Numerical implementation of a lumped parameter model for internal ballistics for the gun chamber

M.Tech. Thesis

By
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DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY INDORE

MAY 2026

Numerical implementation of a lumped parameter model for internal ballistics for the gun chamber

A THESIS

Submitted in partial fulfillment of the requirements for the award of the degree
of
Master of Technology

by
ANEESH



DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY INDORE
MAY 2026

INDIAN INSTITUTE OF TECHNOLOGY INDORE

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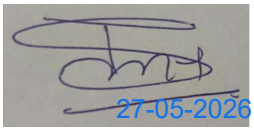
I hereby certify that the work which is being presented in the thesis entitled **NUMERICAL IMPLEMENTATION OF A LUMPED PARAMETER MODEL FOR INTERNAL BALLISTICS FOR THE GUN CHAMBER** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DEPARTMENT OF MECHANICAL ENGINEERING, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July 2025 to May 2026 under the supervision **of Dr. Indrasen Singh**, Associate Professor in mechanical Engineering, Indian institute of technology Indore.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

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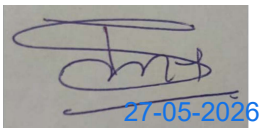
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Signature of Supervisor of M.Tech. thesis
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Convener, DPGC
Date: 27-05-2026

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my thesis supervisor, **Dr Indrasen Singh,** for his valuable guidance and constant encouragement throughout my research work. Every discussion with him was highly informative and helped me gain a sound understanding, knowledge, and awareness of the scientific and technical aspects of my work. I am deeply thankful to him for teaching me how to analyze results and write research articles. I am also grateful to my PSPC members for their continuous support, valuable input, and suggestions throughout this project.

I would also like to thank **Mr. Sanjay Kumar**, Associate Director, ARDE, for his valuable insights into the understanding and implementation of the lumped parameter model for internal ballistics. I sincerely appreciate his professional attitude, which allowed me to work with clarity and confidence.

I am also thankful **to Mr Sumit Chorma, Dr Ramanand Dadhich,** and **all my lab mates** from the **Extreme Mechanics of Advanced Materials and Structures Laboratory** at IIT Indore for their unconditional support and assistance. Their expertise and knowledge in the field of numerical implementation and FEA simulation helped me significantly throughout this work.

The entire M.Tech. program has been a valuable experience for me, not only in terms of acquiring skills in the fields **of internal ballistics, numerical implementation, continuum mechanics,** and **finite element analysis, etc**. but also in developing my understanding of research methodologies, technical writing, interpersonal relationships, and professional ethics. I am thankful to all my friends at IIT Indore for making this journey more enjoyable and memorable.

I would like to express my heartfelt gratitude to my parents and family members for their blessings, continuous support, encouragement, and unwavering faith in me. Their love and motivation constantly inspired me throughout this journey.

Finally, I am grateful to all those who assisted me, directly or indirectly, at various stages of this work.

Thank you everyone!!

ANEESH

(M.Tech in Mechanical Engineering)

Dedicated to my family for their love, care, and blessings…!

Abstract

Internal ballistics deals with everything that happens inside a gun barrel from the moment the propellant is ignited until the projectile leaves the muzzle. Understanding this process is important because it helps design safer, more efficient firearms. The pressure inside the chamber, the burning of the propellant, and the movement of the projectile all directly affect the performance and safety of the weapon system.

In this study, a Lumped Parameter Model (LPM) is developed and implemented using MATLAB to analyze internal ballistics. The LPM is a simplified mathematical model that assumes the propellant burns uniformly and that the pressure is evenly distributed inside the chamber. Although it does not capture detailed effects such as pressure waves or flame spread. It also requires much less computational effort compared to gas-dynamic models.

The model calculates how pressure, temperature, and projectile velocity change with time and displacement after ignition. It also estimates key performance parameters, such as chamber pressure and muzzle velocity. In order to make the predictions more realistic, a multitube propellant grain geometry is included. The results obtained from MATLAB are compared with values reported in the literature, and the comparison shows good agreement. In addition, finite element analysis (FEA) of the gun chamber is performed using Abaqus 2017, and the time-dependent pressure history is calculated using the VUAMP user subroutine by coding LPM in Fortran. Further, a detailed flowchart is also presented to systematically describe each stage of the modelling procedure, from propellant ignition to projectile exit.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

ACRONYMS

Chapter 1: Introduction **15**

1.1 Overview: Description of Ballistic 15

1.2 Configuration: Main components of the gun system 16

1.3 The burning of the propellant 17

1.4 Engraving process 17

1.5 Organization of the Thesis 18

Chapter 2: Literature review **20**

2.1 Review of pertinent literature 20

2.2 Research objective 23

Chapter 3: Lumped parameter model **24**

3.1 Burning of the propellant 24

3.2 The energy balance equations 26

3.3 Equation of motion of the projectile 26

3.4 Estimation of shot base pressure 36

3.5 Governing equations 38

Chapter 4: Numerical implementation of the Lumped parameter model **39**

39

4.1 Introduction 39

4.2 Parameters 40

4.3 Material properties 41

4.4 Flow charts

Chapter 5: Results and discussion **43**

5.1 Pressure variation 43

5.2 Temperature variation 44

5.3 Fraction of ballistic size and mass fraction of ballistic size variation 44

5.4 Velocity variation 45

5.5 Benchmarking with available literature 46

Chapter 6: Conclusions and Future Scope **48**

6.1 Introduction 48

6.2 Conclusions 48

6.3 Future Scope 49

REFERENCES **50**

LIST OF FIGURES

Fig.1.1 Type of internal ballistic 15

Fig.1.2 Schematic of the main components of the artillery gun system 16

Fig.1.3 The ballistic size D of some grain shapes 17

Fig.1.4 Schematic of Engraving Process 18

Fig 2.1 (a) Comparison of pressure histories and
(b) Comparison of Temperature histories. 20

Fig 2.1 (c) Schematic representation of propellant grain geometry and regression process. 21

Fig 2.1 (d) Schematic representation of a general gun system indicating its main parts. 22

Fig 3.1 Burning of the tube-type propellant. 24

Fig 3.3 Type of energy. 26

Fig 3.4 (a) Gun Barrel with Rifling. 32

Fig 3.4 (b) Free body diagram of forces acting on the surface of the band of the shell. 32

Fig 3.5 (a) Representation of Shot Base Pressure Acting on the Base of the Projectile 36

Fig 3.5 (b) variation of velocity with distance. 36

Fig 4.3 Flow chart of lumped parameter model. 42

Fig 5.1 (a) Variation of shot base pressure with time and 43

(b) Variation of shot base pressure with time. 43

Fig 5.1 (c) Variation of shot base pressure with Current position and 43

(d) Variation of shot base pressure with Current position. 43

Fig 5.1: (e) Variation of breech pressure, mean gas pressure, and base pressure with time and 44

(f) Variation of breech pressure, mean gas pressure, and base pressure with current position. 44

Fig 5.2 (a) The variation of projectile velocity with respect to current position. 44

Fig 5.2 (b) The variation of projectile velocity with respect to time. 44

Fig 5.3 (a) The variation of the fraction of ballistic size with respect to time. 45

Fig 5.3 (b) The variation of mass fraction of ballistic size with respect to time 45

Fig 5.4 The variation of temperature with respect to time 45

Fig 5.5 (a)Comparison of Mean Pressure and Time History Between Literature Data and MATLAB Lumped Parameter Model 46

Fig 5.5 (b) Comparison of Mean Temperature and Time History Between Literature Data and MATLAB Lumped Parameter Model. 47

LIST OF TABLES

Table 3.1 Value of the Form function coefficient for different 2-propellant shapes. 25

Table 4.3 Input Parameters and Material Properties Used in MATLAB Implementation of the Lumped Parameter Model. 40

Table 5.5 Comparison of parameter options from literature and determined from the present study. 47

NOMENCLATURE

Symbols:

Symbol	Description
A	Bore area
A_b	Breech face area
C	Charge mass
D	Ballistic size
D_p	Characteristic of granular propellant particle size
Dr	Interphase drag
E	Young's modulus
F	Force constant
K	Chamber volume
M_G	Mass of gun (in recoil)
Nu	Nusselt number
Pr	Prandtl number
Q	Mass flow rate
R	Gas constant
Re	Reynolds number
R_G	Force on groove wall
S	Surface area of propellant grain at time t
S_0	Initial surface area of propellant grain
T	Mean temperature
T_a	Ambient temperature
T_g	Initial temperature of gun
T_0	Adiabatic flame temperature
T_w	Barrel wall temperature
V	Volume per unit mass of gas
V_G	Velocity of gun (in recoil)
a	Rifling rate
c	Speed of sound
c_p	Mean specific heat at constant pressure
c_v	Mean specific heat at constant volume
e	Internal energy
f	Fraction of ballistic size remaining at time t
h	Heat transfer coefficient
k	Polar radius of gyration of projectile
k_c	Thermal conductivity at film temperature
m	Mass of projectile
m_p	Mass rate of injection of gas
$\dot{m}$	Mass rate of addition of gas
n	Number of grooves in rifling and number of gram molecules per unit mass of gas
p	Pressure
p_a	Ambient pressure

q	Heat transfer from propellant gas to solid propellant
s	Distance from breech to projectile base at time t
t	Time
v	Velocity
v_m	Muzzle velocity
v_p	Projectile velocity
x	Distance
z	Mass fraction of ballistic size remaining at time t
α	Rifling angle; pressure index
β	Burning rate constant
γ	Ratio of specific heats (c_p/c_v)
δ	Temperature coefficient of the burning rate
ϵ	Porosity
ϵ_0	Settling porosity
η	Covolume
θ	Form function constant
μ	Coefficient of sliding friction
μ_f	Viscosity of gas at film temperature
ν	Kinematic viscosity
ρ	Density
σ	Poisson's ratio; intergranular stress
τ_0	Skin friction coefficient

Chapter 1

Introduction

1.1 Ballistics

Ballistics is the branch of science that studies the motion and behavior of a projectile during its entire lifecycle, from firing to impact, and its effects on the target. It is broadly divided into three main types: internal ballistics, external ballistics, and terminal ballistics, as shown in Fig.1.

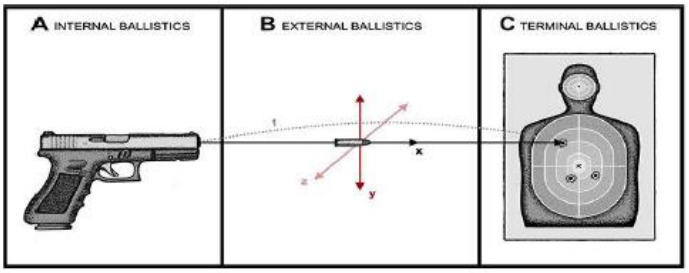


Fig.1.1: Type of internal ballistics.
(Source- https://wholeeartheducation.com)

Internal ballistics focuses on the processes occurring within the firearm, including the burning of the propellant and the motion of the projectile inside the gun barrel until it exits.
External ballistics examine the projectile's flight after leaving the barrel, analyzing factors such as trajectory and the influence of forces acting during motion.
Terminal ballistics studies what happens when the projectile strikes a target, including the resulting impact and damage. Together, these three areas provide a comprehensive understanding of projectile dynamics and their practical implications.

