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On

Stochastic modelling and analysis of hybrid FSO/RF terrestrial communication systems

BY

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CANDIDATE'S DECLARATION

We hereby declare that the project entitled **"Stochastic modelling and analysis of hybrid FSO/RF terrestrial communication systems"** submitted in partial fulfillment for the award of the degree of Bachelor of Technology in 'Electrical Engineering' completed under the supervision of **Dr. Swaminathan R., Assistant Professor, Discipline of Electrical Engineering,** IIT Indore is an authentic work.

Further, I declare that I have not submitted this work for the award of any other degree elsewhere.

Signature and name of the student with date

CERTIFICATE by BTP Guide(s)

It is certified that the above statement made by the students is correct to the best of my knowledge.

Signature of BTP Guide with dates and their designation

Preface

This report on “Stochastic modelling and analysis of hybrid FSO/RF terrestrial communication systems" is prepared under the guidance of Dr. Swaminathan R..

In this report, I have written elaborately about the analysis of terrestrial hybrid FSO/RF system using various performance metrics like outage probability and average symbol error rate. In particular, the switching scheme of the hybrid FSO/RF system, namely, adaptive combining, has been analysed in this report. I have also included the asymptotic analysis of this scheme. Finally, I have also written about the results obtained from the derived expressions using various plots and figures. I have tried my best to explain the content in a concise yet profound manner.

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Abstract

Radio frequency (RF) is predominantly used in most of today’s wireless communication systems. But this frequency band will not be able to meet the growing demand for higher data rates and bandwidth. Free space optics (FSO) systems can serve as a good alternative to RF systems as they offer high data rates and are easily deployable. But, these systems are vulnerable to fog and haze. This drawback is rectified using a back-up RF link. Such FSO systems with a back-up RF link are called hybrid FSO/RF systems, which provide better performance than single-link FSO systems. FSO links are also susceptible to pointing errors. These systems are characterized by the switching schemes they employ. Some of the switching schemes include hard-switching and adaptive combining.

This thesis is concerned about the analysis of adaptive combining using performance metrics like outage probability and average symbol error rate (SER). The theoretical closed form expressions of outage and average SER of adaptive combining are derived with and without pointing errors. These expressions are validated using Monte-Carlo simulations. Asymptotic analysis of the derived expressions has also been carried out along with the determination of diversity order. The performance of adaptive combining has been compared with that of hard-switching and single-link FSO systems. The results show that adaptive combining provides better performance compared to hard-switching and single-link FSO.

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Chapter 1

Introduction

The advent of wireless communication systems is one of the most important advancements in modern technology. It has found uses in many sectors, owing to its sophistication, at an unprecedented rate. Most of the wireless systems used today are at their core RF (Radio-Frequency) technologies. But the RF band of the electromagnetic spectrum has limited capacity. The rate at which data is being consumed around the world is increasing. This means that the supply of sub-bands of RF will fall short in the near future.

Optical Wireless Communication (OWC) refers to transmission in unguided propagation media through the use of optical carriers [1] in visible, infrared (IR) and ultraviolet (UV) frequency band. One of the categories of OWC is the outdoor terrestrial OWC. This has been called Free Space Optical (FSO) communication in the literature. One of the advantages FSO offers over RF is that it doesn't require any license fees because it operates at a frequency of the order of 300 GHz, which is unlicensed. Furthermore, FSO offers higher data rates as well as bandwidth compared to RF. In comparision to fibre optic cables, FSO offers greater immunity to electromagnetic interference due to spatial confinement using narrow laser beams [1]. Also, the costs of installation of FSO are lesser than that of fibre optic networks [2].

An FSO communication system transmits information by modulating the data onto an optical carrier. The data bits are first encoded then modulated. The carrier passes through an amplifier to enhance its optical intensity. The optical carrier is then radiated through the atmosphere to the target destination. At the reception, a photo-detector senses changes in the optical intensity of the received beam to decode information. Such systems are called Intensity Modulation Direct-Detection (IM/DD) systems. The received beam consists of several noises like transmitter noise, thermal noise and photocurrent shot-noise. The received noise is modelled as additive white Gaussian noise (AWGN).

