Study of Change in Neutrino Flux during its Journey through Dark Matter

M.Sc. Thesis

By

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Discipline of Physics

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A THESIS

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for the award of the degree of

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CANDIDATE'S DECLARATION

I hereby declare that the work presented in the thesis entitled **Study of Change in Neutrino Flux during its journey through Dark Matter** in the partial fulfillment of the requirements for the award of the Degree of **Master of Science** and submitted in the **Discipline of Physics, Indian Institute of Technology Indore** is an authentic record of my own work carried out during a period from July 2018 to June 2019 under the supervision of **Prof. Subhendu Rakshit**, IIT Indore. The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institute.

MEGHA JAIN

This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

Prof. Subhendu Rakshit

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MEGHA JAIN has successfully given her M.Sc. Oral Examination held on 21 June, 2019.

Prof. Subhendu Rakshit	**Convener, DPGC**
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Dr. Raghunath Sahoo	**Dr. Abhirup Datta**
Date:	**Date:**

Dedicated to my family

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ABSTRACT

Dark matter and neutrinos provide the two most compelling pieces of evidence for new physics beyond the Standard Model of particle physics, but they are often treated as two different sectors. In this thesis, dark matter is coupled to active neutrinos. Interaction of DM with neutrinos is very intriguing because it involves particles which provide the only evidence of new physics beyond the Standard Model (SM) so far.

Neutrinos are the best messengers from distant objects revealing a lot of information about various astrophysical phenomena. The purpose of this project is to study the change in the spectrum of neutrino as they travel from source to observer. We want to account for the effect of redshift from freestreaming as well as due to their interaction with particles, for example with dark matter particles, as they travel through very long distances i.e., redshift $z \geq 1$.

CONTENTS

List of Figures

Chapter 1

Introduction

This Chapter provides a brief description of the very abundant elementary particle "neutrino" and its discovery, its properties which make it a suitable candidate to study various astrophysical phenomena happening in the universe, its astrophysical sources and the different states in which it can exist. A very brief introduction to dark matter is also given at the end of this chapter.

1.1 Neutrino

Neutrinos are neutral, weakly interacting particles. They are the lightest known massive particles. Neutrinos are one of the most abundant particles in the universe. Because they have very little interaction with matter, however, they are incredibly difficult to detect.

In the standard model of particle physics, neutrinos are neutral leptons, represented by fermionic fields of left chirality, existing in isospin doublets with the corresponding charged leptons. Thus there are three flavours of neutrinos, one for each flavour of charged lepton namely, the electron neutrino (ν_e), the muon neutrino (ν_μ), and the tau neutrino (ν_τ). They carry a weak hypercharge (Y_W) of -1 and a weak isospin (T_3) of $+\frac{1}{2}$. Their electrical charge, given by the expression $Q = T_3 + \frac{Y_W}{2}$ is consequently zero and hence they do not take part in electromagnetic interactions. They carry no colour charge either and are hence uninvolved in strong interactions. They are affected by the weak force and can interact with other particles through processes mediated by the W+, W- and Z bosons. Each neutrino is assigned a lepton family number similar to the corresponding lepton. For example, the electron neutrino ν_e has an electron number of 1 (-1 for $\bar{\nu_e}$), the muon neutrino ν_μ has a muon number of 1 (-1 for $\bar{\nu_\mu}$) and so on. Under the standard model, neutrinos are assumed to be massless and the lepton family numbers are conserved exactly. However a series of experimental observations, beginning with an observation of a deficit in the number of electron neutrinos coming from the Sun with respect to the number predicted by the Standard Solar Model and culminating in a series of dedicated experiments searching for neutrino oscillations have confirmed that neutrinos oscillate from one flavour to another even in vacuum, something which would not be possible if they were massless. As a result, neutrinos are a frontier of Physics beyond the Standard Model.

1.2 Motivation for the present research work

The field related to neutrino particles is rich, diverse and developing rapidly. Neutrinos are by far the most abundant particles in the universe. About 100 trillion neutrinos pass through your body every second without interacting with any of the particles in your body. It is the feeble interaction of neutrinos with matter that makes them uniquely valuable as astronomical messengers. Unlike photons or charged particles, neutrinos can emerge from deep inside their sources and travel across the universe without interference. They are not deflected by interstellar magnetic fields and are not absorbed by intervening matter [figure 1.1]. The observation of astrophysical neutrinos makes it possible to probe the innermost regions of stars, dense regions from which light cannot escape. So, this makes neutrinos an interesting candidate to explore about what is happening in the far universe.

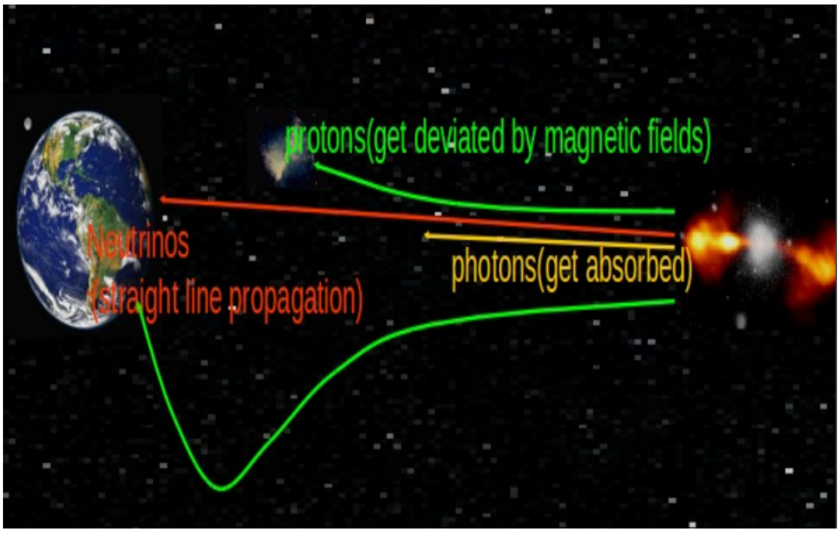

Figure 1.1: Neutrinos as the best messenger

1.3 Evidence for existence of Neutrinos

Wolfgang Pauli first postulated the existance of the neutrino in 1930. At that time, a problem arose because it seemed that both energy and angular momentum were not conserved in beta decay. But Pauli pointed out that if a non-interacting, neutral particle- a neutrino, were emitted, one could recover the conservation laws.

The first detection of neutrinos did not occur until 1955, when Clyde Cowan and Frederick Reines recorded anti neutrinos emitted by a nuclear reactor. Cowan and Reines used a set up consisting of two large tanks of water acting as a target material, allowing antineutrinos from a nearby nuclear reactor to interact with the protons within the water to produce a neutron and a positron. The resultant positron would quickly annihilate with an electron, producing two γ rays which were detected by sandwiching the water tanks between tanks of liquid scintillator. For added certainty, they also detected the γ ray produced by the absorption of the neutron in a layer of Cadmium Chloride. The detection of all three of these γ rays - the third delayed by 5 μs with respect to the others combined with the fact that such events were observed only when the reactor was turned on, demonstrated the existence of the neutrino.

1.4 Astrophysical Sources of Neutrinos

The neutrinos in the universe come from weak interactions (like beta decays in atomic nuclei). Three rivers can be distinguished: the neutrinos from space, the neutrinos from the earth, the neutrinos from mankind activity. Astrophysical sources of neutrinos mainly include:

- Supernovae - Supernovae are the most powerful cosmic sources of MeV neutrinos. When a massive star at the end of its life collapses to a neutron star, it radiates almost all of its binding energy in the form of neutrinos, most of which have energies in the range of few to tens of MeV. These neutrinos come in all flavors.

- Gamma Ray Bursts - Gamma-ray bursts (GRBs) are short-lived bursts of gamma-ray light, the most energetic form of light. It can last anywhere from a few milliseconds to several minutes, GRBs shine hundreds of times brighter than a typical supernova and about a million trillion times as bright as the Sun. When a GRB erupts, it emits cosmic gamma-ray photons and neutrinos at high energies in the Universe.

- Active Galactic Nuclei - An active galactic nucleus (AGN) is a compact region at the center of a galaxy with luminosity much higher than normal luminosity over at least some portion of the electromagnetic spectrum with characteristics indicating that the luminosity is not produced by stars. Such excess non-stellar emission has been observed in the radio, microwave, infrared, optical, ultra-violet, X-ray and gamma ray wavebands. AGNs are high energy neutrino sources.

1.5 Neutrino emission from Supernovae

Type II Supernovae explosions mark the end of the life of massive stars (with $M \geq M_o$) that have developed an iron core surrounded by several onion-like burning shells and an outer envelope of hydrogen and helium. Iron is the most strongly bound nucleus in nature, and no additional burning can generate energy to support the star core. A stellar iron core is supported by the electron degeneracy pressure, a quantum mechanical effect related to the Heisenberg uncertainty principle: $\Delta p \Delta x \simeq \hbar$, that gives momentum to fermions squeezed in a small volume. When enough nuclear ash has accumulated, and the iron core of the star reaches the Chandrasekhar limit of $\sim$ 1.4 solar masses, it becomes unstable, and collapses. The collapse is very rapid and lasts only a small fraction of a second. The compressed core heats, boiling the iron nuclei into separate nucleons, then it becomes energetically favorable to capture the electrons on free protons in the neutronization process,

$$e^{-1} + p \rightarrow n \rightarrow \bar{\nu}_e$$

that converts nearly all protons in the collapsing core into neutrons. The ν_e produced in these reaction rapidly escape from the core generating a neutronization burst of ν_e's. When the collapsing core reaches nuclear density (at a radius R $\sim$ 10 Km) the implosion is halted because of the stiffness of nuclear density matter. At this point a shock wave is formed that propagates outward ejecting the outer layers of the star and producing the spectacular visible explosion. The newly formed proto-neutron star has a radius R $\simeq$ 10 Km (and therefore the density is of the same order of nuclear matter), and contains a kinetic energy of order

$$E_{kin} = -E_{grav} = \frac{GM^2}{R} \simeq 3 \times 10^{53} ergs \qquad (1.5.1)$$

Nearly all (99 %) of the energy is radiated away in the form of neutrinos, with only $\sim 1\%$ going into producing the spectacular explosion as kinetic energy of the ejected layers and electromagnetic radiation. It is likely that the neutrinos emitted by the proto neutron star play a crucial role in the explosion, depositing enough energy near the outward propagating shock to push it out of the star, generating the explosion.

