure 2.2: Same Initial phase given to both the layers in the multiplex network, at time=0.

2.3 Result

We have earlier discussed that the second layer coupled in two different ways. Further, we want to investigate the impact of the negatively coupled second layer on the positively coupled first layer.

2.3.1 Second layer is attractively coupled

We present results for the multiplex network with both the layers represented by 1D lattices and the nodes of the second layer is attractively coupled. Order parameter plot for both the layers with the same degree equal to 65 is shown below. Coupling strength varies from 0 to 5, with step size h=0.025.

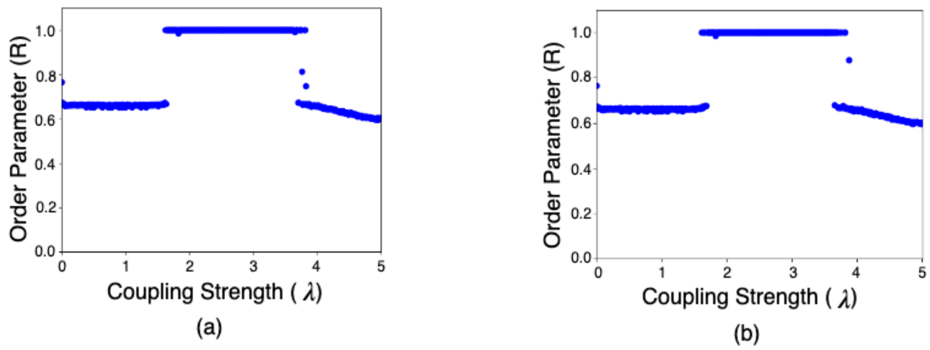


Figure 2.3: Order parameter plot with the coupling strength for the two layers in the multiplex network.

We plot phases of the oscillators and study the interdependence of the oscillators with each other. Chimera pattern is observed for small and high range value of coupling strength, and order parameter(R) obtained for these λ values lies between 0 and 1. In the mid-range values of λ, R equals to 1. As a result, we expect the same phase for all the oscillators, and hence, they are in a perfectly synchronized state.

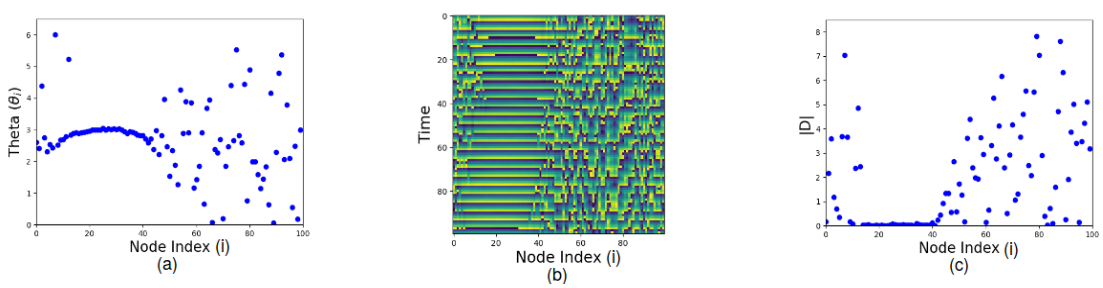


Figure 2.4: (a) Snapshot of the phase plot for the first layer of the multiplex network consisting of two attractively coupled layers, (b) Spatio-temporal pattern of the phase plot for the last 200 transient time, and (c) Laplacian distance measure $|D|$ of the phase plot. Parameters: Network size N=N^1=N^2=100, node degree=$\langle k^1\rangle$=$\langle k^2\rangle$=64, coupling strength λ=1.29, natural frequency ω=0.5, and lag parameter α=1.45 [6].

2.3.2 Multiplexing with a repulsively coupled layer:

We consider a repulsively coupled layer to observe its impacts on the dynamics of the Kuramoto oscillators in the first layer. As expected, the dynamics of the nodes in the first layer is different from the section:2.3.1.

These plots depict chimera state in the dynamics of Kuramoto oscillators of the positive layer multiplexed with a negative layer at a particular transient time.

