# REDUCTION OF CROSS TERMS IN THE WIGNER-VILLE DISTRIBUTION USING VARIATIONAL MODE DECOMPOSITION

**M.Tech. Thesis**

By

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**DISCIPLINE OF ELECTRICAL ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY INDORE**

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# REDUCTION OF CROSS TERMS IN THE WIGNER-VILLE DISTRIBUTION USING VARIATIONAL MODE DECOMPOSITION

**A THESIS**

*Submitted in partial fulfillment of the requirements for the award of the degree*
***of***
**Master of Technology**

*with specialization in*

**Communication and Signal Processing**

*by*

**PREETI MEENA**



**DISCIPLINE OF ELECTRICAL ENGINEERING**
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# INDIAN INSTITUTE OF TECHNOLOGY INDORE

## CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**REDUCTION OF CROSS TERMS IN THE WIGNER-VILLE DISTRIBUTION USING VARIATIONAL MODE DECOMPOSITION"** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DISCIPLINE OF ELECTRICAL ENGINEERING, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July 2018 to July 2019 under the supervision of Prof. Ram Bilas Pachori, Indian Institute of Technology Indore, India.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

**Signature of the student with date**
**(PREETI MEENA)**

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This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

Signature of thesis Supervisor with date

**(Prof. RAM BILAS PACHORI)**

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**PREETI MEENA** has successfully given his/her M.Tech. Oral Examination held on 01/07/2019.

Signature(s) of the thesis Supervisor(s)
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Convener, DPGC
Date:

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# Abstract

This thesis focuses on the analysis of the non-stationary signals using the time-frequency distribution (TFD). The non-stationary signals contain time-varying parameters and conventional Fourier transform is not suitable for analysis of such signals. For such time-varying signals, TFD plays an important role for analysis. The TFDs provide information about the time-varying frequency components present in the signal. In this thesis, the Wigner-Ville distribution (WVD) method is studied to obtain the TFD of the signal. Theoretically, the WVD provides an infinite resolution in time and frequency domains. Due to bilinear nature of the WVD, it contains cross terms in time-frequency representation (TFR) for non-stationary signals.

This thesis presents two methodologies to reduce the cross terms in WVD for the analysis of non-stationary signals. These proposed methodologies are suitable for reducing cross terms in the analysis of non-stationary signals when their monocomponents are well separated in frequency domain for linear and non-linear frequency modulated (NLFM) signals, respectively. In the first proposed method, variational mode decomposition (VMD) decomposes a multicomponent, non-stationary signal into a number of sub-signals. These sub-signals have been converted into analytic sub-signals using the Hilbert transform. After this WVD has been computed to obtain the TFD of each analytic sub-signal. The summation of these computed TFD of all sub-signals provide cross terms free TFD of non-stationary signals.

In signals where the monocomponents are NLFM signals include intra cross terms in its TFD. To remove these cross terms, we need to introduce another method called time domain decomposition (TDD). The TDD further segments the modes obtained after decomposition of the NLFM signals using VMD in time domain and after that WVD is applied on each obtained sub-band to obtain the TFD of each component of the segment. At last, the final WVD has been obtained by summation of all the obtained WVDs. Now, this obtained WVD does not contain cross terms.

The results of the proposed method imply that cross terms are successfully suppressed in TFD of a multicomponent non-stationary signal. These results are compared with different methods such as smoothed pseudo Wigner-Ville distribution (SPWVD), Born-Jordan distribution (BJ), and Choi-Williams distribution (CWD). The proposed method have provided better TFR for the studied multicomponent non-stationary signals. The result are also presented in terms of Renyi information for the studied signals.

# Contents

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# List of Abbreviations

| | |
|---|---|
| **TFD** | Time-frequency distribution |
| **TFSAP** | Time-frequency signal analysis and processing |
| **STFT** | Short-time Fourier transform |
| **FT** | Fourier transform |
| **WT** | Wavelet transform |
| **WVD** | Wigner-Ville distribution |
| **FBSE** | Fourier-Bessel series expansion |
| **TQWT** | Tunable-Q wavelet transform |
| **HHT** | Hilbert-Huang transform |
| **PWVD** | Pseudo Wigner-Ville distribution |
| **SPWVD** | Smoothed pseudo Wigner-Ville distribution |
| **LFM** | Linear frequency modulated |
| **NLFM** | Non-linear frequency modulated |
| **TFR** | Time-frequency representation |
| **TFA** | Time-frequency analysis |
| **IMF** | Intrinsic mode function |
| **ADMM** | Alternate direction method of multipliers |
| **EMD** | Empirical mode decomposition |
| **EVD** | Eigenvalue decomposition |
| **EVDHM** | Eigenvalue decomposition of the Hankel matrix |
| **EWT** | Empirical wavelet Transform |
| **HT** | Hilbert transform |
| **EEG** | Electroencephalography |
| **EMG** | Electromyogram |
| **PPG** | photoplethysmogram |
| **TM** | Time marginal |
| **FM** | Frequency marginal |
| **TD** | Time delay |
| **IF** | Instantaneous frequency |
| **TF** | Time-frequency domain |
| **TS** | Time support |

| | |
|---|---|
| **FS** | Frequency support |
| **VMD** | Variational mode decomposition |
| **TDD** | Time domain decomposition |
| **mEMD-VMD** | modified EMD and VMD |
| **NBC** | Narrow band component |

# Chapter 1

# Introduction

In today's life, we find the non-stationary signals in many fields including radar, biomedical engineering, communication, speech analysis, sonar, heart sound analysis, and many others. Non-stationary signals are the signals which have parameters as a function of time. These signals can be represented as a combination of monocomponent non-stationary signals [1].

## 1.1 The need for a time-frequency distribution (TFD)

In general, there are two types of representation for signals: time-domain ($t$) and frequency-domain ($f$). Fourier transform helps us to obtain frequency domain from time domain and vice-versa. In this transform, variables $t$ and $f$ are treated as mutually exclusive. To obtain representation in terms of one variable, the other variable is "desegregated out". In time-frequency representation (TFR), time and frequency are not mutually exclusive but are present together. Therefore, the time-frequency distribution (TFD) based representation is localized in time domain and frequency domain.

The evaluation of these signals in frequency domain or time domain do not give the complete information. Along these domains, the investigation in time-frequency (TF) domain called time-frequency signal analysis and processing (TFSAP) is required for getting information of non-stationary signals [2] and such signals are represented by using TFD [3]. TFD shows the distribution of energy of the signal over the two-dimensional TF plane.

Various TFR based techniques are known [4, 5, 6] and are applied in many areas such as communications [7], speech signal processing [8], feature extraction for classification [9], biomedical signal processing [10, 11], motor diagnosis [12], seismology [13], and heart sound analysis [14]. The time-frequency toolbox is used for the implementation of studied TFD methods which has been is given in [15].

## 1.2 Overview of the existing techniques

There are different methodologies used to observe the TFD of signals [16] for example short-time Fourier transform (STFT) [17, 18], Hilbert-Huang transform (HHT)[19], wavelet transform (WT)[20, 21], modified empirical mode decomposition (EMD) and variational mode decomposition (VMD) (mEMD-VMD), Wigner-Hough transform [22], empirical wavelet transform (EWT) [13] and Wigner-Ville distribution (WVD). Since, Fourier transform (FT) has zero resolution in time domain and very high resolution in frequency domain. The STFT can be used to overcome this problem by windowing in TFD but provides fixed resolution in both time and frequency domain [17, 23]. A short frequency Fourier transform based fast technique is used to obtain the resolution similar to STFT laterally [24]. HHT based method for TFR is empirical in nature [25, 26]. The time-scale representation of the signal using WT is additionally different type of TFR [27]. In WVD, the autocorrelation is performed over signal before applying FT. It is used to analyze the time-varying signal in TF plane [28]. Recently, a TFR technique is presented in which eigenvalue decomposition of Hankel matrix (EVDHM) is taken into consideration for signal decomposition [29] which is suitable for complex signals in [30].

WVD has extremely high resolution in both time and frequency domain but its performance is degraded because of occurrence of cross terms [31, 32]. The cross terms appear because of its quadratic nature [33]. The appearance of these cross terms makes it difficult to recognize the auto-terms present in the signal. Hence, it is required to suppress the cross terms present in the signal. Two kinds of cross terms take place in WVD: one is intra cross terms and other one is inter cross terms. Cross terms occurs in non-linear frequency modulated (NLFM) signals which are known as intra cross terms and cross terms occurs in linear frequency modulated (LFM) multicomponent signals which are known as inter cross terms [33] [34]. Cross terms can not be removed totally, but it can be suppressed using various techniques [35]. Several applications of WVD are in signal classification [36, 37], gear fault diagnosis [38], detecting the patterns of ventricular late potentials [39], electromigration noise analysis [40], calibration of power frequency harmonic analyzers [41], identification of machine noise [42], speech processing [43, 44], measurement of transient signals [45], and physiological data [46].

Various techniques are given for the cross terms elimination in WVD [47, 48, 49, 50]. The Fourier-Bessel series expansion (FBSE) based technique for cross terms elimination in WVD is introduced in [28] [51]. A filtering technique using tunable-Q wavelet transform (TQWT) is applied for the elimination of cross terms in WVD [33]. A pseudo WVD (PWVD) based cross-term removal technique is given for signals having only two monocomponents signals [52]. The issues of PWVD are resolved in smoothed pseudo WVD (SPWVD) method but compromising the resolution [53]. A method based on EWT is also presented [54]. A method based on EMD and HHT is also introduced for cross-term reduction.

In this thesis, the aim is to remove cross terms in multicomponent LFM and NLFM signals. The VMD [55, 56, 57] method is used for subdividing, which results in narrow-band components (NBC) for a multicomponent non-stationary signal. These NBCs need to satisfy the conditions of a intrinsic mode function (IMF) [58] and the gained NBC, are concentrated at the center frequencies. It shows that VMD segments the signal and also provides the information about the center frequencies. Various applications of VMD are in enhancement of speech signals [59], instantaneous voiced/non-voiced detection in speech signals [58], denoising the signals [60, 61, 62, 63, 64], specific emitter identification [65], analysis of bearing fault [66, 67], rotor system fault [68], in seismic TFR [69]. VMD depends on the constrained variational optimization problem and it can be changed into an unconstrained optimization problem and afterward the solution of it can be given by using a strategy called alternate direction method of multipliers (ADMM) [56].

This VMD based method does not generate accurate results for NLFM signals. Therefore, a new method time domain decomposition (TDD) is introduced and an improved method is proposed in this thesis to suppress the cross terms based on the both methods VMD and TDD. This improved method successfully reduces the cross terms occurred in WVD of NLFM signals.

## 1.3 Motivation

The analysis of a non-stationary signal helps in understanding the properties of the system. Therefore, an efficient method is required for analyzing the signal and extracting all the information carried by the non-stationary signal. The TFR provides information about the changing pattern of the frequency of the signal. This information helps to understand the properties of a system. To find the frequency components of a signal FT is used, but FT fails to find any information regarding variations in the signal characteristics [70, 71, 72]. Therefore, it is not suitable for analysis of the non-stationary signals. Hence, a method is required to analyze these signals. The STFT is used for the analysis of non-stationary signals. In STFT, signal is separated into a small time intervals with the help of a window function and Fourier analysis is done for each segment. Then, the energy density over the TF plane is plotted [73]. The result of STFT strongly depends on the choice of window function used in it. The resolution provided by the STFT is fixed. Since, the nature of the spectrogram is quadratic, it suffers from the cross terms for the closely placed frequency components [74]. The fractional Fourier transform is also introduced for the analysis of these signals [75].

Another suitable method for the analysis of non-stationary signals is WVD. In WVD, windowing is not required. Theoretically, WVD provides an infinite resolution in time and frequency domain. It is not linear in nature which introduces cross terms in its output with significant amplitude and when there is more than one component present in the input signal, which distort the transform domains. This effect of cross terms might have serious effects in some applications, such as speech analysis because speech can be modelled as a sum of the amplitude modulated (AM) and frequency modulated (FM) signals corresponding to center frequencies. Therefore, an effective method to suppress the cross terms and improvement of the frequency resolution, without disturbing the desired properties of TFD is required.

Due to this problem of the occurrence of cross terms, it is necessary to decompose the non-stationary signal into small sub-signals (modes). So in this thesis, a method is used to reduce the cross terms of WVD, and a need to find an efficient methodology for the analysis of non-stationary signals, served as the motivation for this work.

## 1.4 Organisation of the thesis

The thesis contains five chapters that start with Chapter 1, which presents a brief introduction about time-frequency analysis (TFA) and the need for the analysis of signals, then an overview about the existing methods followed by the motivation for the research work. In Chapter 2, a detailed explanation of different TFR methods and VMD is discussed. Chapter 3, includes the proposed method to reduce the cross terms in WVD based TFR of non-stationary signals. In Chapter 4, simulation results and discussion have been provided for different signals. In this chapter performance of the proposed method is also compared with other method using Renyi information. Finally the last chapter of the thesis, Chapter 5 consists of the conclusion and the future work.

