MODELING OF TELESCOPIC HYDRAULIC CYLINDER

M.Tech. Thesis

By

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DEPARTMENT OF MECHANICAL ENGINEERING

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MODELING OF TELESCOPIC HYDRAULIC CYLINDER

A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **CONDITION MONITORING AND MODELING OF TELESCOPIC HYDRAULIC CYLINDER** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DEPARTMENT OF MECHANICAL ENGINEERING Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July, 2023 to May, 2025 under the supervision of **Dr. Pavan Kumar Kankar, Professor, Department of Mechanical Engineering, Indian Institute of Technology Indore.**

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

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This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

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Signature of the Supervisor
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PRATHAMESH GANESH JADHAO has successfully given his M.Tech Oral Examination held on **23rd of May, 2025**.

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Prathamesh Jadhao

Dedicated to the almighty God

and my beloved family

Abstract

Hydraulic cylinders are used in many fields, such as aerospace, construction, and automotive systems. Therefore, understanding the behaviour of hydraulic cylinders is crucial for enhancing the efficiency of hydraulic systems. This study addresses the problem of predefined libraries being unable to simulate real-world conditions, and they are ‘black boxes’, i.e., provide the result without a transparent understanding. This research employs a combination of both experimental and analytical methodologies. A hydraulic cylinder in MATLAB (Simscape library) is simulated and validated experimentally by comparing the pressure variation with time. After the validation of the model, impact load conditions are considered, and the results obtained from it shed light on the dynamic response of the hydraulic cylinder. A comprehensive model of the hydraulic cylinder was developed considering the effects of piston loading conditions like friction (Coulomb friction model) and inertial forces. Matlab has a predefined library of custom hydraulic fluid using which we can define our custom fluid by specifying the values of kinematic viscosity, density etc. The entire hydraulic cylinder setup can be defined by a certain set of governing equations. The findings discuss the dynamic response of the hydraulic cylinder subjected to impact load, when subjected to such loading there is a significant pressure fluctuation of around 160% indicating that the hydraulic cylinder response is highly sensitive to rapid load changes and the system experiences substantial stress during impact which could lead to issues like seal damage or even failure of critical components. This work emphasises the necessity to integrate real-world factors into hydraulic simulations. Future research could involve incorporating additional factors such as temperature, fatigue, wear, etc. Continuous refinement of these models can help the industry to address the challenge associated with hydraulic system performance in practical applications.

LIST OF PUBLICATIONS

Modeling of the Hydraulic Cylinder Using Custom Matlab-Simscape Component

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Acronyms

Uinput	Input voltage
Uspool	Output voltage(Spool position)
Csef	Semi-empirical flow rate constant
Tc	Time constant
VO	Dead volume
X1˙	Displacement of piston
Ẋ1	Velocity of piston
QPA	Flow through port A(when port A is connected to pump)
QBT	Flow through port B(when port B is connected to tank)
QPB	Flow through port B(when port B is connected to pump)
QAT	Flow through port A(when port A is connected to tank)
PA	Pressure at port A
PB	Pressure at port B
PT	Pressure at tank
μs	Static friction coefficient
μd	Dynamic friction coefficient
F	Net force on piston
FNORMAL	Normal force on the piston due to hydraulic oil
A1	Area on port A side
A2	Area on port B side
E	Young’s modulus
E*	Modified Young’s modulus
R	Radius of sphere
KCONTACT	Contact stiffness
C	Contact damping
αCONTACT	Damping coefficient
δ	Indentation Depth
ν	Poisson’s ratio

Chapter 1

Introduction

1.1 Overview

Hydraulic cylinders are vital components in a wide array of mechanical systems. From earthmoving equipment and construction machinery to industrial manufacturing lines, their role in providing controlled linear motion and force cannot be overstated. These cylinders translate the energy of pressurised fluid into mechanical force, enabling the movement of heavy loads with high precision and efficiency.

While most existing studies and design considerations focus heavily on hydraulic systems under static loading conditions, the reality is that many real-world applications involve dynamic loading. Machinery used in mining, excavation, or industrial presses often faces transient or impact loads, which can introduce significant stress and rapidly alter the pressure within the hydraulic circuit. If these dynamic behaviours are not well understood and accounted for, the reliability and safety of the entire system can be compromised.

This section explores the significance of understanding hydraulic cylinder dynamics under such load conditions and summarises prior research efforts that have contributed to this field.

1.2 Background

The reliability and efficiency of hydraulic cylinders are critical in industries such as construction, mining, agriculture, and manufacturing. Over time, extensive research has been dedicated to enhancing their performance under various operating scenarios. Most of this research, however, has primarily concentrated on static load conditions—where the forces acting on the cylinder are steady or change slowly over time.

In practical applications, hydraulic cylinders are frequently exposed to varying loads. These may be due to fluctuating operational demands or external disturbances such as shocks, vibrations, or impact forces. For instance, a backhoe excavator operating on uneven terrain or a pile driver striking with repetitive force experiences intense transient load cycles that push the system beyond static considerations.

To address these challenges, researchers have turned to simulation tools to model the behaviour of hydraulic cylinders under dynamic conditions. One such effort is by Ding et al. [1], who conducted a simulation study using the AMESim platform to analyse a three-stage synchronous hydraulic cylinder. Their work focused on evaluating how the cylinder behaves under dynamic operations involving simultaneous contraction and expansion. The simulation played a key role in identifying and mitigating extraction forces that could compromise the cylinder's performance or synchronisation.

In another important study, Zhai et al. [2] modelled a hydraulic prop subjected to an impact load using AMESim and validated the findings with a transient dynamic simulation performed in ANSYS Workbench. They found that under impact loading, the pressure within the hydraulic prop increased sharply, highlighting the vulnerability of such components to sudden load variations and the necessity for robust design strategies.

Luo et al. [3] contributed further by simulating a four-stage hydraulic cylinder, focusing on both the bore geometry and real-time actuation performance. Their integrated modelling approach allowed for a highly realistic representation of the cylinder's dynamic response. Such studies underline the importance of combining accurate physical modelling with simulation tools to anticipate and mitigate operational issues in hydraulic systems.

Together, these studies reinforce the growing consensus that understanding the dynamic characteristics of hydraulic cylinders is not merely academic—it's essential for practical design and operational safety.

1.3 Working Principle

A hydraulic cylinder operates on the principle of Pascal's Law, which states that pressure exerted on a fluid in a confined space is transmitted uniformly in all directions. Inside a hydraulic cylinder, this principle allows pressurised hydraulic fluid to act on a piston and generate a linear force that moves a load.

Typically, a double-acting hydraulic cylinder has two ports: one for fluid to enter and push the piston forward, and another for fluid to push it back. This bidirectional action is crucial in machines requiring forward and reverse motion.

When the control valve directs fluid into the cap end of the cylinder, the piston rod extends. Conversely, directing fluid into the rod end causes retraction. The force generated by the cylinder is a function of the fluid pressure and the effective piston area. During operation, seals, rods, and valves must all function perfectly to prevent leakage and ensure smooth motion.

In dynamic conditions, especially those involving sudden or impact loads—this entire system is subjected to rapid changes in pressure and flow rate. If the cylinder or associated components are not designed for such stress variations, it could lead to performance issues or failure.

1.4 Applications

Hydraulic cylinders are found in nearly every heavy-duty application that involves lifting, pushing, or controlling movement. In construction, they are used in excavators, bulldozers, cranes, and dump trucks. These machines rely on hydraulic cylinders for precise control and substantial power delivery.