All six neutrino species contribute approximately equal to the energy outflow, since they are produced in the hot core by flavor blind processes like

$$\gamma\gamma \to \nu_\alpha \bar{\nu}_\alpha$$

The theory of neutrino emission in supernova explosions has had a dramatic confirmation the 23rd february 1987, when the neutrinos and the radiation of supernova (SN1987A) that had exploded 170,000 years before in the Large Magellanic Cloud (a small satellite galaxy of our MilkyWay) reached the Earth. Two detectors: Kamiokande in Japan and IMB in the US detected a few events (11 Kamiokande, 7 IMB) in coincidence with each other and in a time interval of 13 seconds. These events that can be interpreted as the detections of positrons from the reaction

$$\bar{\nu}_e + p = e^+ + n$$

From the number and energy spectrum of the observed events, it is possible to extract (with large statistical errors) a fluence and a temperature for the $\bar{\nu}_e$ emitted by the supernova with results in reasonable agreement with the theoretical predictions.

1.6 Neutrino States

The leptons, like all matter particles in the Standard Model, exhibit the peculiar feature that the states in which they undergo charged-current weak interactions, flavour states, are not mass eigenstates. Instead the three flavour states are a superposition, or mixture, of the three mass states. Experimentally we have the freedom to define the reference point from which the flavour states are measured. Conventionally in quarks, we define the flavour states in terms of the up-type quarks, such that all the mixing occurs in the down-type quarks. Similarly in neutrinos, the convention is to define the flavour states in terms of the charged leptons with all the mixing confined to the neutrinos. There is good reason for this choice: the fact that neutrinos interact only weakly and that the differences between their masses are tiny, make it impossible to measure which mass state has been created. One will sometimes find one or other of the flavour/mass states being described as the physical states (usually mass in quarks and flavour in neutrinos) but such a description is misleading. It is the measurement being made which determines which states are important, and any perception that one set is more real than another is simply an artefact of the view the experimenter has. It is natural to assume that like the other fermions each neutrino flavour ν_α has an anti-matter equivalent $\bar{\nu}_\alpha$, i.e. that they are Dirac particles. However because of their neutral charge it is possible that neutrinos could be Majorana particles. If this were to be the case then neutrinos would be their own anti-particles and they could have both Majorana and Dirac masses. There are three neutrino flavour states which are defined by the charged lepton they interact with at charged-current weak vertices: ν_e, ν_μ and ν_τ. There are also three mass states ν_1, ν_2, ν_3 with masses m_1, m_2 and m_3 respectively.

The mass and flavour states are related by a mixing matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{1.6.1}$$

This unitary matrix is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix. The expression can also be inverted to express the mass states, i, in terms of the flavour states, α, highlighting the fact that they are both mixtures of each other:

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_\alpha\rangle \tag{1.6.2}$$

Neutrino oscillations is a phenomenon which occurs as a result of the mixing of neutrino flavour and mass states discussed above. Experimentally it is the observation of a change in the flavour composition of neutrinos as they travel away from their source.

1.7 Dark Matter

The existence of dark matter is a problem beyond the scope of the standard model of particle physics. The motivation for existence of dark matter comes from mainly a few powerful astrophysical observations. The most famous evidence that suggests that there is non-luminous matter in our universe is the anomalous galactic rotation curves. To reproduce the curve, a large amount of mass is required other than that which is visible to us. At first it was thought that dark matter is some non-luminous astronomical object referred to as MACHOs (MAssive Compact Halo Objects). But two collaborations, MACHO and EROS looking into galactic halo by gravitational microlensing, concluded that MACHOs can not make up 100 % of the galactic halo. So the paradigm of dark matter shifted to heavy elementary particles. The current paradigm for explaining dark matter is WIMPs (Weakly Interacting Massive Particles).

Chapter 2

Effect of Neutrino-Dark Matter Interaction on Neutrino Flux

There are four kinds of fundamental interactions.The oldest known one is the gravitational interaction, known since the time of Newton in the 17th century. In the 19th century, electricity and magnetism were unified into the electromagnetic theory, and this is the second kind of interaction that we recognize to be fundamental. In the early part of the 20th century, with the discovery of the atomic nucleus, it was necessary to introduce two more kinds of fundamental interactions. One of them is the strong interaction, required to explain the stability of the atomic nuclei despite the fact that the protons in the nuclei exert repulsive Coulomb forces on one another. The other is the weak interaction, needed to explain the phenomenon of beta radioactivity, in which an electron or a positron comes out of a nucleus.

2.1 Weak Interaction

The weak interaction is responsible for radioactive decays. It is characterised by long lifetimes, and small cross sections. All fermions (spin $\frac{1}{2}$ particles) feel the weak interaction. Of special note are the neutrinos. Neutrinos feel only the weak interaction, which is what makes them so difficult to study. They are the only particles to experience just one of the fundamental forces.

Fermi was motivated by nuclear beta decays, which are processes in which a nucleus with atomic number Z transforms into another nucleus with atomic number (Z + 1), emitting an electron and an antineutrino. At the nucleon level, this implies the decay of a neutron into a proton, an electron and an antineutrino:

$$n \rightarrow p + e^{-1} + \bar{\nu} \tag{2.1.1}$$

Obviously, it involves four fermions. The process can be explained by an interaction where all four fermion fields interact at a point as can be seen in figure 2.1.

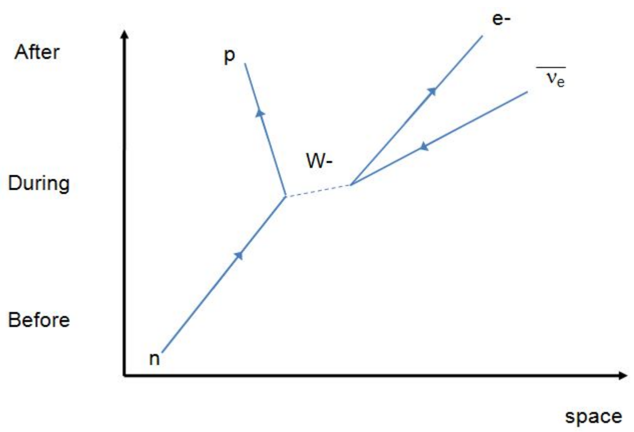

Figure 2.1: Fermi Diagram for beta decay

Here, it is assumed that the current between two fermions interacts with the current between the other two fermions: a current-current interaction. The interaction is written as,

$$\mathcal{L}_{int} = -G \left[\bar{\psi}_p \gamma_\mu \psi_n\right] \left[\bar{\psi}_e \gamma_\mu \psi_\nu\right] \tag{2.1.2}$$

where (n) denotes the neutron field, (p) the proton field, and so on, G is a constant.

2.2 Cross-section of neutrino-DM interaction

We consider the case of resonant interaction [1] described by the Lagrangian,

$$L \supset gN^{\dagger}\nu\phi \tag{2.2.1}$$

where ν is the neutrino hitting the scalar dark matter particle ϕ, N is the new fermion particle produced at resonance and g is the coupling constant between neutrino and dark matter particle. We have considered the interaction

$$\nu_i\phi \rightarrow \ N \rightarrow \nu_j\phi,$$

where i,j $\rightarrow$ 1,2,3 and ν_1, ν_2, ν_3 are the mass eigen states of neutrinos. Here, a neutrino of mass m_i hits a DM particle ϕ and resonantly produces N which further decays into another neutrino of mass m_j and dark matter particle ϕ.

We consider the process of neutrinos scattering off DM particles. If the incoming neutrino has an energy E_1, the energy of the recoiled neutrino is given by, [2]

$$\begin{aligned} E_3 = & \frac{E_1 + m_{DM}}{2}(1 + \frac{m_\nu^2 - m_{DM^2}}{s}) \\ & + \frac{\sqrt{E_1^2 - m_\nu^2}}{2}\left[\left(1 - \frac{(m_\nu + m_{DM})^2}{s}\right)\left(1 - \frac{(m_\nu - m_{DM})^2}{s}\right)\right] \end{aligned} \tag{2.2.2}$$

where θ is the scattering angle of the neutrino. The relevant Mandelstam variables are, [3]

$$\begin{aligned} s &= (p_1^\mu + p_2^\mu)^2 \\ &= m_\nu^2 + m_{DM}^2 + 2E_1 m_{DM} \\ &\sim m_{DM}^2 + 2E_1 m_{DM} \end{aligned} \tag{2.2.3}$$

$$
\begin{aligned}
t &= (p_1^\mu - p_3^\mu)^2 \\
&= 2m_\nu^2 + 2(E_1E_3 - p_1p_3cos\theta) \qquad (2.2.4)\\
&\sim 2m_\nu^2 + 2E_1E_3(1 - cos\theta)
\end{aligned}
$$

The energies of incoming neutrinos are such that, $E_1 \sim p_1$ holds well. The scattering angle θ in the centre-of-momentum frame can take all values between 0 to π, whereas that is the case in the laboratory frame only when $m_\nu < m_{DM}$.

The differential cross-section in the laboratory frame is given by, [2]

$$
\frac{d\sigma}{d\Omega} = \frac{p_3^2}{64\pi^2 m_{DM} p_1 p_3 (E_1 + m_{DM}) - p_1 E_3 cos\theta} \sum_{spin} \mid M \mid^2 \qquad (2.2.5)
$$

where, $d\Omega = sin\theta d\theta d\phi$.

2.3 Diffusion Equation in the Expanding Universe

The propagation of the extragalactic high-energy neutrino flux towards Earth, as they traverse the diffuse DM halo, can be described by the diffusion equation. For propagation of high-energy particles from a single source at a point $\mathbf{r}_g$, the diffusion equation reads, [4]

$$
\frac{\partial n_p(E,\mathbf{r},t)}{\partial t} - div[D(E,\mathbf{r},t)\nabla n_p] - \frac{\partial[b(E,\mathbf{r},t)n_p]}{\partial E} = Q(E,\mathbf{r},t)\delta^3(\mathbf{r}-\mathbf{r}_g) \qquad (2.3.1)
$$

where $n_p(E,\mathbf{r},t)$ is the space density of particles p with energy E at time t and at the point $\mathbf{r}$, $D(E,\mathbf{r},t)$ is the diffusion coefficient, $b(E,\mathbf{r},t) = -\frac{dE}{dt}$ describes the continuous energy losses, and Q(E,$\mathbf{r}$,t) is the source generation function.