The reliability of FSO links depends on atmospheric conditions. These are particularly vulnerable to fog, haze and scintillation; in the presence of which their performance declines [2]. An innovative solution to this shortcoming is the utilisation of another RF link as a back-up link to the FSO link. Under fog and haze, RF can be activated as per a switching scheme to ensure optimum performance. Under rain and snow, FSO can be used as usual. Such a system that uses RF link to enhance and preserve the performance of FSO link is called a hybrid FSO/RF system. A hybrid system enables in saving costs when compared to an only-FSO based system [2] and is easier to integrate in the existing communication infrastructure.

1.1 Literature survey

The basic channel modelling of FSO link begins by taking into account atmospheric-turbulence-induced fading, which happens when the optical power of the FSO link is affected due to solar heating and wind [2]. This has been modelled using Gamma-Gamma distribution [3]. The small scale fading in case of RF link has been modelled using Nakagami-m distribution [4]. The channel modelling of FSO can be extended to take into consideration effects like pointing errors and atmospheric losses. Pointing errors crop up due to divergence of the beam while propagating through atmosphere. These errors become particularly important in long link distances. Atmospheric losses occur when pollutants, dust, aerosol etc. scatter the optical beam and hence attenuate the optical power.

The switching scheme of a hybrid system is an important aspect that decides the system performance. One of the proposed switching schemes involves switching between FSO and RF in a complementary fashion, wherein only one of the links is active at any point of time depending upon the atmospheric conditions [5]. However, this scheme involves frequent hardware switching. In [6], the performance analysis of various diversity-combining-based FSO/RF systems has been investigated. Here, both FSO and RF links transmit simultaneously all the time. The drawback here is that RF stays active even when the FSO link has good transmission quality which means that the RF trasmission power is wasted in this scenario. In addition, the data rate of FSO link is also reduced to that of RF link due to diversity combining employed at destination.

A promising switching scheme has been proposed by Tamer et al. [7]. According to the scheme, only the FSO link stays active as long as its instantaneous Signal-to-Noise Ratio (SNR) is above a switching threshold. When the instantaneous SNR of FSO link drops below the switching threshold, the RF link is also activated and both the links are then combined at the reception through Maximal Ratio Combining (MRC). This technique is called adaptive combining. This scheme circumvents all the drawbacks of the previously discussed switching schemes.

1.2 Motivations

The motivations behind the proposed work are as follows:

- The analysis of adaptive combining is restricted only to outage probability [7] to the best of our knowledge. Also, the final expressions derived for outage probability are complex and there is a scope for simplification.
- To the best of our knowledge, no research has been done on the average symbol error rate (SER) of adaptive combining. Average SER analysis of MRC-based FSO/RF system has been investigated [6], but only an approximate expression for average SER has been derived. Also, there are no prior works on outage & SER analysis of adaptive combining taking pointing errors & atmospheric losses into account.
- This project aims to carry out rigorous analysis of a hybrid FSO/RF system which employs adaptive combining and derive exact expresssions for average SER and outage probability with and without pointing errors & atmospheric losses. In addition, asymptotic analysis has also been carried out to obtain the diversity gain of the proposed system.
- There is also a need to determine a range for switching threshold for optimum performance of the hybrid FSO/RF system.

1.3 Contributions

The contributions of the proposed work are given as follows:

- A part of the outage probability expression derived in [7] has been simplified to an expression involving single summation as opposed to the expression involving triple summation in [7]. Using single link FSO system and the switching scheme in [5] as benchmarks, the system performance under adaptive combining is also studied.

- Exact expressions of outage probability and average SER have been derived for adaptive combining scheme with and without pointing errors.
- Asymptotic analysis has been carried out for expressions of both outage and average SER and diversity gain has been obtained for both the cases of adaptive combining, i.e. with and without pointing errors.
- A constraint on setting an optimal switching threshold with respect to outage probability has also been worked out. This has been achieved by plotting outage probability as a function of switching threshold.
- The effect of atmospheric attenuation due to different weather conditions on the system performance has also been investigated.