For designing a gun system, it is essential to calculate critical parameters such as chamber pressure, muzzle velocity, and stress distribution within the barrel. This requires a detailed analysis of internal ballistics, which examines the processes occurring inside the firearm from ignition to the projectile's exit. Studying internal ballistics ensures safety by evaluating high-pressure generation and maintaining the structural integrity of the barrel and chamber. A proper understanding also helps optimize muzzle velocity, improve shot consistency, and support efficient ammunition design, ultimately enhancing overall firearm performance and reliability.
Internal ballistics can be analyses using different numerically implemented models, mainly the gas dynamic model and the lumped parameter model.
The gas dynamic model provides a detailed description of spatial gradients and combustion phenomena such as ignition, flame propagation, and pressure waves, offering high accuracy at the expense of greater computational cost.

The lumped parameter model assumes uniform and instantaneous ignition of the propellant and neglects spatial variations within the chamber. Due to its simplicity and lower computational requirements, it is widely used for predicting key performance parameters such as mean chamber pressure and muzzle velocity, making it suitable for system-level design and rapid performance estimation, though it is less appropriate for studying ignition-induced pressure wave effects.

1.2 Configuration

Figure 1.2 shows the schematic configuration of the artillery gun system and its major components involved in the firing process. The system mainly consists of the breech end, chamber, propellant, projectile, driving band, barrel, and muzzle. Each component performs a specific function to ensure proper projectile acceleration and firing performance.



Fig.1.2: Schematic of main components of the artillery gun system.
(Source- https://www.army.mil)

Breech end: The rear portion of the barrel where the cartridge is loaded and where high-pressure gases are generated during firing.
Chamber: The chamber is the rear section of the barrel where the propellant and projectile are positioned before ignition.
Projectile: The moving component (bullet/shell) that is propelled through the barrel toward the target.
Propellant: The propellant is an energetic material that burns to generate high-pressure gases for accelerating the projectile.
Cartridge case: The casing that holds the propellant and primer and houses the projectile before firing.
Rifled barrel: The barrel with spiral grooves that impart spin to the projectile for stability in flight.

Gun barrel: The main tubular component of the weapon system that contains the firing process.
Driving band: A soft metal ring on the projectile that engages the rifling and seals the gas pressure.
Muzzle: The muzzle is the front end of the barrel from which the projectile exits into free flight.
Rifling: Rifling consists of helical grooves inside the barrel that impart rotational motion (spin) to the projectile for improved stability and accuracy.

1.3 The burning of the propellant

Propellant burning starts when the grain reaches ignition temperature, generating high-temperature gases that rapidly raise chamber pressure. The ballistic size represents the minimum distance between opposite burning surfaces for complete combustion. According to Piobert's law, the grain burns in parallel layers with a uniform regression rate. The grain geometry and form function control the change in burning surface area and thus influence pressure development. Ballistic size is the distance between two burning surfaces. The ballistic size D of some grain shapes is shown below in Fig.1.3.

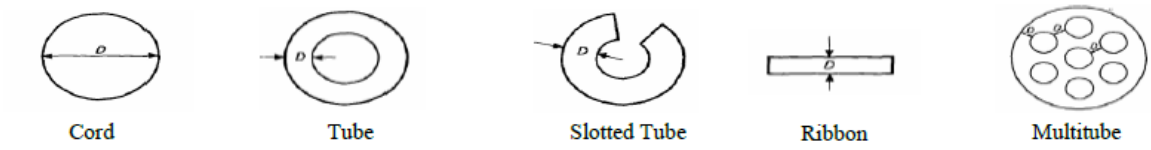


Fig.1.3: The ballistic size D of some grain shapes [4].

1.4 Engraving process

After completing the loading and safety procedures, the projectile moves through the barrel due to the high-pressure gases created by the combustion of the propellant inside the chamber. As the gases expand, they push the projectile forward along the barrel bore. In the initial stage of its motion, the driving band of the projectile contacts the rifling grooves on the inner surface of the barrel. This interaction causes the driving band material to deform and gradually engrave into the rifled profile. Figure 1.4 shows a diagram of the engraving process inside the gun barrel.
The engraving process is important for the firing mechanism because it transfers rotational motion from the rifled barrel to the projectile. The spin generated during this process improves the aerodynamic stability and accuracy of projectiles without fins during flight. Additionally, the driving band creates a good gas seal, stopping high-pressure gases from leaking around the projectile. If engraving goes wrong, it can cause gas leaks, barrel wear, and poor ballistic

performance. Thus, proper engagement between the driving band and rifling is crucial for achieving efficient projectile motion and the desired firing performance.

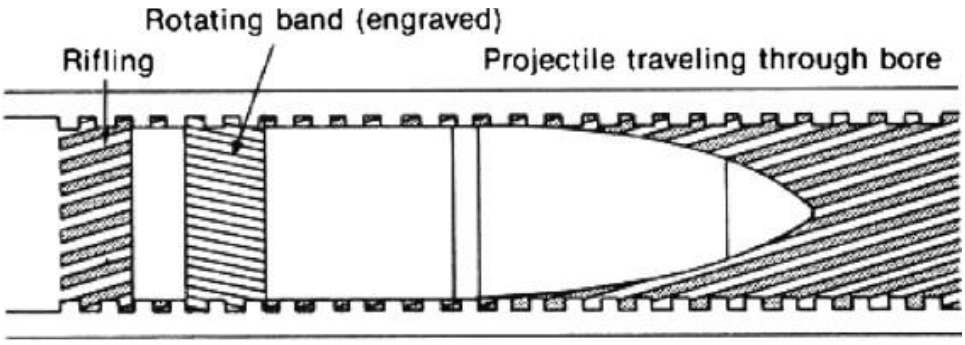


Fig.1.4: Schematic of engraving process [5].

1.5 Thesis organisation

Based on the discussion in the previous sections, it is clear that predicting internal ballistic characteristics is crucial for understanding the firing process and performance of gun systems. Implementing the lumped parameter model provides an efficient method for estimating chamber pressure, projectile velocity, and other ballistic parameters while maintaining low computational cost. The detailed formulation, implementation, and analysis in this work are presented in the following chapters. Below is a brief overview of the thesis organization to explain the contents covered in this study.

Chapter 2. **Literature review:** This chapter reviews the relevant literature on internal ballistics, propellant combustion, and lumped parameter modelling. It identifies existing research gaps and formulates the objectives of this work.

Chapter 3. **Lumped parameter model:** This chapter provides the theoretical formulation of the lumped parameter model for internal ballistic analysis. It discusses the governing equations related to propellant burning, energy balance, projectile motion, and shot-base pressure estimation in detail.

Chapter 4. **Numerical implementation of lumped parameter model in MATLAB:** This chapter outlines the numerical implementation steps for the developed lumped parameter model in MATLAB. It presents details on the Euler Forward method, model parameters, flowchart methodology, and implementation steps. It also discusses integrating the model with Abaqus using user subroutines for finite element simulations.

Chapter 5. **Results and discussion:** This chapter presents the results obtained from the numerical model. It analyzes variations in chamber pressure, temperature, projectile velocity, ballistic size fraction, and mass fraction, comparing them with existing literature for validation.

Chapter 6. **Conclusions and future scope:** This chapter summarizes the main findings from this work and highlights the conclusions drawn from the numerical implementation. It also offers recommendations and possible directions for future research.

Chapter 2

Literature review

2.1 Review of pertinent literature

The DST Group Technical Note 1655 (2017) develops a fast lumped parameter interior ballistic model to assess novel propellant grain geometries, such as those from additive manufacturing, by simulating key gun chamber dynamics including pressure, temperature, and projectile trajectories. The model relies on simplified conservation of mass and energy equations with assumptions like simultaneous propellant ignition, neglect of heat losses and recoil, and initial gas from fully consumed primer, gas temperature derives from an energy balance subtracting projectile kinetic and rotational energy, burning follows the steady-state law with shared libraries for grain form functions.

The code, written in C, uses simple time-stepping to start with primer gases, then calculates burning and projectile movement after pressure builds up, including base pressure effects, barrel twist rotation, and losses from drag and friction. matches IBHVG2 results on the AGARD gun test (132 mm, 7-hole grains)[1].

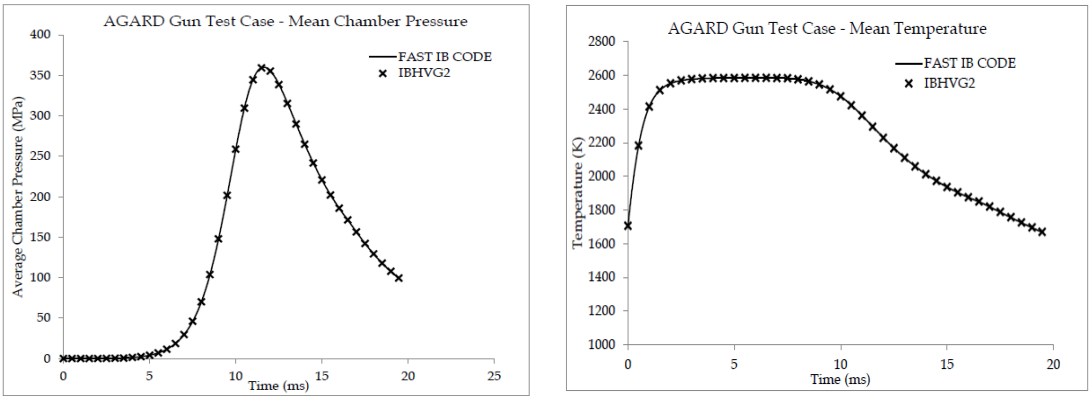


Fig 2.1: (a) Comparison of pressure histories [1], and **(b)** Comparison of Temperature histories [1].

Guermonprez et al. (2025) proposed a combined method to simulate interior and intermediate ballistics using a 0-D Lumped Parameter model (LPM) together with a 2D CFD model. The advantage of LPM lies in its fast computation, however, it cannot capture pressure waves and flame-spread. Whereas the CFD models are more accurate but require a large computation time. To overcome these issues, the authors developed a new Lumped Parameter Code (LPC) that uses the Noble-Abel equation of state and Vieille's law for propellant burn rate. A major improvement in their work is an image-processing-based method for calculating propellant

grain regression for complex geometries (such as 7-hole and 19-hole grains). This method is faster and more flexible than the reported analytical and cellular-automata methods. [2].