We use the Friedmann-Robertson-Walker metric for the flat space and radial direction,

$$ds^2 = c^2dt^2 - a^2(t)d\mathbf{x}^2 = -g_{\mu\nu}dx^{\mu\nu} \tag{2.3.2}$$

where diag $g_{\mu\nu} = (-1, a^2, a^2, a^2)$ and diag $g^{\mu\nu} = (-1, 1/a^2, 1/a^2, 1/a^2)$, $\mathbf{x}$ is the spatial coordinate, corresponding to comoving distance, and a(t) is the scaling factor of the expanding universe, normalized as $a(t_0) = 1$ at the present age of the universe t_0. The redshift z is given by $1 + z = \frac{1}{a(t)}$, and $\frac{dt}{dz}$, by

$$\frac{dt}{dz} = -\frac{1}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Lambda}} \tag{2.3.3}$$

where H_0 is the Hubble parameter at z= 0 and Ω_m and Λ are the cosmological mass density and vacuum energy in units of the critical density. The physical and proper distances are locally determined as $\mathbf{dr} = a(t)\mathbf{dx}$. For the proper distance it is assumed that a(t) is not changed in the process of distance measurement between $\mathbf{x} = 0$ and $\mathbf{x}$, and thus the proper distance between these two coordinates is $\mathbf{r}_{prop} = a(t)\mathbf{x}$, and the velocity of the universe expansion is $u = \dot{a}x = H(t)\mathbf{r}_{prop}$.
For the physical distance it is assumed that measurement is performed with the help of the light signal $[ds^2 = c^2dt^2 = a^2(t)dx^2 = 0]$, and the distance between the object with redshift z and observer with z=0 is

$$\begin{aligned} r_{ph} &= \int_0^x a(t)dx = c\int_0^z dz \mid \frac{dt}{dz} \mid \\ &= c\int_0^z \frac{dz}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Lambda}} \end{aligned} \tag{2.3.4}$$

The local observer sees the particle flux density produced by diffusion,

$$j_k = -D\frac{\partial}{\partial x^k}n(\mathbf{x}, t) \tag{2.3.5}$$

In the case of isotropic diffusion (the diffusion coefficient is rotation invariant), j_k is the covariant space vector, which together with the proper space density of particles n forms the covariant 4-vector $j_\mu(\mathbf{x}, t) = (n, j_k)$ (the Latin indices run over 1-3, and the Greek ones over 0-3).

The conservation of current j^μ can be written as

$$\frac{\partial}{\partial x^\mu}(\sqrt{g} j^\mu) = 0 \tag{2.3.6}$$

where $g =\mid detg_{\mu\nu} \mid$ and $\sqrt{g} = a^3(t)$. Differentiating the current conservation equation using the definition of the Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$, then transforming the contravariant component j^k into the covariant one as $j_k = g^{km} j_m$, where diag $g^{km} = 1/a^2(t)$, and finally using the expression for particle flux density for

$$\frac{\partial}{\partial x^k} j^k = -D g^{ik} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^k} n(\mathbf{x}, t) \tag{2.3.7}$$

$$= -\frac{D}{a^2} \nabla_x^2 n(\mathbf{x}, t) \tag{2.3.8}$$

one obtains the diffusion equation,

$$\frac{\partial}{\partial t} n(\mathbf{x}, t) + 3H(t) n(\mathbf{x}, t) - \frac{D}{a^2} \nabla_x^2 n(\mathbf{x}, t) \tag{2.3.9}$$

where the 3H(t)n term describes expansion of the universe.

Operating here and everywhere below with a particle density n in the expanding volume using two sets of coordinates $(\mathbf{r}, t)$ and $(\mathbf{x}, t)$. This density is given by

$$n(\mathbf{r}, t) = n(\mathbf{x}, t) \tag{2.3.10}$$

where $\mathbf{r}(t) = a(t)\mathbf{x}$. Differentiating $n(\mathbf{r}, t)$,

$$\frac{d}{dt} n(\mathbf{r}, t) = \left(\frac{\partial n}{\partial t}\right)_r + \frac{\partial n}{\partial \mathbf{r}} \frac{d\mathbf{r}}{dt} \tag{2.3.11}$$

$$= \left(\frac{\partial n}{\partial t}\right)_r + H(t)\mathbf{r}\frac{\partial n}{\partial \mathbf{r}} \tag{2.3.12}$$

and using

$$\frac{dn(\mathbf{r},t)}{dt} = \frac{\partial n(\mathbf{x},t)}{\partial t} \tag{2.3.13}$$

one obtains

$$\left(\frac{\partial n}{\partial t}\right)_x = \left(\frac{\partial n}{\partial t}\right)_r + H(t)\mathbf{r}\nabla_r n \tag{2.3.14}$$

where subscripts x and r indicate bases $(\mathbf{x}, t)$ and $(\mathbf{r}, t)$ respectively. The diffusion equation is obtained from conservation of the number of particles. Consider a sphere of radius x, which expands in the basis $(\mathbf{r}, t)$ as $r(t) = a(t)x$. The number of particles inside this sphere is changing only due to diffusive flux, which is defined as $j = -D(\mathbf{r}, t)\nabla_r n(\mathbf{r}, t)$. The corresponding equation reads

$$\begin{aligned} \frac{d}{dt}\int_{V(t)} dV n(\mathbf{r},t) &= -\int_{S(t)} \mathbf{jds} \\ &= \int_{V(t)} dV div[D(\mathbf{r},t)\nabla_r n(\mathbf{r},t)] \end{aligned} \tag{2.3.15}$$

where S(t) is the expanding sphere and V(t) is the volume inside it.In above equation, the Gauss theorem was used.

Performing differentiation on the left-hand side of above equation with respect to time, taking into account expansion of the elemental volume dV with time,

$$\frac{d}{dt}\delta V = \frac{d}{dt}[a^3(t)(\delta V)_{comov}] = 3H(t)\delta V \tag{2.3.16}$$

and using equation for dn/dt, one obtains

$$\left(\frac{\partial n}{\partial t}\right)_r + 3H(t)\mathbf{r}\nabla_r n + 3H(t)n - div[D(r,t)\nabla_r n(\mathbf{r},t)] = 0 \tag{2.3.17}$$

This is the diffusion equation in the physical basis $(\mathbf{r}, t)$. One may add to the right-hand side the source term $Q_0\delta^3(\mathbf{r} - \mathbf{r}_g)$.

The sum of the second and third terms in the above equation merges into $\nabla_r(n\mathbf{u})$, where in our case $\mathbf{u} = H(t)\mathbf{r}$ is the expansion velocity of the universe.

In the basis $(\mathbf{x}, t)$ the first two terms in equation give $\left(\frac{\partial n}{\partial t}\right)_x$ and we arrive at the diffusion equation in the form

$$\left(\frac{\partial n}{\partial t}\right)_x + 3H(t)n(\mathbf{x}, t) - \frac{D}{a^2}\nabla_x^2 n(\mathbf{x}, t) = \frac{Q_0}{a^3(t)}\delta^3(\mathbf{x} - \mathbf{x}_g) \quad (2.3.18)$$

On introducing energy loss term as $-\frac{\partial}{\partial E}(nb)$ the diffusion equation for the expanding universe in the basis $(\mathbf{x}, t)$ becomes,

$$\frac{\partial n}{\partial t} - b(E,t)\frac{\partial n}{\partial E} + 3H(t)n - n\frac{\partial b}{\partial t}(E,t) - \frac{D(E,t)}{a^2(t)}\nabla_x^2 n = \frac{Q(E,t)}{a^3(t)}\delta^3(\mathbf{x} - \mathbf{x}_g) \quad (2.3.19)$$

where the total energy losses $\frac{dE}{dt} = -b(E,t)$ may be presented as a sum of collisional energy losses $b_{int}(E,t)$ and adiabatic energy losses $H(t)E$ and $Q(E,t)$ is the number of particles with energy E produced at time t per unit time.

2.4 Neutrino Oscillations

Neutrino oscillation is a phenomenon which arises from mixing between the flavor and mass eigenstates of neutrinos. That is, the three neutrino states that interact with the charged leptons in weak interactions are each a different superposition of the three (propagating) neutrino states of definite mass. Neutrinos are emitted and absorbed in weak processes in their flavor eigenstates but travel as mass eigenstates.

As a neutrino superposition propagates through space, the quantum mechanical phases of the three mass states advance at slightly different rates, due to the slight differences in their respective neutrino masses. This results in a changing superposition mixture of mass eigenstates as the neutrino travels; but a different mixture of mass eigenstates corresponds to a different mixture of flavor states. So a neutrino born as, say, an electron neutrino will be some mixture of e,

μ, and τ neutrino after traveling some distance. Since the quantum mechanical phase advances in a periodic fashion, after some distance the state will nearly return to the original mixture, and the neutrino will be again mostly electron neutrino. The electron flavor content of the neutrino will then continue to oscillate as long as the quantum mechanical state maintains coherence. Since mass differences between neutrino flavors are small in comparison with long coherence lengths for neutrino oscillations, this microscopic quantum effect becomes observable over macroscopic distances.