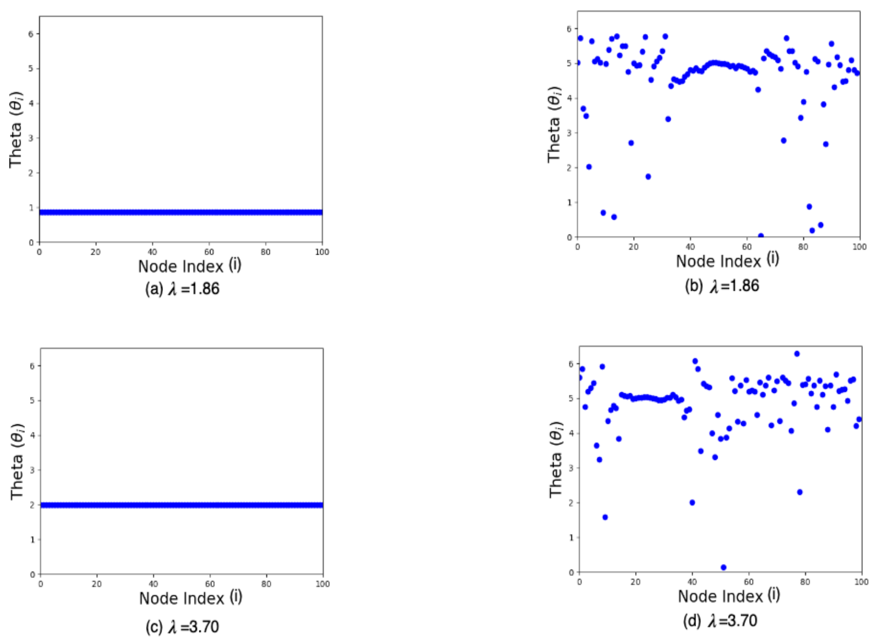


Figure 2.5: Snapshots of the phase plot show chimera state in the positive layer upon multiplexing with a negative layer. (a) and (c) is in a completely synchronized state for the multiplex network consisting of two positively coupled layers. (b) and (d) shows chimera state for the positive layer which is multiplexed with a negative layer [6].

In the case of the multiplex network consisting of two positive layers, coherent state is highly stable with time for the mid coupling range ($2 \lesssim \lambda \lesssim 4$). This behavior observed due to the high dependence of a node to its neighboring nodes in a network. Whereas in the second case, the positive layer shows chimera state for the same range value of coupling strength.

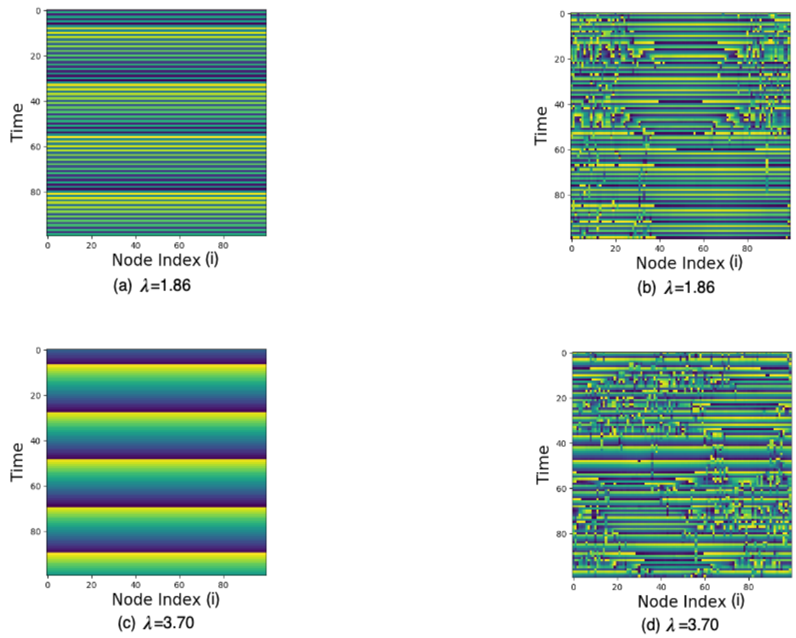


Figure 2.6: Spatio-temporal patterns of the phase plot show chimera state in the positive layer upon multiplexing with a negative layer. (a) and (c) is in a completely synchronized state for the multiplex network consisting of two positively coupled layers. (b) and (d) shows chimera state for the positive layer, which is multiplexed with a negative layer [6].

2.4 Conclusion

From the above studies discussed in the this chapter, we observed that chimera state only appears for some specific initial conditions. We also report for the range of coupling strength and different parameters for which chimera state is seen. Upon multiplexing with a negative layer, the coherent state observed in the positive layer for case(1) is destroyed. Hence chimera state is enhanced in the positive layer by multiplexing with a negative layer in the multiplex network.

Chapter 3

Chimera in Logistic map model

Modeling the real world complex systems into the multiplex network framework is one of the advancement in network science. With the help of such models we were able to understand the dynamics of the system. The mathematical model in this chapter we are going to study is based on the logic map. Population growth is studied using the logistic map. We studied the emergence of chimera states by considering regular network. We also investigate chimera in both single as well as in multiplex network. We also studied the range of coupling strength displaying chimera states for both the layers. We considered two cases, (i) a single layer network (ii) a multiplex network consisting of two structurally identical layers [3].