In agriculture, cylinders are found in tractors, harvesters, and other mechanised farming equipment. The ability to perform repetitive heavy lifting with minimal manual input makes them indispensable.

Industrial machinery, such as presses and injection moulding machines, utilise hydraulic cylinders for exact force application. In marine and aerospace sectors, they are part of steering and stabilising systems, where reliability is paramount.

Given these applications, the importance of ensuring that hydraulic cylinders can perform reliably under all load conditions—especially dynamic ones—becomes even more pressing.

1.5 Various Defects in Hydraulic Cylinders

Despite their robust design, hydraulic cylinders are prone to several defects over time, especially when exposed to dynamic and impact loads:

- **Seal Failure**: One of the most common issues. Under high pressure and frequent motion, seals can wear out or get damaged, leading to internal leakage and reduced efficiency.
- **Rod Bending or Buckling**: Sudden loads can exceed the buckling capacity of the piston rod, particularly in long-stroke cylinders.

- **Scoring or Scratching of Cylinder Walls**: Contaminants in hydraulic fluid or metal-to-metal contact can damage the cylinder bore.
- **Fluid Contamination**: Dynamic operations can introduce dust, water, or debris into the system, degrading performance.
- **Overheating**: Excessive or rapid cycling can cause temperature buildup, impacting fluid viscosity and seal integrity.

Identifying and addressing these defects through timely maintenance and robust design is key to maintaining long-term operational reliability.

1.6 Problem Statement

While hydraulic systems are generally well understood under controlled static conditions, their behaviour under dynamic loading is complex and less explored in practice. The response of a hydraulic cylinder to impact or transient loads involves rapid changes in pressure, flow rate, and mechanical stress—all of which can compromise the structural integrity of the cylinder or degrade performance if not properly accounted for.

This gap in understanding poses challenges for industries where reliability is non-negotiable. Engineers need better predictive tools and simulation models to assess and enhance the dynamic response of hydraulic cylinders. This study aims to contribute to that effort by modelling a hydraulic cylinder under impact load and evaluating its performance characteristics.

1.7 Research Objectives

The following objectives guide this research:

1. **To model a hydraulic cylinder under dynamic load conditions** using a suitable simulation platform (e.g., Simscape or AMESim).
2. **To evaluate the system response**—including pressure spikes, piston displacement, and force transmission—when subjected to impact or rapidly changing loads.
3. **To analyse the influence of material properties, hydraulic fluid characteristics, and design parameters** on the cylinder's dynamic performance.
4. **To assess the system's ability to absorb and dissipate impact energy** while returning to a steady-state operation without failure.
5. **To recommend design improvements or operational guidelines** that could enhance the reliability of hydraulic cylinders in real-world dynamic conditions.

Chapter 2
Literature Review

In recent years, significant attention has been paid to understanding the dynamic behaviour of hydraulic cylinders, especially as their applications in construction, mining, and industrial equipment become more demanding. While traditional analysis often focuses on static load scenarios, a growing body of research highlights the critical need to evaluate how these systems respond to dynamic and impact loads, which are more representative of real-world conditions. Below is a summary of three key studies that have shaped the foundation for such investigations.

2.1. Simulation of Synchronous Hydraulic Cylinders– Ding et al. [1]

Ding and colleagues conducted a detailed simulation of a three-stage synchronous hydraulic cylinder using the AMESim platform. Their primary goal was to study the cylinder's performance during synchronous contraction and expansion operations—common in heavy-duty equipment where multiple stages must move in coordination.

One of the key takeaways from this study was the identification of undesirable extraction forces, which can arise when synchronisation is imperfect. Through their model, the researchers were able to fine-tune the system's parameters, reducing these unwanted forces and improving the overall efficiency of the hydraulic mechanism.

What stands out in their work is not just the application of AMESim as a simulation tool, but the practical implications of their findings. In many real-world systems, synchronisation errors can lead to jerky motion, increased wear, or even component failure. Ding et al.'s study provides a useful framework for mitigating such risks through simulation-based design refinement.

2.1 Impact Load Analysis of Hydraulic Props – Zhai et al. [2]

In another important contribution to the field, Zhai and his team focused on how hydraulic props behave when exposed to impact loads—a situation commonly encountered in mining supports and similar structures. Their approach combined the strengths of two powerful tools: AMESim for hydraulic system modelling and ANSYS Workbench for transient dynamic structural analysis.

By comparing the two simulations, they were able to cross-verify their results and gain deeper insights. One of their major findings was the sudden and sharp rise in internal pressure when the prop was subjected to impact. This is particularly concerning because such spikes can lead to seal damage, fluid leakage, or even structural failure if not properly accounted for.

Their study reinforces the importance of anticipating non-linear responses in hydraulic systems. While static load design might ignore these pressure surges, Zhai's findings make it clear that dynamic impact scenarios demand careful modelling and robust component design.

2.2 Four-Stage Cylinder Modelling – Luo et al. [3]

Luo and colleagues carried out a simulation of a four-stage hydraulic cylinder, aiming to explore both its geometric configuration (bore design) and real-time actuation dynamics. Their study stands out for integrating mechanical design with dynamic system modelling, thereby providing a more comprehensive view of how these cylinders behave in motion.

The use of detailed simulation allowed them to replicate real-world conditions more accurately, accounting for changes in pressure, velocity, and force over time. The authors highlighted the benefit of using an integrated approach that blends hydraulic simulation with actuation dynamics to enhance realism and predictive accuracy.

This study is especially relevant for industries where multi-stage cylinders are used, such as telescopic lifts or cranes, where both compactness and extension length are critical. Luo et al. showed how the right simulation strategy can help engineers understand the subtle interdependencies in such complex systems.

2.3 Synthesis of Insights

Collectively, these three studies offer valuable insights into different aspects of hydraulic cylinder dynamics:

- Ding et al. focused on synchronisation and force optimisation in multi-stage systems.
- Zhai et al. highlighted the risks posed by impact loads and how they cause sharp pressure surges.
- Luo et al. presented a holistic modelling approach that combines design and real-time simulation to predict cylinder behaviour more accurately.

All three point to a common theme: traditional static analysis is no longer sufficient. As equipment continues to operate in unpredictable environments, there is a clear need for simulation-based design and testing under dynamic and transient conditions. These studies not only provide the theoretical foundation but also demonstrate practical methodologies that can be built upon in future research.

This literature review sets the stage for the current study, which aims to further investigate the dynamic response of hydraulic cylinders, particularly their ability to absorb and dissipate impact loads, using integrated simulation tools.

Chapter 3
Experimental Setup

To thoroughly investigate the behaviour of hydraulic cylinders under different loading conditions, a detailed experimental setup was developed. This setup was specifically designed to monitor system performance, detect potential faults, and gather real-time data for validation against simulation results. The setup includes both a standard hydraulic cylinder and a telescopic hydraulic cylinder, operated in conjunction, as shown in **Figure 1**.