# Chapter 2

# Time-Frequency Distribution and Variational Mode Decomposition

In this chapter WVD, VMD and other TFD techniques which have been used for comparison are explained.

## 2.1 Wigner-Ville distribution (WVD)

In WVD, the autocorrelation operation is performed before applying FT [28]. Theoretically, it provides infinite resolution and practically very high resolution [31].

Now, for a non-stationary signal $r(t)$, the mathematical model for the WVD is given as follows [76, 77, 78, 79, 8]:

$$WVD(t,\omega) = \int_{-\infty}^{+\infty} r\left(t+\frac{\tau}{2}\right) r^*\left(t-\frac{\tau}{2}\right) e^{-j\omega\tau} d\tau \tag{2.1}$$

In eq. (2.1), $r^*(t)$ represents complex conjugate of $r(t)$. In the above equation it can be seen that WVD is bilinear in behaviour and because of this, cross terms are introduced in WVD for the NLFM monocomponent signal or multicomponent signal. Suppose signal $r(t)$ contains $(n_0)$ modes, where $n_0 = 1, 2, 3, \ldots .N_0$. The signal $r(t)$ is expressed as follows:

$$r(t) = \sum_{n_0=1}^{N_0} r_{n_0}(t) \tag{2.2}$$

The WVD of the signal $r(t)$ is given as follows [80]:

$$WVD(t,\omega) = \sum_{n_0=1}^{N_0} WVD(t,\omega) + 2\sum_{k=1}^{N_0-1}\sum_{l=k+1}^{N_0} \Re[WVD_{r_k r_l}(t,\omega)] \tag{2.3}$$

In (2.3), the first term represents auto-terms and second term represents cross terms in WVD. Total '$N_0$' auto-terms and $\binom{N_0}{2}$ cross terms. The occurrence of cross terms in WVD makes it difficult to retrieve auto-terms of signal. Furthermore, it makes the transform domain complex to explain [81, 82, 83]. In this manner, it is necessary to remove cross terms for getting auto-terms of signal.

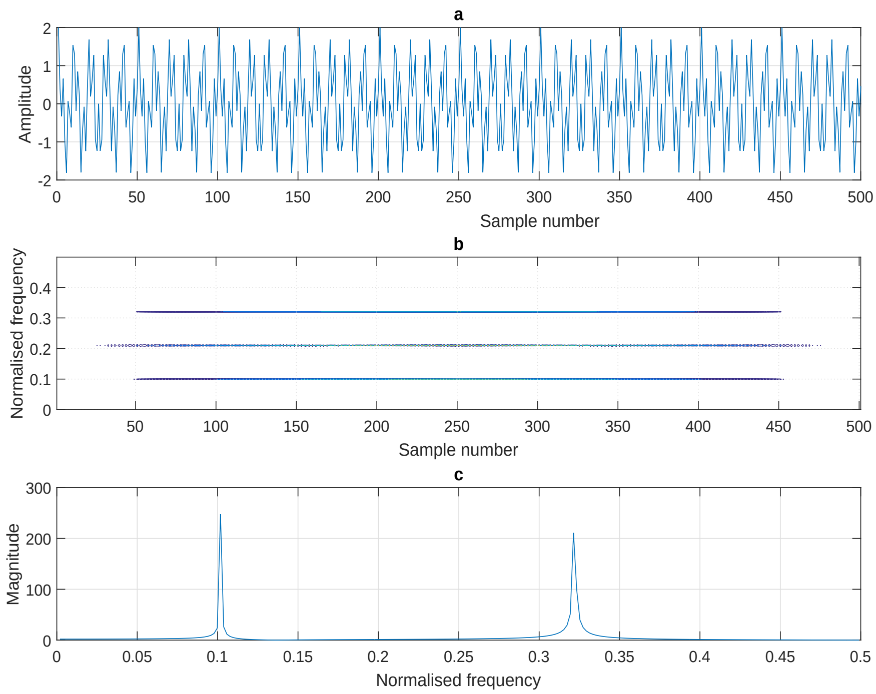


Figure 2.1: Plots of the: (a) Signal $x[n]$ in time domain, (b) WVD of signal $x[n]$, (c)Magnitude spectrum of $x[n]$

In order to explain WVD, we have considered following signal for study:

$$x[n] = \cos\left(\frac{200\pi n}{1000}\right) + \cos\left(\frac{640\pi n}{1000}\right) \tag{2.4}$$

The signal $x(n)$ contain two components whose frequency are constant over time, and separated in frequency domain. The range of $n$ varies from 0 to 500. The graphical representation of $x[n]$ in time-domain is shown in Fig. 2.1(a). Whereas its WVD is shown in Fig. 2.1(b) and the magnitude spectrum of the signal is shown in Fig. 2.1(c).

### 2.1.1 Properties of WVD

WVD is the most widely used TFD because of its properties. WVD satisfies several key properties of a joint energy density, but it is not guaranteed that the values are non-negative for all the signals in the TF plane. Positive values can be enforced by applying smoothing operation, this also removes the cross terms and perhaps lead to known positive quadratic distributions.

The properties of WVD are listed below [91].

1. Real-valuedness: $WVD_i(t, w)$ is real-valued for all $i$, $t$ and $w$. Further, WVD of the real signals is symmetric.

2. Marginality: The WVD satisfies the marginals in time and frequency domain.

   Time marginal (TM): The instantaneous power of WVD at a given time is the integration of WVD of a signal along the time axis.

   Frequency marginal (FM): The energy spectrum is given by integrating of WVD of a signal along the time axis.

3. Global energy: The total energy of the signal is given by integrating WVD over both time and frequency.

4. Non-negativity: It is the main property (or rather a drawback) of WVD. In WVD positivity is not guaranteed because it satisfies the marginals and there is no positive quadratic distribution exists that satisfies the time and frequency marginal integrals. To achieve the non-negativity, marginality has to be sacrificed.

5. Covariance: The two types of covariances are as follows:

Time-shift covariance: Shifting of time by some amount in the signal creates the shifting of time by same amount in the WVD of the signal.

Frequency-shift covariance: Shifting of frequency by some amount in the signal creates the shifting of frequency by same amount in the WVD of the signal.

6. Time delay (TD): The TD is given by the mean of the WVD with respect to the time.

7. Instantaneous frequency (IF): For an analytic signal, the IF is given by the mean of the WVD with respect to the frequency.

8. Finite support: WVD satisfies the support as both time support (TS) and frequency support (FS)

   The TS of $WVD_i(t, w)$ is limited by the duration of the signal $i(t)$ which is non-zero over a finite interval $t_1 < t < t_2$ and zero outside an interval i.e for $t < t_1$ and $t > t_2$. then $WVD_i(t, w)$ is also zero for $t < t_1$ and $t > t_2$.

   The FS of $WVD_i(t, w)$ is limited by the bandwidth of the signal $i(t)$. i.e if the FT of $i(t) = i(w)$ is non-zero over a finite interval $w_1 < w < w_2$ and zero outside an interval i.e for $w < w_1$ and $w < w_2$. then $WVD_i(t, w)$ is also zero for $w < t_1$ and $w < t_2$.

9. Unitarity property: The WVD satisfies the unitary property. i.e it preserves the inner products.

### 2.1.2 Drawbacks of WVD

The three main drawbacks of WVD are [91]:

1. Non-local nature of WVD: Uniform weighting is given to instants in future and past.

2. Lack of positivity: The values of WVD may be positive or may be negative in TF plane.

3. Cross terms: If the signal have more than one frequency then its WVD is constitute of cross-components.

## 2.2 Variational mode decomposition (VMD)

The VMD is a non-recursive method which is applied for multicomponent signal decomposition. It is an adaptive technique. An input signal $G(t)$ is decomposed into a set of '$i$' number of segments $g_i(t)$ are called modes. This analysis uses frequency related insights. In addition to the decomposition of the signal, it also calculates center frequencies $\omega_i$, where $i = 1, 2, 3, ..., I$ and the bandwidth in spectral domain is considered as a particular sparsity property of every mode while reconstructing the original signal. For calculating the bandwidth of a sub-signal, VMD method uses the constrained optimization problem. Constrained optimization algorithms use the single-variable and multi-variable optimization algorithms repeatedly and simultaneously maintain the search effort inside the feasible search region. Region is decided by the lower and upper bounds. Steps required to compute the bandwidth of a component are explained as follows:

1. The positive frequency spectrum of the $g_i(t)$ mode is obtained by applying Hilbert transform (HT).

2. Shift the frequency spectrum of a mode to baseband by multiplying with an exponential function which is tuned to the corresponding computed center frequencies.

3. The $H^1$ Gaussian smoothness of the demodulated signal is applied to obtain the bandwidth of each mode, i.e. the $L^2$-norm of the gradient [66, 84].

The resulting constrained variational problem are defined as given below:

$$\begin{aligned} &\min_{\{g_i\},\{\omega_i\}} \left\{ \sum_{i=1}^{I} \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) * g_i(t) \right] e^{-j\omega_i t} \right\|_2^2 \right\} \\ &\text{Where} \quad \sum_{i=1}^{I} g_i(t) = G(t) \end{aligned} \tag{2.5}$$

The lagrangian multiplier $\lambda$ and the penalty factor $\alpha$ are applied to transform the constrained optimization problem Eq. 2.5 into unconstrained optimization problem. The unconstrained optimization problem can be written as follows:

$$\begin{aligned}\mathcal{L}\left(\{g_i\},\{\omega_i\},\lambda\right):=\alpha\sum_i\left\|\partial_t\left[\left(\delta(t)+\frac{j}{\pi t}\right)*g_i(t)\right]e^{-j\omega_i t}\right\|_2^2\\+\left\|G(t)-\sum_i g_i(t)\right\|_2^2+\left\langle\lambda(t),G(t)-\sum_i g_i(t)\right\rangle\end{aligned}\tag{2.6}$$

Solution of Eq. 2.5 is calculated as the saddle point of the Eq. 2.6 using the strategy ADMM. Next mode and corresponding updated center frequency can be calculated using the following expressions:

$$\hat{g}_i^{n+1}(\omega)=\frac{\hat{G}(\omega)-\sum_{k\neq i}\hat{g}_k(\omega)+\frac{\hat{\lambda}(\omega)}{2}}{1+2\alpha(\omega-\omega_i)^2}\tag{2.7}$$

$$\omega_i^{n+1}=\frac{\int_0^\infty\omega|\hat{g}_i(\omega)|^2d\omega}{\int_0^\infty|\hat{g}_i(\omega)|^2d\omega}\tag{2.8}$$

In Eq. 2.7 and 2.8, $\hat{G}(\omega)$, $\hat{g}_k(\omega)$, $\hat{\lambda}(\omega)$, and $\hat{g}_i^{(n+1)}(\omega)$ are the FT of $G(t)$, $g_k$(t), $\lambda$(t), and $g_i^{(n+1)}$ (t) respectively. Eq. 2.7 includes the Wiener filter structure [56, 58, 59, 66, 84, 85, 86].

The MATLAB program for VMD method is provided in [87]. Information parameters in this VMD code given as an input are the required number of modes ($i$), total DC components, the signal in time domain whose decomposition requires, the balancing parameter of the data-fidelity constraint ($\alpha$), the convergence criterion tolerance (tol), time step of the dual ascent (tau), and the center frequency initialization $\omega$ (init). To converge, this given strategy utilized 500 iterations [58].

### 2.2.1 Input parameters in VMD method

1. Input signal ($s$): The signal in the time domain whose decomposition into sub-signals/modes is needed.

2. Penalty factor ($\alpha$): The penalty factor in the VMD method is inversely proportional to the bandwidth of the components present in the input signal.

There is no sharp rule to choose the value of $\alpha$ since the value essentially depends on the amplitude and the noisiness of the data. The larger value of $\alpha$ is not effective to estimate the center frequencies of the components with precision and the lower value of $\alpha$ is not good for noise robust components analysis. To extract the fewer components with the accurate value of center frequencies, a lower value of $\alpha$ is preferred.

3. The time step of the dual ascent ($\tau$): The parameter $\tau$ in the VMD method is used to control the Lagrangian multiplier $\lambda$ i.e $\tau$ governs how quickly the $\lambda$ accumulates the reconstruction error. Widely used values of $\tau$ is either 0 or 0.1. a high value of $\tau$ is not preferred because it may lead to fast freezing of the modes. The relation between $\tau$ and $\lambda$ can be given by mathematical expression shown below [56]:

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^{n}(\omega) + \tau \left( \hat{m}(\omega) - \sum_{k} \hat{y}_k^{n+1}(\omega) \right), \qquad \text{for all } \omega \geq 0 \tag{2.9}$$

If the exact reconstruction of a signal is not required for the analysis of the signal using VMD, then the outcome of the $\lambda$ can be nullified by taking the value of $\tau$ equal to zero.