3.1 Theoretical model

Dynamics of the i^{th} node in the network is studied by using a parameter $z_t(i)$ at time t using the mathematical model discussed in the first chapter. It is mandatory to chose μ value in the chaotic region to observe chimera. In this case we assume μ value to be equal to 3.8 [3]. We choose the initial states $z_{t=0}(i)$ as a random number chosen such that it lies between 0 and Gaussian function given below,

$$\exp[\frac{-(i-\frac{N}{2})^2}{2\sigma^2}] \tag{3.1}$$

3.2 Network Architecture

3.2.1 Single layer network

First, we considered a single layer network with 100 non-locally coupled identical oscillators, and average degree k equals to 64 for each node. We developed a code in C++, to get $z_t(i)$ values after some time t for all the oscillators. The time evolution of each node is predicted from equation (1.5). Coupling strength (ε) values lies between 0 and 1. In this case N=100, i and j=1,2,...,100, m=1 and $\sum_{j=1}^{N} A_{ij} = 64$ [8]. The initial states $z_{t=0}(i)$ in a single layer network is plotted with the node index as shown in figure 3.1.

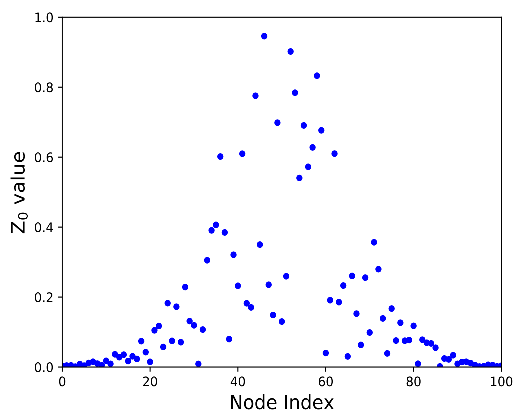


Figure 3.1: Initial states for a single layer network.

These 100 random variables $z_0(i)$ lies between 0 and 1, and these values are taken as the initial condition at time t=0 in the logistic map model to observe the evolution of each node after some transient time t=5000.

3.2.2 Multiplex network(Two layer)

Next, we consider a multiplex network consisting of two identical regular networks, which is a much more realistic representation of the real network model. Adjacency matrix for this two-layer multiplex network is

$$A = \begin{pmatrix} A^1 & I \\ I & A^2 \end{pmatrix}$$

Here A^1 and A^2 is the adjacency matrix for the first and second layer respectively. I is the identity matrix indicating a bi-directional connection between the two layers. We took 100 nodes for each layer. Average degree for each node equals to 64 in the intralayer edges and 1 in the inter layer edges. Each node has an equal number of neighbors in each direction in a layer. To better understand the time evolution of nodes with time, we considered two cases.

(1) First, we give same initial states $z_0(i)$ to the nodes for both the layers.

(2) Second, we considered different initial states to both layers.

Figure 3.2 represent the initial states $z_{t=0}(i)$ for i^{th} node in the network. In the figure 3.2 (a) initial states are the same for both the layers. But in the figure 3.2 (b) initial states for the nodes in the first layer are different from the nodes in the second layer. As mentioned above the initial condition is given and then coupling strength (ε) is varied between 0 and 1 for both the layers. For each coupling strength, the system has been evolved up to 15000 time steps, and at t=15000 the corresponding z_t values are observed. The plot of z_t values with the node index i shows chimera pattern in both the layers as shown in the result section.

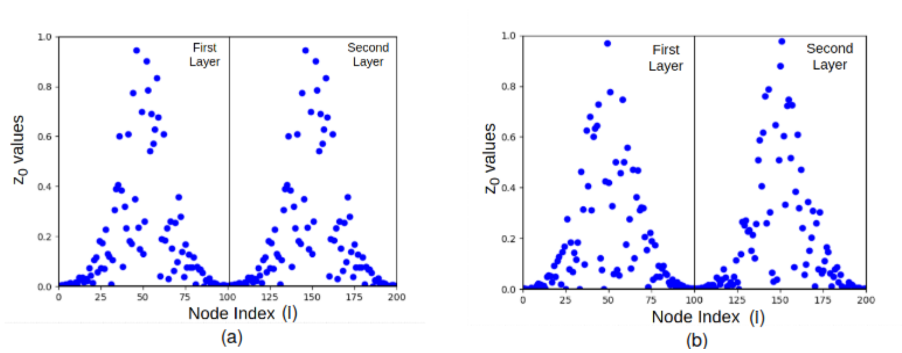


Figure 3.2: Initial states for the two layers in the Multiplex network.