Chapter 2

System model of hybrid FSO/RF system with adaptive combining

In this chapter, we look at the system and channel models that will be used to derive the outage probability and average SER expressions in the coming chapters.

2.1 System model

For RF and FSO we use the following expressions, respectively, for the received baseband signal

$$y_1[k] = h[k]x[k] + n_1[k], \tag{2.1}$$

$$y_2[k] = I[k]x[k] + n_2[k], \tag{2.2}$$

where x is the input signal, n_1 & n_2 are additive white Gaussian noises, y_1 & y_2 are the received signals, h is RF channel gain and I is FSO channel gain. Here, we assume both the links to be slow flat fading links and also that we have the channel state information (CSI) of the links. In the hybrid system, we use FSO and RF links that are activated according to adaptive combining scheme. In this switching scheme, we keep the FSO link active throughout but the RF link is activated only when the instantaneous SNR of FSO link (γ_{FSO}) is less than a switching threshold (γ_T). In this case, the activated RF link is combined with the FSO link using maximal ratio combining (MRC) (as shown in Fig. 2.1). The combining rule for MRC is given by

$$y_{MRC} = \sqrt{\gamma_{RF}}\frac{y_1}{\sigma_{n_1}} + \sqrt{\gamma_{FSO}}\frac{y_2}{\sigma_{n_2}} \tag{2.3}$$

where y_{MRC} is the output signal after MRC, γ_{RF} is the instantaneous SNR of RF and σ_{n_1} & σ_{n_2} are the standard deviations of n_1 & n_2, respectively.

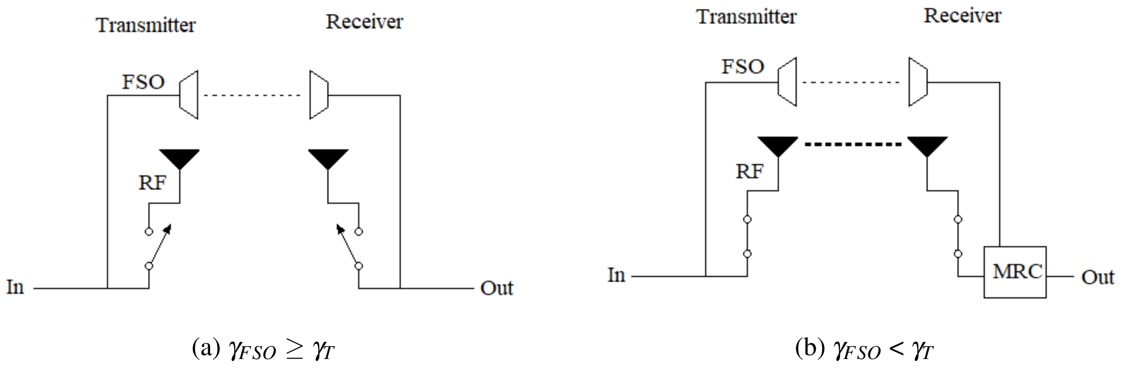

Figure 2.1: Hybrid FSO/RF system model where γ_{FSO} is the instantaneous SNR of FSO link and γ_T is the switching threshold

We consider γ_c to be the instantaneous SNR at the receiver and based on the definition of adaptive combining, we can define γ_c as

$$\gamma_c = \begin{cases} \gamma_{FSO} + \gamma_{RF}, & \gamma_{FSO} < \gamma_T. \\ \gamma_{FSO}, & \gamma_{FSO} \geq \gamma_T. \end{cases} \tag{2.4}$$

where the first case is the resultant SNR after the links are combined using MRC. Using this definition, we can derive the cumulative distribution function (CDF)

$$F_{\gamma_c}(x) = Pr[\gamma_{\mathrm{FSO}} \geq \gamma_{\mathrm{T}}, \gamma_{\mathrm{FSO}} < x] + Pr[\gamma_{\mathrm{FSO}} < \gamma_{\mathrm{T}}, \gamma_{\mathrm{FSO}} + \gamma_{\mathrm{RF}} < x] \tag{2.5}$$