The LPC is validated using two well-known test cases: the AGARD gun and an experimental 40 mm gun. In both cases, the predicted pressure, temperature, and muzzle velocity match very well with established ballistic codes such as IBHVG2, CTA1, MOBIDIC-NG, and also with experimental data. This proves that the LPC gives reliable results at very low computational cost. [2].

A key contribution of the paper is the coupling of the LPC with the ONERA CFD solver (CEDRE) using a dynamic mesh technique. The LPC outputs (pressure, temperature, mass flow rate) are used as boundary conditions in the CFD model, allowing the authors to simulate not only interior ballistics but also the first few milliseconds after the projectile leaves the barrel (intermediate ballistics). With this coupled method, the authors were able to reproduce shock waves, expansion waves, Mach disk formation, and base-bleed depressurization accurately. [2].

Overall, the study provides a fast, accurate, and practical methodology that combines the strengths of both LPM and CFD. It shows how interior and intermediate ballistics can be simulated together in a realistic way, with much lower computational cost than full CFD. [2].

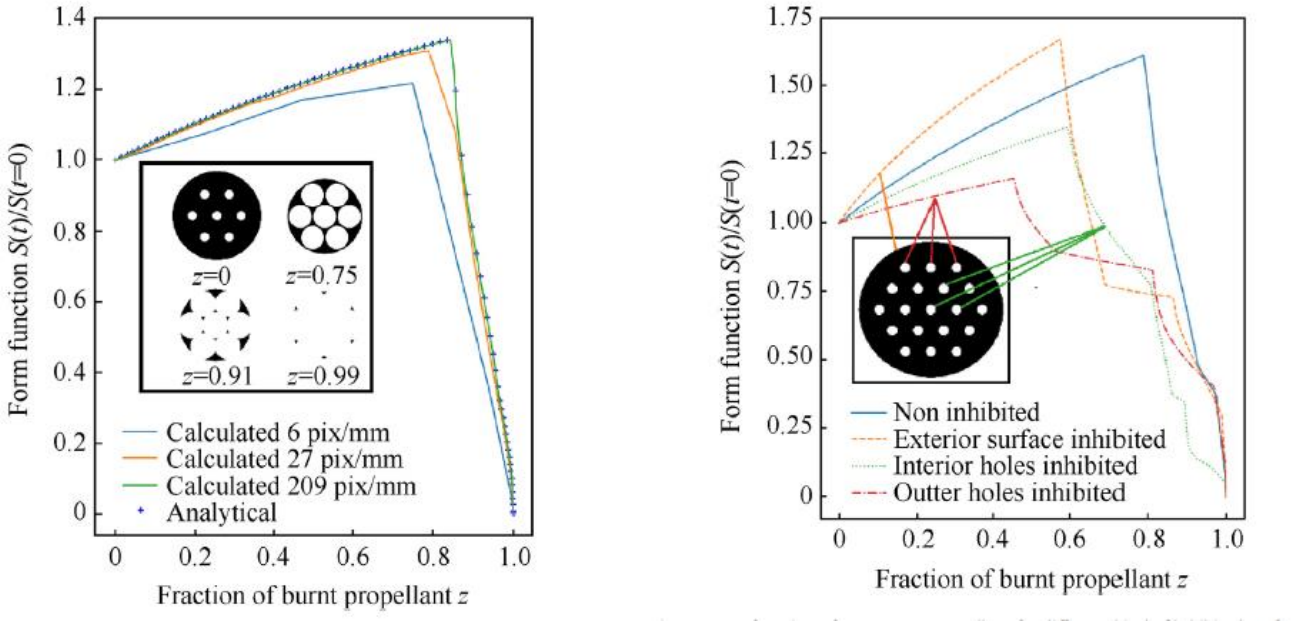


Fig 2.1:(c) Schematic representation of propellant grain geometry and regression process [2].

The paper presents a detailed analysis of internal ballistics with particular emphasis on the combustion behavior of solid propellants and its influence on pressure development inside the gun chamber. It explains how propellant grains burn progressively from the surface once ignition temperature is reached, generating high-temperature gases that significantly increase chamber pressure. The study highlights the importance of ballistic grain geometry, ballistic size, and the form function in determining how the burning surface area evolves during combustion. By applying Piobert's law and surface regression principles, the paper establishes a clear relationship between grain design and pressure and time characteristics. [3].

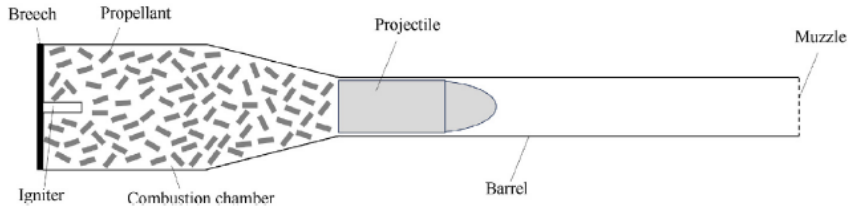


Fig 2.1: (d) Schematic representation of a general gun system indicating its main parts. [3].

Further, the authors compare different mathematical modelling approaches, including lumped parameter and gas dynamic models, to predict pressure, temperature, and projectile motion within the barrel. The work emphasises that accurate modelling of burning rate, energy release, and gas expansion is essential for optimizing muzzle velocity, ensuring structural safety, and improving overall weapon performance. Overall, the paper contributes to a better understanding of how propellant chemistry, grain geometry, and thermodynamic processes interact to control internal ballistic behavior [3].

Finally, from the reviewed literature it is observed that lumped parameter models are widely used for internal ballistic analysis because of their simple formulation, low computational cost and fast execution time. Most researchers preferred lumped parameter approaches for predicting chamber pressure, projectile motion and muzzle velocity due to their suitability for parametric studies and optimisation problems. However, in many published studies, complete modelling details and ballistic parameters are not clearly reported. Therefore, in the present work, a lumped parameter model is also adopted for internal ballistic analysis because it provides an efficient and reliable method for predicting ballistic performance with easier implementation and faster computation.

2.2 Research objective

To implement the Lumped Parameter Model (LPM) in MATLAB for calculating critical internal ballistic parameters, which are essential for the proper and efficient design of a gun system.

To benchmark the developed MATLAB-based LPM results with available literature and utilize the validated parameters as input for subsequent finite element simulations.

To perform finite element simulations of the artillery gun system in Abaqus 2017 using the lumped parameters obtained from the numerical implementation.

Chapter 3

Lumped parameter model

The Lumped Parameter Model (LPM) is a simplified and widely used approach for analyzing internal ballistic phenomena inside a gun barrel. Internal ballistics involves complex interactions among propellant combustion, gas generation, pressure development, and projectile motion. In comparison to gas dynamic models, which capture detailed spatial variations and combustion effects with higher accuracy, the Lumped Parameter Model significantly reduces computational effort while still providing reliable predictions of important ballistic parameters. Therefore, it is commonly used for preliminary design studies and performance evaluation of gun systems.

The Lumped Parameter Model assumes that the propellant charge is ignited instantaneously and burns uniformly, while pressure, temperature, and gas properties remain spatially uniform throughout the chamber. Due to these assumptions, complex effects such as flame propagation and pressure waves are neglected. Despite these simplifications, the model effectively predicts key parameters such as chamber pressure, shot base pressure, projectile velocity, and muzzle velocity. Due to its simplicity and computational efficiency, the Lumped Parameter Model is suitable for system-level analysis, parametric studies, and quick estimation of ballistic performance.

3.1 Burning of the propellant

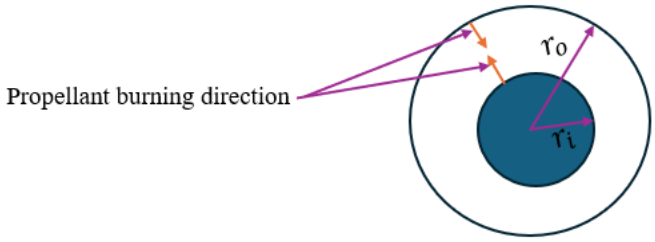


Fig 3.1: Burning of the tube-type propellant.

When the propellant starts burning, as illustrated in Fig. 1, combustion takes place progressively along the inner surface of the propellant grain. As a result, the inner radius of the propellant increases with time due to inward burning of the material. Simultaneously, the thickness of the propellant layer decreases, causing the outer radius of the remaining propellant section to reduce. This continuous regression of the burning surface governs the combustion process and influences the rate of gas generation inside the chamber[4].

Let r_i and r_o denote the initial inner radius and outer radius of the propellant grain, respectively. The wall thickness of the propellant, commonly referred to as the ballistic size, is defined as the difference between the outer and inner radius and is expressed as,[4]

$$D = r_o - r_i$$

Let tube length be λD, inner radius (r_i') and outer radius r_o' at time 't'

$$r_i' = r_i + \frac{1}{2}(1-f)D \qquad r_o' = r_o - \frac{1}{2}(1-f)D$$

Hence z is the mass fraction of ballistic size at the time "t"

$$z = \frac{original\ \ volume - Present\ volume}{original\ volume} = 1 - \frac{original\ volume}{original\ volume}$$

$$Present\ volume = \pi\left\{\left(r_o - \frac{1}{2}(1-f)D\right)^2 - \left(r_i + \frac{1}{2}(1-f)D\right)^2\right\}(\lambda - 1 + f)D$$

$$original\ \ volume = \pi\{(r_o)^2 - (r_i)^2\}(\lambda D)$$

$$z = (1-f)(1+\theta f) \tag{1}$$

where θ is the **Form function coefficient**, which **depends** on the type of propellant shape. As shown in the table below.[4]

Table 3.1: Value of the Form function coefficient for different 2-propellant shapes [4].

Propellant shape	**Burning surface**	**Form function coefficient**	**Ballistic size**
Long cord	Decreasing	1	Diameter
Long tube	Constant	0	Wall thickness
Slotted tube	Slowly decreasing	D^2/S_O	Wall thickness
Long ribbon	Slowly decreasing	$1/\lambda$	Thickness
Tube	Slowly decreasing	1, $1/\lambda$	Wall thickness
Multitube 7	Increasing	-0.17	1.15x ballistic size

For the modelling of internal ballistics, it is required to take the fraction of ballistics size (f) at time as a function of gas pressure. The fraction of size helps to implement the lumped parameter model. This may be expressed as (empirical relation)[4].

$$D\frac{df}{dt} = -\beta p^{\alpha} \tag{2}$$

Where β is the burning rate coefficient, which is not constant; the burning rate coefficient is a function of the temperature. The expression of the burning rate coefficient in terms of gas temperature is expressed as [4].

$$\beta = \beta_{Ta}(1 + \delta(T - T_a)) \quad (3)$$

Where

β_{Ta} is the burning rate for a charge initially at ambient temperature (T_a).