4. The number of modes to be extricate ($K$): To sub-divide a signal using VMD method, the initial information about the number of modes $K$ is needed.

5. The numbers of DC components ($DC$): The input parameter $DC$ in VMD method is used to find or extricate the DC components present in the signal.

6. The tolerance for convergence criterion ($tol$): The input parameter $tol$ is used for the convergence in VMD method. It manage the reconstruction error of the mode.

7. The initial frequencies for the extracted modes ($\omega_{\text{init}}$): The $\omega_{\text{init}}$ has two initial values $\omega_{\text{init}} = 1$ and $\omega_{\text{init}} = 0$ for initialization of frequencies. $\omega_{\text{init}} = 1$ represents the initialization based on uniform distribution. Whereas $\omega_{\text{init}} = 0$ represents the zeros.

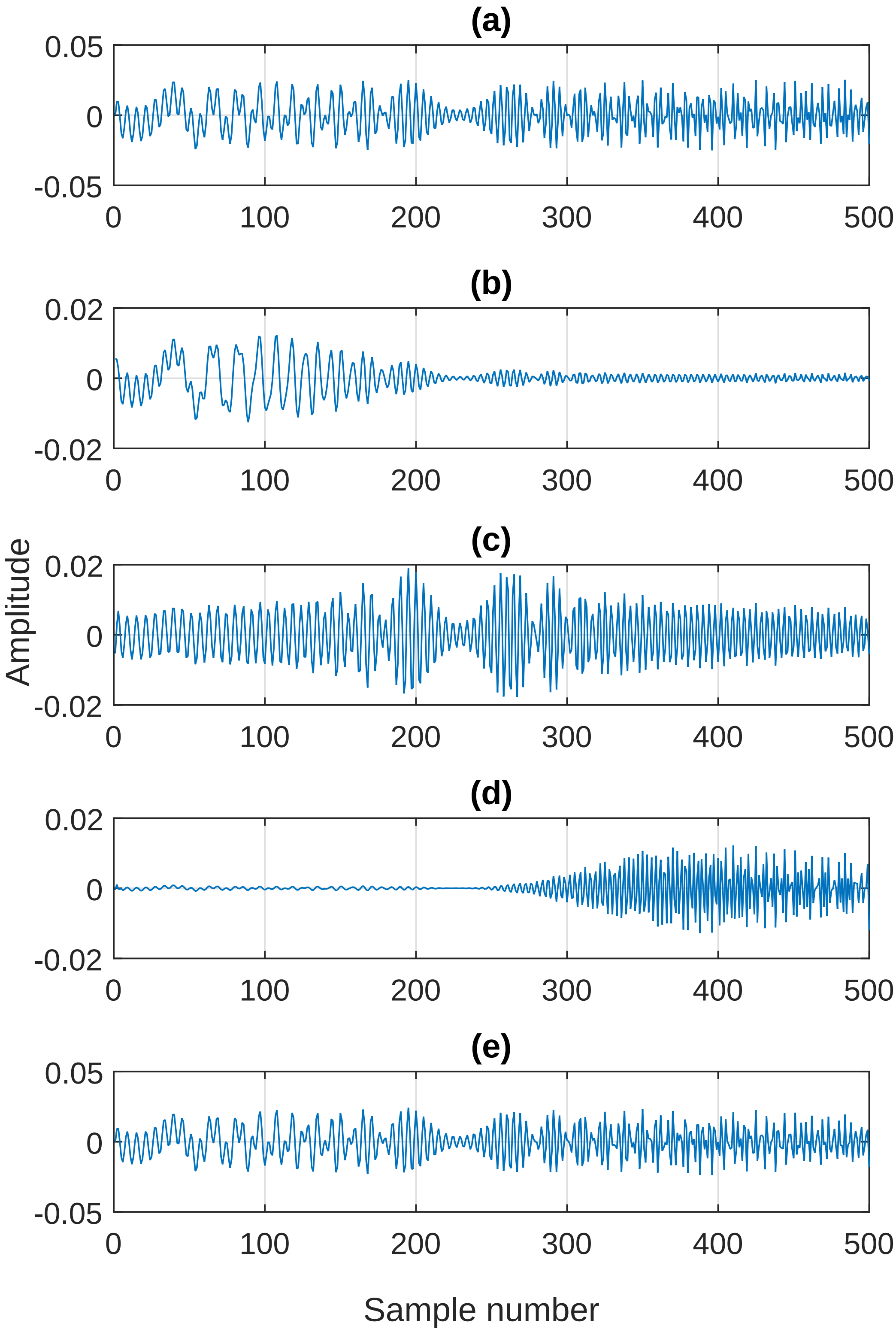


Figure 2.2: (a) Signal, (b) Decomposed mode 1, (c) Decomposed mode 2,(d) Decomposed mode 3, (e) Reconstructed signal

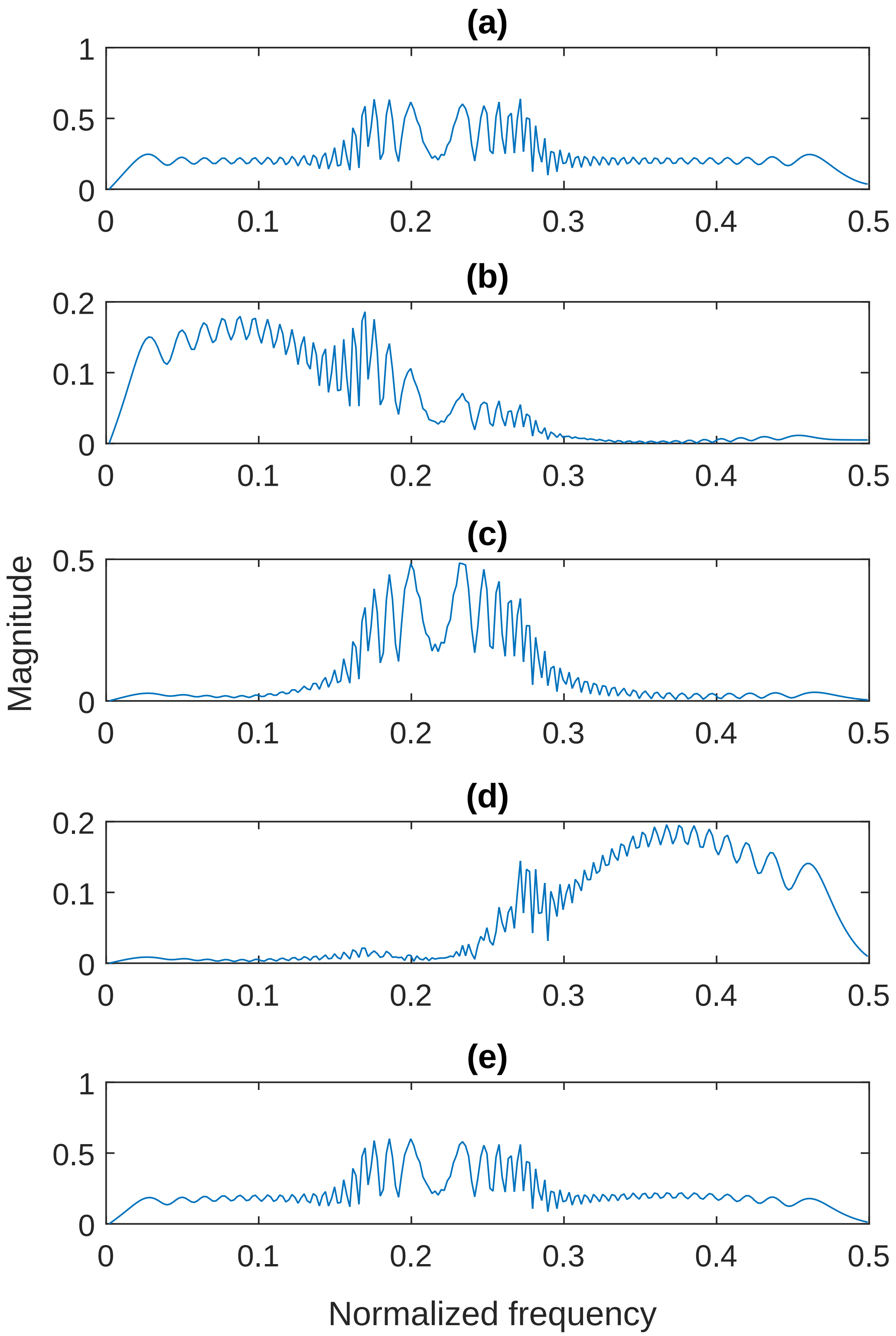


Figure 2.3: Magnitude spectrum of (a) Signal, (b) Decomposed mode 1, (c) Decomposed mode 2,(d) Decomposed mode 3, (e) Reconstructed signal

### 2.2.2 Example for VMD method

The process of VMD is explained earlier. Now the results of VMD are shown by considering an example of a signal shown in Fig. 2.2(a). The mathematical expression is given by below equation:

$$f = \frac{1}{79}\left(sin\left(\left(\frac{\pi n}{40}\right) + 155\right)\frac{2n}{50}\right) + \frac{1}{79}\left(sin\left(\left(\frac{2\pi n}{80}\right) + 99\right)\frac{n}{100}\right) \tag{2.10}$$

Input parameters namely $\alpha$, $\tau$, $K$, $DC$, $\omega_{\text{init}}$ and $tol$ have been fixed to 60, 0, 3, 0, 1, $10^{-7}$. This means VMD decomposes this signal into three modes. Mode 1 is represented by Fig. 2.2(b), mode 2 is represented by Fig. 2.2(c) and mode 3 is represented by the Fig. 2.2(d). After decomposition, the signal is reconstructed by adding all three modes together. The reconstructed signal is shown in Fig. 2.2(e). On observing the reconstructed signal we observe that it is similar to the original signal with negligible error. Fig. 2.3(a) shows the magnitude spectrums of signal, Figs. 2.3(b-d) show the magnitude spectrums of all three modes and magnitude spectrum of reconstructed signal is shown in Fig. 2.3(e)

## 2.3 Smoothed pseudo Wigner-Ville distribution

It is also called modified WVD. In WVD expression (2.1), the range of $\tau$ is from $-\infty$ to $+\infty$ which is not practical in real life applications. So, the integral in the WVD is modified to include a window function. When a smoothing window is moving forward over the WVD of a signal in the direction of increasing frequency then it gives a new distribution known as PWVD [88, 89, 90]. Smoothing window is moves forward in the direction of both increasing time and frequency, then it gives another distribution known as SPWVD [91, 92, 93].

The equation for PWVD of the signal $b(t)$ in the time domain is given as:

$$PWVD_b(t,\omega) = \int_{-\infty}^{+\infty} b\left(t+\frac{\tau}{2}\right) b^*\left(t-\frac{\tau}{2}\right) w\left(\frac{\tau}{2}\right) w^*\left(\frac{\tau}{2}\right) e^{-j\omega\tau} d\tau \tag{2.11}$$

Where $w(t)$ is the window. It helps in weakening the strength of the cross terms in the time domain by performing the frequency-smoothing operation. However, the cross terms are present in the frequency domain and these existing cross terms can be reduced by using the time-smoothing function resulting in SPWVD and it can be represented by the below expression [94],

$$SPWVD = \int_{-\infty}^{+\infty} \hat{w}(t-t')PWVD(t',\omega)dt' \tag{2.12}$$

where, $\hat{w}(t)$ represent the time smoothing function. In the above equation, an infinite length expression is converted into a finite length expression.

### 2.3.1 Drawback of SPWVD

The window function used in SPWVD helps in removing the cross terms. The drawback of the windowing is that the more you smooth the WVD, the resolution will suffer more and more in time domain, frequency domain and TF domain. Therefore, the resolution of the SPWVD is poor.