After evaluating and expanding the terms, we get the following

$$F_{\gamma_c}(x) = \begin{cases} F_1(x), & x \leq \gamma_{\mathrm{T}}. \\ F_2(x) - F_{\gamma_{\mathrm{FSO}}}(\gamma_T) + F_{\gamma_{\mathrm{FSO}}}(x), & x > \gamma_{\mathrm{T}}. \end{cases} \tag{2.6}$$

where $F_{\gamma_{FSO}}$ is the CDF of γ_{FSO} and

$$F_1(x) = \int_0^x f_{\gamma_{FSO}+\gamma_{RF}}(t)dt \tag{2.7}$$

and

$$F_2(x) = \int_0^{\gamma_T} f_{\gamma_{FSO}}(t) F_{\gamma_{RF}}(x-t)dt \tag{2.8}$$

where $f_{\gamma_{FSO}+\gamma_{RF}}(t)$ is the probability distribution function (PDF) of $\gamma_{FSO} + \gamma_{RF}$, $f_{\gamma_{FSO}}(t)$ is the PDF of γ_{FSO} and $F_{\gamma_{RF}}(t)$ is the CDF of γ_{RF}. We can derive $f_{\gamma_{FSO}+\gamma_{RF}}(t)$ using the fact that both the links are statistically independent

$$f_{\gamma_{FSO}+\gamma_{RF}}(x) = \int_0^x f_{\gamma_{FSO}}(t) f_{\gamma_{RF}}(x-t)dt \tag{2.9}$$

where $f_{\gamma_{RF}}(t)$ is the PDF of γ_{RF}.

We obtain the PDF of γ_c by derivating its CDF

$$f_{\gamma_c}(x) = \begin{cases} f_{\gamma_{FSO}+\gamma_{RF}}(x), & x \leq \gamma_T. \\ f_{\gamma_{FSO}}(x) + G(x), & x > \gamma_{\mathrm{T}}. \end{cases} \tag{2.10}$$

where

$$G(x) = \int_0^{\gamma_T} f_{\gamma_{FSO}}(t) f_{\gamma_{RF}}(x-t)dt \tag{2.11}$$

2.2 RF channel model

The envelope of RF channel gain h can be characterised using a Nakagami-m distribution which is given by

$$f_{|h|}(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\sigma^m} e^{-\frac{m}{\sigma}x^2}, \tag{2.12}$$

where m is the fading severity parameter and σ is the standard deviation ($\sigma = \sqrt{E[|h|^2]}$). We assume $|h|$ to remain constant for at least one symbol duration (slow fading). Now, the relation between γ_{RF} and $|h|$ is

$$\gamma_{RF} = \bar{\gamma}_{RF}|h|^2, \tag{2.13}$$

where $\bar{\gamma}_{RF}$ is the average SNR of the RF link. Taking expectation on both sides we get

$$\begin{aligned} E[\gamma_{RF}] = \bar{\gamma}_{RF} = \bar{\gamma}_{RF} E[|h|^2], \\ E[|h|^2] = 1 \end{aligned} \tag{2.14}$$

Using power transformation of random variables, we obtain the PDF of γ_{RF} as

$$f_{\gamma_{RF}}(x) = \frac{C^m x^{m-1}}{\Gamma(m)} e^{-Cx}, \tag{2.15}$$

where $C = m/\bar{\gamma}_{RF}$. To derive the CDF of γ_{RF}, we integrate its PDF

$$F_{\gamma_{RF}}(x) = \frac{\gamma(m, Cx)}{\Gamma(m)}, \tag{2.16}$$

where $\gamma(\cdot)$ is the lower incomplete Gamma function.