δ is the temperature coefficient of the burning rate $(1.8 - 3.6\ \%\ \text{per}\ 10^0\text{C})$

3.2 The equation of the state

Equation of the state used to determine the value of the mass and volume of the gas, also, this equation relates the pressure and temperature of the gas. But in the case of internal ballistics, the most commonly used modified Vander Waals equation [4].

$$p(V_{gas} - \eta) = nRT \quad (4)$$

η It is known as the Covolume. This will be the constant. For most of the propellant, it is in the range 0.87 to 1.05 cc/gm.[4].

$$c_p - c_v = nR \qquad \frac{c_p}{c_v} = \gamma$$

3.3 The energy balance equation

When the propellant started to warm, many types of energy were liberated, but for the lumped parameter model analysis, we are considering the energy liberated by a single change and the way in which energy is absorbed. In the line figure, we can see the energy that is liberated [4].

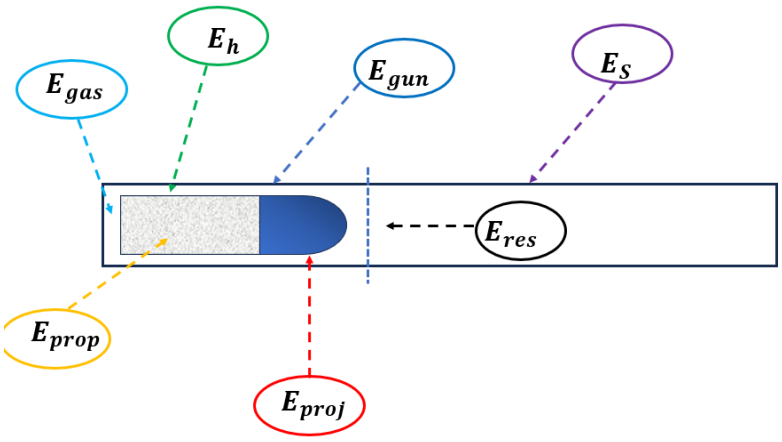


Fig 3.3: Type of energy.

$$Energy\ liberated\ by\ burning\ of\ propellant = Internal\ energy\ of\ propellant\ gases + intial\ air + External\ work \quad (5)$$

$$External\ work = Czc_vT_0 - (Cz + M_a)c_vT$$

$$Energy\ liberated\ by\ burning\ of\ propellant = Czc_vT_0$$

$$Internal\ energy\ of\ propellant\ gases + intial\ air = (Cz + M_a)c_vT$$

Thus, at any instant, the external work done is given by

$$External\ work = Czc_vT_0 - (Cz + M_a)c_vT$$

As we know, the value of specific heat at constant volume c_v from the above relation

$$C_v = \frac{nR}{(\gamma - 1)}$$

And hence

$$External\ work\ done = \frac{nR}{(\gamma - 1)}[\ Cz(T_0 - T) - M_aT\ ]$$

So, assuming that the specific heat and molecular weight of air and propellant gases are the same so for calculating gas temperature using the equation of state. The value of T putting in above equation

So above equation can be written as

$$(\gamma - 1)\ External\ work\ done = [\ CzF(Cz + M_a)p(V - \eta)\ ] \quad (6)$$

Where $nRT_0 = F$ is the force constant of the propellant and is a measure of the energy released per unit mass of the solid propellant.

External work is the summation of the total energy which is generated at the time of internal ballistics [4].

1. The kinetic energy of the projectile (combination of the rotational and the translational energy)
2. The kinetic energy of the gas
3. The kinetic energy of the unburnt propellant
4. The heat energy lost to the gun
5. The strain energy of the gun
6. The energy to overcome the resistance force
7. The recoil energy of the gun

Where V is the volume occupied by unit mass of gas, and hence Czv is the total volume occupied by the gas at the instant being considered. This is given by [4]

$$CzV = chamber\ vol. + barrel\ vol. - vol. occupied\ by\ unburnt\ propellant$$

that is,

$$CzV = K + A(s - s_0) - \frac{C(1-z)}{\rho_{prop}}$$

3.3.1 The kinetic energy of the projectile

As the projectile moves inside the barrel, it undergoes both translational and rotational motion, thereby possessing translational kinetic energy as well as rotational kinetic energy [4].

$$Translational\ energy = \frac{1}{2}mv_p^2$$

$$Rotational\ energy = \frac{1}{2}I\omega^2 = \frac{1}{2}mk^2a^2v_p^2$$

$$The\ kinetic\ energy\ of\ the\ projectile\ (E_p) = \frac{1}{2}m(1 + k^2a^2)v_p^2 \qquad (7)$$

Where m is the mass of the projectile and v_p is the velocity of the projectile, K is the polar radius of gyration and a rifling rate.

3.3.2 The kinetic energy of the gas

The kinetic energy of the propellant gas is evaluated by assuming a linear variation of gas velocity along the barrel, increasing from zero at the breech to the projectile velocity at its base. and that the cross-sectional areas of the bore and the chamber are the same [4].

Thus, the kinetic energy of the mass of gas *($m = \rho A \delta x$)* at some point *'x'* is

$$\frac{1}{2}\rho\left(\frac{x}{s}\right)^2 v_p^2 A$$

Assuming that the gas density is constant in space and integrating from $x = 0\ to\ x = s$ yields

$$E_{gas} = \frac{1}{2}\int_0^s \rho\left(\frac{x}{s}\right)^2 v_p^2 A\, dx = \frac{1}{6}\rho s A v_p^2$$

Where ρsA = mass of the gas = $Cz + m_a$

$$E_{gas} = \frac{1}{6}(Cz + m_a)v_p^2 \tag{8}$$

3.3.1 The kinetic energy of the unburnt propellant

The kinetic energy of the unburnt propellant is evaluated using an empirical model in which the propellant particle velocity is assumed to be proportional to the local gas velocity. Thus, at any location x in the chamber, the propellant velocity is given by [4].

$$v_2 = z^k v_1 = z^k\left(\frac{x}{s}\right)v_p$$

where z is the mass fraction of burnt propellant and k is an empirical constant.

The total kinetic energy of the unburnt propellant is obtained by integrating along the chamber length as

$$E_{prop} = \frac{1}{2}\int_0^s \rho A Z^{2k}\left(\frac{x}{s}\right)^2 v_p^2 dx = \frac{1}{6}\rho s A z^{2k} v_p^2$$

Using the relation for unburnt propellant mass $C(1 - z) = \rho sA$, the expression reduces to

$$E_{prop} = \frac{1}{6}c(1 - z)z^{2k}v_p^2 \tag{9}$$

Although this model accounts for propellant motion, it is based on an empirical proportionality constant k and therefore provides only an approximate estimation of the propellant kinetic energy.

3.3.4 Heat energy lost to the gun

The heat energy loss to the gun barrel is an important component of the internal ballistics' energy balance. For large caliber guns, the heat loss is typically about 5%of the total energy, while for small caliber weapons it may reach up to 20%. A semi-empirical formulation relates the heat loss to the shot kinetic energy as [4].

$$E_h = \frac{0.38\, d^{1.5}(s - s_0 + K/A)(T_0 - T_g)\, v_p^2}{1 + 0.6\, d^{2.175}/v_m^{0.8375}} \tag{10}$$

where d is the gun caliber, T_0 is the initial gun temperature, and $v_m$is the muzzle velocity.

Muzzle velocity is the velocity (speed) of a projectile at the instant it leaves the muzzle (open end) of a gun barrel.
An alternative empirical relation expresses heat loss as a fraction of the projectile kinetic energy:

$$E_h = \frac{1}{2}\, b(m + \frac{1}{3}(cz + m_a)\, v_p^2 \tag{11}$$

where $b$is the heat loss factor. value of the heat loss factor =0.17 (experimental value) [4].

3.3.5 The strain energy of the gun

The strain energy stored in the gun barrel due to internal pressure can be evaluated using linear elasticity theory. For a thick-walled cylindrical shell subjected to internal pressure, the volumetric expansion per unit initial volume is given by [4].

$$\nu = \frac{3}{3\lambda + 2\mu}\left(\frac{d_e^2}{d_e^2 - d_i^2}\right) + \frac{1}{\mu}\left(\frac{d_i^2}{d_e^2 - d_i^2}\right)$$

where $d_i$and $d_e$are the internal and external diameters of the cylindrical shell.

The Lami's constants $\lambda$and μ are related to the material properties such as (Young's modulus E and Poisson's ratio σ).

$$\lambda = \frac{\sigma E}{(1+\sigma)(1-2\sigma)}, \qquad \mu = \frac{E}{2(1+\sigma)} \lambda = \frac{\sigma E}{(1+\sigma)(1-2\sigma)}, \qquad \mu = \frac{E}{2(1+\sigma)}$$

where E is Young's modulus and σ is Poisson's ratio.

The work done due to elastic expansion gives the strain energy per unit volume as $\frac{1}{2}\nu p^2$. Considering the chamber volume $V = K + A(s - s_0)$, the total strain energy of the gun is expressed as [4].

$$E_s = \frac{1}{2}\nu[K + A(s - s_0)]p^2 \tag{12}$$

3.3.6 Energy to overcome resistance forces

The work done in engraving the driving band is a complex phenomenon and is generally small compared to the total kinetic energy of the projectile. It is typically estimated to be about 4–5% of the projectile's kinetic energy. Hence, in early analyses, the engraving work was approximated as [4].

$$E_{res} = 0.04 \times E_p \tag{13}$$

where E_P is the kinetic energy of the projectile.

In more refined models, the engraving resistance is considered as a variable force. Let $R(x)$denote the resistance force acting on the driving band during engraving. The work done against this resistance is then expressed as [4].

$$E_{res} = \int_{s_0}^{s} R(x)\, dx \tag{14}$$

where s_0 is the initial position and s is the instantaneous position of the projectile inside the barrel.

3.3.7 The recoil energy of the gun

The recoil energy of the gun is determined based on the principle of conservation of momentum. Neglecting the contribution of unburnt propellant, the momentum balance between the projectile and the gun can be expressed as [4].

$$M_G V_G = \left[m + \frac{1}{2}(Cz + m_a)\right] v_p$$

where $M_G$and $V_G$are the mass and recoil velocity of the gun, respectively, and $v_p$is the projectile velocity.

The corresponding recoil energy of the gun is given by

$$E_{gun} = \frac{\left[m + \frac{1}{2}(Cz + m_a)\right]^2}{2M_G} v_p^2 \tag{15}$$

In the limiting case where the mass of propellant gases and entrained air is negligible $(Cz + m_a \ll m)$, The ratio of recoil energy to the projectile kinetic energy reduces approximately to the inverse of the mass ratio of the gun to the projectile. This highlights that heavier gun systems experience relatively lower recoil energy for a given projectile mass and velocity.[4]

3.4 Equation of motion of the projectile



Fig 3.4: (a) Gun Barrel with Rifling.
(Source-https://www.emag.com/industries-solutions)

Figure 3.4 shows that the inner surface of the barrel contains helical grooves (rifling), which are designed to impart rotational motion (spin) to the projectile as it travels along the bore. Therefore, the projectile undergoes two types of motion: translational motion and rotational motion.