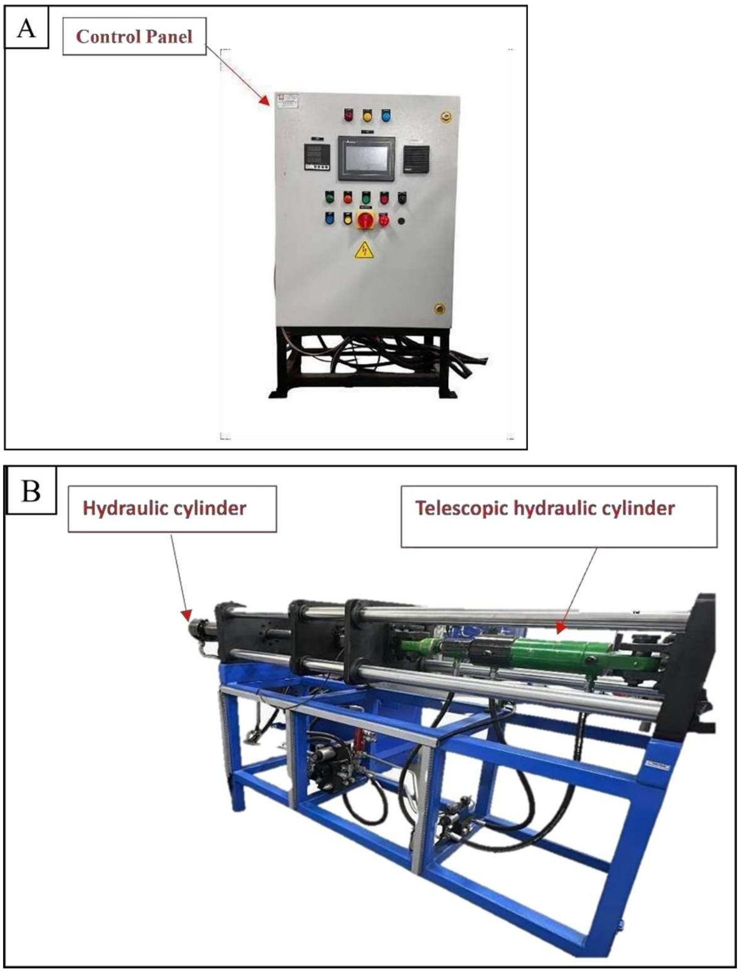

Figure 1: A) Control panel & B) Hydraulic cylinder setup

3.1 System Components and Layout

At the core of the setup is a hydraulic circuit powered by HP ENKLO 68 hydraulic fluid, a widely used lubricant known for its stability under high pressure and temperature. The fluid is circulated using a fixed displacement gear pump, which ensures a constant volume of fluid delivery per revolution. This pump is driven by a 3-phase electric motor rated at 15 H.P., providing sufficient power for the actuation of the cylinders.

The hydraulic cylinder in the system is equipped with two ports: A and B. During the extraction stroke, hydraulic oil flows from the pump into port A, while port B is connected to the return tank. This flow direction is reversed during retraction, with port B receiving pressurised fluid from the pump and port A acting as the return path to the tank. This configuration allows precise bidirectional control of the cylinder movement.

In terms of geometry, the hydraulic cylinder has:

- An outer tube diameter of 95 mm
- A bore diameter (inner diameter) of 80 mm
- A piston diameter of 50 mm

The total stroke length—the maximum distance the piston can travel—was determined by measuring the length of the cylinder in both fully extended and retracted states. In the extended position, the cylinder measures 1143 mm, and in the retracted position, it measures 743 mm. Therefore, the effective stroke length is calculated as:

$$\text{Stroke} = 1143\text{mm} - 743\text{mm} = 400\text{mm}$$

3.2 Instrumentation and Data Acquisition

To monitor the pressure variations within the cylinder during operation, a pressure transducer is installed at the start of the cylinder tube as shown in Figure 2. This sensor is crucial for capturing the real-time pressure response of the system, especially under changing load conditions.

The key specifications of the pressure sensor are as follows:

- Maximum pressure measurement range: up to 600 bar
- Sampling rate: 10 kHz (10,000 samples per second)

This high sampling rate ensures that even rapid fluctuations in pressure, such as those caused by transient or impact loading, are accurately captured.

The sensor is connected via cable to a HYDAC portable data recorder, a specialised device used for real-time data logging. HYDAC is known for its reliability in mobile and field-based hydraulic measurements, making it suitable for this type of experimental work.

3.3 Test Conditions and Procedures

In the initial phase of testing, the hydraulic cylinder is manually operated under dead-load or no-load conditions. This means that the cylinder extends and retracts without lifting or resisting any significant external force. The purpose of this test is to understand the system's baseline pressure behaviour in the absence of added stress.

The recorded pressure data during these operations is plotted as a function of time. This forms a reference point for evaluating how the cylinder behaves in its simplest operational state.

3.4 Simulation and Model Validation

To complement the experimental observations, a custom Simscape model of the hydraulic cylinder was developed using MATLAB/Simulink. This virtual model replicates the physical characteristics and control logic of the experimental system.

Using the same input parameters (such as pump flow rate, cylinder dimensions, and fluid properties), the model generates a pressure vs time graph that represents the simulated behaviour of the cylinder.

The simulated pressure profile is then compared to the experimentally acquired data under dead-load conditions. This comparison serves as a validation step to ensure the simulation model accurately represents the real system.

3.5 Dynamic Load Testing

Once the model is validated for no-load conditions, it is subjected to impact or dynamic loading scenarios. These tests are crucial to studying the cylinder’s transient behaviour, especially how quickly it responds to sudden forces and how it manages pressure spikes.

By observing the pressure fluctuations and system recovery in both experimental and simulated conditions, insights can be drawn about:

- The resilience of the hydraulic system
- The accuracy of the simulation model
- The system’s ability to absorb and dissipate impact energy

Such analysis is key in improving cylinder design for high-performance industrial applications, where reliability under extreme conditions is critical.

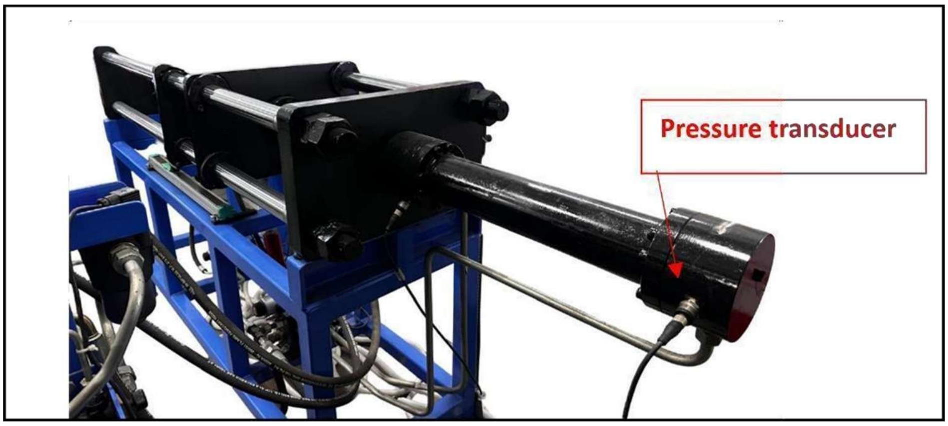

Figure 2: Hydraulic cylinder

Chapter 4

Model and Parameters

To understand the behaviour of the hydraulic system under various loading conditions, a detailed and physically accurate simulation model was developed using MATLAB Simscape. This model replicates the experimental setup and provides a controlled environment to analyze the hydraulic system's performance under different scenarios, including no-load and impact conditions.

The simulation model was constructed using a modular approach, dividing the hydraulic system into five major components: the pump, tank, hydraulic fluid, valve, and actuator. Each component was either sourced from MATLAB's Simscape library or custom-designed using governing physical principles to closely replicate real-world performance. This chapter presents each of these components, the mathematical relationships governing their behaviour, and how they were implemented in the simulation environment.