2.3 FSO channel model

Similar to RF channel model, we start with the envelope of FSO channel gain I which is characterised by Gamma-Gamma distribution

$$f_{|I|}(x) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}} x^{\frac{\alpha+\beta}{2}-1}}{\Gamma(\alpha)\Gamma(\beta)} K_{\alpha-\beta}(2\sqrt{\alpha\beta x}), \tag{2.17}$$

where α & β are small and large scale scattering parameters and $K_\nu(\cdot)$ is the modified Bessel function of the second kind. Now, α & β are determined by link distance (L) and air refractive index (C_n^2) and are given by [8]

$$\alpha = \left[\exp\left(\frac{0.49\chi^2}{(1+1.11\chi^{12/5})^{7/6}}\right) - 1\right]^{-1}, \tag{2.18}$$

$$\beta = \left[\exp\left(\frac{0.51\chi^2}{(1+0.69\chi^{12/5})^{5/6}}\right) - 1\right]^{-1}, \tag{2.19}$$

where $\chi^2 = 0.5C_n^2 k^{7/6} L^{11/6}$ is the Rytov variance, $k = 2\pi/\lambda$ and $\lambda = 1550 \times 10^{-9}$m is the wavelength of the optical beam. The relation between γ_{FSO} and $|I|$ is given by

$$\gamma_{FSO} = \bar{\gamma}_{FSO}|I|^2, \tag{2.20}$$

where $\bar{\gamma}_{FSO}$ is the average SNR of the FSO link. Using the Meijer G representation of $K_\nu(\cdot)$ and power transformation of random variables we can derive the PDF of γ_{FSO} as

$$f_{\gamma_{\mathrm{FSO}}}(x) = \frac{x^{-1}}{2\Gamma(\alpha)\Gamma(\beta)} G_{0,2}^{2,0}\left(Dx^{\frac{1}{2}} \;\middle|\; \begin{matrix} - \\ \alpha, \beta \end{matrix}\right), \tag{2.21}$$

where $D = \alpha\beta(\bar{\gamma}_{FSO})^{-1/2}$. We can also derive the CDF of γ_{FSO} by integrating its PDF, which gives us

$$F_{\gamma_{FSO}}(x) = \frac{2^{\alpha+\beta-2}}{\pi\Gamma(\alpha)\Gamma(\beta)} G_{1,5}^{4,1}\left(\frac{D^2 x}{16} \;\middle|\; \begin{matrix} 1 \\ b_1, b_2, b_3, b_4, b_5 \end{matrix}\right) \tag{2.22}$$

where $b_1 = \frac{\alpha}{2}, b_2 = \frac{\alpha+1}{2}, b_3 = \frac{\beta}{2}, b_4 = \frac{\beta+1}{2}$ & $b_5 = 0$.

Chapter 3

Performance analysis of hybrid FSO/RF system without pointing errors

In this chapter, we look at the outage and average symbol error rate (SER) analysis of hybrid FSO/RF system without pointing errors using the adaptive combining scheme. We also look at the asymptotic expressions of outage & average SER and derive the diversity order.

3.1 Outage probability

In adaptive combining, the system is said to be in outage when γ_c is less than an outage threshold, γ_{out}, which can be expressed as

$$P_{AC} = Pr[\gamma_c < \gamma_{out}] \tag{3.1}$$

where P_{AC} is the outage probability. This expression can be further simplified to

$$P_{AC} = F_{\gamma_c}(\gamma_{out}) \tag{3.2}$$

To obtain $F_{\gamma_c}(x)$, we start with $f_{\gamma_{FSO}+\gamma_{RF}}(x)$ as given in (2.9). After substituting (2.15) and (2.21) in (2.9), we get

$$f_{\gamma_{FSO}+\gamma_{RF}}(x) = \frac{C^m}{2\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \int_0^x \left[t^{-1} G_{0,2}^{2,0}\left(Dt^{\frac{1}{2}} \,\middle|\, \begin{matrix} - \\ \alpha, \beta \end{matrix} \right) \right] \left[(x-t)^{m-1} e^{-C(x-t)} \right] dt \tag{3.3}$$