For the analysis of the projectile motion, both effects are considered. When the projectile moves inside the barrel, the driving band comes into contact with the barrel surface, resulting in the development of normal and frictional forces, as shown in the free body diagram in Fig 3.4.

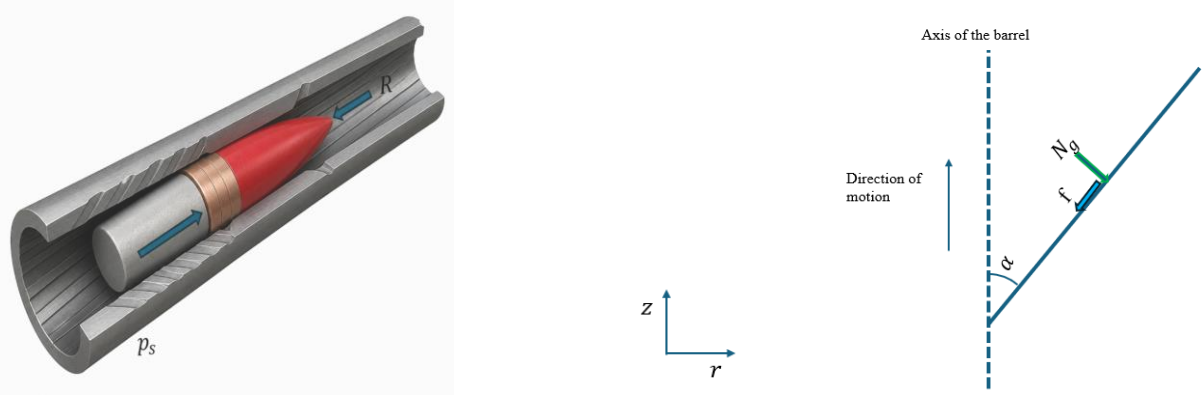


Fig 3.4: (b) free body diagram of forces acting on the surface of the band of the shell.
(Source-https://www.thefirearmblog.com)

Let s denote the axial displacement of the projectile base from the breech at time t. The projectile velocity (v_p) and acceleration (a) are therefore given by [4]

$$\frac{ds}{dt} = v_p \qquad\qquad a = \frac{dv_p}{dt}$$

To determine the projectile's acceleration, both longitudinal (translational) and rotational effects must be considered. Let ψ represent the angle through which the shot has turned at this instant [4], then

$$\psi = as \qquad \omega = \frac{d\psi}{dt} \qquad\qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\psi}{dt^2}$$

Where

a is the rifling rate.
ω is the angular velocity.
α is the angular acceleration.

as we know from above relation

$$\psi = as$$

Above equation differentiating with respect to 't'

$$\frac{d\psi}{dt} = a\frac{ds}{dt}$$

Again, on differentiating

$$\frac{d^2\psi}{dt^2} = a\frac{d^2s}{dt^2} = a\frac{dv_p}{dt}$$

By applying Newton's second law of motion, the forces acting in the direction of motion (transverse direction) are balanced, as illustrated in Fig 3.4.

$$F = ma = m\frac{dv_p}{dt}$$

$$F = A(p_s - p_a) - R - N_g sin\alpha - \mu N_g cos\alpha$$

$$m\frac{dv_p}{dt} = A(p_s - p_a) - R - N_g sin\alpha - \mu N_g cos\alpha \quad (16)$$

Where
N_g - Force applied by band on barrel
f - friction force exerted on a driving band by the barrel groove wall
R - Resistance force
p_s - Shot base pressure (pressure exerted by the gas behind the projectile)

Balancing of the forces in the transverse direction for rotational motion corresponds to the radial direction, as illustrated in Fig 3.4.

$$T = I\alpha$$

Where I is the polar moment of inertia $I = mk^2$,k is the radius of gyration and m is the mass of projectile.

$$T = F_r\, r = \left(N_g cos\alpha - \mu N_g sin\alpha\right) r \quad (17)$$

When the projectile moves inside the barrel, it undergoes rotational motion due to the torque generated by the interaction between the driving band and rifling. Using Eq. (17), the generated torque acting on the projectile can be determined. The final torque balance equation for the torsional motion is therefore expressed as follows.

$$T = F_r\, r = \left(N_g cos\alpha - \mu N_g sin\alpha\right) r = mk^2\frac{d^2\psi}{dt^2}$$

$$mk^2\frac{d^2\psi}{dt^2} = mk^2\alpha\frac{d^2s}{dt^2} = \left(N_g cos\alpha - \mu N_g sin\alpha\right) r$$

$$mk^2\alpha\frac{d^2s}{dt^2} = mk^2\alpha\frac{dv_p}{dt} = \left(N_g cos\alpha - \mu N_g sin\alpha\right) r$$

$$mk^2\alpha\frac{dv_p}{dt} = \left(N_g cos\alpha - \mu N_g sin\alpha\right) r$$

From the above force balance equation in the radial direction, the value of N_g can be

$$N_g = \frac{mk^2\alpha}{(cos\alpha - \mu sin\alpha)r}\frac{dv_p}{dt}$$

$$ar = tan\alpha \qquad\qquad a = \frac{tan\alpha}{r}$$

Where, α represents the rifling angle and a denotes the rifling rate.

Substituting the values of $N_g$and αinto Eq. (16), the force balance equation in the z-direction, i.e., the transverse direction, can be obtained.

$$m\frac{dv_p}{dt} = A(p_s - p_a) - R - \frac{(sin\alpha + \mu cos\alpha)}{(sin\alpha - \mu cos\alpha)}mk^2\frac{\alpha}{r}\frac{dv_p}{dt}$$

$$m\frac{dv_p}{dt} = A(p_s - p_a) - R - \frac{tan\alpha}{r}\frac{(1 + \mu cot\alpha)}{(1 - \mu tan\alpha)}mk^2 a\frac{dv_p}{dt}$$

Where,

$$\epsilon = \frac{(1 + \mu cot\ \alpha)}{(1 - \mu tan\ \alpha)}$$

represents the permeability factor. Thus, the rotational motion of the projectile produces an effect equivalent to an additional mass term in the linear acceleration equation [4].

$$m\frac{dv_p}{dt} = A(p_s - p_a) - R - \frac{tan\alpha}{r}\frac{(1 + \mu cot\alpha)}{(1 - \mu tan\alpha)}mk^2 a\frac{dv_p}{dt}$$

$$m\frac{dv_p}{dt} = A(p_s - p_a) - R - \epsilon m k^2 a^2 \frac{dv_p}{dt}$$

$$m(1 + k^2 a^2 \epsilon)\frac{dv_p}{dt} = A(p_s - p_a) - R \tag{18}$$

Equation (18) represents the final equation of motion of the projectile. However, this equation is valid only under the condition that the pressure force acting on the projectile, $A(p_s - p_a)$, is greater than the engraving resistance force, R[4].

3.5 Estimation of shot base pressure

Although the lumped parameter model neglects spatial gradients and it is primarily essential to estimate the shot base pressure separately from the mean chamber pressure to accurately represent the projectile motion.

Shot base pressure is the pressure exerted by the propellant gases on the base surface of the projectile during its motion inside the gun barrel.

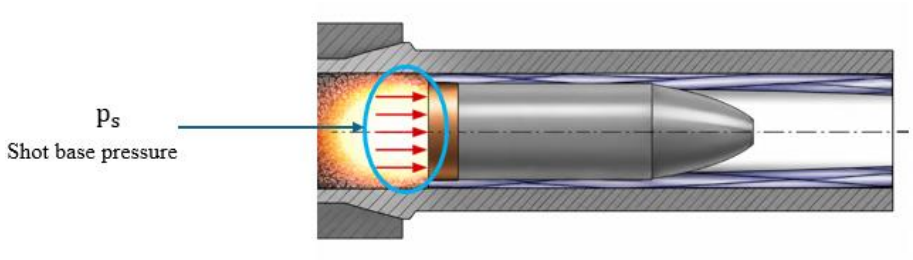


Fig 3.5: (a) Representation of shot base pressure acting on the base of the projectile.
(Source-https://www.hunter-ed.com)

To achieve this, it is assumed that the gas density and the distribution of solid propellant are spatially uniform. However, the velocities of the gas and propellant are assumed to increase linearly from zero at the breech to their respective values $v_p$and $v_2$at the shot base. Furthermore, for simplification, the chamber and bore are assumed to have the same cross-sectional area [4].

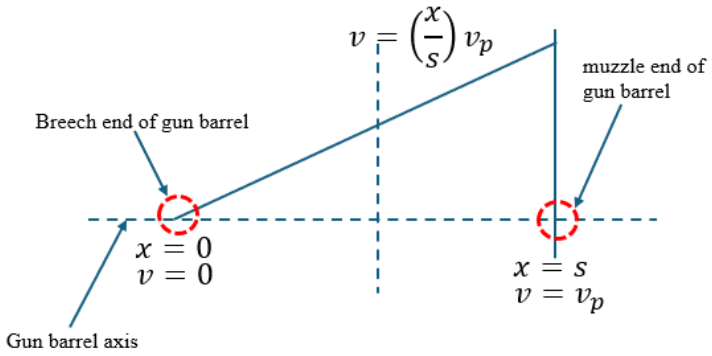


Fig 3.5:(b) variation of velocity with distance.

Let p(x)represent the pressure at a distance $x$from the breech. By applying the conservation of momentum for the gas and propellant mixture in an elemental volume of length dx. the following relation is obtained [4].

$$-A\frac{\partial p(x)}{\partial x}\,dx = \frac{\partial}{\partial t}\left[Cz(\frac{x}{s})\,v_p + C(1-z)\frac{x}{s}v_2\right]dx$$

Integrating from x to s, the following expression is obtained

$$A\big(p(x)-p(s)\big) = \frac{1}{2s^2}(s^2-x^2)\frac{\partial}{\partial t}\big(Czv_p + C(1-z)v_2\big) \tag{19}$$

Equation (19) is a significant relation, as it enables the calculation of pressure at any location along the barrel length. By determining the pressure distribution at a distance x From the breech, the pressure acting inside the gun barrel can be evaluated accurately. This pressure data can further be utilized for numerical simulations and for developing an optimized and safer barrel design [4]

The mean pressure is calculated using the energy balance equation, Eq. (6). With the help of the mean pressure value, the shot base pressure can be determined, since the shot base pressure is a function of both the mean pressure and the resistance pressure. As the resistance pressure is assumed to be constant, the shot base pressure becomes dependent only on the mean pressure. The base pressure acting on the projectile is calculated using Eq. (20) [1].

$$p_{base} = \frac{3p + \left(\frac{C}{m}(p_{res} + p_{air})\right)}{3 + (\frac{C}{m})} \tag{20}$$

Breech end pressure is defined as the pressure exerted by the propellant gases on the breech portion of the gun system during the firing process. It acts on the rear end of the barrel and breech mechanism due to the rapid combustion of the propellant inside the chamber. This pressure plays an important role in determining the structural strength, recoil behavior, and overall performance of the gun system.