## 2.4 Born-Jordan (BJ) distribution

The Born-Jordan distribution is also a type of bilinear TFD introduced to suppress the cross terms present in WVD due to its quadratic nature. It satisfies the marginal property. The kernel function used in it can be given as follows [95]:

$$p(\theta,\tau) = \frac{\sin(\frac{\theta\tau}{2})}{\frac{\theta\tau}{2}} \tag{2.13}$$

This distribution gives the high resolution in TFR and its mathematical expression is given as [95],

$$TFD_B J(t,f) = \frac{1}{2\pi}\int \frac{1}{\mid\tau\mid} e^{-j2\pi f\tau} \int_{t-\frac{|\tau|}{2}}^{t+\frac{|\tau|}{2}} r\left(u+\frac{\tau}{2}\right) r^*\left(u-\frac{\tau}{2}\right) du d\tau \tag{2.14}$$

## 2.5 Choi-Williams distribution

The Choi-Williams distribution (CWD) is a type of bilinear TFD similar to WVD. The distribution used the exponential kernel to remove the cross terms therefore it also called as exponential distribution. The kernel function used in CWD can only filter out the cross terms that result from the components that differ in both time and frequency center [96]. To obtain the determining function $A(u, \tau)$ the kernel function can be modified by multiplying an exponential term as follows [54]:

$$A(u, \tau) = \int_{-\infty}^{+\infty} a(\Theta, \tau) e^{j\pi\Theta u} d\Theta \tag{2.15}$$

The expression of CWD is as follows:

$$CWD(t, f) = \int \int_{-\infty}^{+\infty} r\left(u + \frac{\tau}{2}\right) r^* \left(u + \frac{\tau}{2}\right) A(u - t, \tau) e^{-j2\pi f\tau} du d\tau \tag{2.16}$$

The kernel function and determining function can be expressed as:

$$a(\Theta, \tau) = e^{\frac{\Theta^2 \tau^2}{\sigma}} \tag{2.17}$$

$$A(t, \tau) = \frac{\sqrt{\frac{\sigma}{\pi}}}{2\tau} e^{\frac{\sigma t^2}{4\tau^2}} \tag{2.18}$$

Where $\sigma$ is positive parameter controlling the concentration of $CWD(t, f)$ around the origin of the TF plane. Hence large value of $\sigma$ results in less smoothing.

The above three discussed methods can be explained with the help of the following example, which we used to compare with our proposed method.
Let us consider the signal $x[n]$ taken in WVD section which is mathematically expressed in Eq. 2.4. WVD of $x[n]$ is shown in Fig. 2.1(b). The TFD of $x[n]$ using SPWVD, BJ and CWD are shown in Figs. 2.4(a), 2.4(b) and 2.4(c).

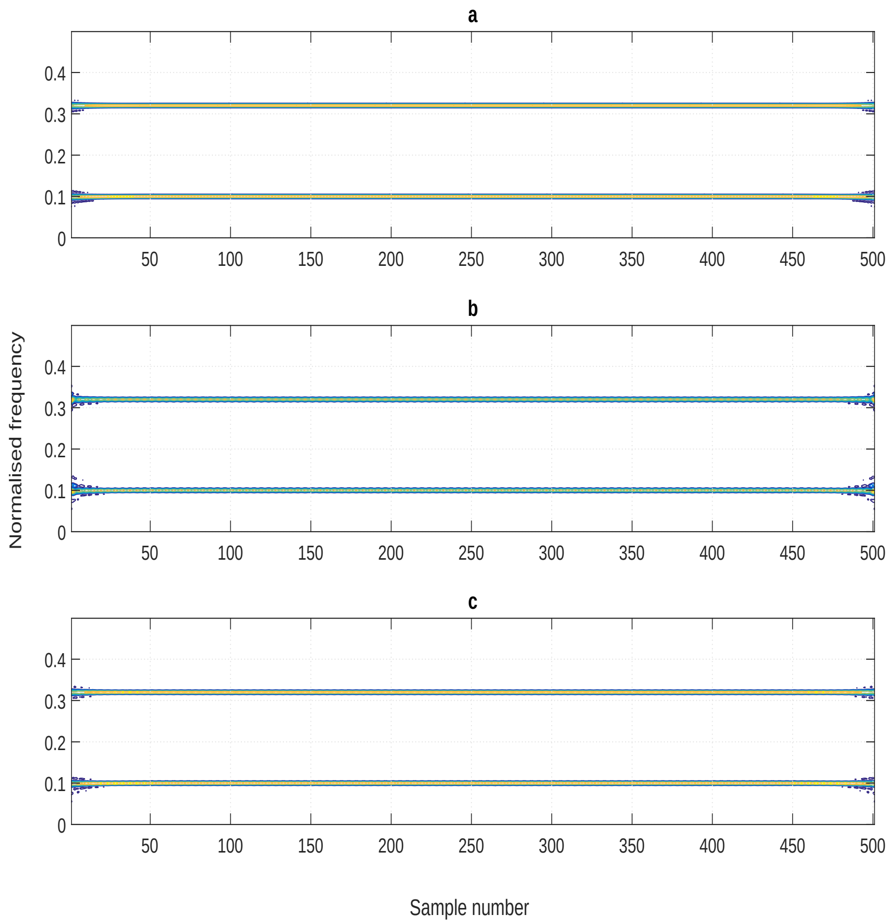


Figure 2.4: Plots of the: (a) TFD of $x[n]$ using SPWVD, (b) TFD of $x[n]$ using BJ, (c) TFD of $x[n]$ using CWD

## 2.6 Summary

In this chapter various TFD methods are explained in detail: WVD, SPWVD, JB and CWD. Along with these, VMD is also discussed, which is used for decomposing the signal into sub-signals. The drawback of WVD is that it provides cross terms in TFR. SPWVD, BJ and CWD are some of the methods used for reducing cross terms in TFR.

In this chapter, the TFR using SPWVD, BJ and CWD is shown by considering two signals. The results obtained using these methods are compared with the result of WVD. By comparing the results, it can be concluded that SPWVD, BJ and CWD provide cross-term free TFR, but the resolution is poor.

# Chapter 3

# Proposed Methods for Cross Terms Free TFR

## 3.1 Introduction

The TFR obtained using WVD has cross terms in it. So in order to suppress these cross terms, method proposed in this thesis which is based on VMD. This method has some limitations, to overcome these limitations another method with some modifications to previous one is used. VMD is used for the decomposition of a signal into NBC. In addition with that, obtained NBC are concentrated around center frequencies. These NBCs need to satisfy the conditions of IMF. This shows that VMD subdivide the signal as well as it additionally provides information about the center frequencies. VMD has numerous uses such as: enhancement of speech signal [59], instantaneous voiced/non-voiced detection in speech signals [58], denoising the signal [60, 61, 62, 63, 64], specific emitter identification [65], analysis of bearing fault [66, 67], fault of rotor system [68], seismic time-frequency analysis [69].

VMD is based on constrained variational optimization problem and to find the solution of this, first problem is changed into an unconstrained optimization problem. The solution of this problem can be solved by using a convex optimization method called ADMM.

In another method a new method TDD is used. TDD is used for sub-dividing sub-

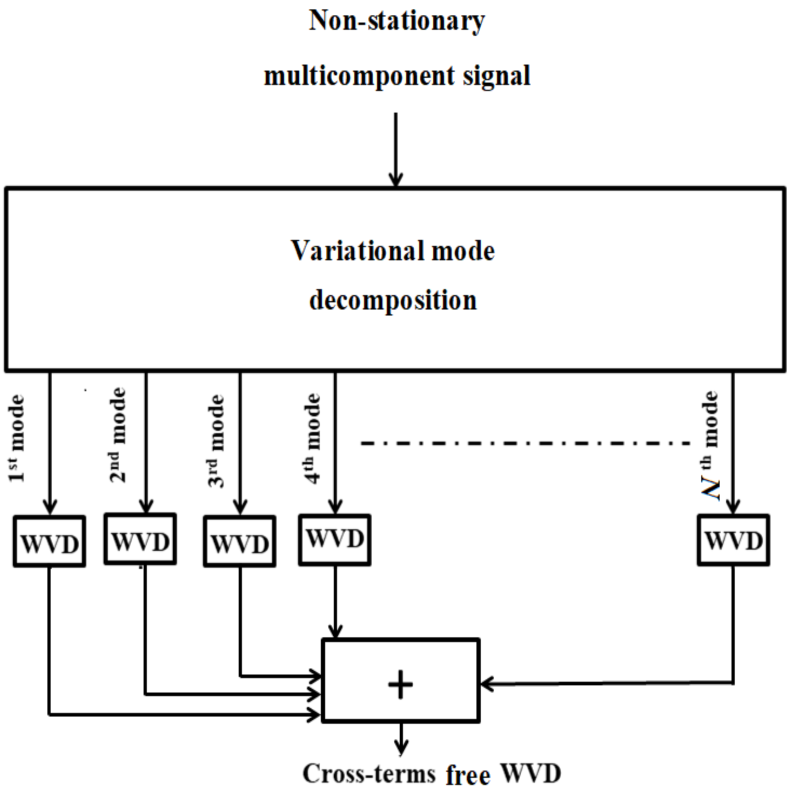


Figure 3.1: Proposed method 1 for disappearance of cross terms in WVD

signal obtained from VMD in the time domain. In both proposed methods, WVD and VMD methods are used. Explanation of these methods are provided in Chapter 2

## 3.2 Proposed method

In this section, two methods are introduced to suppress the cross terms present in TFR of the signal. First method is based on the VMD method and second one is based on the both methods TDD and VMD. The need of introducing second method and the advantage of using second method over the first method are also provided.

### 3.2.1 Proposed method 1

In this proposed method, to get cross terms free TFR, the signal is subdivided into N number of modes with the help of VMD method. Working process of VMD is

explained earlier (Section 2.2). Thereafter WVD of these sub-signals is computed. Finally, by taking the summation of WVD of all the sub-signals, the resultant WVD is obtained which is free from cross terms. In this way, the cross terms in WVD is suppressed with the help of this proposed methodology.
The block diagram of the proposed method 1 is shown in Fig. 3.1

### 3.2.2 Proposed method 2

In this proposed method, above explained method is improved so that it can be also applicable on NLFM signals to remove the intra-interference. In this TDD method is introduced after the VMD method. The sub-signals obtained after applying VMD, contains the component which are not separated in time domain. This non separable components results in cross terms. So to remove these cross terms TDD is used. TDD segments the input sub-signal into monocomponents which are disjoint in time-domain [33]. After the TDD, WVD is applied on each segmented sub-signal and in the end, the WVD of all the obtained segmented-components are added to obtain a cross terms free WVD.
The block diagram of the proposed method 2 is shown in Fig. 3.2
The energy based method is used in TDD. In this energy based method, cumulative energy of the signal at each sample gives information about the presence of the components. Instead of taking square of the signal directly, the cumulative sum of squares of signal is taken. By doing this it can reduce the errors due to the zero-crossings in segmentation of the components of signals. The mathematical expression of the cumulative sum $CS_l[n]$ is given as [33]:

$$CS_l[n] = \sum_{k=0}^{n-1} M_l^2[k] \tag{3.1}$$

Where $M_l$ are the modes obtained from the VMD method. Smoothing of the cumulative energy function of signal can be done with the help of the moving average filter. This can be done to reduce the zero-crossings occur during the disjoint of the components in time domain in same mode of VMD.
The filtered output of the moving average filter is represented by $FCS_l[n]$ and it is

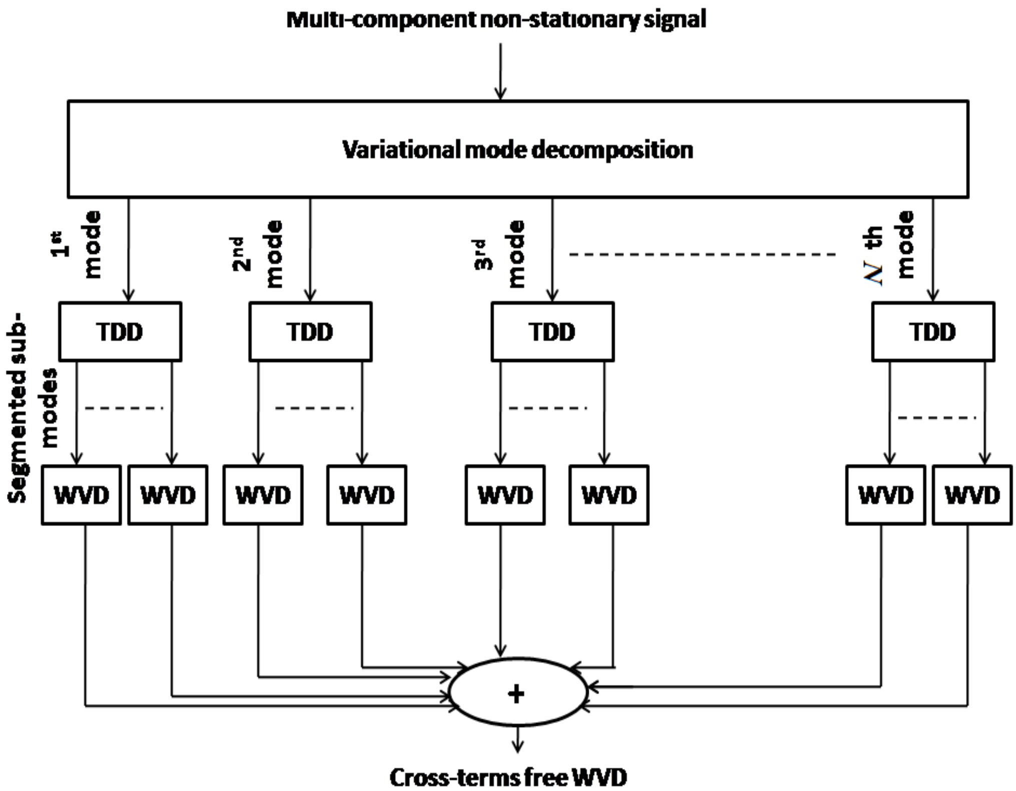


Figure 3.2: Proposed method 2 for disappearance of cross terms in WVD

given by the following expression [33]:

$$FCS_l[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} CS_l[n+k] \tag{3.2}$$

After the filtering operation, differentiation operation has been performed on the filtered output of the moving average filter ($FCS_l[n]$). The derivative of $FCS_l[n]$ is denoted by $DFCS_l[n]$ and it is computed as follows:

$$DFCS_l[n] = FCS_l[n] - FCS_l[n-1] \tag{3.3}$$

Now again the smoothing operation is performed on the $DFCS_l[n]$ and normalization of the smoothed $DFCS_l[n]$ can be done. After performing all these operations, thresholding can be done by comparing the normalized $DFCS_l[n]$ with the chosen threshold value $C_{th}$ as follows.

$$F_l[n] = \begin{cases} 1, & DFCS_l[n] > C_{th} \\ 0, & DFCS_l[n] \leqslant C_{th} \end{cases} \tag{3.4}$$

It is clear from the above expression that $F_l[n] = 1$ when $DFCS_l[n] > C_{th}$ which indicates the presence of the component in the segmented sub-signal (sub-mode) and $F_l[n] = 0$ when $DFCS_l[n] \leqslant C_{th}$ which indicates the absence of the component in the sub-modes. The modes or sub-signals obtained by decomposing the signal with the help of VMD method is given as an input to TDD.