4.1 Hydraulic System Overview

In any hydraulic system, the five key elements that form the backbone of fluid power transmission are:

1. **Pump –** provides the energy required to move fluid through the system.
2. **Tank (Reservoir) –** stores the hydraulic fluid.
3. **Hydraulic Fluid –** the medium through which energy is transmitted.
4. **Valve –** directs the flow of fluid based on control signals.
5. **Actuator (Hydraulic Cylinder) –** converts hydraulic energy into mechanical motion.

In the simulation model, the pump is represented by a constant pressure source block, and the tank by a hydraulic reference block—both standard components in MATLAB's Simscape library. The valve and actuator were custom-coded to include dynamic characteristics observed in the experimental setup. A custom hydraulic fluid block was defined to match the physical properties of HP Enklo 68 oil used in the experiments.

4.2 Custom Hydraulic Fluid Modelling

Simscape allows users to define a custom fluid if the default options do not meet specific experimental requirements. In our experimental setup, we used HP Enklo 68, a mineral oil with excellent lubrication properties and thermal stability. This fluid has the following key properties:

- **Density (ρ)**: 851.8 kg/m^3
- **Kinematic viscosity**: 6.83×10^{-5} m^2/s @ 40°C
- **Operating temperature**: 38°C (no significant change in viscosity)

By creating a custom fluid block, we ensured that all components within the hydraulic loop received consistent and accurate fluid properties. This was essential for modeling pressure drops, fluid inertia, and damping effects with high fidelity. These parameters remained constant throughout the simulation since the operating temperature closely matched the nominal temperature used for the fluid specification.

4.3 Valve Modelling and Control Logic

In the physical setup, flow direction is managed using a 4/3 solenoid-operated directional control valve. This type of valve has four ports (P, T, A, B) and three spool positions. In Simscape, we developed a custom-coded valve block that uses an input control signal (voltage) to simulate spool displacement and consequently direct fluid flow.

4.3.1 Governing Equations and Flow Control

The valve's behaviour is governed by the following differential equation:

$$\frac{dUspool}{dt} = \frac{\text{Uinput} - \text{Uspool}}{\text{Tc}} \quad (1)$$

Where:

- U_input is the voltage input
- U_spool is the instantaneous spool position

Depending on the value of U_spool, the valve routes fluid to different ports:

- When Uspool > 0,

$$Q_{PA} = C_{sef} \times U_{spool} \times \text{sign}\,(P_P - P_A) \times \sqrt{P_P - P_A} \quad (2)$$

$$Q_{BT} = C_{sef} \times U_{spool} \times \text{sign}\,(P_B - P_T) \times \sqrt{P_B - P_T} \quad (3)$$

- When Uspool < 0,

$$Q_{PB} = C_V \times U_{spool} \times \text{sign}\,(P_P - P_A) \times \sqrt{P_P - P_B} \quad (4)$$

$$Q_{AT} = C_V \times U_{spool} \times \text{sign}\,(P_B - P_T) \times \sqrt{P_A - P_T} \quad (5)$$

- Uspool = 0: All flow paths are blocked. Q = 0 for all ports (ideal closed center configuration with no leakage).

This flow modelling approach is semi-empirical and widely used in hydraulic system design. It ensures realistic flow rate representation depending on pressure differential and spool displacement.

4.4 Hydraulic Actuator (Double-Acting Cylinder)

The actuator in this model is a **double-acting hydraulic cylinder**, similar to the one used in the experimental test rig. The actuator receives pressurized fluid from the valve and converts it into linear motion, either extracting or retracting the piston depending on flow direction.

4.4.1 Friction Modelling: Coulomb Model

To replicate real-world motion resistance, friction inside the cylinder is modeled using a Coulomb friction approach. This includes both static (stiction) and dynamic friction.

If $\dot{X}_1 = 0$ then,

$$F_{friction} = \mu_s \times F_{normal} \tag{7}$$

When the piston is in motion:

$$F_{friction} = \mu_d \times F_{normal} \tag{8}$$

$$F = A_1 \times P_A - A_2 \times P_B - F_{FRICTION} \tag{9}$$

Where:

- A_1 and A_2 are piston areas on each side
- P_A and P_B are respective pressures
- $F_{friction}$ is the internal resistive force due to seals and surface interactions

Nitrile rubber seals are considered in this model, which are known for their good oil resistance and moderate dynamic friction.

4.5 Model Architecture and Implementation

As shown in Figure 3, the full simulation model is built in a modular format. Each subsystem (pump, valve, actuator, fluid, and tank) is developed individually and connected through hydraulic ports and signal lines. The control signal for the valve is modeled as a PWM voltage signal, simulating manual switching or microcontroller-driven actuation.

All model parameters—geometrical and material—are matched with the experimental setup. These include:

- **Piston diameter**: 50 mm
- **Bore diameter**: 80 mm
- **Cylinder stroke**: 400 mm
- **Supply pressure**: 200 bar (for initial validation)
- **Sensor sampling rate**: 10 kHz

4.6 Validation and Assumptions

The model is validated using experimental data gathered during no-load conditions. The pressure vs. time response in the simulation is compared to actual sensor readings from the setup. A high degree of correlation ($R^2 > 0.98$) confirms the model's accuracy for baseline testing.

Some simplifications have been made to maintain simulation efficiency:

- Fluid is assumed incompressible within the typical pressure ranges used.
- Flow is laminar, with no internal leakage assumed in valves or cylinders.
- The valve response time is modeled ideally, though real-world actuation lag may introduce minor delay.

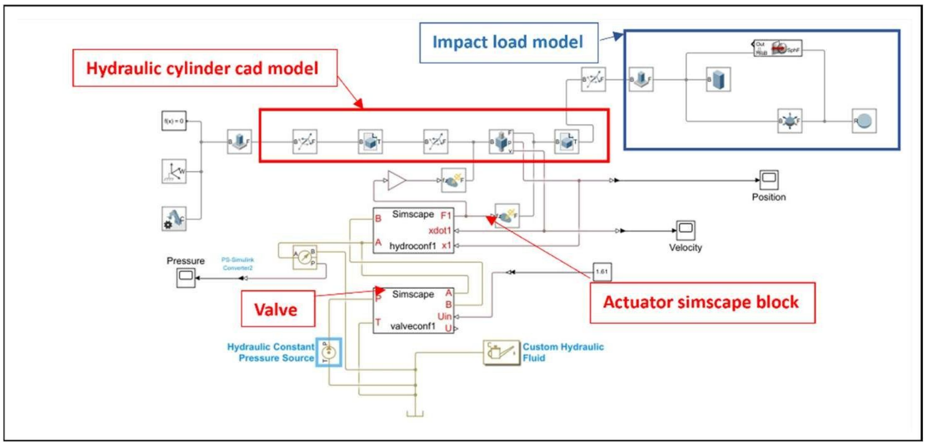

Figure 3: Model architecture

Table 1

Parameter	Symbol	value
Pump pressure	P_P	450 MPa
Semi-empirical flowrate constant	C_{Sef}	$0.85 \times 10^{-7}\ m^3/s\sqrt{Pa}$
Normal Force	F_{normal}	4020.8 N
Pressure at port A	P_A	0.8 MPa
Contact stiffness	$K_{CONTACT}$	1.07×10^{11} N/m
Indentation Depth	δ	0.001m
Poisson's ratio	ν	0.3
Young's modulus	E	210 GPa
Modified Young's modulus	E*	115.38 GPa

Contact damping	C	1.07×10^9 N/(m/s)

The values mentioned in the above table are selected from typical values found in literature and some values are calculated based on the formula listed in equations mentioned in section 3.