The breech end pressure is expressed as a function of the shot base pressure and the resistance pressure. Since the resistance pressure is assumed to remain constant throughout the analysis, the breech end pressure becomes dependent only on the shot base pressure, which is calculated using Eq. (20). Therefore, the breech end pressure can be determined using the following equation, Eq. (21) [1].

$$P_{breech} = P_{base} + \frac{C}{2m}(p_{base} - p_{res} - p_{air}) \tag{21}$$

Where ,

P_{breech} - Breech end pressure

P_{base} - Shot base pressure

p_{res} - Resistance pressure

3.6 Governing equations

Rate of burning equation

$$D\frac{df}{dt} = -\beta p^{\alpha}$$

Form function

$$z = (1 - f)(1 + \theta f)$$

Effective chamber volume

$$CzV = K + A(s - s_0) - \frac{C(1 - z)}{\rho_{prop}}$$

Energy balance equation

$$(\gamma - 1)\ External\ work\ done = [\ CzF(Cz + M_a)p(V - \eta)\ ]$$

Equation of state

$$p(V_{gas} - \eta) = nRT$$

Burning rate coefficient equation

$$\beta\ =\ \beta_{Ta}(1 + \delta(T - T_a))$$

Shot base pressure

$$p_{base} = \frac{3p + \left(\frac{C}{m}(p_{res} + p_{air})\right)}{3 + (\frac{C}{m})}$$

Breech end pressure

$$P_{breech} = P_{base} + \frac{C}{2m}(p_{base} - p_{res} - p_{air})$$

Equation of motion of a shot

$$m(1 + k^2a^2\epsilon)\frac{dv_p}{dt} = A(p_s - p_a) - R$$

$$\frac{ds}{dt} = v_p$$

Chapter 4

Numerical implementation of the lumped parameter model

4.1 Introduction

In the present work, the Euler Forward method is primarily employed due to its simplicity and computational efficiency for solving the coupled differential equations. Using this implementation, the variation of important ballistic parameters such as pressure, temperature, and projectile velocity is evaluated with respect to time and projectile displacement.

Furthermore, critical parameters, including the burning rate coefficient and the initial chamber volume, are calibrated using a trial-and-error approach along with the data reported in the literature, for obtaining realistic ballistic predictions. The results generated from the MATLAB-based lumped parameter model are then benchmarked with corresponding published work [1].

After successful validation, the developed lumped parameter model is incorporated into Abaqus for coupled finite element analysis through user subroutines such as VUAMP, VDLOAD, and VFRIC. These subroutines are used to apply transient pressure loading, define time-dependent loading behavior, and model contact friction effects during projectile-barrel interaction.

The overall numerical implementation procedure is adopted in MATLAB.

4.2 Parameters

The Lumped Parameter Model consists of several input parameters that govern the internal ballistic behavior and significantly influence the prediction of firing characteristics. The accuracy of the model mainly depends on critical parameters such as the burning rate coefficient, chamber volume, charge mass, resistance force, form function, and pressure index. In addition, other geometric, thermodynamic, and projectile-related parameters are also required for the numerical implementation of the model. These parameters collectively affect propellant combustion, pressure generation, gas expansion, and projectile motion inside the barrel.

4.3 Material properties

For the numerical implementation of the Lumped Parameter Model (LPM) in MATLAB, various input parameters and material properties are required. These parameters are associated with the propellant, projectile, and gun system, and they play an important role in predicting the internal ballistic behavior of the gun chamber. The accuracy of the developed model mainly depends on the proper selection of these parameters.

The required properties include geometric parameters, thermodynamic properties, propellant combustion characteristics, projectile properties, and gun system parameters. These parameters are

utilized in the governing equations to calculate important ballistic quantities such as mean chamber pressure, shot base pressure, breech pressure, gas temperature, and projectile velocity.

The values of the material properties and input parameters considered in the present study for the implementation of the Lumped Parameter Model are listed in Table 4.3.

Table 4.3: Input Parameters and Material Properties Used in MATLAB Implementation of the Lumped Parameter Model [2,4].

Propellant			
Parameter name	**Symbol**	**Value**	**UNIT**
Total charge mass	C	9.5255	kg
Density of the propellant	ρ_{prop}	1578	kg/m^3
Form function constant	Θ	-0.172	-
Covolume	η	0.001083	-
Adiabatic flame temperature of the propellant	T_0	2586	Kelvin
Fraction of ballistic size remaining at time 't'	f	0.999999	-
Burning rate constant	β	10.8*10^(-10)	$m/s/pa^{\alpha}$
Heat loss factor	b	0.17	-
Number of gram molecules per unit mass of gas	n	46.948	mol/kg
Web size (ballistic size)	D	0.02	m
Universal gas constant	R	8.314	J/K mol
Temperature coefficient of the burning rate	δ	0.0018	-
Pressure index	α	0.9	-
Projectile and Gun System			
Mass of projectile	m	45.359	kg
Bore Area	A	0.01368	m2
bore dia	d	0.132	m
Initial volume of the chamber	K	0.0104	m3
Mass of gun (in recoil)	M_G	3250	kg
Another parameter			
Ambient Temperature	T_a	293	K
Ambient pressure	p_a	o.101325	MPa
Engraving resistance pressure	p_{rest}	13.79	MPa
Force constant	F	1.04E+06	-

4.4 Flow Chart (Process of implementation of the Lumped parameter model in MATLAB)

The flowchart shown below illustrates the implementation procedure of the lumped parameter model in MATLAB. All the computational steps involved in the numerical implementation are presented sequentially as follows:

1. Initially, all the required input parameters, such as projectile mass, bore diameter, chamber volume, propellant properties, burning rate coefficient, initial pressure, temperature, and time increment (dt)are defined in MATLAB.
2. The burning parameter f at time $(t+dt)$ is evaluated using the governing burning rate equation.
3. The condition for projectile motion is checked using the net accelerating force expression:
$$A(p_s - p_a) - R > 0$$
4. If the above condition is satisfied, the projectile starts moving inside the barrel.
5. The projectile displacement, velocity, and acceleration at the next step are calculated using the equations of motion through the Euler Forward numerical integration method.
6. Based on the updated projectile displacement s and burning parameter f, the chamber volume V and burned propellant fraction z are determined.
7. The mean chamber pressure is calculated using the energy equation of the lumped parameter model.
8. The gas temperature is evaluated using the equation of state whenever required.
9. The shot base pressure and breech pressure are estimated using the corresponding pressure distribution equations.
10. The computed ballistic parameters, such as pressure, temperature, projectile velocity, displacement, and acceleration, are stored for post-processing and analysis.
11. The model checks whether the projectile remains inside the barrel or exits the barrel.
12. If the projectile exits the barrel, the computational procedure is terminated.
13. If the burning parameter reaches its limiting condition $(f < 0)$,The integration of the burning rate equation is stopped by assigning
$$f = 0, z = 1$$
14. The complete numerical procedure is repeated iteratively for the subsequent time steps until the projectile completely leaves the barrel.
15. Finally, the obtained MATLAB results are analysed to study the variation of pressure, temperature, and projectile velocity with respect to time and projectile displacement, and the generated results are benchmarked with available literature data for validation of the developed lumped parameter model.

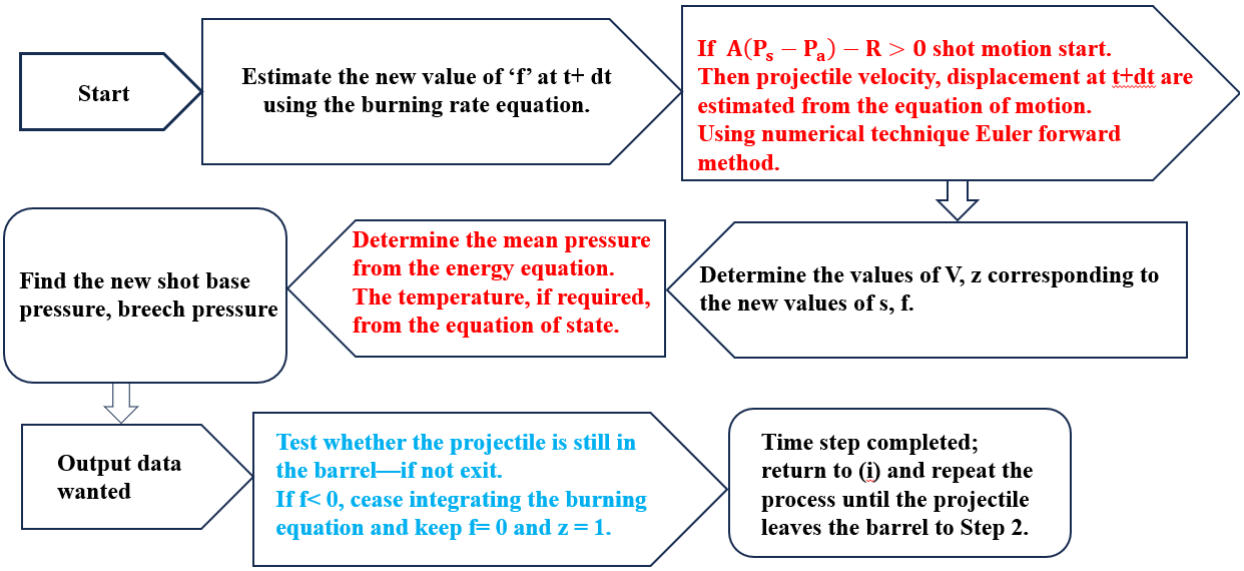


Fig 4.4: flow chart of lumped parameter model.

Chapter 5

Results and discussion

5.1 Pressure variation

When the propellant starts burning inside the gun chamber, a large amount of high-pressure gas is generated, as shown in the figure. The pressure acting on the rear face of the projectile is referred to as the shot base pressure, while the average pressure of the gases within the chamber is known as the mean pressure. Initially, the pressure rises rapidly due to the combustion of the propellant. As the shot base pressure increases, the projectile begins to move along the barrel. During projectile motion, the chamber volume continuously changes, causing variations in pressure with respect to both time and projectile displacement. The pressure evolution throughout the firing process can be observed in Fig. 5 .1.