Sub-signal may carry the components separated in the time domain in the same sub-signal. So, the information about the location of these component can be found by observing the value of $FCS_l[n]$. If the components are present then for that interval the value of $FCS_l[n]$ would be increasing, whereas the value of $FCS_l[n]$ remains unchanged or constant if the components are not present. As we observe that the value of $F_l[n]$ depends on the $DFCS_l[n]$ (derivative of the $FCS_l[n]$). Therefore, the value of $F_l[n] = 1$ is only when the $FCS_l[n]$ is increasing which indicates the existence of the component and the value of $F_l[n] = 0$ is only when the $FCS_l[n]$ is constant which indicates the non-existence of the component.

This sequence of one's and zeros have been used for finding the starting and the terminating locations of the components in the same sub-signal obtained from VMD method in order to separate them in time domain. The value of threshold $C_{th}$ depends on the noisiness of the signal. If the signal is noisy then the value of $C_{th}$ is larger as compared to the value of $C_{th}$ chosen for noise-free signal.

### 3.2.3 Summary

In this chapter, two methods: proposed method 1 and proposed method 2 are discussed. The proposed method 1 to suppress cross terms in VMD based WVD is discussed in the first section of this chapter. This method has some limitation, that this method is not suitable to reduce intra cross terms. The aim of the proposed method 1 is to suppress the cross terms in multicomponent LFM signals. To remove the limitation of the proposed method 1, a new method (proposed method 2) is introduced to suppress cross terms in the WVD using TDD and VMD. The detailed explanation of TDD method with mathematical expressions is given in this chapter. The proposed method 2 is applicable on both multicomponent LFM signals and NLFM signals.

The experimental results of these methods are shown in the next chapter. It can be observed that the performance of both the proposed methods are better than the other methods discussed in Chapter 2.

# Chapter 4

# Experiment Results

The efficiency of the proposed method is observed by considering the following non-stationary multicomponent LFM and NLFM signals. This methodology is also applied on a real bat-echo signal to observe the performance of this method on real signals. Also to see the performance, results of this method are compared with the results of the other methods SPWVD, BJ and CWD. The TFR of all the methods are also shown with the representative examples.

## 4.1 Linearly frequency modulated multi-component signal

The signal $f_1$ having two LFM signals and both the components of $f_1$ are well separated in frequency domain. A signal $f_1$ is expressed as follows:

$$f_1 = V(\cos(W\pi n + 1)Xn) + V(\cos(Y\pi n + Z)Ln) \tag{4.1}$$

Where, $V = \frac{1}{120}$, $W = \frac{1}{2500}$, $X = \frac{12}{8}$, $Y = \frac{3}{200}$, $Z = 190$ and $L = \frac{1}{250}$. In above equation $n$ ranges from 0 to 500. The signal $f_1$ is displayed in Fig. 4.1 and its WVD is displayed in Fig. 4.2. It is noticed that cross terms are occurred in between the auto-terms. The TFR using SPWVD, BJ and CWD are shown in Fig.4.3(a), Fig. 4.3(b), Fig. 4.3(c) respectively. The TFR using proposed method is shown in Fig. 4.3(d) which is cross terms free and only auto-terms are present.

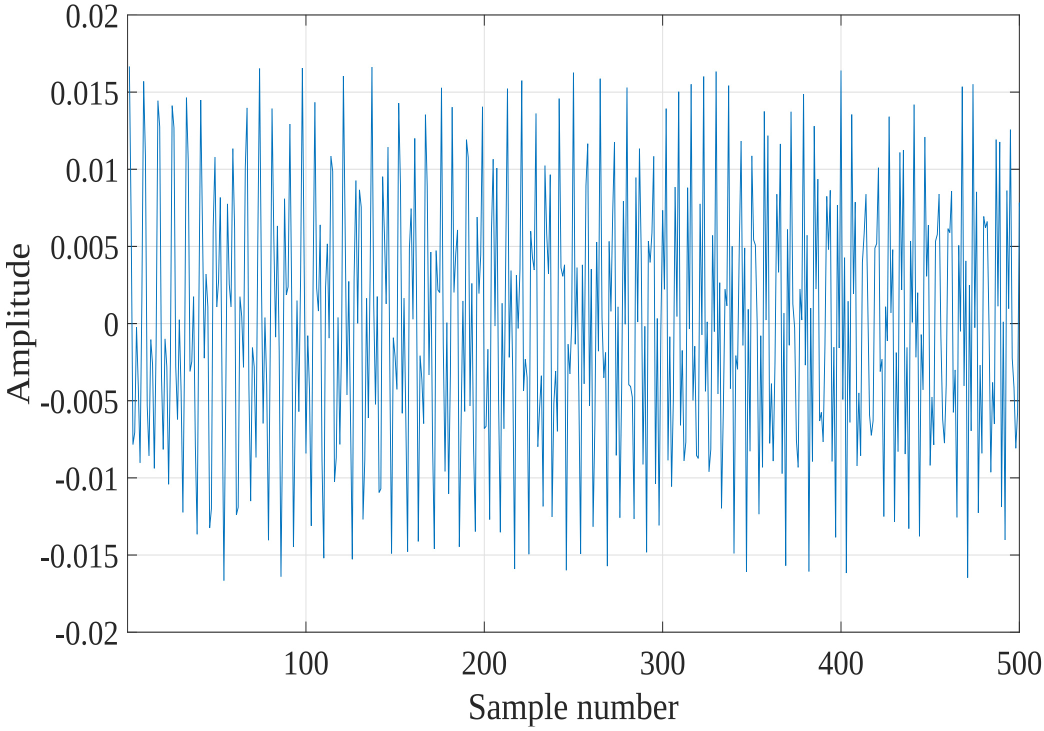


Figure 4.1: Plot of signal $f_1$

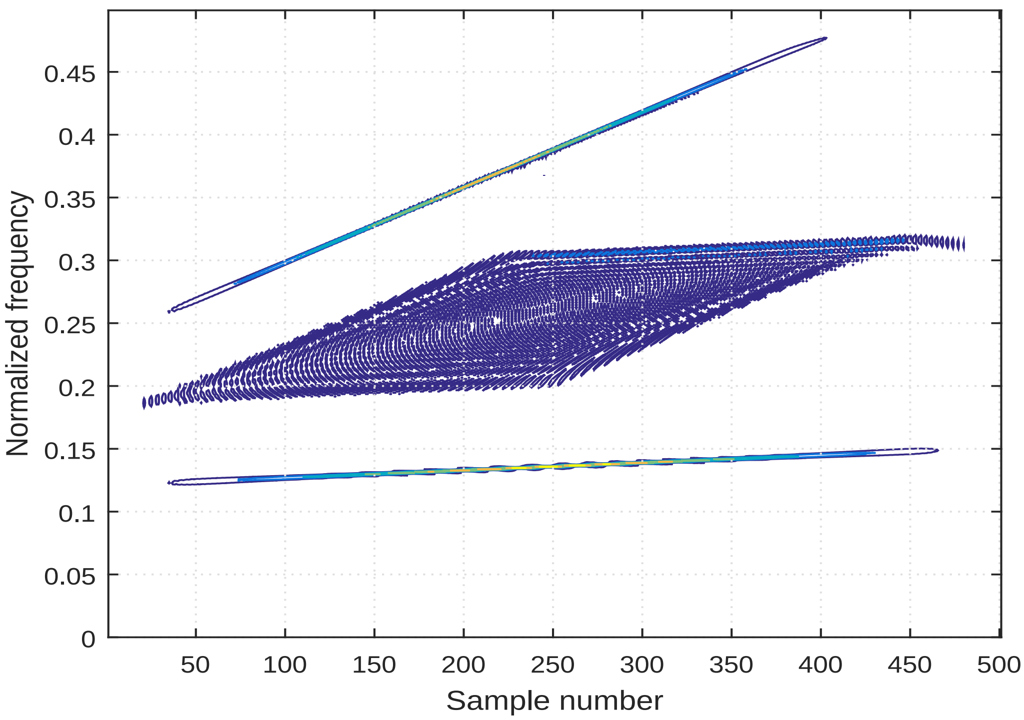


Figure 4.2: WVD of $f_1$

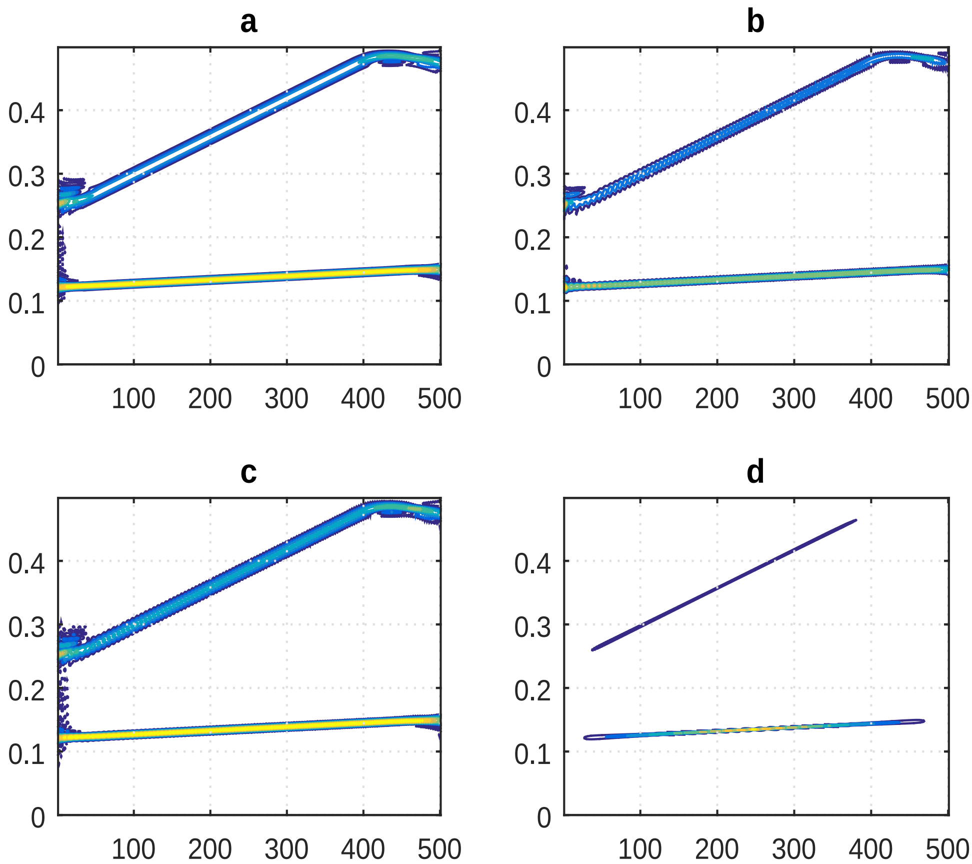


Figure 4.3: The TFD of $f_1$ utilizing (a) SPWVD (b) BJ distribution (c) CWD (d) Presented method

Second signal $f_2$ is also having two LFM signals but the rate of frequency modulation is more for the components of signal $f_2$. The sign $f_2$ is expressed as follows:

$$f_2 = P(\cos(Q_1\pi n + 1)Q_2 n) + P(\cos(Q_3\pi n + Q_4)Q_5 n) \tag{4.2}$$

Where, $P = \frac{1}{99}$, $Q_1 = \frac{1}{200}$, $Q_2 = \frac{3}{50}$, $Q_3 = \frac{3}{100}$, $Q_4 = 188$ and $Q_5 = \frac{1}{100}$. In Eq. (4.2), $n$ varies from 0 to 500. The graph of signal $f_2$ is displayed in Fig. 4.4 and TFD of signal $f_2$ using WVD is displayed in Fig. 4.5. The cross terms is occurred in between the auto-terms. The TFR using SPWVD, BJ and CWD are shown in Fig. 4.6(a), Fig. 4.6(b), Fig. 4.6(c) respectively. The TFD using the proposed method is displayed in Fig. 4.6(d), respectively. in these Figs. it can be observe that the proposed method performs better in comparison with the SPWVD, BJ and CWD method for LFM signals.