3.3 Result

3.3.1 Single layer network

We observed the evolution of $z_t(i)$ after some transient for different values of coupling strength (ε). For small values of coupling strength nodes behave independently of each other. $z_t(i)$ values are scattered as shown in the figure 3.3 (a), which are not in a synchronized state. But for mid-range values of coupling strength (ε), some nodes evolve according to their neighboring nodes, and some are independent of their neighboring nodes. As a result, some nodes are coherent, and some are incoherent. So this gives us the chimera state (figure 3.3 (c) and (d)) For higher coupling strength values (ε) nodes are highly dependent on their neighboring nodes. Thus the nodes are synchronized (figure 3.3 (d)).

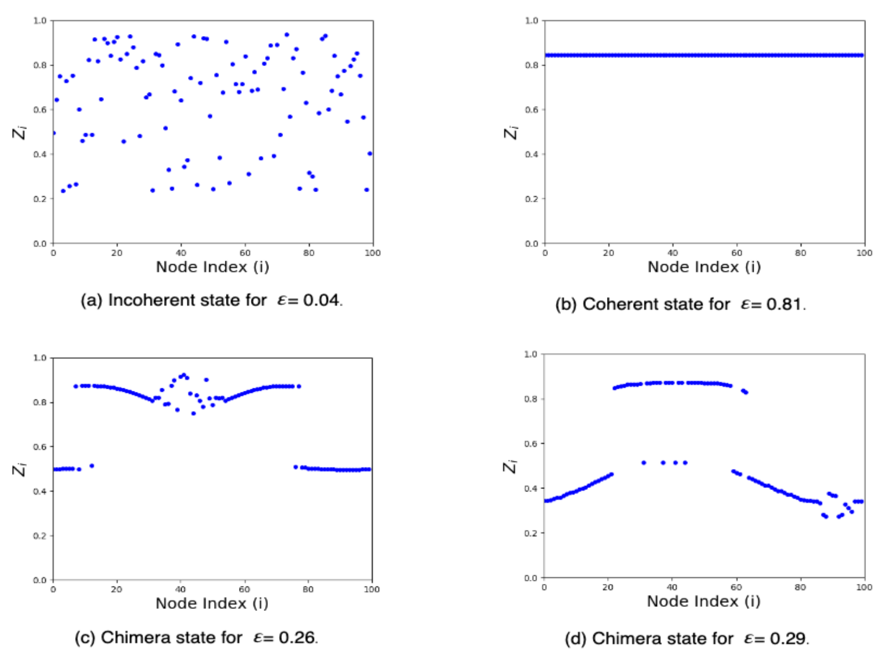


Figure 3.3: The figure illustrates different states for single layer network for different values of coupling strength (ε) [8].

3.3.2 Multiplex network

3.3.2.1 For same initial condition

In this case, we get symmetric plots for both the layers. The mathematical explanation for this behavior is as follows:
Since we give same initial state $z_t(i)$ to both the layers the term $[f(z_t(j)) - f(z_t(i))]$ in the equation(1.5) is same for both the layers. As a result both layers behave symmetrically (figure 3.4).

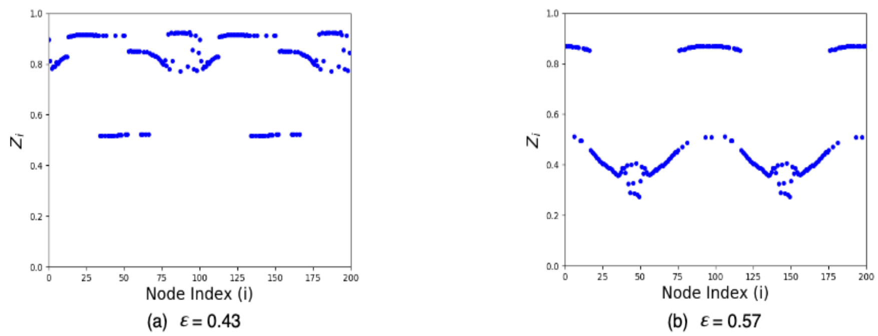
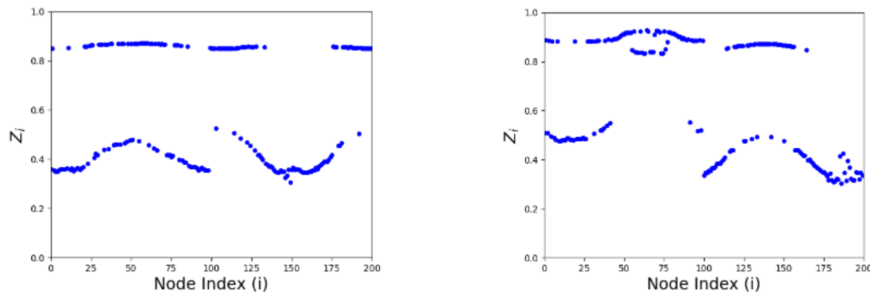


Figure 3.4: Chimera state for multiplex network for different values of coupling strength(ε) [1].