To explain the phenomenon of oscillations, one has to decompose a flavor eigenstate $|\nu_\alpha\rangle$ in the mass eigenstate basis. We suppose that there are n different types of neutrinos. The flavor eigenstate basis and the mass eigenstate basis are related by a unitary matrix U as,

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{n} U^*_{\alpha i} \, |\nu_i(t)\rangle \qquad (2.4.1)$$

where all the $|\nu_i\rangle$'s carry the same momentum p. Now, the energy eigen states evolve with time as,

$$|\nu_i(t)\rangle = e^{-\frac{iE_i t}{\hbar}} \, |\nu_i(0)\rangle \qquad (2.4.2)$$

with

$$E_i = \sqrt{p^2c^2 + m_i^2c^4} \qquad (2.4.3)$$

The probability of transition to the flavor state β is,

$$\begin{aligned} P_{\alpha\beta}(t) &= | \, \langle \nu_\alpha(t) | \nu_\beta \rangle \, |^2 \\ &= | \sum_{i=1}^{n} \sum_{j=1}^{n} U_{\alpha i} U^*_{\beta j} \, \langle \nu_i(t) | \nu_j(0) \rangle \, |^2 \\ &= | \sum_{i=1}^{n} U_{\alpha i} U^*_{\beta i} e^{-\frac{iE_i t}{\hbar}} \, |^2 \end{aligned}$$

The neutrinos being ultrarelativistic, one can expand the energy as,

$$\begin{aligned} E_i &= pc + \frac{m_i^2 c^3}{2p} \\ &= E + \frac{m_i^2 c^4}{2E} \end{aligned} \tag{2.4.4}$$

Finally, the probability of transition after a distance $L \simeq ct$ is,

$$P_{\alpha\beta}(L) = \sum_{i,j=1}^{n} U_{\alpha i} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j} e^{-\frac{i\Delta m_{ij}^2 c^3 L}{2E\hbar}} \tag{2.4.5}$$

with $\Delta m_{ij}^2 = m_i^2 - m_j^2$. From now on, $\hbar = c = 1$, we will be using the natural units. Now, consider a model with two neutrino flavors (e and μ), there is just one mixing angle θ. In this case the mixing matrix reduces to,

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.4.6}$$

and the oscillation probabilities are just,

$$\begin{aligned} P_{e\mu}(L) &= \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)^2 \\ &= \sin^2 2\theta \sin^2 \left(\frac{\pi L}{L_{osc}} \right) \end{aligned} \tag{2.4.7}$$

$$P_{ee}(L) = 1 - P_{e\mu}(L) \tag{2.4.8}$$

where L_{osc} is the usual oscillation length defined as $L_{osc} = \frac{4\pi E}{\Delta m_{21}^2}$
In the standard paradigm, Pontecorvo Maki Sakata Nakagawa (PMNS) matrix accounts for the mixing of the 3 neutrino flavors containing

3 mixing angles and 1 CP violation phase,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \tag{2.4.9}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ is the Dirac phase ($\theta_{ij} \in [0, \pi/2]$ and $\delta \in [0, 2\pi]$). One can distinguish two pictures: the normal hierarchy, with $m_1 < m_2 < m_3$ and the inverted hierarchy, with $m_3 < m_1 < m_2$.

Above section discusses about the neutrino oscillations in vaccuum. Similar neutrino oscillations are observed when they travel through a medium having matter.

2.5 Final Flux

We consider the case of resonant interaction [1] described by the Lagrangian,

$$L \supset gN^{\dagger}\nu\phi \tag{2.5.1}$$

where ν is the neutrino hitting the scalar dark matter particle ϕ, N is the new fermion particle produced at resonance and g is the coupling constant between neutrino and dark matter particle. We have considered the interaction

$$\nu_i\phi \rightarrow \ N \rightarrow \nu_j\phi,$$

where i,j $\rightarrow$ 1,2,3 and ν_1, ν_2, ν_3 are the mass eigen states of neutrinos. Here, a neutrino of mass m_i hits a DM particle ϕ and resonantly produces N which further decays into another neutrino of mass m_j and dark matter particle ϕ.

There are in general eight possibilities where N can be either of Dirac type or of Majorana type, ϕ can be a real or complex scalar and $m_\phi < m_N$ or $m_N < m_\phi$. We focus on the case with N being of pseudo-Dirac type and real ϕ playing the role of DM (ultralight) coupled dominantly to ν_τ i.e., $g_e = g_\mu = 0$ and $g_\tau \neq 0$. For simplicity, we only consider a single ϕ and a single N in this work. We take the incoming differential neutrino flux $\Phi(E)$ to be isotropic. This is not an assumption that all sources are the same. It is rather the statement that, in any given direction, the sum of contributions from neutrino sources along the line of sight is the same as from any other direction.

2.5.1 Neutrino Interactions with Dark Matter

High energy extragalactic neutrinos travel a long distance before reaching Earth, through the isotropic dark matter background. The number flux falls by a factor of e^{-1} over one mean free path at λ, and optical depth measures exactly the number of mean free paths in a given cosmological distance L, which the neutrino travels. In other words, optical depth ($\tau = \frac{L}{\lambda} = n\sigma L$) gives the number of interactions between neutrino and DM during its propagetion from source to observer.
So, in presence of neutrino-DM interaction, the flux of astrophysical neutrinos passing through isotropic DM background falls exponentially as $e^{-\frac{L}{\lambda}} = e^{(-n\sigma L)}$ [1, 2, 5, 6]. Here, L is the distance traversed by the neutrinos in the DM background, n denotes number density of DM particles given by, [1]

$$\begin{aligned} n(z) &= n_0(1+z)^3 \\ &= \frac{\Omega_{DM,0}\rho_c}{m_{DM}}(1+z)^3 \\ &= 1.26\left(\frac{keV}{m_{DM}}\right)(1+z)^3 cm^{-3} \end{aligned} \tag{2.5.2}$$

and σ represents the cross-section of neutrino-DM interaction given as, [1]

$$\sigma_{ij}(s) = \frac{g_i^2 g_j^2}{16\pi} \frac{(m_\Gamma^2 - m_{DM}^2)^2}{m_\Gamma^2 + m_{DM}^2} \frac{1}{(s - m_\Gamma^2)^2 + \Gamma_\Gamma^2 m_\Gamma^2} \tag{2.5.3}$$

where, m_Γ is the mass of particle produced at resonance (here, N), m_{DM} is the mass of dark matter particle, Γ_Γ is the decay width of N given by, [1]

$$\Gamma_\Gamma = \sum_i \frac{g_i^2}{16\pi} \frac{(m_\Gamma^2 - m_{DM}^2)^2}{m_\Gamma^3} \tag{2.5.4}$$

s is the Mandelstam variable given by, [1]

$$s = 2 m_{DM} E_\nu + m_{DM}^2 \tag{2.5.5}$$

For ultralight scalar DM, the dark matter number density is quite large. The cross-section depends on both the structure of the neutrino-DM interaction vertex and the DM mass. So, the neutrino-DM cross-section is significant for lower DM mass and larger coupling constant. Also, cosmological length of the path which the neutrino travels in the isotropic DM background is sufficiently large. So, the number of interactions given by $n\sigma L > O(1)$, so an appreciable flux suppression is produced. Thus, it is essentially the interplay of DM mass and the nature of neutrino-DM interaction that lead to a significant flux suppression.

2.5.2 Cross Section

The cross-section for neutrino-DM scattering through a fermionic mediator (equation 2.4.3) in case of ultralight scalar DM can be approximated as,

$$\sigma_{ij} \sim \frac{g_i^2 g_j^2}{16\pi m_\Gamma^2} \frac{1}{\left(1 - \frac{2 m_{DM} E_\nu}{m_\Gamma^2}\right)^2} \tag{2.5.6}$$

where m_ν, E_ν are the mass and energy of the incoming neutrino respectively, m_{DM} is the mass of the ultralight DM, and m_Γ is the mass of the heavy fermionic mediator.

At lower neutrino energies, the cross section reduces to,

$$\sigma_{ij} \sim \frac{g_i^2 g_j^2}{16\pi m_\Gamma^4}(m_\Gamma^2 + 4m_{DM}E_\nu) \tag{2.5.7}$$

As the energy increases, the $m_{DM}E_\nu$ term becomes more dominant and eventually, the cross-section increases with energy.

At higher neutino energies, the cross section becomes,

$$\sigma_{ij} \sim \frac{g_i^2 g_j^2 m_\Gamma^2}{64\pi m_{DM}^2}\frac{1}{E_\nu^2} \tag{2.5.8}$$

So, as the energy increases, the cross section decreases.

At resonance energy, $E_{res} = \frac{m_\Gamma^2}{2m_{DM}}$, the cross section for ν-DM scattering takes the form,

$$\sigma_{ij} \sim \frac{g_i^2 g_j^2}{16\pi m_\Gamma^2}\delta(1 - \frac{E}{E_{res}}) \tag{2.5.9}$$

So, at resonance energy, the cross section for the process $\nu_i\phi \to N \to \nu_j\phi$ attains its maximum value. This implies that maximum scattering of neutrinos occurs at the resonance energy which leads to a sharp fall in their number.

2.5.3 At energies lower than the resonance energy

In this thesis, we have focused on the case where dark matter (ϕ) is coupled dominantly to ν_τ. Since, in Normal Hierarchy, ν_3 neutrino mass eigenstate is heavier than both the ν_1 and ν_2 mass eigenstates i.e., $m_1 \sim m_2 \ll m_3$, therefore $\Delta m_{31}^2 \sim \Delta m_{32}^2 > 0$. Since the squared-mass difference for the processes, $\nu_3\phi \to N \to \nu_1\phi$ and $\nu_3\phi \to N \to \nu_2\phi$ is positive therefore, ν_τ will oscillate to ν_e and ν_μ flavor states. But the reverse processes are not possible as Δm_{13}^2

and Δm^2_{23} both are negative. The ν_τ's changes their state to ν_e's and ν_μ's as they propagate through dark matter. As a result, there is a decrease in the number of ν_τ's. So, at energies lower than the resonance energy, final flux shows a deviation from the initial flux which shows the decrease in number flux of neutrinos as they travel from source to the observer.

Also since the cross section for neutrino-DM interaction increases continuously for $E < E_{res}$, therefore, larger number of neutrinos are scatterd which leads to a decrease in their number flux.

2.5.4 Dip at the resonance energy

At resonance energy, the cross section for neutrino-DM interaction attains its maximum value. This implies that maximum scattering of neutrinos occurs at E_{res} which leads to a sharp fall in their number. So a dip in the final flux is observed at E_{res}. It is therefore the nonvanishing DM-neutrino interaction that is responsible for producing dips in the diffuse supernova neutrino background caused by their scattering off DM particles.

Chapter 3

Results and Discussion

This chapter provides information about the results obtained during the period of research. At the end of the chapter, discussion regarding the results which could not be reproduced in its exact form, in the given time period, is also given.

3.1 Change in neutrino flux during its journey through Dark Matter

There are many astrophysical sources of neutrinos like Gamma Ray Bursts, Active Galactic Nuclei and Supernovae, which are the most powerful sources of neutrinos When a massive star is terminated by the collapse of its core to a neutron star then neutrinos and antineutrinos of all flavours are emitted.These particles may be

- absorbed in the surroundings of stellar medium.
- transported further in the universe and suffer energy depletion due to redshift.
- further during transport may be scattered due to their interaction with other particles as they travel from the source to Earth.