Chapter 5
Validation

5.1. Introduction and Experimental Setup

In hydraulic system design and analysis, it is essential to understand the performance of actuators under realistic load conditions. In this study, the behavior of a hydraulic cylinder is analyzed under *dead load conditions*, meaning the system operates without dynamic external forces acting during motion—only gravity and internal resistance affect performance. This allows for a clearer assessment of pressure buildup, fluid behavior, and component interaction.

The experimental setup is manually operated with a constant input signal to the hydraulic valve. Manual operation at dead load conditions helps replicate steady-state usage as often encountered in maintenance and field operations. Similarly, a mathematical model of the system was developed and run under the same conditions with a fixed control input, allowing for direct comparison between the physical system and the simulation.

Figure 4 illustrates the experimental setup and provides key sections of interest in pressure response during the operation. These insights help diagnose discrepancies, evaluate performance, and suggest improvements in system design or modeling accuracy.

5.2. Pressure Build-up and Initial Response

As shown in Section 1 of Figure 4, one of the first notable characteristics observed during the experiment is a delay in pressure build-up after the input signal is applied. This delay is typical in hydraulic systems, particularly those controlled manually. The pressure does not spike instantly because the hydraulic fluid requires time to travel through the circuit, overcome the inertia of the system, and compress slightly before acting on the cylinder piston.

The delay can also be influenced by minor factors like internal leakage, valve response time, and slight slack in mechanical linkages. This initial lag highlights the importance of accounting for real-world system latency when designing hydraulic controls, especially for precision applications like industrial lifting or robotic systems.

Once the internal pressure overcomes static resistance, the cylinder begins to extract at a relatively steady rate. However, as seen in Section 2 of the figure, the pressure during this phase is not perfectly uniform. Small fluctuations or oscillations appear in the pressure graph. These variations are minor but consistent and are likely caused by long-term wear and tear on the hydraulic components—most commonly on seals, piston rings, and internal valve parts. Daily usage leads to microscopic changes that, over time, influence system smoothness and responsiveness.

These oscillations, while not immediately alarming, serve as an early indicator of potential maintenance needs. In heavily used hydraulic systems, they may point to the need for re-calibration or part replacement to avoid future failures or reduced efficiency.

5.3. Comparison Between Model and Experimental Setup

One of the most significant findings from the experiment is the difference in total extraction time. The experimental hydraulic cylinder completes its extraction in approximately 13 seconds, while the simulated model achieves the same in 11 seconds. This 2-second gap may seem small at first glance, but in time-sensitive operations or synchronized multi-cylinder systems, such discrepancies can have major consequences.

The primary reason for this difference is rooted in system interaction. The real experimental setup includes a telescopic hydraulic cylinder that functions in tandem with the main cylinder. As the main hydraulic cylinder extends, the telescopic cylinder contracts, creating a mechanically coupled system. This coupling introduces additional resistances and force demands that are not captured in the simplified model.

Specifically, at around the 9.3-second mark, the pressure graph reveals a sudden and sharp increase in pressure, marked as Section 4 in Figure 4. This pressure spike aligns with the point at which the first stage of the telescopic cylinder begins to retract. The first stage typically has a larger cross-sectional area, meaning more force is required for movement. According to Pascal's Law, the pressure in a hydraulic system is transmitted equally in all directions, so a larger area results in higher force for the same pressure—or, conversely, more pressure is needed for the same force.

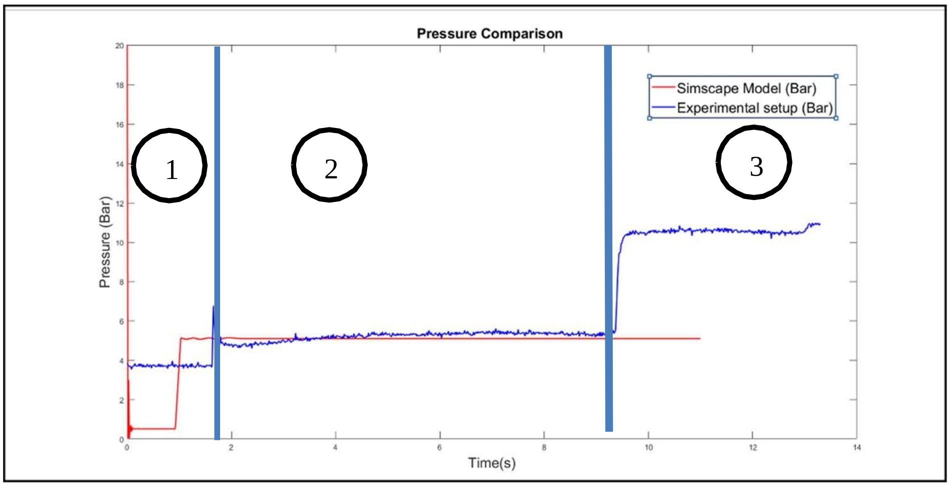

Figure 4: Pressure vs. Time (Comparison)

This interaction causes a sudden load on the hydraulic pump and valve system, which results in the spike observed in the experimental data. Since the simulation does not account for the telescopic cylinder, this load increase is not replicated, explaining the faster extraction time in the model.

Chapter 6

Results and discussion

This comparative study between the experimental and simulated behavior of a hydraulic cylinder under dead load conditions yields several key insights:

- Pressure build-up is delayed initially due to system inertia and fluid dynamics, which is expected in real-time manual operation.
- Pressure fluctuations during extraction suggest routine component degradation. These serve as useful diagnostic indicators for preventive maintenance.
- Modeling discrepancies, particularly in extraction time, underscore the importance of accurately simulating mechanical couplings. In this case, the omission of the telescopic cylinder interaction in the model leads to faster-than-realistic performance predictions.
- The sharp pressure spike at 9.3 seconds in the real system highlights how real-world load-sharing between hydraulic components impacts system dynamics.

To enhance future modeling efforts, it is recommended that:

- The model incorporate coupled system dynamics, including interactions with telescopic cylinders or other actuators.
- Degradation parameters such as wear, leakage, or changing fluid properties over time be introduced to reflect realistic performance drift.
- Validation efforts continue with varied load scenarios to understand how the system behaves beyond dead load conditions.

Ultimately, while simulations offer a fast and flexible tool for design and control strategy development, field validation remains critical to ensuring system safety, reliability, and performance accuracy.

6.1 Impact load

To accurately simulate real-world operating conditions of hydraulic systems, it is essential to account not only for static or gradual loads, but also for sudden, dynamic forces—commonly referred to as impact loads. In this study, an impact scenario has been designed to mimic such a condition using a controlled and physically meaningful setup within a simulation environment.

The simulated impact involves a steel sphere with a mass of 261 kg and a diameter of 0.2 meters, falling freely from a height of 1 meter onto a rigid steel plate. The dimensions of the steel plate are 1 meter by 1 meter, with a thickness of 0.05 meters. This plate is rigidly welded to the base of a hydraulic piston, thereby transmitting the impact load directly to the hydraulic system upon contact. The configuration is visually represented in Figure 5, which provides a clear illustration of the setup geometry and connection points.