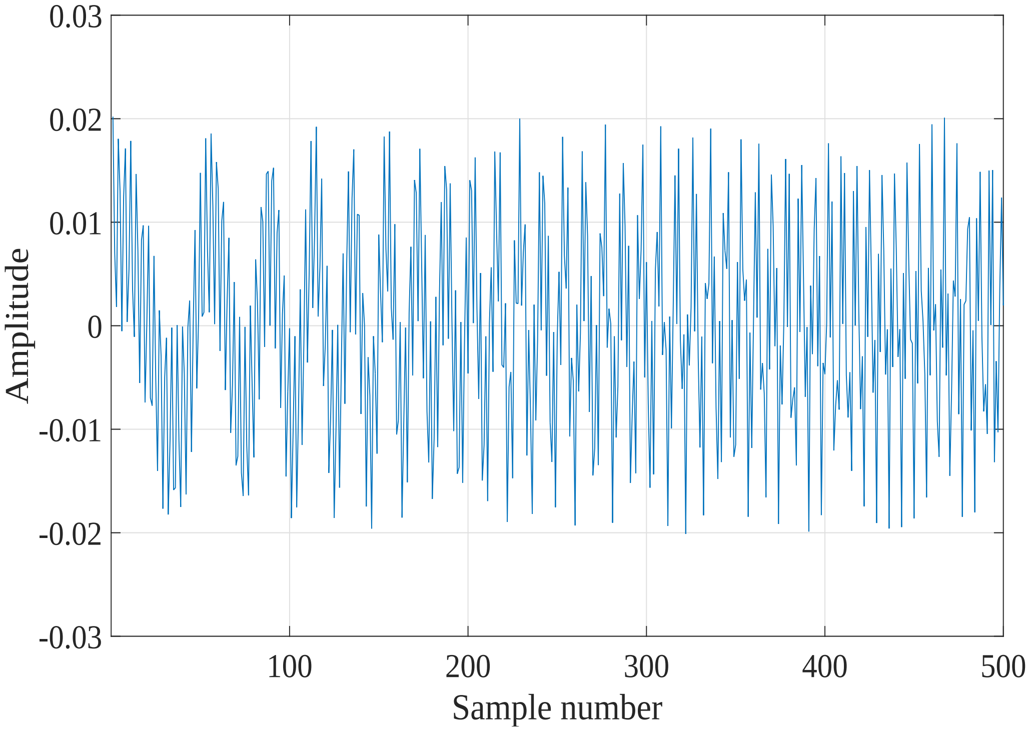


Figure 4.4: Plot of signal $f_2$

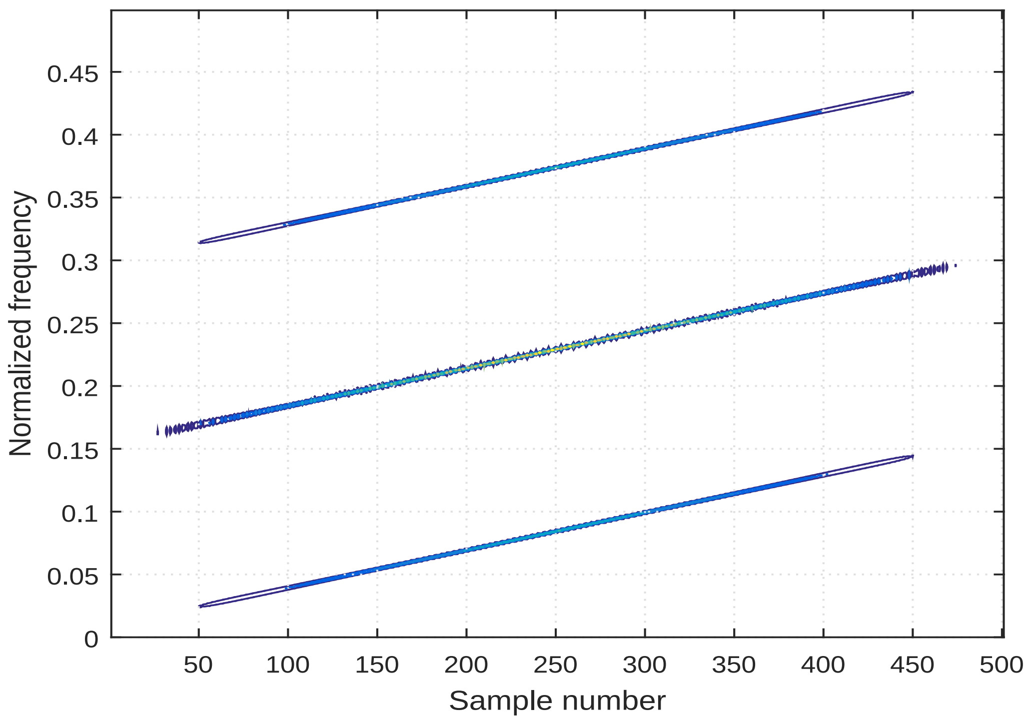


Figure 4.5: WVD of $f_2$

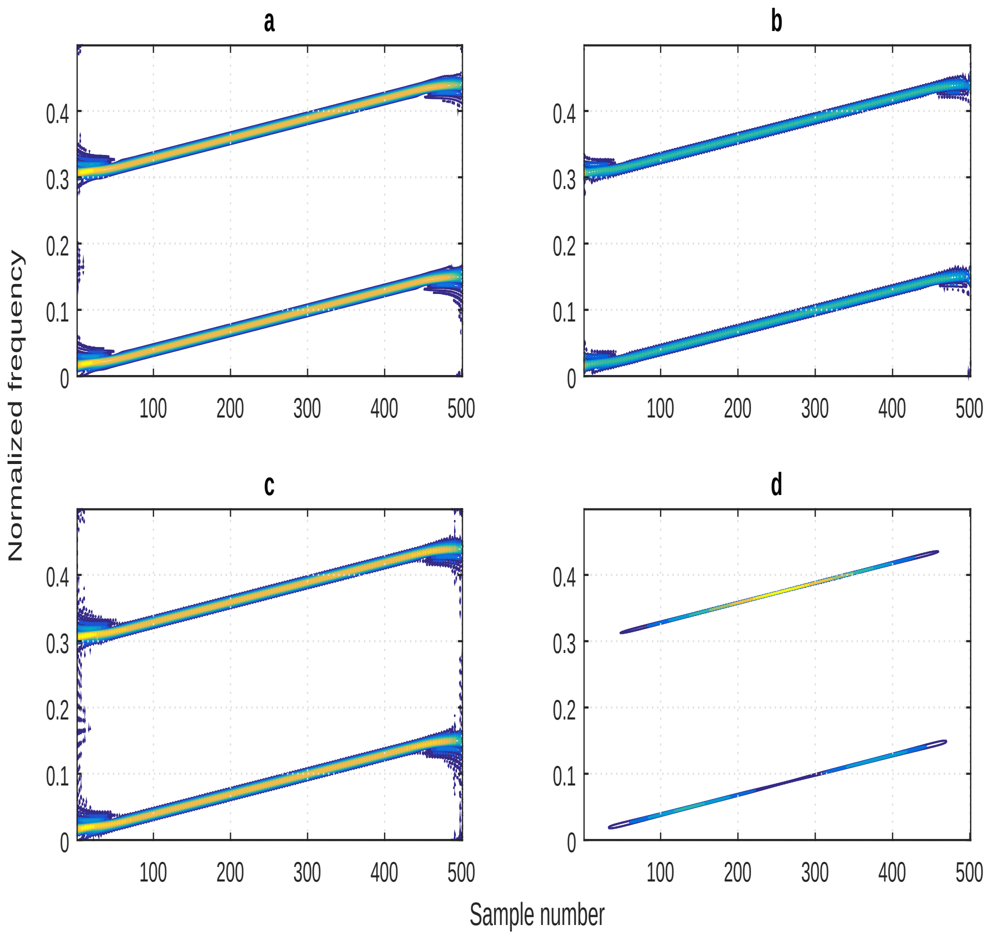


Figure 4.6: The TFD of $f_2$ utilizing (a) SPWVD (b) BJ distribution (c) CWD (d) Presented method

## 4.2 Time-limited constant frequency signal

The signal $f_3$ is made of two time limited constant frequency monocomponents and it is given by,

$$f_3 = \frac{p[t]}{7}\left[\cos\left(\frac{87\pi t}{166}\right) + \cos\left(\frac{125\pi t}{166}\right)\right] \tag{4.3}$$

Where, function $p[t]$ bounds the signal $f_3$ in time and it can be expressed as follows:

$$p[t] = \begin{cases} 0, & t < 101 \\ 1 - \left[\frac{(t-166)^2}{5000}\right], & 102 \le t \le 211 \\ 0, & 212 \le t \end{cases}$$