In this project, we only want to see the change in the spectrum of neutrino as they travel from source to observer.We want to account for the effect of redshift from freestreaming as well as due to their interaction with particles, for example with dark matter particles, as they travel through very long distances i.e. redshift z $\geq$ 1.

3.1.1 Effects Of Interaction

We consider the case of resonant interaction [1] described by the Lagrangian,

$$LgN^{\dagger}\nu\phi,$$

where, ν is the neutrino hitting the scalar dark matter particle ϕ, N is the new fermion particle produced at resonance, g is the coupling constant between neutrino and dark matter particle. We have considered the interaction

$$\nu_i\phi \rightarrow \; N \rightarrow \nu_j\phi,$$

where i,j $\rightarrow$ 1,2,3, ν_1, ν_2, ν_3 are the mass eigen states of neutrinos. Here, a neutrino of mass m_i hits a DM particle and resonantly produces N which further decays into another neutrino of mass m_j and dark matter particle . We want to see the effect of this interaction on the neutrino flux as they pass through dark matter. The weak interactions of neutrinos make them unique messengers for studying astrophysical systems. ICE CUBE has seen various ultra high energy neutrinos, whose flux can be very different from the source. We want to see the effects of such interactions on neutrino [7]. At lower energies, the transport as well as the neutrino oscillations accompanied by interactions can also change the neutrino flavour ratio [1].

3.2 Differential Cross Section for Neutrino-DM interaction

Starting from the equation,

$$\frac{d\sigma}{dt} = \frac{\pi}{pp'}\frac{d\sigma}{d\Omega_{CM}} \tag{3.2.1}$$

where p,p' are the magnitudes of 3-momentum of neutrinos in incoming and outgoing states in the lab frame, t is mandelstam variable given by [3]

$$dt = 2m_{DM}dE' \tag{3.2.2}$$

m_{DM} is mass of dark matter particle ϕ (here we take $m_{DM} = 1$MeV) and E' is the outgoing neutrino energy in lab frame, $d\Omega_{CM} = 2\pi dcos\theta_{CM}$ is the solid angle in center of mass frame and θ_{CM} is the angle between incoming and outgoing neutrino in center of mass frame. We got the relation between differential cross section in lab frame and the differential cross section in center of mass frame as

$$\frac{d\sigma_{ij}}{dE'} = \frac{d\sigma_{ij}}{dcos\theta}\frac{m_{DM} + 2E_\nu}{(E_\nu)^2}, \tag{3.2.3}$$

Here, $\frac{d\sigma_{ij}}{dE'}$ is the differential cross section in lab frame, $\frac{d\sigma_{ij}}{dcos\theta}$ is the differential cross section in center of mass frame. Since we are taking $m_\nu \approx 0MeV$, so $\theta_{CM} = \theta_{LAB}$ (here we have taken $\theta_{LAB} = \theta$). The differential cross section is given by, [1]

$$\frac{d\sigma_{ij}}{dcos\theta} = \frac{g_i^2 g_j^2}{32\pi}\frac{(m_\Gamma^2 - m_{DM}^2)^2}{(m_\Gamma^2 + m_{DM}^2)}\frac{1 + cos\theta}{((s - m_\Gamma^2)^2 + \Gamma_\Gamma^2 m_\Gamma^2)} \tag{3.2.4}$$

where, θ is the angle between incoming and outgoing neutrino in lab frame (as $\theta_{CM} = \theta_{LAB}$), s is the Mandelstam variable given by, [1]

$$s = 2m_{DM}E_\nu + m_{DM} \tag{3.2.5}$$

E_ν is the energy of neutrino at interaction, Γ_Γ is the decay width for the particle N decaying by the process $N \to \nu_j\phi$ given by, [1]

$$\Gamma_\Gamma = \sum_{i=1}^{3}(g_i)^2\frac{(m_\Gamma^2 - m_{DM}^2)^2}{16\pi m_\Gamma^3} \tag{3.2.6}$$

g_i(or g_j) is coupling constant in mass basis (here $g_1 = 0.04025$, $g_2 = 0.059250$, $g_3 = 0.06845$) from neutrino oscillations, m_Γ is the

mass of the particle produced at resonance i.e. m_N(here we take $m_\Gamma = 6.5$ MeV).
We see from figure 3.1 that $\dfrac{d\sigma_{ij}}{dcos\theta}$ increases as cosθ increases. Since, $\dfrac{d\sigma_{ij}}{dcos\theta}$ tells us the probability of interaction, so it can be concluded that the interaction between ν and ϕ is more favourable at higher values of cosθ i.e. at lower scattering angles.

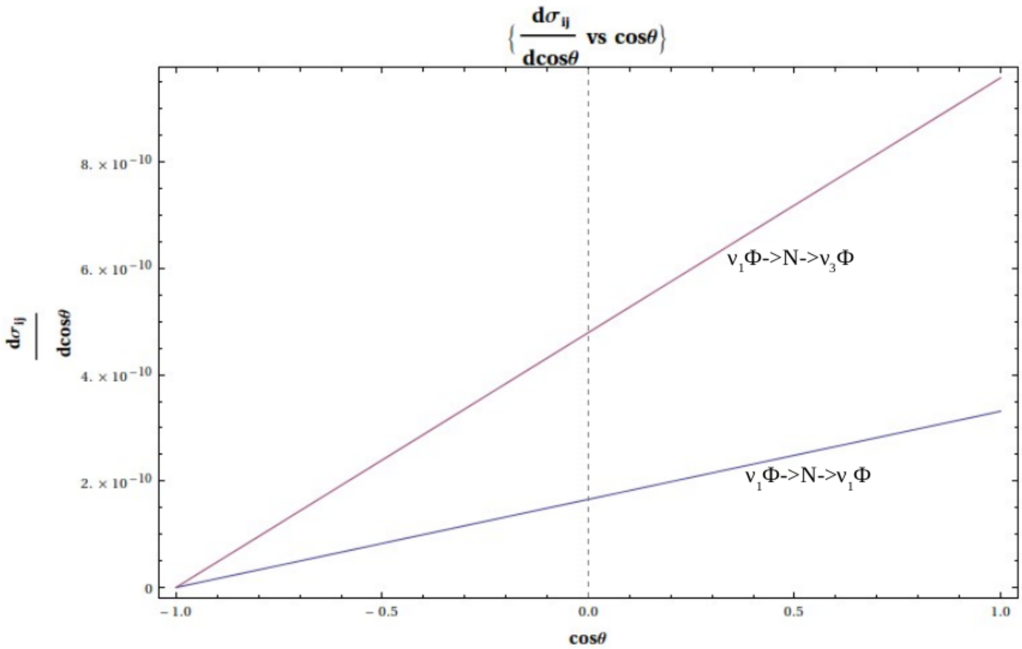

Figure 3.1: Plot of Differential Cross-Section v/s Energy

Here, ν_1, ν_2, ν_3 are the mass eigen states of neutrinos. $\nu_1\phi \to N \to \nu_3\phi$ implies that neutrino ν_1 on hitting the dark matter particle produces N at resonance which decays to neutrino ν_3 and dark matter particle. $\nu_1\phi \to N \to \nu_1\phi$ implies that neutrino ν_1 on hitting the dark matter particle produces N at resonance which decays to neutrino ν_1 and dark matter particle.

3.3 Cross Section for Neutrino-DM interaction

If neutrino of mass m_i interacts with a dark matter particle ϕ of mass m_{DM} and produces another particle N at resonance, which further decays into another neutrino of mass m_j and dark matter particle, i.e.

$$\nu_i\phi \rightarrow \ N \rightarrow \nu_j\phi,$$

then close to resonance,the total cross section is given by, [1]

$$\sigma_{ij} = \frac{g_i^2 g_j^2}{16\pi} \frac{(m_\Gamma^2 - m_{DM}^2)^2}{(m_\Gamma^2 + m_{DM}^2)((s - m_\Gamma^2)^2 + \Gamma_\Gamma^2 m_\Gamma^2)} \tag{3.3.1}$$

Here, s is the Mandelstam variable, E_ν is the energy of neutrino at interaction, Γ_Γ is the decay width for the particle N decaying by the process $N \rightarrow \nu_j\phi$, g_i (or g_j) is coupling constant in the neutrino mass basis.

As we can see from figure 3.2 that the cross section of the above process peaks at 20.625 MeV. Also, the peak is at same value of energy i.e. resonance energy is same for all interactions (since N is same).

$$\nu_i\phi \rightarrow \ N \rightarrow \nu_j\phi,$$

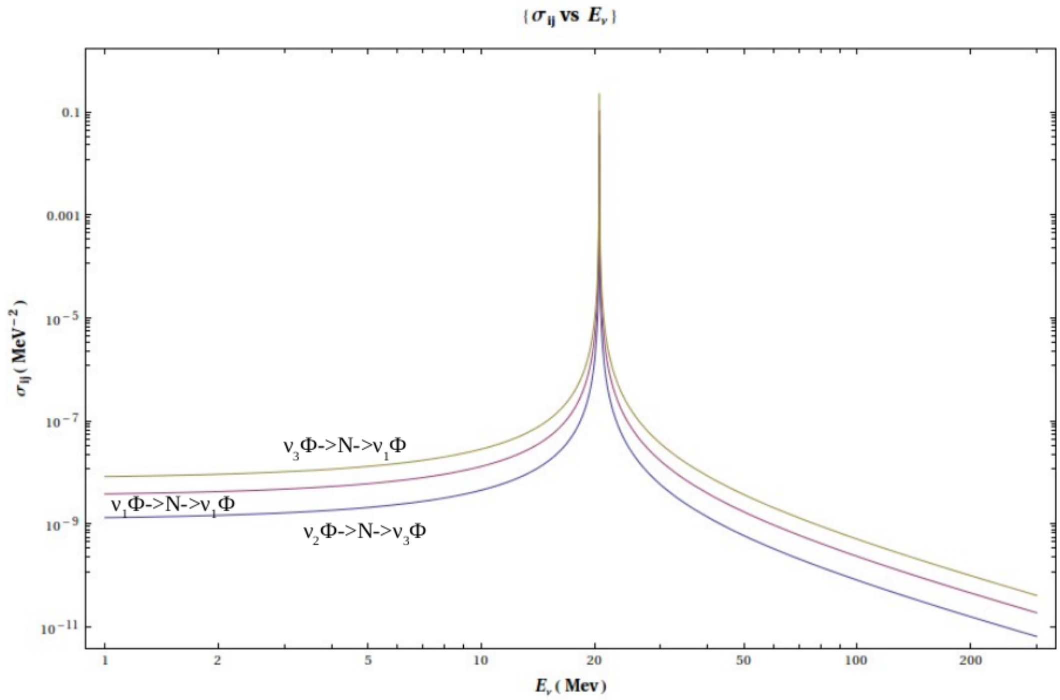

Figure 3.2: Plot of Cross-section v/s Energy

3.4 Neutrino Flux Change in Neutrino-Dark Matter Scattering

The time evolution of flux is governed by, [1]