To model this event in a realistic manner, the impact force resulting from the collision between the falling sphere and the steel plate is simulated using the Multi-Body Contact Force block available in the Simscape Multibody platform in MATLAB/Simulink. This block enables the modeling of collisions between rigid bodies by accounting for both contact stiffness and damping effects. Two key parameters must be defined to characterize the contact interaction accurately:

- Kcontact – Contact stiffness
- **C** – Contact damping

These parameters essentially govern how "hard" or "soft" the contact appears in the simulation and how much energy is dissipated during the collision. An accurate estimation of these values is essential to capturing realistic peak forces and dynamic responses.

To determine appropriate values for $K_{contact}$ and C, the Hertzian contact theory is employed. This well-established theory describes the stress distribution and deformation at the point of contact between two curved or flat bodies when they collide. For the purposes of this simulation, the following assumptions are made:

- The collision between the steel sphere and the steel plate is perfectly elastic, meaning no energy is lost to heat, sound, or permanent deformation.
- Both bodies are assumed to be homogeneous, isotropic, and linearly elastic, which is consistent with steel properties under moderate impact forces.

The duration of contact is short, but long enough for Hertzian assumptions to hold (i.e., quasi-static contact pressure $K_{contact}$ can be derived based on the material properties (such as Young's modulus and Poisson's ratio of steel), the radii of curvature of the impacting bodies (in this case, a sphere and a flat plate), and the impact geometry. **C**, the damping coefficient, is more complex to calculate analytically, but it can be estimated using empirical methods or literature-based approximations to reflect the slight energy dissipation that occurs even in near-elastic impacts.
In summary, this simulation setup serves to replicate the transient effects of an impact load on a hydraulic actuator system. By applying a controlled and theoretically grounded impact force, the response of the piston and the broader hydraulic system can be studied under high-stress, dynamic

conditions. This is particularly useful for understanding shock resistance, stress wave propagation, and potential design vulnerabilities under sudden loads. Furthermore, the use of Hertzian theory ensures the modeling remains rooted in well-validated physical principles, providing confidence in the simulation outcomes.

$$E^* = \frac{E}{2(1-\nu^2)} \qquad (10)$$

$$K_{CONTACT} = \frac{4E^*R}{3(1-\nu^2)[\delta^{\frac{1}{2}}]} \qquad (11)$$

$$C = \alpha_{CONTACT} \times K_{CONTACT} \qquad (12)$$

The values thus calculated can be seen in Table 1 and the pressure vs time graph is plotted below.

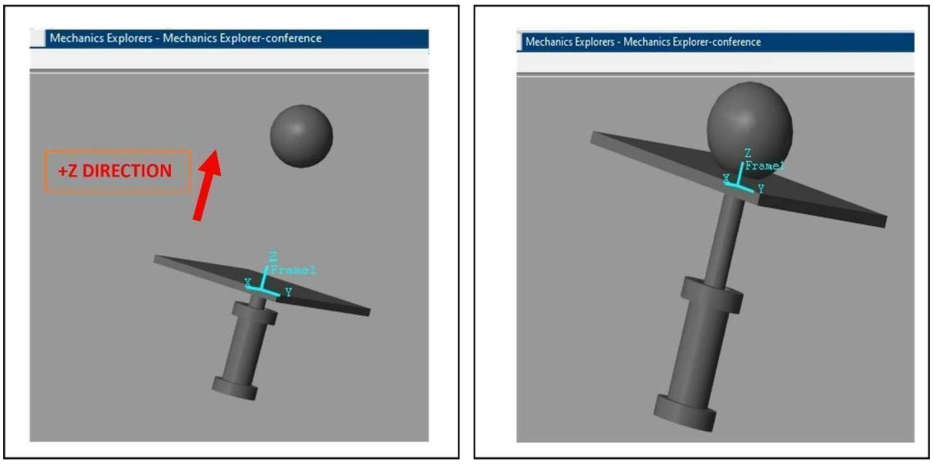

Figure 5: Mechanics explorer (window for showing real-time actuation)

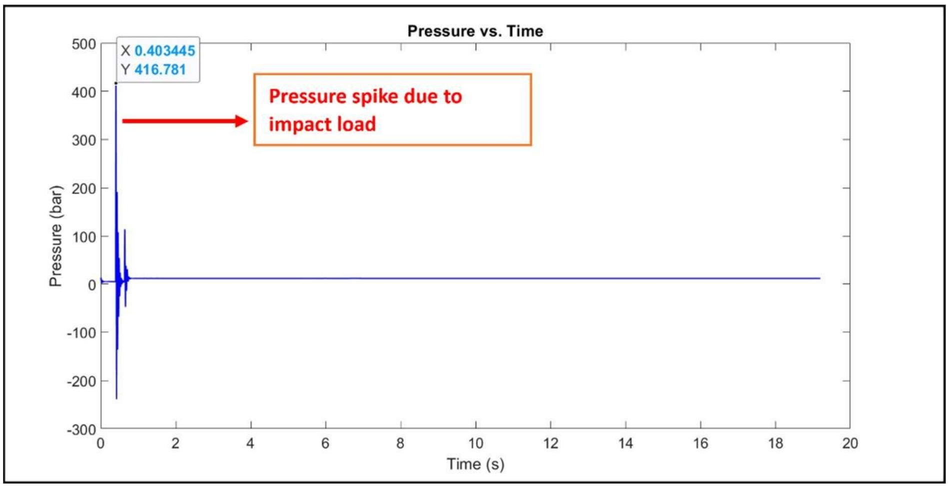

Figure 6: Pressure vs. Time (Impact load)

As you can see in Figure 4 the pressure suddenly decreases when the steel ball strikes the flat plate, showing the dynamic response of the hydraulic cylinder to the impact load. The sphere upon contact with the plate results in a rapid deceleration of the ball, generating a sudden load, which is compensated by the hydraulic cylinder as the pressure decreases to around -237 bar. After the sharp decrease, the pressure rapidly increases to around 416 bar, to compensate for the extra impact load of the sphere. The pressure sensor measures the pressure difference at port A and tank, and the positive and negative value signifies the direction of the flow the negative pressure of -237 bar indicates that the piston retracts inside the cylinder bore i.e. the piston moves in the –Z direction shown in figure 5 causing a reversal of flow of hydraulic fluid. After the initial impact and pressure fluctuation, the system eventually reaches a steady state as the sphere ball rests on the steel plate, signifying that the model is no longer subjected to dynamic forces.