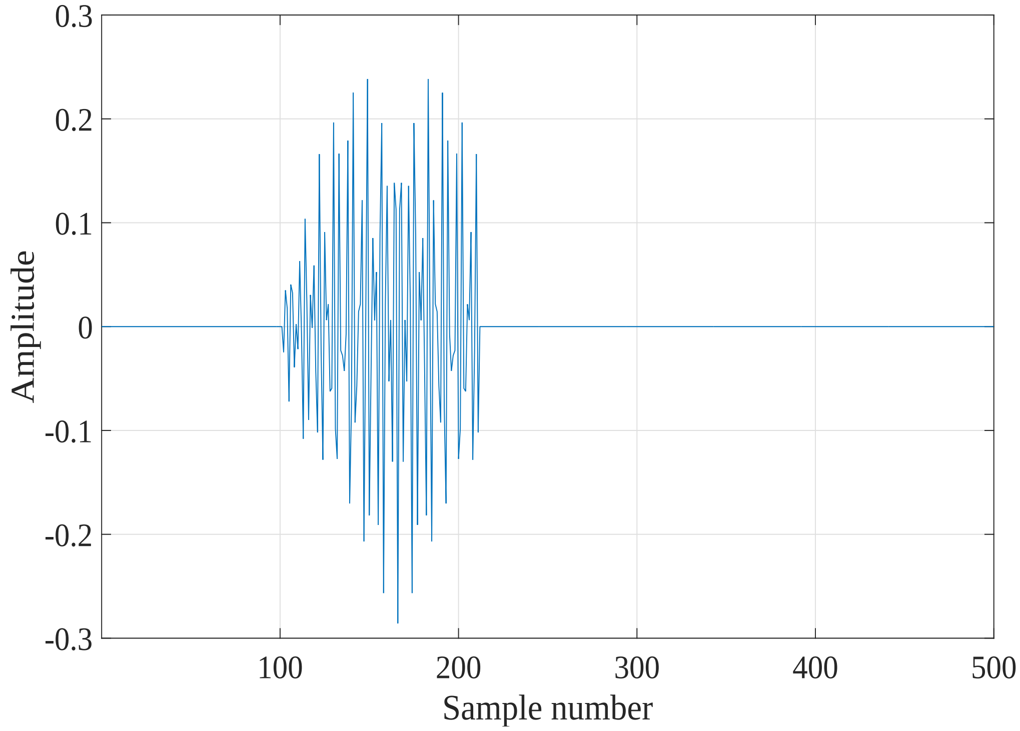


Figure 4.7: Plot of signal $f_3$

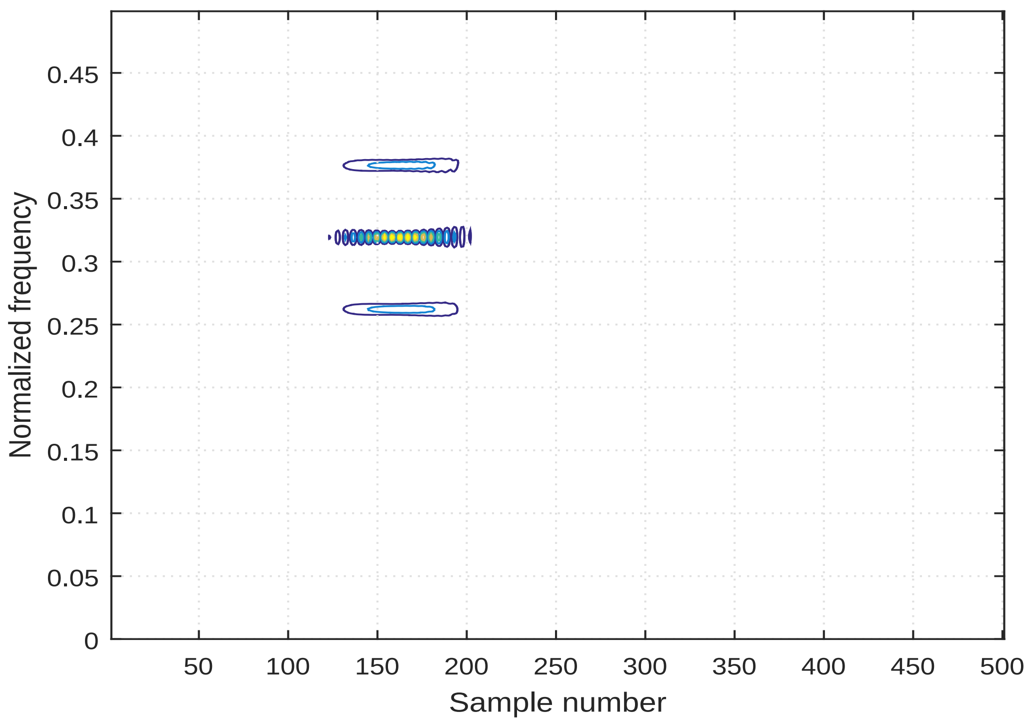


Figure 4.8: WVD of $f_3$

The graph of signal $f_3$ is shown in Fig. 4.7 and TFR of signal $f_3$ using WVD is displayed in Fig. 4.8. The cross terms free TFD using SPWVD, BJ, CWD and proposed method for signal $f_3$ are shown in Fig. 4.9(a), Fig. 4.9(b), Fig. 4.9(c), and Fig. 4.9(d), respectively. All the signals taken for simulation shows the performance of the proposed methodology in comparison with the WVD and SPWVD, BJ and CWD method.

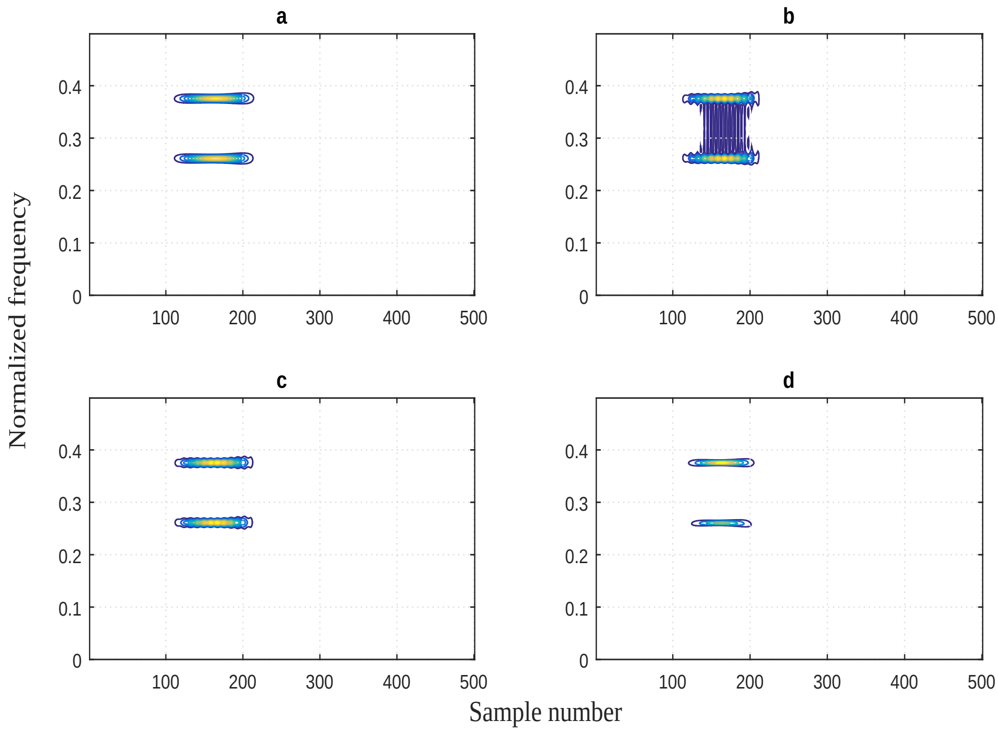


Figure 4.9: The TFD of $f_3$ using (a) SPWVD (b) Born-Jordan distribution (c) Choi-Williams distribution(d) Presented method

## 4.3 Bat-echo signal

Another signal is a multicomponent bat-echo signal and this is a natural signal generated by a large brown bat called Eptesicus fuscus. The components of this signal are well separated in TF domain. The duration of the bat signal taken into consideration in proposed method is 2.5 ms, with the sampling period of 7 micro seconds. The TFA of the bat sonar signal is done in [97].

The plot of the bat-echo is given in Fig. 4.10 and WVD of bat-echo signal is displayed in Fig. 4.11. The TFR of the bat-echo using the SPWVD, BJ and CWD method is given in Fig. 4.12(a), Fig. 4.12(b), Fig. 4.12(c) respectively and the cross terms free TFR obtained by applying proposed method is given in Fig. 4.12(d).

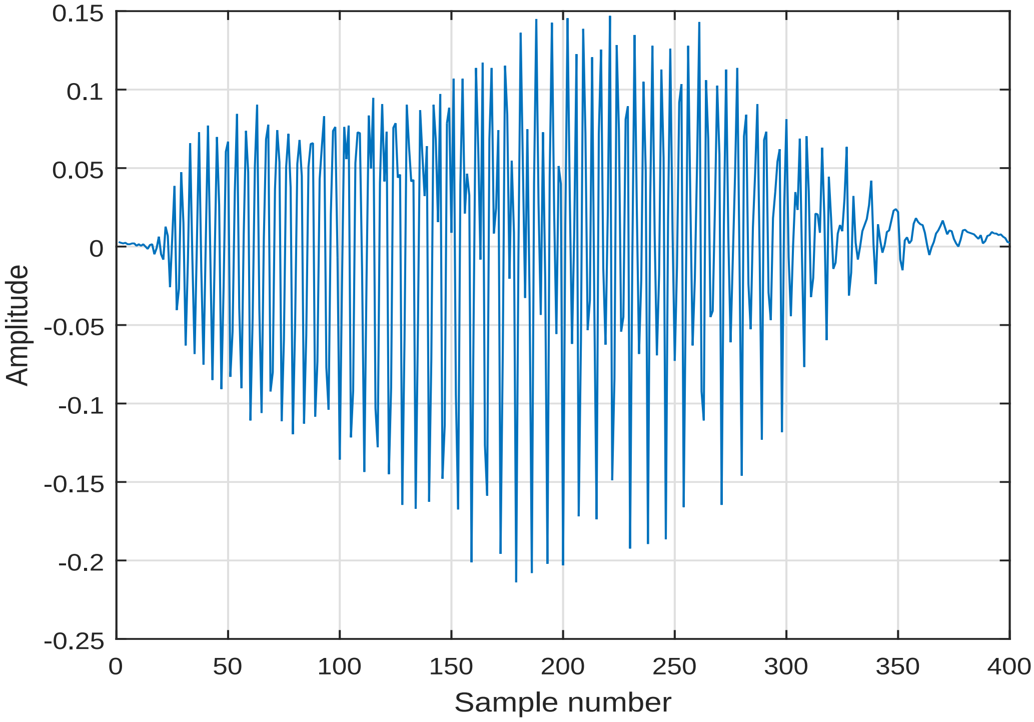


Figure 4.10: Graph of bat-echo signal

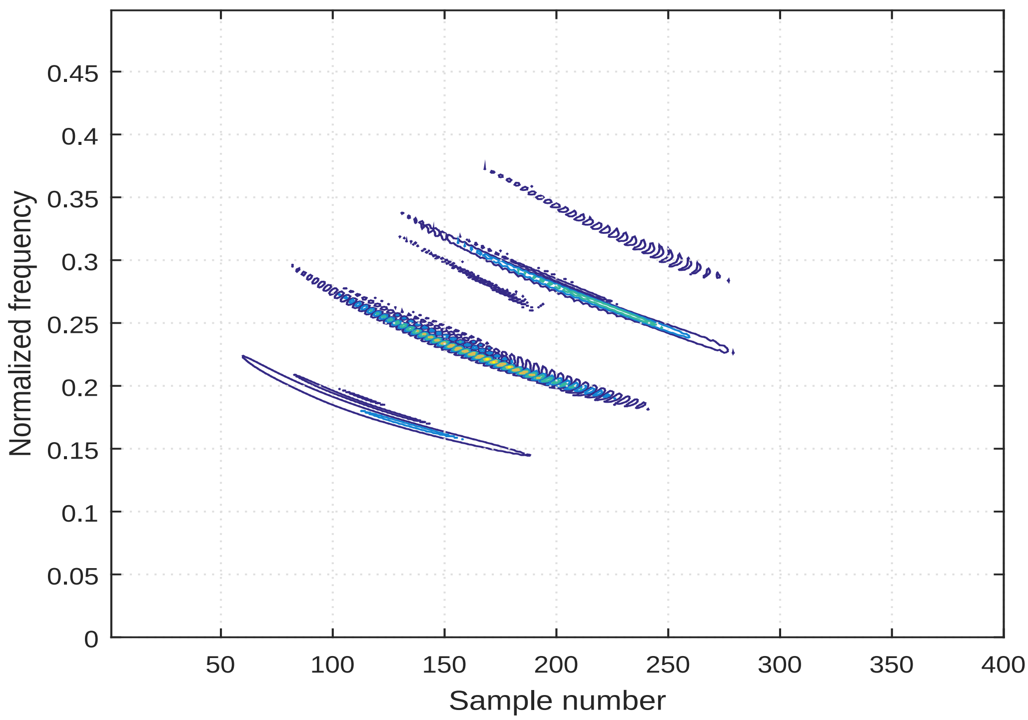


Figure 4.11: WVD of bat-echo signal

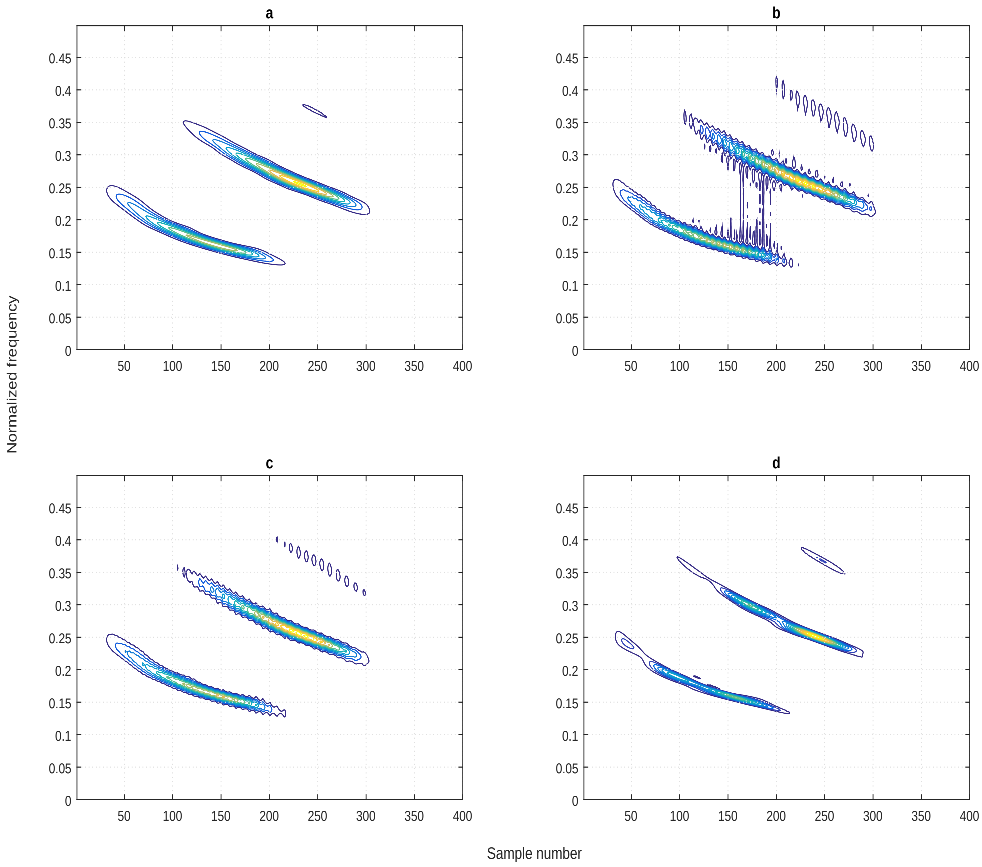


Figure 4.12: TFD of bat-echo signal using (a) SPWVD (b) BJ distribution (c) CWD (d) Presented method

## 4.4 Linear frequency modulated and non-linear frequency modulated signal

A two component signal $f_5$ whose one component is LFM and other component is NLFM. Both the components are well separated in TF domain. The mathematical expression of signal $f_5$ is given below

$$f_5 = \frac{1}{70}(\cos(\frac{4\pi n}{490} + 487)\frac{n}{480} + 28\cos(\frac{\pi n}{256})) + \frac{1}{80}(\cos(\frac{\pi n}{1200} + 27)\frac{n}{10}) \tag{4.4}$$

The plot of the $f_5$ is given in Fig. 4.13 and WVD of signal is displayed in Fig. 4.14. The TFR of the $f_5$ using the SPWVD, BJ and CWD method is given in Fig. 4.15(a), Fig. 4.15(b), Fig. 4.15(c) respectively and the cross terms free TFR obtained by applying proposed method is given in Fig. 4.15(d).