$$\frac{\partial F_i(t,E_\nu)}{\partial t} = -3H(t)F_i(t,E_\nu) + \frac{\partial(H(t)E_\nu F_i(t,E_\nu))}{\partial E_\nu} - \frac{F_i(t,E_\nu)}{\lambda_i(t,E_\nu)} + \frac{L_i(t,E_\nu)}{a(t)^3} \sum_{j=1}^{3}\int_{E'_\nu}^{\infty} dE'_\nu [T_{ji}(t,E'_\nu,E_\nu)F_j(t,E'_\nu)] \quad (3.4.1)$$

where [6], $F_i(t,E_\nu)$ represents the flux of neutrinos of mass m_i, $H(t)$ represents the Hubble parameter and is given as a function of red-shift z as, [1]

$$H(z) = H_o\sqrt{\Omega_\lambda + \Omega_{m0}(1+z)^3} \quad (3.4.2)$$

Ω_λ=0.685, $\Omega_{m,0}$=0.315 and $H_0 = 2.181 * 10^{-18}s^{-1}$, $\lambda_i(t,E_\nu)$ represents is the mean free path of neutrinos of mass m_i given by, [1]

$$\lambda_i(t,E_\nu) = \frac{1}{\sum_{j=1}^{3} n(t)\sigma_{ij}(E_\nu)} \quad (3.4.3)$$

$n(t)$ is the number density of dark matter particles at time t given by, [1]

$$\begin{aligned} n(z) &= n_0(1+z)^3 \\ &= \frac{\Omega_{DM,0}\rho_c}{m_{DM}}(1+z)^3 \\ &= 1.26\left(\frac{keV}{m_{DM}}\right)(1+z)^3 cm^{-3} \end{aligned} \quad (3.4.4)$$

$\sigma_{ij}(E_\nu)$ is the cross section for the process $\nu_i\phi \to \ N \to \nu_j\phi$ given as, [1]

$$\sigma_{ij}(s) = \frac{g_i^2 g_j^2}{16\pi}\frac{(m_\Gamma^2 - m_{DM}^2)^2}{m_\Gamma^2 + m_{DM}^2}\frac{1}{(s-m_\Gamma^2)^2 + \Gamma_\Gamma^2 m_\Gamma^2} \quad (3.4.5)$$

E_ν is the energy of incoming neutrino ν_i, $L_i(t,E_\nu)$ represents the luminosity of the source of neutrinos of mass m_i, a(t) is the scale factor of universe related to redshift by $a(t) = \frac{1}{(1+z)}$, term responsible for redistribution of flux to lower energy is given by, [1]

$$T_{ji}(t, E'_\nu, E_\nu) = n(t)\frac{d\sigma_{ji}(E'_\nu, E_\nu)}{dE_\nu} \tag{3.4.6}$$

where, E'_ν is the energy of incoming neutrino and E_ν is the energy of outgoing neutrino.

1^{st} and 2^{nd} terms originate due to the energy losses which occur due to the expansion of the Universe. 3^{rd} term arises due to interaction of neutrino and accounts for the attenuation of incoming neutrino at a given energy due to scattering. 4^{th} term accounts for particle regeneration from one energy to another, including when an incoming particle of energy E'_ν is down-scattered to a lower energy E_ν but not lost and when a target particle with their rest mass energy is upscattered to higher energy. 5^{th} term represents the comoving luminosity of the sources of neutrinos of mass m_i.

The solution to above equation is given by

$$F_i(t, E_\nu) = (1+z)^2 \int_z^{z_{max}} dz' \frac{1}{H(z')} e^{-\int_z^{z_{max}} dz'' \frac{1}{(1+z'')H(z'')\lambda_i(z'', E''_z)}} \left\{ L_i(z', E'_z) + \sum_{j=1}^{3} \int_{E'_z}^{\infty} dE'_{z'} T_{ji}(z', E'_{z'}, E_{z'}) F_j(z', E'_{z'}) \right\} \tag{3.4.7}$$

Neutrinos with energies in the range of tens of MeV are produced in large quantities after the explosions of core-collapse supernovae (SN) of type II, Ib or Ic. The above equation governs the transport of neutrinos through dark matter cloud, taking into account the redshifting of energies of neutrino.

3.4.1 Initial Flux

We start with the SN flux without the new interactions. The diffuse SN flux is a remnant of neutrinos emitted from all the supernova that have occurred in the Universe. In the absence of neutrino absorption and without taking into account oscillations, the initial DSNB flux is formulated as, [1]

$$F_i(t, E_\nu) = (1+z)^2 \int_z^{z_{max}} dz' \frac{L_i(z', E'_z)}{H(z')} \tag{3.4.8}$$

From figure 3.3, we can see that the initial flux falls continuously with energy.

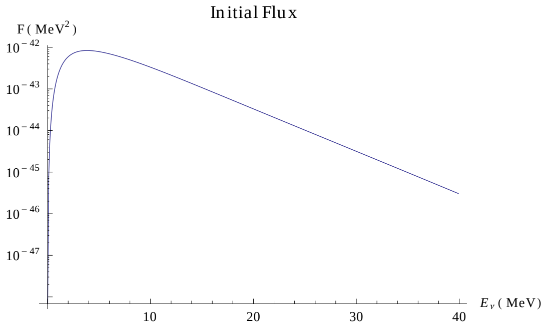

Figure 3.3: Plot of Initial Flux v/s Energy

3.4.2 Attenuation Factor

To study the variation of attenuation factor with energy and redshift, we have used [1]

$$(1+z)^2 \int_z^{z_{max}} dz' \frac{L_i(z', E'_z)}{H(z')} e^{-\int_z^{z_{max}} dz'' \frac{1}{(1+z'')H(z'')\lambda_i(z'', E''_z)}} \tag{3.4.9}$$

Here, $e^{-\int_z^{z_{max}} dz'' \frac{1}{(1+z'')H(z'')\lambda_i(z'', E''_z)}}$ represents the attenuation factor, H(t) represents the Hubble parameter and is given as a function of redshift z as $H(z) = H_0\sqrt{(\Omega_\lambda + \Omega_{m,0}(1+z)^3)}$ where, $\Omega_\lambda = 0.685$, $\Omega_{m,0} = 0.315$ and $H_0 = 2.181 * 10^{-18} s^{-1}$, $\lambda_i(t, E_\nu)$ represents is the mean-free path of neutrinos of mass m_i,

$$\lambda_i(t, E_\nu) = \frac{1}{\sum_{j=1}^{3} n(t)\sigma_{ij}(E_\nu)} \tag{3.4.10}$$

where, n(t) is the number density of dark matter particles at time t, $\sigma_{ij}(E_\nu)$ is the cross section for the process $\nu_i\phi \rightarrow\ N \rightarrow \nu_j\phi$ and

E_ν is the energy of incoming neutrino ν_i. We have taken z = 0 and $z_{max} = 6$. From figure 3.4, we can see that attenuation factor experiences a dip between energy values 10 MeV-16 MeV at z = 0. The dip broadens as the value of redshift increases. Attenuation factor takes care of the attenuation of neutrino flux as it propagates through the dark matter.

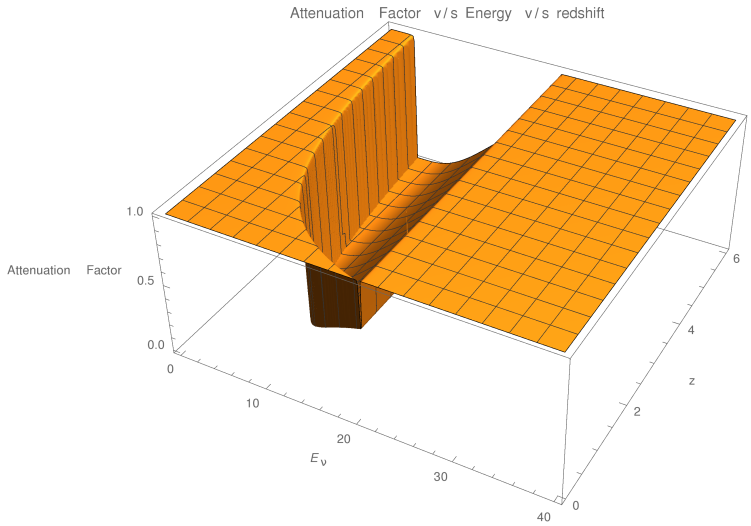

Figure 3.4: Plot of Attenuation factor v/s Energy v/s Redshift

3.4.3 Flux(w/o regeneration)

To study the effect of attenuation factor on the neutrino flux as it propagates through the dark matter, we have used [1]

$$F_i(t, E_\nu) = (1+z)^2 \int_z^{z_{max}} dz' \frac{L_i(z', E'_z)}{H(z')} e^{-\int_z^{z_{max}} dz'' \frac{1}{(1+z'')H(z'')\lambda_i(z'', E''_z)}} \tag{3.4.11}$$

where, $F_i(t, E_\nu)$ represents the attenuated flux, $e^{-\int_z^{z_{max}} dz'' \frac{1}{(1+z'')H(z'')\lambda_i(z'', E''_z)}}$ represents the attenuation factor, H(t) represents the Hubble parameter and is given as a function of redshift z as $H(z) = H_0\sqrt{(\Omega_\lambda + \Omega_{m,0}(1+z)^3)}$ where, $\Omega_\lambda = 0.685$, $\Omega_{m,0} = 0.315$ and $H_0 = 2.181 * 10^{-18} s^{-1}$, $\lambda_i(t, E_\nu)$ represents is the mean-free path of neutrinos of mass m_i given by

$$\lambda_i(t, E_\nu) = \frac{1}{\sum_{j=1}^{3} n(t)\sigma_{ij}(E_\nu)} \tag{3.4.12}$$

Here, n(t) is the number density of dark matter particles at time t, E_ν is the energy of incoming neutrino ν_i, $\sigma_{ij}(E_\nu)$ is the cross section for the process $\nu_i\phi \rightarrow\ N \rightarrow \nu_j\phi$.