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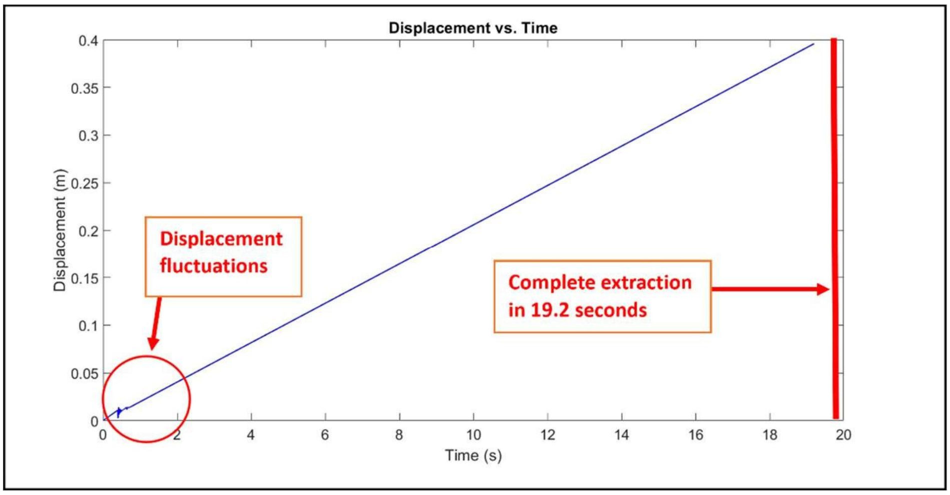

Figure 7: Displacement vs. Time

In Figure 7 we can see that the complete extraction time is 19.2 seconds the increase in time can be attributed to compensating for the impact load as well as the extra weight of the sphere when it finally comes to a steady state and rests on the steel plate. Displacement fluctuations can also be observed in Figure 8, which are primarily driven by rebound of the hydraulic fluid within the cylinder, the initial impact momentarily causes the reversal of hydraulic fluid back to the valve resulting in a rapid reduction in displacement, once the impact energy is absorbed the system begins to counteract the load and there is typically a rebound effect as the system tries to stabilize and the fluctuation gradually decreases and then the displacement stabilizes. As the steel sphere strikes the piston, a retraction occurs which can be observed in the displacement graph (Figure 8), this results in a negative pressure value of -237 bar as the direction of retraction of the piston is in the –Z direction. After the initial decrease, the pressure rises as the system absorbs the energy of the impact. This rise in pressure is a result of the hydraulic valve working to counteract retraction and stabilize the system.

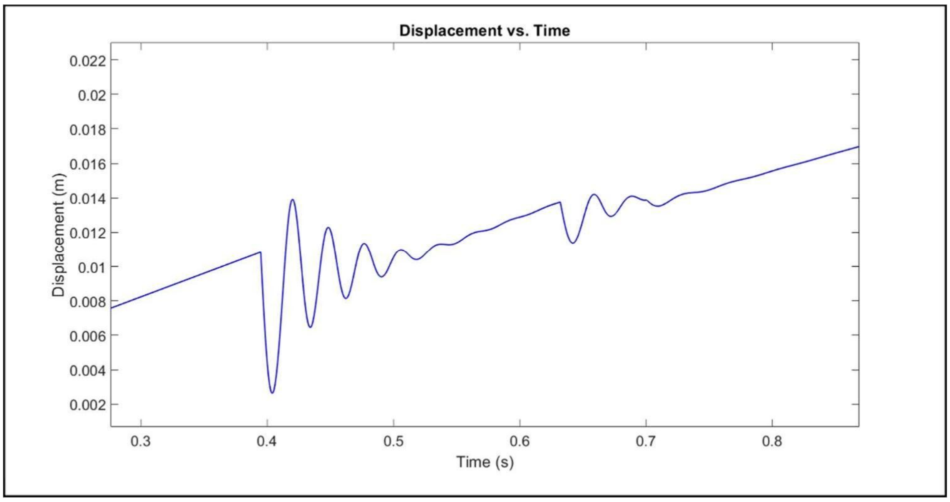

Figure 8: Displacement fluctuations

As the pressure increase is too rapid and frequent it could lead to damage of seals and cause leakage and also lead to catastrophic failure of the entire setup but the subsequent stabilisation of the pressure indicates that the system is capable of handling the impact load successfully absorb and dissipate the energy as you can see in Figure 7.

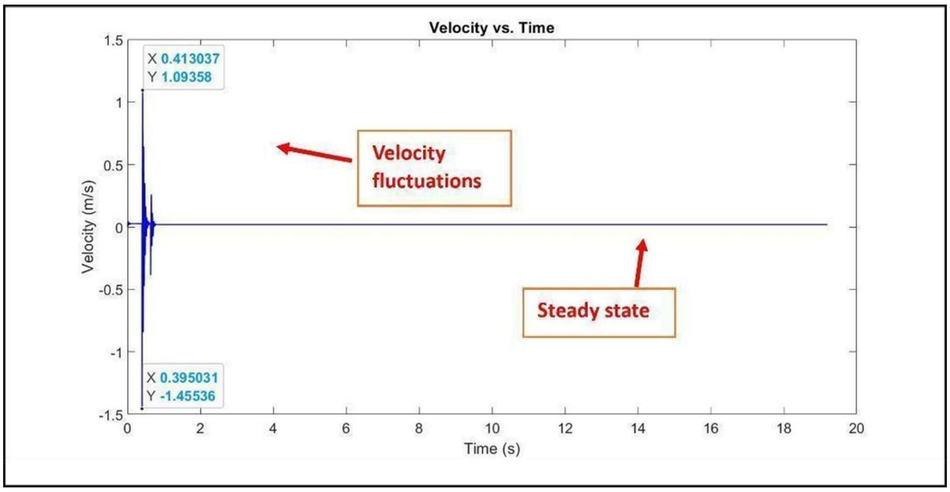

Figure 9: Velocity vs. Time

The velocity fluctuates significantly from 1.09m/s to -1.45m/s as shown in Figure 9, indicative of high energy interaction, the sudden deceleration is attributed to the transfer of kinetic energy of the sphere to the plate, once the sphere comes to rest the velocity stabilizes to a value of 0.0206m/s.

Chapter 7

Conclusion

This study successfully developed and validated a dynamic model of a double-acting hydraulic cylinder using MATLAB Simulink and the Simscape Multibody platform. The modeling process included two major phases: (1) evaluation under static (dead load) conditions, and (2) simulation under dynamic impact loading. Through both scenarios, the hydraulic system's behavior was carefully analyzed with respect to pressure, displacement, and response time.

During the first phase, the cylinder was operated under constant dead load conditions using a manual control input. Both the experimental setup and the model were evaluated for extraction time, pressure development, and overall stability. The results confirmed that the model closely emulated the behavior of the physical setup, achieving near-identical displacement responses and similar pressure fluctuations, validating the fidelity of the simulation.

Subsequently, the system was tested under more demanding dynamic conditions by simulating the impact of a 261 kg steel sphere falling onto a rigid plate mounted on the cylinder piston. This scenario tested the system's ability to absorb and respond to a sudden external force, thereby evaluating the true dynamic performance of the hydraulic actuator.

7.1 Observations on Dynamic Behaviour

The pressure and displacement graphs from the simulations offered valuable insights into the transient response of the system under impact loading. The moment the steel sphere struck the plate, a sharp negative pressure spike of approximately -237 bar was recorded, indicating the retraction of the piston due to the initial impact force. The piston moved in the –Z direction, pushing hydraulic fluid back through the control valve and resulting in a reversal of flow.

This was followed by a sharp pressure increase up to 416 bar, as the hydraulic valve system compensated for the external load and attempted to re-extend the piston. The displacement profile reflected these dynamics clearly, showing momentary retraction, rebound, and eventual stabilization. These fluctuations diminished over time as the system reached steady-state, with the piston now supporting both the dead load and the additional static load from the resting sphere.

The complete extension time increased to 19.2 seconds, compared to 13 seconds under dead load, highlighting the additional work the actuator must perform under combined static and dynamic loading. The rebound effects due to hydraulic fluid inertia and flow reversal played a crucial role in this extended actuation period.