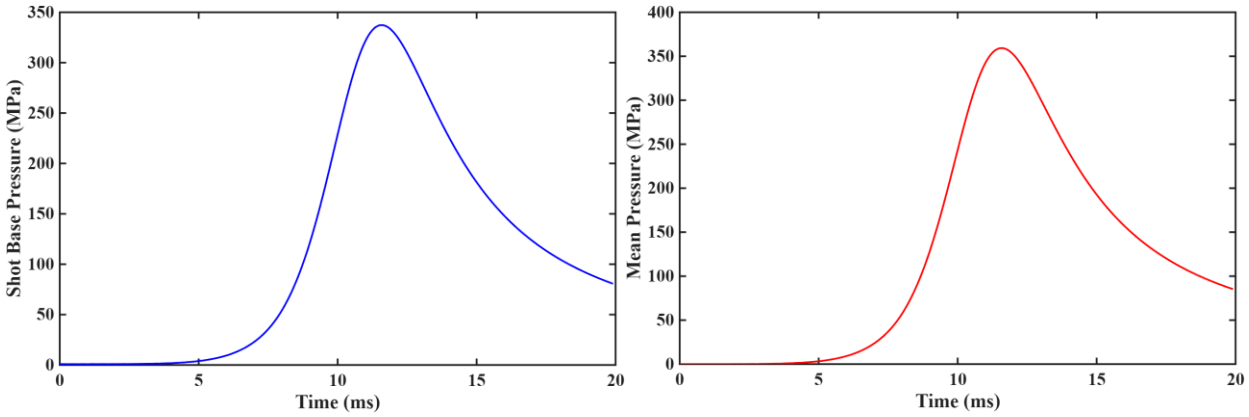


Fig 5.1: (a) Variation of shot base pressure with time and (b) Variation of shot base pressure with time.

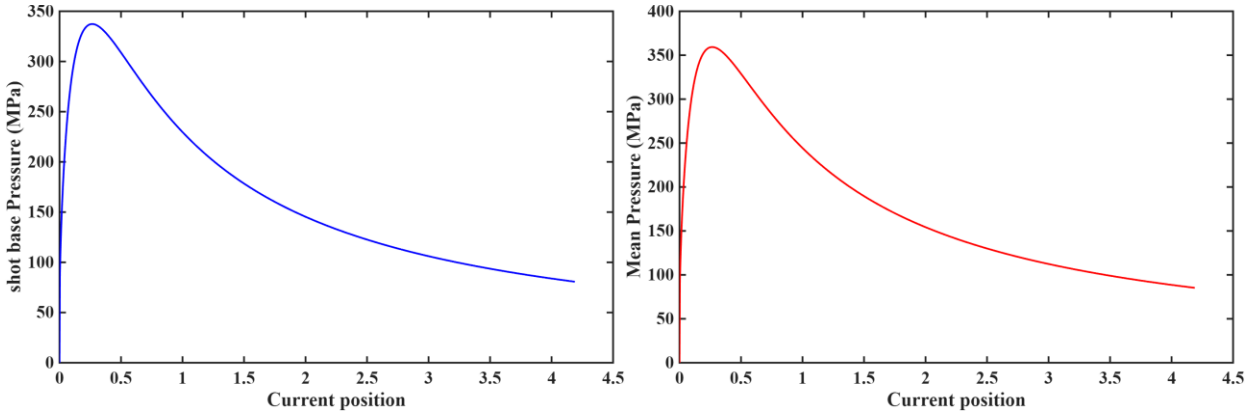


Fig 5.1: (c) Variation of shot base pressure with current position and (d) Variation of shot base pressure with current position.

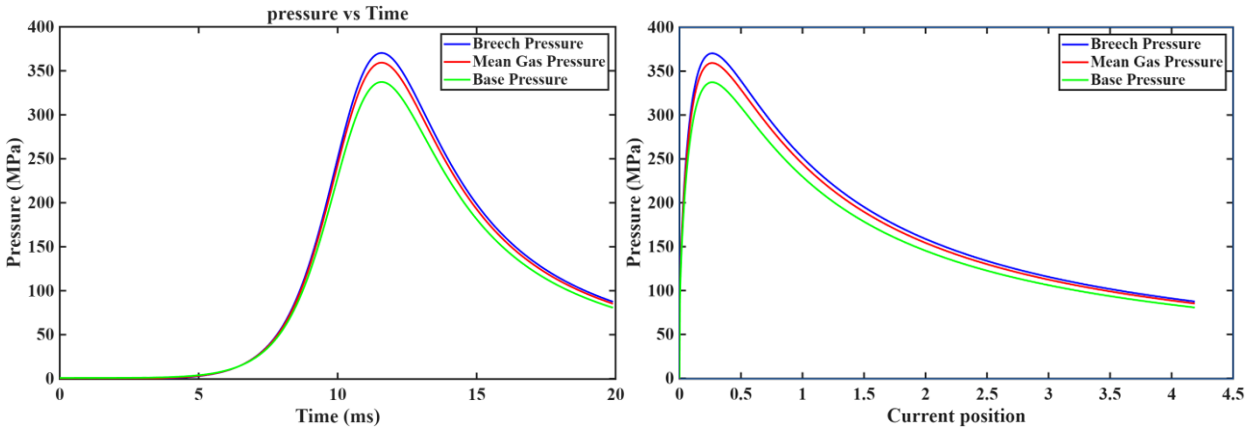


Fig 5.1: (e) Variation of breech pressure, mean gas pressure, and base pressure with time, and (f) Variation of breech pressure, mean gas pressure, and base pressure with current position.

5.2 Velocity variation

In internal ballistics, the variation of projectile velocity with respect to time and displacement is shown in Figure 5.2. The graph indicates that the projectile velocity increases continuously as it travels along the barrel, reaching its maximum value at the muzzle end.

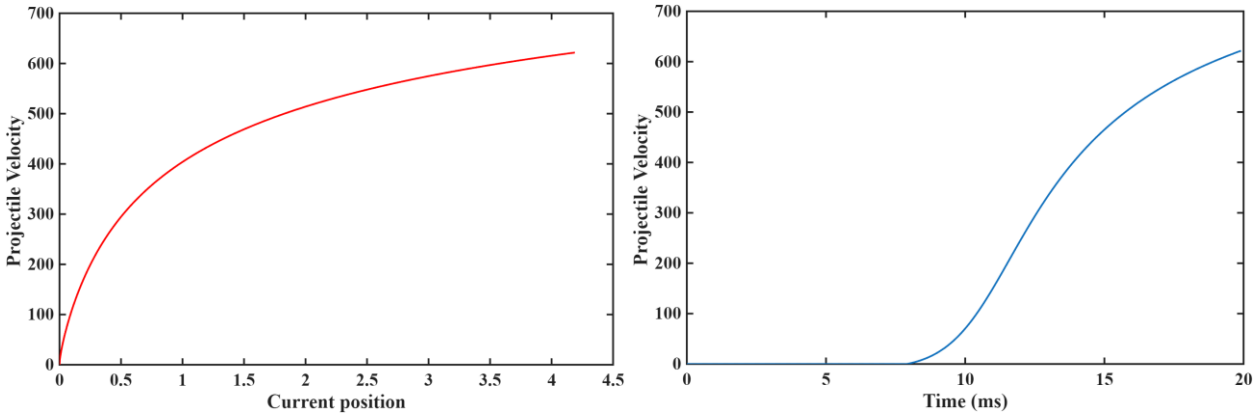


Fig 5.2: (a) The variation of projectile velocity with respect to current position, and (b) the variation of projectile velocity with respect to time.

5.3 Fraction of ballistic size and mass fraction of ballistic size variation

Fig 5.3 shows the variation of the fraction of ballistic size (f) and the mass fraction of propellant(z) with respect to time. The plot illustrates the progressive regression of the propellant grain and the corresponding reduction in mass during combustion.

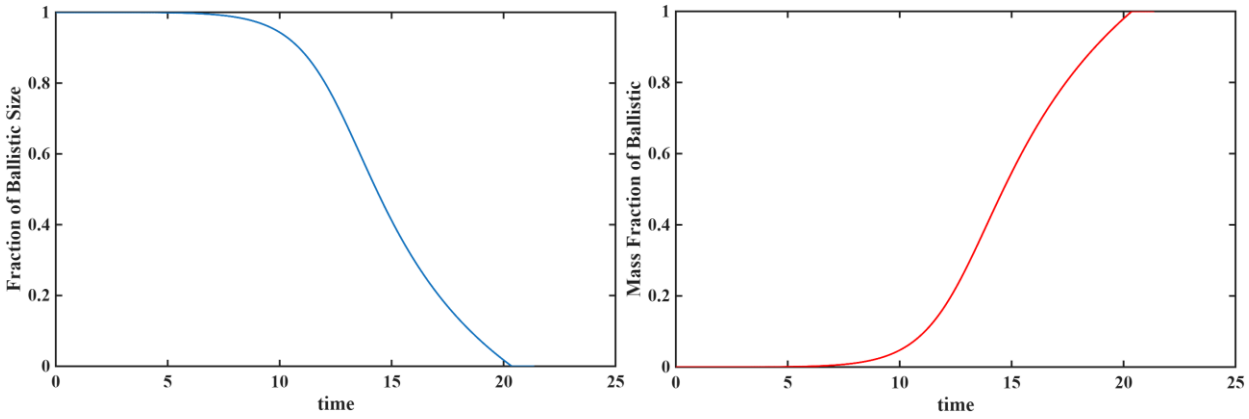


Fig 5.3: (a) The variation of fraction of ballistic size with respect to time, and (b) The variation of mass fraction of ballistic size with respect to time.

5.4 Temperature variation

Initially, the igniter starts burning and generates a high temperature of about 2585 K. This temperature ignites the main propellant charge. During this early stage, the temperature stays nearly constant because the propellant continues to burn and release heat. As the projectile begins to move inside the barrel, the chamber volume continuously increases. This expansion of the combustion gases leads to a change in temperature over time. In Figure 5.4, the gas temperature gradually decreases in the later stages due to chamber expansion and the conversion of thermal energy into the projectile's kinetic energy. You can clearly see the temperature change over time in the figure.

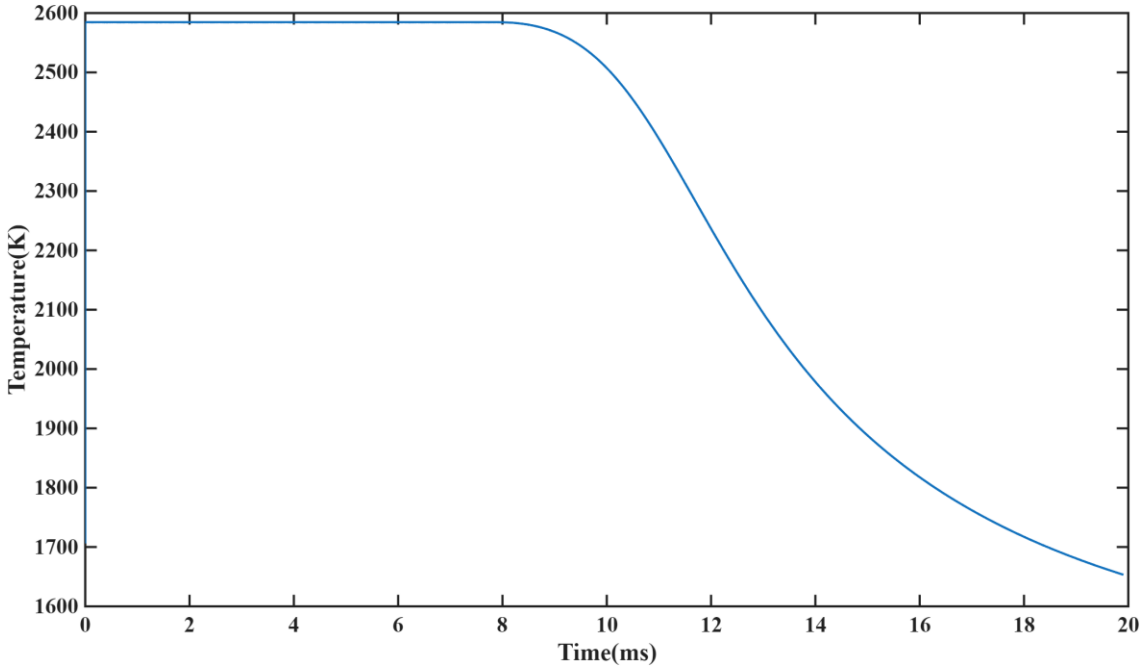


Fig 5.4: The variation of temperature with respect to time.