3.3.2.2 For different initial condition

For different initial states, the term $[f(z_t(j)) - f(z_t(i))]$ in the equation(1.5) is different for both the layers. As a result we get different values of $z_{t+1}(i)$ for each node in both the layers. Hence both layer behave differently and they are independent of each other (figure 3.5).



(a) $\varepsilon = 0.44$ (b) $\varepsilon = 0.50$

Figure 3.5: Chimera state for multiplex network for different values of coupling strength(ε) [3].

3.4 Conclusion

Based on the observations, we conclude that the chimera state is highly dependent on the initial condition. For both single layer and multiplex network, chimera pattern is observed for mid coupling strength(ε) values. For small values of coupling strength(ε), the system is in a desynchronized state, i.e., the nodes are independent of each other. But for high values of coupling strength(ε), the nodes are completely dependent on each other. As a result, the system is in a synchronized state.

Chapter 4

Designing chimera by introducing heterogeneous delay

Likewise, we discussed in the previous chapters, where we have ignored the effects of delay on the system. It has been proved that the oscillators behave differently with delay and without delay. Hence delay plays a crucial role in the dynamics of the system. It is defined as the time taken, reaching the information from one node to another. In real life, we have many examples of such a phenomenon. Consider the case of fans sitting in a football stadium. Even if everyone were successfully clapping in perfect synchrony, it would not sound that way to the fans themselves, as the applause coming from far across the field would be significantly delayed, because of the finite speed of sound [9]. Next, we are going to study the impact of delay on the given network. The investigations resulted in engineering single-cluster and the multi-cluster chimera state. Here, in this report, we approach the problem of managing the chimera states by introducing heterogeneous delay on the edges of a network. The presence of heterogeneous delays in a network is more realistic in the context of real-world networks where interactions (edges) between the pairs of nodes are subjected to heterogeneous perturbations from its surroundings [10].

4.1 Theoretical Model

After introducing delay, the dynamics of the network defined by the new modified mathematical model turns out to be

$$z_i(t+1) = f[z_i(t)] + \frac{\varepsilon}{\sum_{j=1} A_{ij}} \sum_{j=1}^{mN} A_{ij} \times \{f[z_j(t - D_{ij})] - f[z_i(t)]\} \tag{4.1}$$

Where ε is the coupling strength and varies between 0 and 1, m is the number of layers in the network, i and j are the indices of the oscillators. N is the total number of nodes in a layer. A is the adjacency matrix which gives us information about node connection in a network. Elements (D_{ij}) of the delay matrix D is the time delay given to the edges of the network.

$$D_{ij} = \begin{cases} \tau & if\ A_{ij}\ is\ delayed. \\ 0 & otherwise \end{cases}$$

The existing problems based on delay could be better solved by considering the heterogeneous delay in a model. So we took τ value from a random uniform distribution bounded by $0 \leq \tau \leq \tau_{max}$ and τ_{max}=10 [10]. The initial values of z for each node is a random variable lies between 0 and 1. We were able to reproduce successfully the results that are reported in the paper "Engineering chimera patterns in networks using heterogeneous delays."

4.2 Spatial inverse participation ratio (sIPR)

Till now, there has been numerous technique adopted to identify chimera. In this section, we are presenting one such technique known as spatial inverse participation ratio (sIPR) borrowed from the eigenvector localization concepts. sIPR defined as

$$sIPR = \frac{\sum_i (\langle d_i \rangle_t)^4}{[\sum_i (\langle d_i \rangle_t)^2]^2} \tag{4.2}$$

Where d_i is in laplace form and is a second order differentiation with respect to time. ($\langle d_i \rangle_t$) is the time average of d_i.

$$d_i = |(z_{i+1}(t) - z_i(t)) - (z_i(t) - z_{i-1}(t))| \tag{4.3}$$

d_i is interpreted as relative spatial distances between neighboring node. Using equation (4.3), we could compute the spatial distance of a node with respect to its neighboring nodes [10].

4.3 Result

Primarily it has been observed that those particular nodes attached to the delay edges evolve differently with that of the neighboring nodes without delay. These delay nodes with delayed edges pop out from the symmetric profile and break the symmetry of the system. Thus we were able to promote chimera and design the incoherent region in the system.

The various figures shown below reveals that heterogeneous delay added to the nodes leads to chimera state in the system. By appropriately placing delay to the desired nodes, we were able to engineer chimera state in a specific region.