7.2 Influence of Material and Model Parameters

One of the key strengths of this study lies in the depth of physical accuracy embedded in the simulation model. Several factors contributed to the system's realistic dynamic behavior:

- **Material Properties**: Steel was chosen for both the piston plate and the impacting sphere. The elastic and inertial properties of steel were critical in defining the energy transfer during impact and the subsequent response of the system.
- **Hydraulic Oil Characteristics**: The behavior of the fluid—specifically its compressibility, viscosity, and response to rapid pressure changes—was instrumental in shaping the dynamics observed. Hydraulic oil with a high bulk modulus responded quicker to pressure changes, enhancing the accuracy of the simulation.

- **Friction Modeling**: The Coulomb friction model was integrated into the simulation to more accurately reflect the interaction between moving components. This model accounts for the reduction in friction with increasing velocity, a phenomenon observed in real systems. This ensured that the displacement behavior during high-speed transitions (such as during impact) was accurately captured.
- **Inertia and CAD Integration**: The use of CAD models imported into MATLAB's Solid Library allowed for the precise definition of each component's geometry and mass distribution. By assigning realistic density values, the model accurately incorporated inertial forces, which are particularly important during acceleration and deceleration phases like impact and rebound.

7.3 Simscape as a Modelling Tool

The choice of Simscape Multibody as the modeling environment proved highly beneficial. The visual approach to system construction allowed for better intuitiveness and clarity in defining physical relationships. Moreover, the platform supports bidirectional coupling between mechanical and hydraulic domains, which is essential for modeling fluid-structure interactions like those present in hydraulic actuation.

Simscape's modularity also allowed for easy testing of different configurations and parameters, facilitating quick iterations and tuning. This capability is valuable not only for academic research but also for industrial prototyping, where such simulations can significantly reduce the cost and time associated with physical testing.

7.4 Practical Applications and Design Utility

The validated model developed in this study serves as an effective design and diagnostic tool. Engineers can use the model to:

- Test various hydraulic valve configurations and control schemes
- Evaluate failure modes, such as seal failure, fluid leakage, or component fatigue
- Simulate load scenarios before real-world deployment
- Optimize response times and damping characteristics

In industrial contexts such as construction equipment, aerospace landing systems, or automated manufacturing, such validated simulations can help engineers preemptively identify issues, improve performance, and reduce unnecessary component wear through optimized control strategies.

7.5 Future Scope and Enhancements

While the present study offers a robust foundation for understanding hydraulic cylinder behavior under dynamic loads, several opportunities exist to enhance the model further:

- **Environmental Conditions**: Incorporating temperature effects could allow the simulation to reflect fluid property changes over time. Oil viscosity and seal performance are known to vary with temperature, especially in outdoor or high-load environments.
- **Fluid Aging and Contamination**: Over time, hydraulic fluid degrades due to oxidation, water absorption, or particulate contamination. Modeling these changes could improve long-term performance prediction.
- **Seal Wear and Leakage**: Introducing wear models for piston and valve seals would allow for the simulation of performance degradation over time, crucial for lifecycle assessments.

- **Real-time Hardware-in-the-Loop (HIL) Testing**: By integrating the simulation with a real-time control system, researchers could use this model as part of a hardware-in-the-loop setup, enabling dynamic controller testing and system validation in hybrid physical-virtual environments.
- **Alternative Friction and Contact Models**: Exploring viscous or Stribeck friction models, or nonlinear damping functions, could yield more accurate dynamic responses, especially for lower-speed or micro-actuation systems.

Chapter 8

Appendix A

8.1 CODE FOR VALVE

```
component valve

nodes

    P = foundation.hydraulic.hydraulic; % P:left
    T = foundation.hydraulic.hydraulic; % T:left
    A = foundation.hydraulic.hydraulic; % A:left
    B = foundation.hydraulic.hydraulic; % B:left


end

inputs
    Uin = {0, 'V'}; % Uin:right
end

outputs
    U = {0, 'V'};
end

parameters

    Cv = {7e-8, 'm^3/(s*V*Pa^0.5)'};
    tau = {1e-4, 's'};
end

variables

     p_PA = {0, 'Pa'}; % Pp-PA
     p_AT = {0, 'Pa'}; % PA-PT
     p_PB = {0, 'Pa'}; % Pp-PB
     p_BT = {0, 'Pa'}; % PB-PT
     Q_PA = {0, 'm^3/s'}; % flow from P to A
     Q_AT = {0, 'm^3/s'}; % flow from A to T
     Q_PB = {0, 'm^3/s'}; % flow from P to B
     Q_BT = {0, 'm^3/s'}; % flow from B to T
end

branches

    Q_PA: P.q -> A.q;
```

```
    Q_AT: T.q -> A.q;
    Q_PB: P.q -> B.q;
    Q_BT: T.q -> B.q;

end

equations

    U.der == (Uin - U)/tau;
    if U>0
        Q_PA == Cv*U*sign(p_PA)*sqrt(abs(p_PA));
        Q_BT == Cv*U*sign(p_BT)*sqrt(abs(p_BT));
        Q_PB==0;
        Q_AT==0;
    elseif U<0
        Q_PA==0;
        Q_AT == Cv*U*sign(p_AT)*sqrt(abs(p_AT));
        Q_PB == Cv*U*sign(p_PB)*sqrt(abs(p_PB));
        Q_BT==0;
    else
        Q_PA==0;
        Q_AT==0;
        Q_PB==0;
        Q_BT==0;
    end
end
end
```

8.2 CODE FOR HYDRAULIC ACTUATOR

```
component hydro2
    % Custom hydraulic component

    % Nodes (ports)
    nodes
        A = foundation.hydraulic.hydraulic; %  A
        B = foundation.hydraulic.hydraulic; %  B
        C = foundation.hydraulic.hydraulic; % C
    end

    % Inputs
    inputs
        x1 = {0, 'm'};       % x1
        xdot1 = {0, 'm/s'}; % xdot1
        x2 = {0.044, 'm'};  % x2
        xdot2 = {0, 'm/s'}; % xdot2
```

```
    end

    % Outputs
    outputs
       F1 = {0, 'N'}; % F1


    end

    % Parameters
    parameters
       A1 = {0.004948, 'm^2'}; % STAGE 1 RIGHT SIDE
       A2 = {0.001909, 'm^2'}; % STAGE 1 LEFT SIDE

       l1 = {0.133, 'm'};      % Length of bore of first stage

       VO = {8e-4, 'm^3'};     % Dead volume
       Be = {900e6, 'Pa'};     % Bulk modulus
       eta = {0.9, '1'};       % Efficiency
       t= {0,'s'};             % time
    end

    % Variables
    variables
       pA = {80e5, 'Pa'}
       qA = {0, 'm^3/s'}; % Flow rate at port A
       pB = {0, 'Pa'};    % Pressure at port B
       qB = {0, 'm^3/s'}; % Flow rate at port B

    end

    variables (Access = private)
       % Private variables
    end

    % Branches
    branches
       qA : A.q -> * ; % Flow from A to B

    end

    % Equations
    equations
       % Define the relationships between pressures, flow rates, and
displacements
```

```
    let
      VA = if A1 * x1 <= VO, VO else A1 * x1 end;
      VB = if A2 * (l1 - x1) <= VO, VO else A2 * (l1 - x1) end;

    in
      qA == (pA.der * VA) / Be + A1 * xdot1;
      qB == (pB.der * VB) / Be + A2 * xdot1;

    end

    % Pressure relationships
    pA == A.p;
    pB == B.p;


    % Force equations
    F1 == (A1 * pA - A2 * pB) - xdot1 / {1, 'm/s'} * abs(A1 * pA - A2 *
pB) * (1 - eta);

  End
end
```

Chapter 8
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