Using the series expansion of e^x and expanding $e^{-C(x-t)}$, we get

$$f_{\gamma_{FSO}+\gamma_{RF}}(x) = \frac{C^m}{2\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty} \frac{(-1)^n C^n}{n!} \int_0^x t^{-1}(x-t)^{n+m-1} G_{0,2}^{2,0}\left(Dt^{\frac{1}{2}} \,\middle|\, \begin{matrix} - \\ \alpha, \beta \end{matrix} \right) dt \tag{3.4}$$

Using [9, Eq. (07.34.21.0084.01)] we obtain

$$f_{\gamma_{FSO}+\gamma_{RF}}(x) = \frac{2^{\alpha+\beta-2}C^m x^{m-1}}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty} (-1)^n \frac{C^n}{n!} \Gamma(m+n) x^n G_{1,5}^{4,1}\left(\frac{D^2 x}{16} \mid \begin{matrix} 1 \\ \mathscr{B}_1 \end{matrix} \right), \tag{3.5}$$

where $\mathscr{B}_1 = [\mathscr{B}_{1,1}, \mathscr{B}_{1,2}, \mathscr{B}_{1,3}, \mathscr{B}_{1,4}, \mathscr{B}_{1,5}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 1-n-m]$. To obtain $F_1(x)$, we substitute (3.5) in (2.7) and use [9, Eq. (07.34.21.0084.01)] to obtain

$$F_1(x) = \frac{2^{\alpha+\beta-2}C^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty} (-1)^n \frac{C^n}{n!} \Gamma(m+n) x^{m+n} G_{2,6}^{4,2}\left(\frac{D^2 x}{16} \mid \begin{matrix} \mathscr{B}_2 \\ \mathscr{B}_3 \end{matrix} \right), \tag{3.6}$$

where $\mathscr{B}_2 = [\mathscr{B}_{2,1}, \mathscr{B}_{2,2}] = [1-n-m, 1]$ and $\mathscr{B}_3 = [\mathscr{B}_{3,1}, \mathscr{B}_{3,2}, \mathscr{B}_{3,3}, \mathscr{B}_{3,4}, \mathscr{B}_{3,5}, \mathscr{B}_{3,6}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 1-n-m, -n-m]$. Now, we derive $F_2(x)$ by substituting (2.21) & (2.16) in (2.8), which gives us

$$F_2(x) = \int_0^{\gamma_T} \left[\frac{t^{-1}}{2\Gamma(\alpha)\Gamma(\beta)} G_{0,2}^{2,0}\left(Dt^{\frac{1}{2}} \mid \begin{matrix} - \\ \alpha, \beta \end{matrix} \right) \right] \left[\frac{\gamma(m, C(x-t))}{\Gamma(m)} \right] dt \tag{3.7}$$

We expand $\gamma(m, C(x-t))$ using the series expansion in [10, Eq. (8.352.1)] along with binomial expansion and series expansion of exponential. After carrying out these steps we get

$$\gamma(m, C(x-t)) = (m-1)! \left[1 - e^{-Cx} \sum_{n=0}^{\infty} \frac{(Ct)^n}{n!} \sum_{k=0}^{m-1} \frac{C^k}{k!} \sum_{j=0}^{k} \binom{k}{j} x^{k-j} (-t)^j \right] \tag{3.8}$$

Now we substitute (3.8) in (3.7) and use the relation $\Gamma(m) = (m-1)!$ to obtain

$$\begin{aligned} F_2(x) = & \left[\int_0^{\gamma_T} \frac{t^{-1}}{2\Gamma(\alpha)\Gamma(\beta)} G_{0,2}^{2,0}\left(Dt^{\frac{1}{2}} \mid \begin{matrix} - \\ \alpha, \beta \end{matrix} \right) dt \right] - \left[\frac{e^{-Cx}}{2\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{(C)^n}{n!} \sum_{k=0}^{m-1} \frac{C^k}{k!} \right. \\ & \left. \sum_{j=0}^{k} \binom{k}{j} x^{k-j} (-1)^j \int_0^{\gamma_T} t^{n+j-1} G_{0,2}^{2,0}\left(Dt^{\frac{1}{2}} \mid \begin{matrix} - \\ \alpha, \beta \end{matrix} \right) dt \right] \end{aligned} \tag{3.9}$$