5.5 Benchmarking

In this report, the results obtained from the developed MATLAB code are presented and compared with available literature. The MATLAB program is based on the Lumped Parameter Model (LPM) for internal ballistics. A similar LPM approach has been used in the study by Dolman (2017), Weapons and Combat Systems Division, Defence Science and Technology Group (DST-Group-TN-1655). However, in that report, the governing equations and critical parameter values were not explicitly detailed, and only the final LPM results were presented. Therefore, a comparative analysis has been carried out between the present model predictions and the results reported by Dolman (2017) [1].

Fig.1.4 illustrates the variation of mean chamber pressure and temperature with respect to time, along with their comparison. It can be observed that the predicted trends from the present MATLAB model closely match the literature results. The percentage error between the two sets of results is minimal, as shown in Table 1.2.

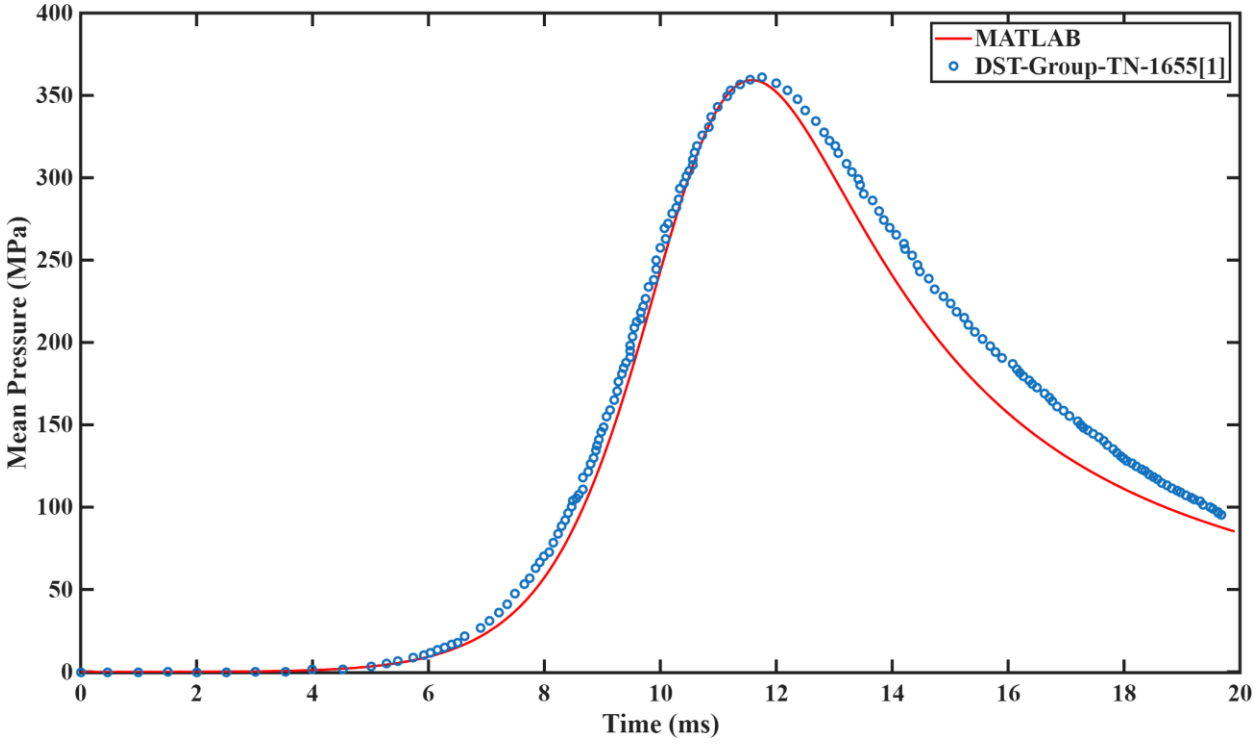


Fig 5.5: (a)Comparison of mean pressure and time history between literature data [1] and MATLAB Lumped Parameter Model.

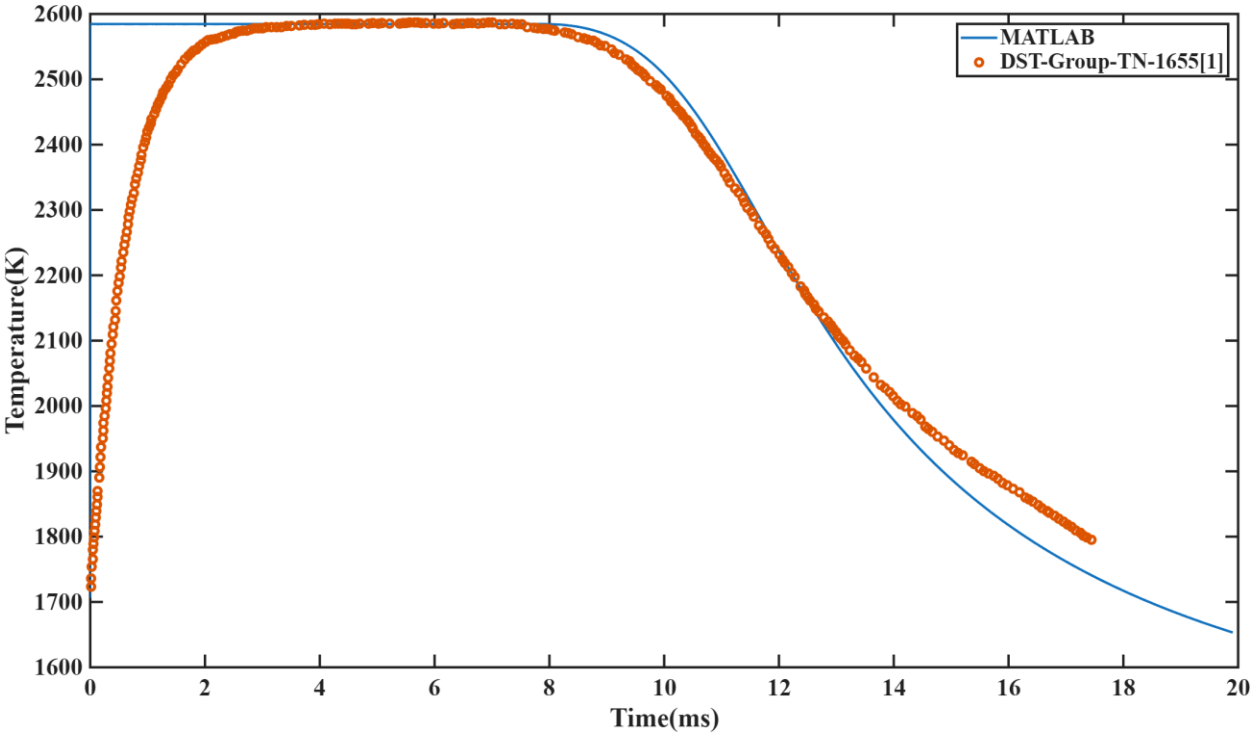


Fig 5.5: (b)Comparison of mean temperature and time history between literature data [1] and MATLAB Lumped Parameter Model.

Table 5.5: Comparison of parameter optioned from the literature and determined from the present study.

Parameters	**IBHVG2**	**DST IB CODE**	**MATLAB Code**	**% error**
Maximum breech Pressure (MPa)	371.8	371.5	370.3	0.40
Maximum projectile base Pressure (MPa)	337.8	337.6	336.5	0.37
Time of maximum breech pressure (ms)	11.61	11.63	12.01	3.44

Chapter 6

Conclusions and future scope

6.1 Conclusions

In the present work, a Lumped Parameter Model (LPM) for internal ballistics was successfully developed and numerically implemented in MATLAB to predict the firing characteristics of a gun chamber. The model incorporated governing equations for propellant burning, energy conservation, projectile motion, and shot base pressure estimation. The Euler Forward numerical integration technique was employed to solve the coupled equations and obtain the transient ballistic response during the firing cycle.

The developed model successfully predicted the variation of important ballistic parameters such as mean chamber pressure, breech pressure, shot-base pressure, gas temperature, projectile velocity, and propellant burning characteristics with respect to time and projectile displacement. The pressure profile showed a rapid rise during propellant combustion followed by a gradual decay due to chamber expansion. Similarly, the temperature initially reached approximately 2585 K and later decreased because of gas expansion and conversion of thermal energy into projectile kinetic energy. The projectile velocity continuously increased along the barrel and attained a maximum value near the muzzle, demonstrating expected internal ballistic behaviour.

The variation of the ballistic size fraction and mass fraction of the propellant successfully captured the progressive burning process and regression characteristics of the propellant grain. The implemented numerical model effectively represented the coupled interaction between propellant combustion and projectile motion inside the barrel.

The MATLAB results were benchmarked with available literature data and showed good agreement. The comparison of mean pressure and temperature histories demonstrated close matching with previously reported results. The maximum percentage error remained relatively small, with peak breech pressure and projectile base pressure errors below 1%, confirming the accuracy and reliability of the developed model.

Furthermore, the developed LPM formulation provides a computationally efficient approach for predicting internal ballistic characteristics with lower computational cost compared to detailed gas dynamic models. The validated model can therefore be used as a useful tool for preliminary gun system design, performance estimation, and further coupled finite element analysis using Abaqus user subroutines.

6.2 Future Scope

The developed Lumped Parameter Model (LPM) can be further extended for finite element analysis (FEA) of the gun system using Abaqus 2017. The implemented user subroutines such as VUAMP, VFRIC, and VDLOAD can be utilized to apply transient pressure loading and study the structural response of the barrel during firing.

Using the developed numerical approach, the strain and stress distribution on the outer surface of the gun barrel can be evaluated. In addition, the plastic deformation behavior of the rotating (driving) band during the engraving process can also be analyzed.

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