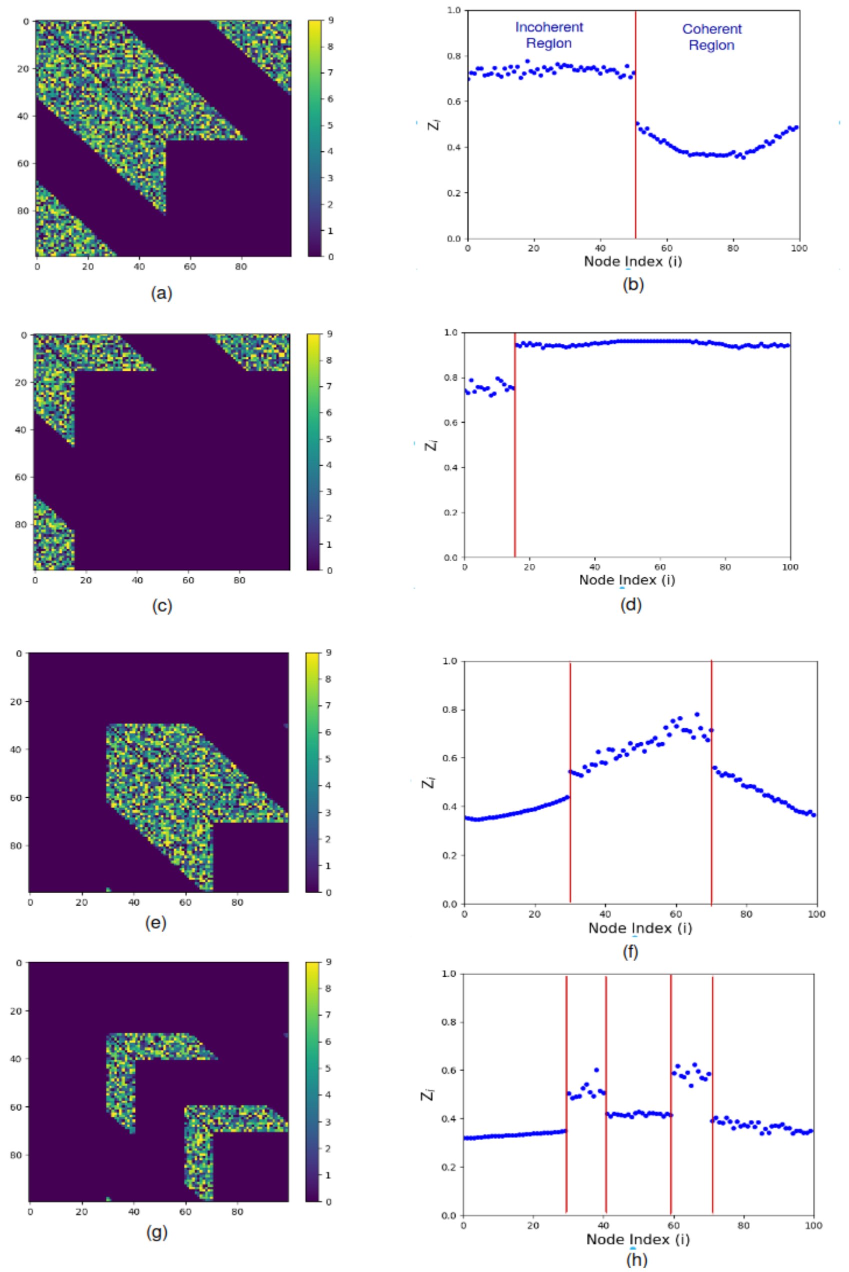


Figure 4.1: The set of figures illustrates the delay matrices and their corresponding chimera state observed after introducing heterogeneous delay. Parameters: Network size N= 100, node degree=k= 64, coupling strength= $\varepsilon = 0.77$ [10].

Figures 4.1 (a) and (c) are the Delay matrices representing a regular network with a large and relatively smaller cluster of the delayed nodes, respectively. The consequence of such delay configuration turns out to be with large ICR in (b) and smaller ICR in (d). The delay configuration in (e) elicits the in-between nodes are incoherent. Introducing two different clusters of delayed nodes separated by the undelayed nodes produces the delay matrix shown in figure (g). The outcome of such configuration displays two ICR in (h), and as a result, we obtain a multi-chimera state.

Figure 4.2 illustrates sIPR values obtained for different coupling strength values for an undelayed regular ring network. We get a clear understanding of the various state of the system as a function of coupling strength.