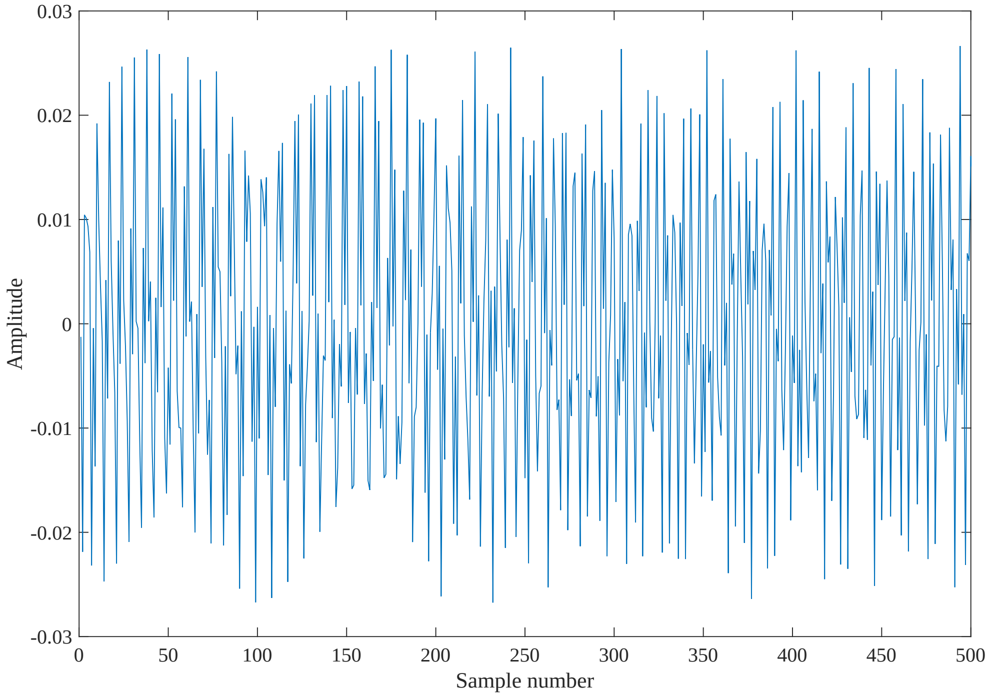


Figure 4.13: Graphical representation of signal $f_5$

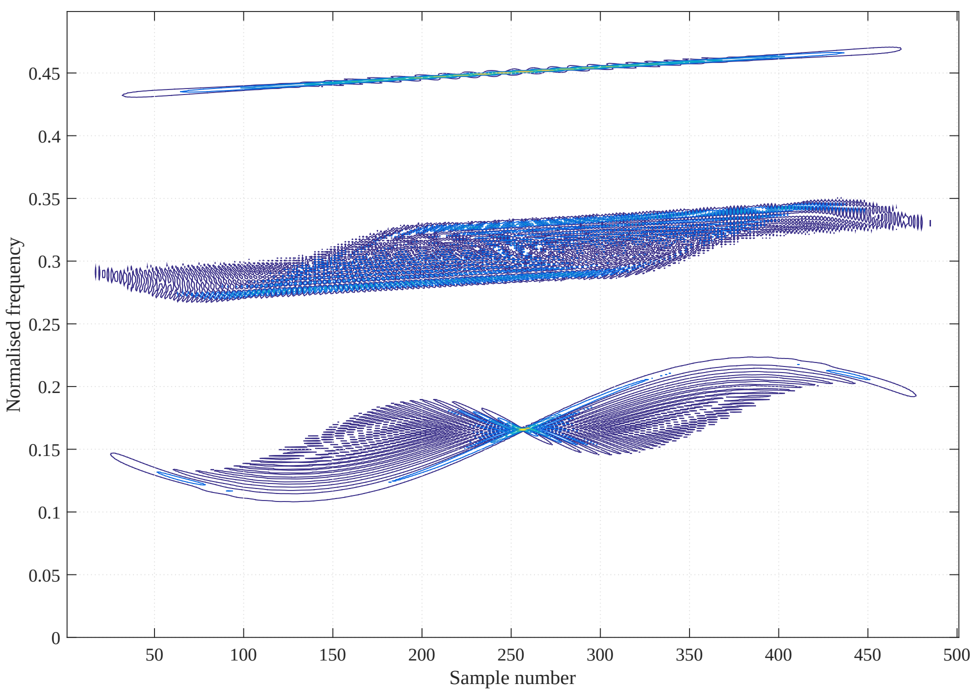


Figure 4.14: The TFD of $f_5$ using WVD

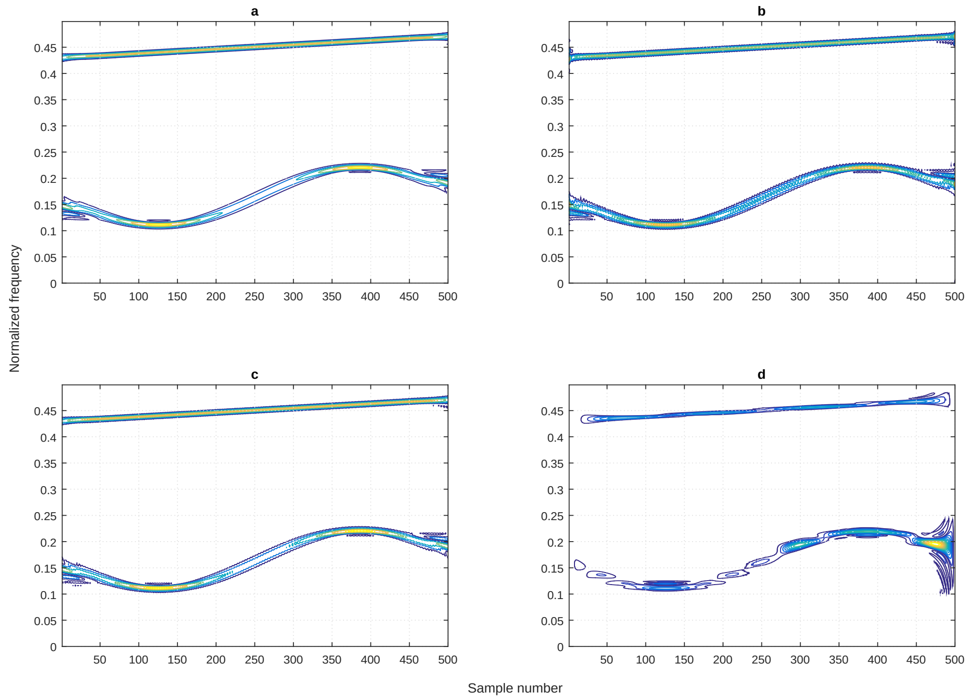


Figure 4.15: TFD of $f_5$ using (a) SPWVD (b) BJ distribution (c) CWD (d) Presented method

## 4.5 Performance evaluation

To illustrate the performance of the proposed method, Renyi information measure is taken into consideration. By comparing the value of Renyi information for TFR of different signals obtained from the different methods and proposed method, performance of the proposed method can be judged. The mathematical expression for Renyi information is as follows [98, 99, 100, 101, 102, 103]:

$$R_\alpha = \frac{1}{1-\alpha}\log_2\left(\sum_{i=-I}^{I}\sum_{j=-J}^{J}[C_s(i,j)]^\alpha\right) \tag{4.5}$$

In above expression, $C_s(i,j)$ is a TFD of Cohen's class and $\alpha$ is the order of information. The value of the alpha chosen is 3. This measure gives the complexity and information content in time-varying signals in its TF plane. Lower the value of Renyi information results in the better TFD. The computed value of Renyi information for TFR obtained from the SPWVD, BJ, CWD and proposed method are given in the Table 4.1.

Table 4.1: The Renyi information for TFR obtained using different methods

| **Methods** | **Renyi information** | | | | |
|---|---|---|---|---|---|
| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ |
| SPWVD | 3.60 | 3.78 | 1.36 | 2.6836 | 3.6370 |
| Born-Jordan distribution | 3.53 | 3.70 | 1.28 | 2.62 | 3.58 |
| Choi-Williams distribution | 3.59 | 3.77 | 1.34 | 2.66 | 3.62 |
| Proposed method | 1.02 | 0.99 | 0.76 | 1.2826 | 1.7503 |

The values of the Renyi information measure is low for the presented methodology when compared with the values of SPWVD, BJ and CWD. It means that the proposed methodology gives better TFD as compared to the SPWVD, BJ and CWD which is also shown by the graphical representation of the signals in experiment results.

## 4.6 Summary

In this chapter the experimental results are discussed and the performance of the proposed method is also shown using the Renyi information.
The results of the proposed methods are compared to the results of SPWVD, BJ and CWD method and Renyi information is calculated for all the methods. From the obatained results it is observed that, the proposed method gives better results and low value of the performance measure parameter.

# Chapter 5

# Conclusion and Future Work

## 5.1 Conclusion

The non-stationary signal has many applications in real life. It is necessary to find the information in it and to extract the information or message carried by the signal, TFR are used. Many methods are discovered for TFR of the signals as discussed in Chapter 1. The WVD is one of them which is used for TFR of the signal as discussed in this thesis. The WVD has advantages over the other methods like it gives good resolution, no need of window function. But, there is always an issue with this method due to the quadratic nature of WVD. The quadratic terms present in WVD rises the problem of occurring interference between the components. These components result from the interference are known as cross terms. Another terms present in the WVD are called as auto-terms and the required information is present in the auto-terms of the WVD. The analysis of auto-terms in TF domain is an important factor for signal understanding. The existence of cross terms mislead the auto-terms. Therefore, to get the auto-terms, it is necessary to remove the cross terms from the WVD of the signals so that message carried by that signal can be extracted. This issue of the WVD is removed in SPWVD, BJ and CWD but these methods provide poor resolution.

In this thesis two methods are proposed to reduce the cross terms present in the WVD of the signal. Proposed method 1 is based only on the VMD method. Whereas, proposed method 2 is based on VMD as well as TDD. VMD is used for the segmentation of the signal whose information have to be extracted. WVD and VMD methods are explained in detail in Chapter 2 and the proposed methods are explained in Chapter 3. In proposed method 2, TDD is used for the decomposition of the sub-signals obtained from the VMD into the segments in time domain. Thereafter, WVD is applied on every segments and add to get the cross terms free TFR.

The simulation results given in the Chapter 4 of this thesis shows that the methods proposed in this thesis successfully eliminates the cross terms of the WVD, which is the main aim of the proposed methods of the thesis. Results obtained from the proposed methods are compared with the results of the SPWVD, BJ and CWD. It shows the methods proposed in the thesis give the good resolution and better performance as compared to other TFR method. To judge the performance of the proposed methods, Renyi information is used. It can be observed that proposed methods give the lowest value of Renyi information which proves that proposed methods are better.

On observing the experiment results it can be concluded that the methodology presented in this thesis to suppress the cross terms in WVD of non-stationary multicomponent signals works well. Experimental examination has been conducted on signals in order to show the effectiveness.

## 5.2 Future work

In this thesis, it can be seen that the proposed methods is introduced for the suppression of the cross terms occurred in the WVD of the multicomponent non-stationary signals whose components are well separated in time-frequency domain. In future expand the application of the proposed methods by using it for the analysis of multicomponent non-stationary signals whose components are not well separated in time-frequency domain i.e signals having the intersection of the components in time-frequency domain. In future, this method can be extended for analysis of noisy signals.

This method can further be used for the classification of the signals like classification of normal and abnormal classes of physiological signals, seizure and non-seizure electroencephalography (EEG) signals, photoplethysmogram (PPG) signals, electromyogram (EMG) signals, etc. In the classification of signals, the proposed methods can extract the features of the signals and based on these features classification can be performed.

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