The attenuation factor will be multiplied by $\dfrac{L_i(z', E'_z)(1+z)^2}{H(z')}$ to give the final attenuated flux. So, this term takes care of dark matter-neutrino interaction while the neutrino is propagated. After propagation, the final flux of neutrinos can be seen in figure 3.5, when the effect of redshift is not considered. Here, we have considered attenuation only and no regeneration term, so the final flux of neutrinos shows a dip at resonance energy= 20.625 MeV and then falls continuously with energy.

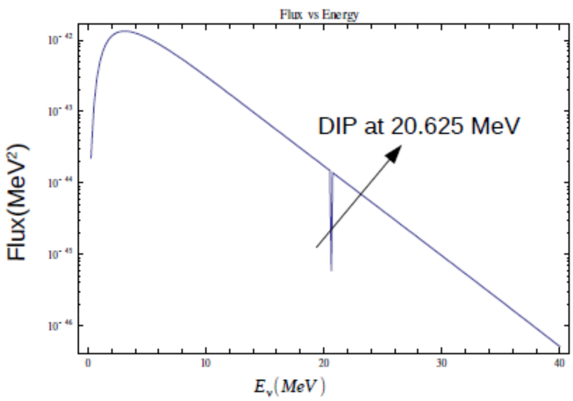

Figure 3.5: Plot of Attenuated Flux v/s Energy (without redshift)

From figure 3.6, we can see that the dip has broadened because of the redshift taken into account.

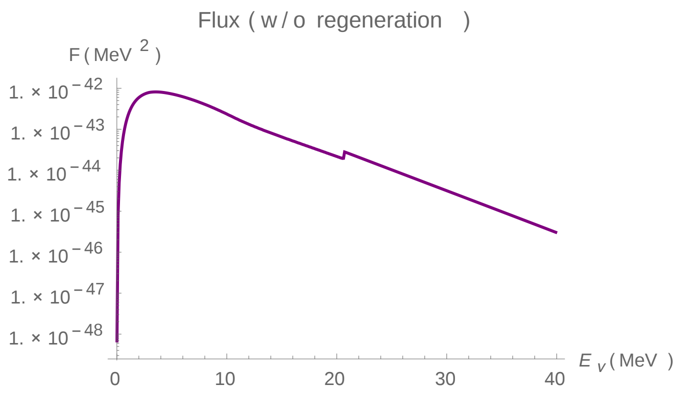

Figure 3.6: Plot of Flux(w/o regeneration) v/s Energy

We can see the distortion of the energy spectrum of DSNB within scenarios with DM (with mass in the MeV range), coupled to ordinary neutrinos ν as $gN^{\dagger}\nu\phi$ with the lighter new particle (a scalar ϕ or a heavy neutrino N) playing the role of DM. We see that such a coupling could give rise to a resonance scattering of neutrinos off the ambient DM background. Although the resonance would be in general very narrow, but the cumulative effect of resonance scattering of neutrinos at different redshifts could lead to a significantly wide dip in the spectrum at the detectors.

3.5 Freestreaming Flux

In the free streaming limit, the evolution of differential number density of neutrinos is described by the equation, [6]

$$\frac{\partial \tilde{n}(t,E)}{\partial t} = \frac{\partial (b\tilde{n}(t,E))}{\partial E} + L(t,E) \tag{3.5.1}$$

where, $\tilde{n}(t,E)$ is the differential (in energy E) number density of neutrinos given by, [6]

$$\tilde{n}(t,E) = \frac{dn(t,E)}{dE} \tag{3.5.2}$$

The first term on the right takes into account the continuous energy loss rate $b = H(t)E$, due to redshift, the second term is the differential number luminosity density of the sources given by, [6]

$$L(z,E) = W(z)L_0(E) \tag{3.5.3}$$

$L_0(E)$ is the differential number luminosity for each source given by, [8]

$$L_0(E) = q_0 E^{-\gamma} e^{-\frac{E}{E_{cut}}} \tag{3.5.4}$$

W(z) is the redshift evolution of the source density, assumed to follow the star formation rate (SFR) given by, [8]

$$W(z) = \begin{cases} (1+z)^{3.4} & (0 \leq \ z < 1) \\ (1+z)^{-0.3} & (1 \leq z \leq 4) \end{cases} \tag{3.5.5}$$

The observable neutrino number flux is given by, [6]

$$J(E) = \frac{c}{4\pi}\tilde{n}(0, E) = \int_0^{z_{max}} dz \frac{cL(z, E(1+z))}{4\pi H(z)} \tag{3.5.6}$$

H(z) is Hubble's parameter given as, $H(z) = H_0\sqrt{\Omega_\lambda + \Omega_{m,0}(1+z)^3}$, $\Omega_\lambda = 0.7$, $\Omega_{m,0} = 0.3$ and $H_0 = 2.181 * 10^{-18}s^{-1}$, c is a constant taken as 1(natural units). In this case, neutrinos from cosmic sources travel to the detector without interaction. Here, the spectrum of neutrino energies simply reflects the distribution of source redshifts through the relation $E = \frac{10^3}{(1+z)}$GeV.

Figure 3.7 shows that the normalised free streaming flux attains value = 1 at energy value = 10^3 GeV, remains almost constant till energy value = 10^6 MeV, then starts to fall as the energy increases and then becomes zero at energy value = 10^8 GeV. This shows that maximum number of neutrinos have energies in the range $(10^3 - 10^6)$MeV and a less number of them have higher energies. So, due to the freestreaming effect, neutrinos of lower energy will be more abundant than neutrinos with higher energies. This is due to the expansion of the universe.

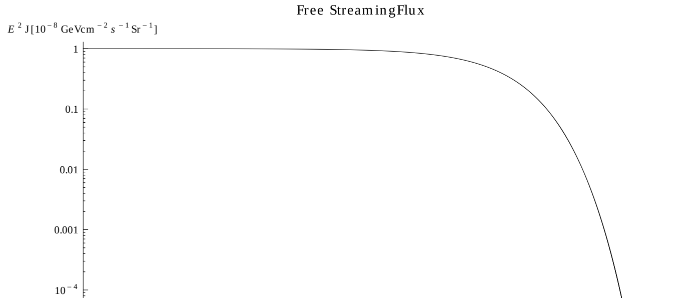

Figure 3.7: Plot of Flux(no interaction) v/s Energy

3.6 Discussion

3.6.1 Flux (with regeneration)

To see the effect of regeneration term on the neutrino flux as it propagates through the dark matter, we have used, [1]

$$F_i(t, E_\nu) = (1+z)^2 \int_z^{z_{max}} \frac{dz'}{H(z')} e^{-\int_z^{z_{max}} dz'' \frac{1}{(1+z'')H(z'')\lambda_i(z'', E''_z)}}$$
$$\left\{ \sum_{j=1}^{3} \int_{E'_z}^{\infty} dE'_{z'} T_{ji}(z', E'_{z'}, E_{z'}) F_j(z', E'_{z'}) \right\} \quad (3.6.1)$$

Above term represents the repopulation of the spectrum at energies lower than the resonance energy where, the differential DSNB flux of flavor a is formulated as, [9]

$$F_a(E_\nu) = \int_0^{z_{max}} dz R_{SN}(z) \frac{dN_a}{dE'_\nu} \frac{(1+z)}{H(z)} \quad (3.6.2)$$

$R_{SN}(z)$ represents the SN rate per comoving volume at redshift z given by, [1]

$$R_{SN}(z) = \frac{0.0088}{M_0} \dot{\rho}_o [(1+z)^{a\zeta} + (\frac{1+z}{B})^{b\zeta} + (\frac{1+z}{C})^{c\zeta}]^{\frac{1}{\zeta}} \quad (3.6.3)$$

with constants given as $\dot{\rho}_o = 0.02 M_O yr^{-1} MPc^{-3}$, a = 3.4, b = -0.3, c = -2.5, $\zeta = -10$, $B = (1+z_1)^{1-\frac{a}{b}}$, $C = (1+z_1)^{\frac{b-a}{c}}(1+z_2)^{1-\frac{b}{c}}$, $z_1 = 1$, $z_2 = 4$. Term responsible for redistribution of number flux of neutrinos to lower energy is given as, [1]

$$T_{ji}(t, E'_\nu, E_\nu) = n(t) \frac{d\sigma_{ji}(E'_\nu, E_\nu)}{dE_\nu} \quad (3.6.4)$$

where, E'_ν is the energy of incoming neutrino and E_ν is the energy of outgoing neutrino. The relation between differential cross section in lab frame and the differential cross section in center of mass frame as is given as, [1]

$$\frac{d\sigma_{ji}(E'_\nu, E_\nu)}{dE'_\nu} = \frac{d\sigma_{ij}}{dcos\theta}\frac{2E_\nu + m_{DM}}{E_\nu^2} \tag{3.6.5}$$

The differential cross section is given as, [1]

$$\frac{d\sigma_{ij}}{dcos\theta} = \frac{g_i^2 g_j^2}{32\pi}\frac{1}{(m_\Gamma^2 + m_{DM}^2)}\frac{1+cos\theta}{((s-m_\Gamma^2)^2 + \Gamma_\Gamma^2 m_\Gamma^2)} \tag{3.6.6}$$

Here, we consider the modifications to the SN flux due to the resonance interaction of a SN with dark matter particle. In this process a supernova neutrino with energy E_ν^{SN} will go through the resonance when the kinematic condition,

$$E_\nu^{SN} = \frac{(m_\Gamma^2 - m_{DM}^2)^2}{2m_{DM}} = E^{Res} \tag{3.6.7}$$

is satisfied. More specifically, a neutrino observed with energy E_ν^{Obs} will have gone through resonance if its energy lies in the region,

$$\frac{E^{Res}}{1+z} < E_\nu^{Obs} < E^{Res}$$

where z is the redshift. After the neutrinos that have energies in the region given by above equation go through the resonance they are redistributed to lower energies when the produced scalar decays back to neutrino mass eigenstates. In particular, neutrinos after interaction will be redistributed with a flat energy distribution from zero energy up to the original energy of the incident supernova neutrino.