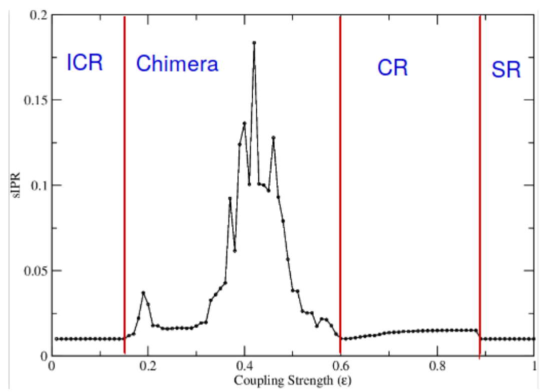
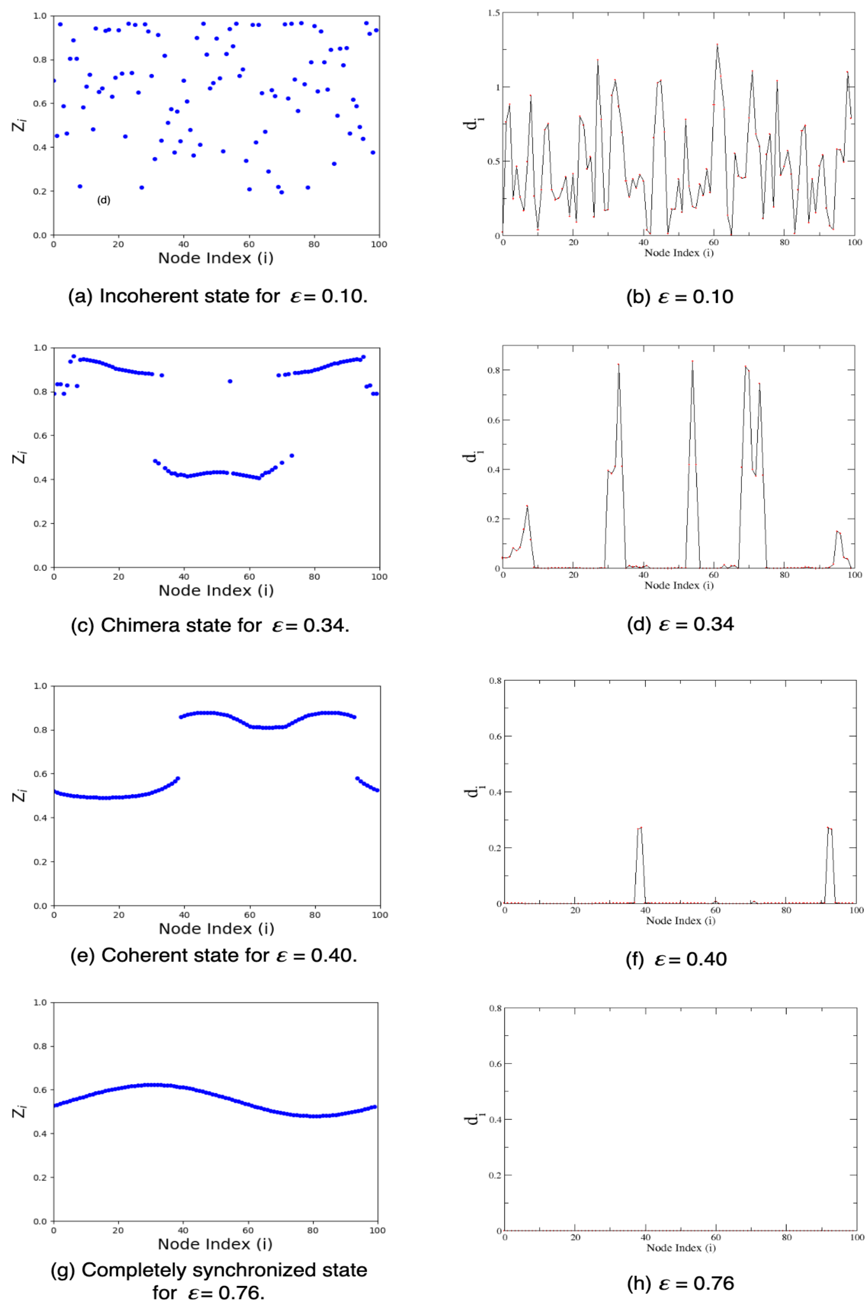


Figure 4.2: Coupling strength correlating the state of the system (i.e., the system can either be in a state of incoherent, chimera, coherent, or synchronized state) for an undelayed regular network [10].

For the coherent and incoherent state, d_i takes low and high value, respectively. Whereas in chimera state, d_i can take both high and low value. As a result, sIPR value is high for chimera state and low value for the coherent and incoherent state. The synchronous region in figure 4.2 shows a smooth spatial profile and the corresponding d_i values for this region equal to zero. sIPR value for this region cannot be defined. Hence we assume the sIPR value to be equals to $\frac{1}{N}$.



(a) Incoherent state for ε = 0.10.

(b) ε = 0.10

(c) Chimera state for ε = 0.34.

(d) ε = 0.34

(e) Coherent state for ε = 0.40.

(f) ε = 0.40

(g) Completely synchronized state for ε = 0.76.

(h) ε = 0.76

Figure 4.3: snapshots and Laplacian profiles of the dynamical states for the regular network corresponding to various points in the sIPR profile plotted in 4.2. [(a) and (b)] ε=0.10 and sIPR = 0.0100175, [(c) and (d)] ε=0.34 and sIPR = 0.0359414, [(e) and (f)] ε= 0.4 and sIPR = 0.136304, [(g) and (h)] ε=0.76 with sIPR = 0.0146935. Other parameters are network size (N) = 100, node degree=k=64 [10].

Figure 4.4 (b) depicts delay is given to all the nodes. As a result, this type of delay configuration shows a transition from a coherent state to incoherent state without showing any chimera state in the high coupling region. This different approach of introducing delays to all nodes destroys the chimera state.

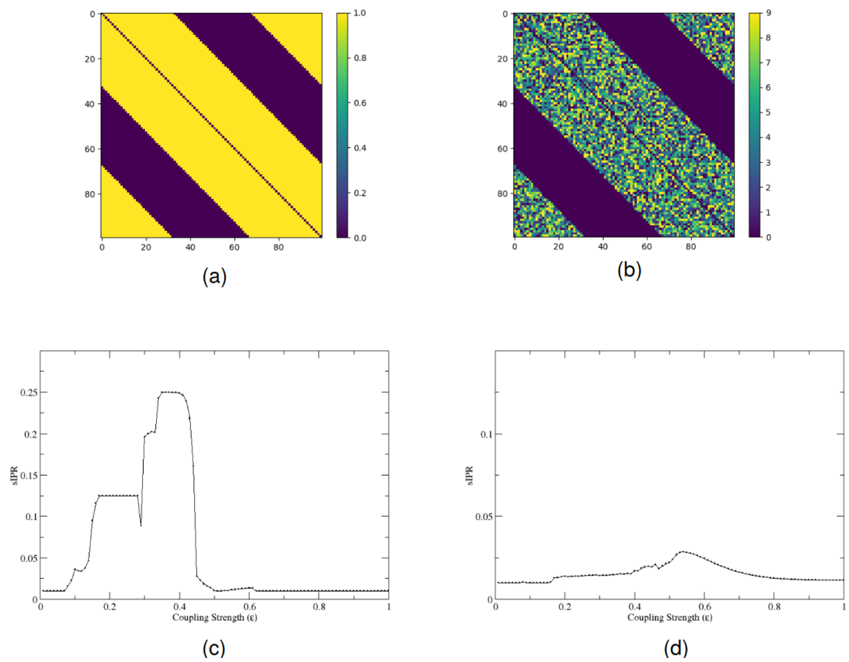


Figure 4.4: Delay matrices representing the heterogeneous delay induced in the network and corresponding sIPR profile for the delay configuration [10].

4.4 Conclusion

Based on our studies, we were able to engineer chimera state by inducing the ICR after suitably placing the heterogeneous delay to the set of nodes in a network. We also observed for low coupling strength values, the heterogeneous delay produces an incoherent state. In the mid coupling strength region, we noticed the occurrence of chimera state, but such delay configuration cannot control the location of the incoherent region. Whereas for high coupling strength values the location of the incoherent region coincides with the edges having the heterogeneous delays. By appropriately placing delays to the nodes, we were able to design both the single-cluster and the multi-cluster chimera state. The number of nodes with delays is also an important factor for the observation of the chimera state. We explored sIPR value could help us to identify the chimera state. But we cannot distinguish between the ICR and CR corresponding to their sIPR values.

Chapter 5

Summary

To summarize, we have observed the occurrence of chimera state for both the Kuramoto and logistic map model in case of single and multiplex networks. In chapter 2, we reported multiplexing with a repulsively coupled layer enhance chimera in the positively coupled layer. We also studied how does the state of a system vary with coupling strength and the other parameters like phase lag parameter. In chapter 3, we observed the emergence of a chimera state for both single and multiplex network. A specific initial condition is used to find chimera state. We also investigated how does coupling nature of the network affect the system to be in the coherent or in chimera state. Eventually, in chapter 4, we were able to design ICR for the high coupling strength region by introducing heterogeneous delays to specific nodes in a network. We also adopted a new technique known as sIPR to identify chimera state. By appropriately placing heterogeneous delays to the cluster of nodes enable us to engineer both single and the multi-cluster chimera state.

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