To find the effect of these interactions on the flux of the SN, we note that the neutrinos leaving a supernova at redshift z emerge as the mass eigenstates. However, these mass eigenstate fluxes are now modified through interaction with the dark matter particles as they propagate to the Earth. To illustrate how the interactions modify the neutrino mass eigenstate flux we consider as an example the flux of the ν_1 mass eigenstate, F_{ν_1} . We start by defining the modified flux of the ν_1 eigenstates as $\tilde{F_{\nu_1}}$. For each redshift z, the ν_1 eigenstates that satisfy the condition given by above will have resonance interaction with the dark matter particles, producing the intermediate scalar. We label the absorbed flux as $F_{\nu_1}^{Res}$. Naively, the modified flux would be given by,

$$\tilde{F_{\nu_1}} = F_{\nu_1} - F_{\nu_1}^{Res} \tag{3.6.8}$$

However, above expression does not take into account that the scalar decays back into neutrino mass eigenstates. We need to add this contribution to above equation. The scalar can decay to any of the neutrino mass eigenstates. The probability that a scalar decays to the neutrino mass eigenstate ν_1 is P_1. These decays result in redistribution of the neutrino energies from zero energy up to the energy of the incident SN with a flat energy distribution. We define $P_1 \times F_{1\to 1'}^{Res}$ as the fraction of the flux of ν_1 that initiates a resonance, producing a scalar which then decays back into a ν_1 eigenstate with degraded energy (indicated by the notation $1'$). Then,

$$\tilde{F_{\nu_1}} = F_{\nu_1} - F_{\nu_1}^{Res} + P_1 \times F_{1\to 1'}^{Res} \tag{3.6.9}$$

We still need to take into account the contributions from the decays of scalars produced by other neutrino mass eigenstates. Therefore, there should be a sum over all of the initial states, and

$$\tilde{F_{\nu_1}} = F_{\nu_1} - F_{\nu_1}^{Res} + P_1 \times \sum_{i=1,2,3} F_{i\to 1'}^{Res} \tag{3.6.10}$$

In more general notation, for the jth neutrino mass eigenstate,

$$\tilde{F_{\nu_j}} = F_{\nu_j} - F_{\nu_j}^{Res} + P_j \times \sum_{i=1,2,3} F_{i\to j'}^{Res} \tag{3.6.11}$$

Above expression gives the modified flux for jth neutrino mass eigen state after regeneration and absorption.

Since, the process we are interested in is elastic scattering, so the number of neutrinos should remain conserved. As some neutrinos are facing absorption (with energies in narrow interval $(E^{Res} - \frac{\Gamma_\Gamma m_\Gamma}{2m_{DM}}, E^{Res} + \frac{\Gamma_\Gamma m_\Gamma}{2m_{DM}}))$, so some neutrinos should go in lower energy bin so that the neutrino number remains conserved. This is redistribution of of the flux to lower energies after the interaction $\nu_i\phi \to \ N \to \nu_j\phi$.

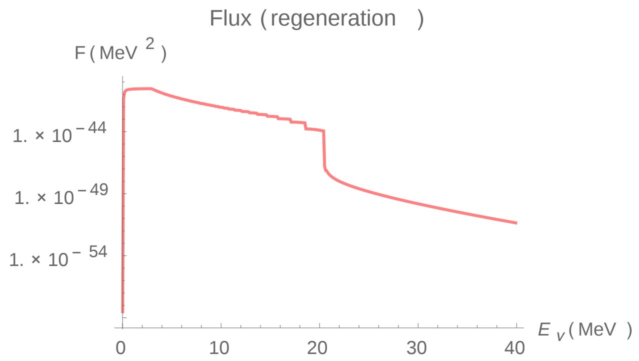

Figure 3.8: Plot of Flux(with regeneration) v/s energy

We can see from figure 3.8 that the more number of neutrinos have been shifted to energies lower than the resonance energy. So, regeneration moves particles to lower energies while increasing their numbers.

3.6.2 Comparison of fluxes

Figure 3.9 shows the initial flux, attenuated flux and the final flux. Final flux is the flux after cosmological absorption and redistribution to lower energies and with absorption, but without flux redistribution. The figure shows that as expected, the distortion of the spectrum starts at the resonance energy. The dip is broadened due to the absorption at different redshifts. We have focused on the case with real ϕ and Dirac N, for this is the only case for which the annihilation cross section of the DM pair remains below the thermal limit (i.e., ∼1 pb) even for couplings as large as g ∼ O(1). We have considered the case of the scalar interacting with three neutrino mass eigenstates and have shown the very interesting effect on the observed SN spectrum for the normal mass hierarchy.

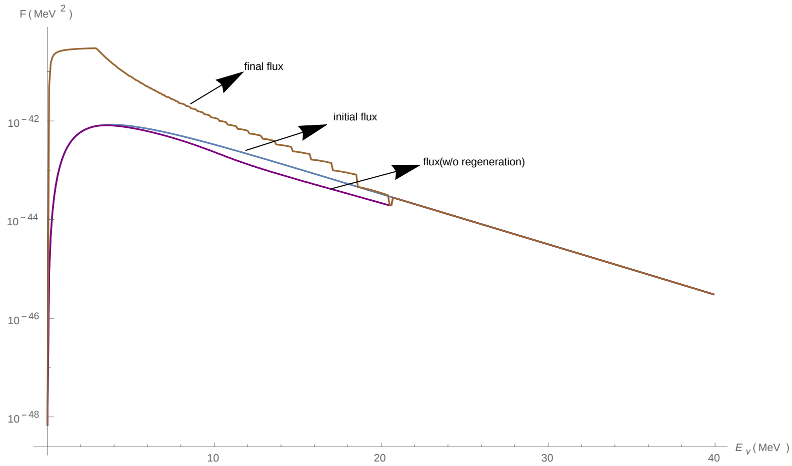

Figure 3.9: Plot for comparison of fluxes

Chapter 4

Conclusion and Future Scope

During the course of research work, I have been able to gather some knowledge in the fields of programming, cosmology and particle physics. It has been a pleasant and enlightening first experience in the field of academic (theoretical) research. Most importantly, it has made me further aware of how little I know and how much remains to be found out.

4.1 Conclusion

We started by considering the emission of neutrinos from the source (supernova) and got the initial neutrino flux. Then, we got the cross section of neutrino-dark matter interaction, with mass of the DM in the MeV range coupled to ordinary neutrinos as $gN_R^\dagger \nu \phi$. We found that such a coupling could give rise to a resonant scattering of neutrinos off the ambient DM background. We then studied the distortion of the energy spectrum of DSNB where it was shown that initial neutrino flux then gets attenuated as the neutrinos travel away from the source in the universe. Also, a dip was observed in the neutrino spectrum which could be explained as due to the cumulative effect of resonant scattering of neutrinos at different redshifts .

4.2 Scope for future study

The flux equation governs the transport of neutrinos through dark matter cloud, taking into account the redshifting of energies of neutrinos. One can consider the interaction of high energy astrophysical neutrinos with dark matter particles (of heavier masses or a complex scalar) or interaction of high energy astrophysical neutrinos with relic neutrino background (neutrino secret interactions) and thus can see such effects for different interactions of neutrinos . One can see such effects for different flux (apart from the star formation rate) like a source distribution with a constant comoving number density out to z_{max}, beyond which no sources exist. [10]

Neutrino-neutrino interaction has been studied for Waxman-Bahcall like fluxes before (i.e. E^{-2}). One can study the same for more realistic fluxes like Blazar and non-Blazar AGNs and SFRs which are supposed to be seen by IceCube. Hence some of the features of the neutrino flux spectrum above 20 TeV is expected to be explained by IceCube.

In this thesis, s-channel cross section is considered. One can extend this work by considering a t-channel cross section. One can take different astrophysical sources of neutrinos like Gamma Ray Bursts and Active Galactic Nuclei and study the effect of different interactions like neutrino-neutrino interaction for various fluxes at IceCube. [7]

The present research work considers neutrino flux in the energy range of few to tens of MeVs. It can be extended to high energy neutrino flux as already ICE CUBE has seen various ultra high energy neutrinos, whose flux can be very different from the source. So, one can take very high energy neutrino fluxes and see its impact at ICE CUBE. Also, one can extend this work by taking a complicated interaction to this study in future.

Nonvanishing DM neutrino interactions have several important cosmological and astrophysical consequences. They can explain the observed DM relic density if DM has been thermally produced and annihilations into neutrinos are the dominant channel.

Bibliography

[1] Yasaman Farzan and Sergio Palomares-Ruiz. Dips in the Diffuse Supernova Neutrino Background. *JCAP*, 1406:014, 2014.

[2] Sujata Pandey, Siddhartha Karmakar, and Subhendu Rakshit. Interactions of Astrophysical Neutrinos with Dark Matter: A model building perspective. *JHEP*, 01:095, 2019.

[3] P.B. Pal. *An Introductory Course of Particle Physics*. Taylor & Francis, 2014.

[4] Veniamin Berezinsky and A. Z. Gazizov. Diffusion of cosmic rays in expanding universe. *Astrophys. J.*, 643:8–13, 2006.

[5] Andrs Olivares-Del Campo, Cline Bhm, Sergio Palomares-Ruiz, and Silvia Pascoli. Dark matter-neutrino interactions through the lens of their cosmological implications. *Phys. Rev.*, D97(7):075039, 2018.

[6] Kenny C. Y. Ng and John F. Beacom. Cosmic neutrino cascades from secret neutrino interactions. *Phys. Rev.*, D90(6):065035, 2014. [Phys. Rev.D90,no.8,089904(2014)].

[7] M. G. Aartsen et al. Observation of High-Energy Astrophysical Neutrinos in Three Years of IceCube Data. *Phys. Rev. Lett.*, 113:101101, 2014.

[8] Takeshi Araki, Fumihiro Kaneko, Toshihiko Ota, Joe Sato, and Takashi Shimomura. MeV scale leptonic force for cosmic neutrino spectrum and muon anomalous magnetic moment. *Phys. Rev.*, D93(1):013014, 2016.

[9] Yu Seon Jeong, Sergio Palomares-Ruiz, Mary Hall Reno, and Ina Sarcevic. Probing secret interactions of eV-scale sterile neu-

trinos with the diffuse supernova neutrino background. *JCAP*, 1806(06):019, 2018.

[10] Anthony DiFranzo and Dan Hooper. Searching for MeV-Scale Gauge Bosons with IceCube. *Phys. Rev.*, D92(9):095007, 2015.