On observation, we can see that the first term is the integral of PDF of γ_{FSO}, which is its CDF. The integral in the second term can be solved using [9, Eq. (07.34.21.0084.01)]

$$\begin{aligned} F_2(x) = F_{\gamma_{FSO}}(\gamma_T) - & \left[\frac{2^{\alpha+\beta-2}e^{-Cx}}{\pi\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{(C\gamma_T)^n}{n!} \sum_{k=0}^{m-1} \frac{(Cx)^k}{k!} \sum_{j=0}^{k} \binom{k}{j} \left(\frac{-\gamma_T}{x} \right)^j \right. \\ & \left. G_{1,5}^{4,1}\left(\frac{D^2 \gamma_T}{16} \mid \begin{matrix} \mathscr{B}_4 \\ \mathscr{B}_5 \end{matrix} \right) \right], \end{aligned} \tag{3.10}$$

where $\mathscr{B}_4 = [\mathscr{B}_{4,1}] = [1-n-j]$ and $\mathscr{B}_5 = [\mathscr{B}_{5,1}, \mathscr{B}_{5,2}, \mathscr{B}_{5,3}, \mathscr{B}_{5,4}, \mathscr{B}_{5,5}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, -n-j]$. Now to obtain P_{AC} we substitute (2.22), (3.6) and (3.10) in (2.6) and plug in γ_{out} in $F_{\gamma_c}(x)$.

3.2 Average SER

Average SER is the mean number of symbols that are received erroneously at the reception. It is obtained by averaging the conditional error probability for a particular modulation scheme (MPSK in this case) over the PDF of the instantaneous SNR of the link. So

$$\bar{P}_e = \int_0^\infty P(e|x) f_{\gamma_c}(x) dx \tag{3.11}$$

where $\bar{P}_e$ is the average SER and P(elx) is the conditional error probability for the MPSK modulation scheme. We have

$$P(e|x) = \frac{A}{2} erfc(\sqrt{x}B) \tag{3.12}$$

where

$$A = \begin{cases} 1, & M = 2 \\ 2, & M > 2 \end{cases} \tag{3.13}$$

$B = sin\left(\frac{\pi}{M}\right)$ and erfc($\cdot$) is the complementary error function. From (2.10), we have $f_{\gamma_c}(x)$ but we need to determine $G(x)$ to completely obtain $f_{\gamma_c}(x)$. Substituting (2.21) and (2.15) in (2.11), we get

$$G(x) = \int_0^{\gamma_T} \left[\frac{t^{-1}}{2\Gamma(\alpha)\Gamma(\beta)} G_{0,2}^{2,0}\left(Dt^{\frac{1}{2}} \mid \begin{matrix} - \\ \alpha, \beta \end{matrix} \right) \right] \left[\frac{C^m (x-t)^{m-1}}{\Gamma(m)} e^{-C(x-t)} \right] dt \tag{3.14}$$

We expand $e^{-C(x-t)}$ using the series expansion of the exponential, apply binomial expansion on powers of $(x-t)$ and use [9, Eq. (07.34.21.0084.01)] to solve the resultant integral. After these steps, we get

$$G(x) = \frac{2^{\alpha+\beta-2}(Cx)^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{i=0}^{\infty} \frac{(-Cx)^i}{i!} \sum_{j=0}^{m+i-1} \frac{(-\gamma_T)^j}{x^{j+1}} \binom{m+i-1}{j} G_{1,5}^{4,1}\left(\frac{D^2\gamma_T}{16} \mid \begin{matrix} \mathscr{B}_6 \\ \mathscr{B}_7 \end{matrix} \right) \tag{3.15}$$

where $\mathscr{B}_6 = [\mathscr{B}_{6,1}] = [1-j]$ and $\mathscr{B}_7 = [\mathscr{B}_{7,1}, \mathscr{B}_{7,2}, \mathscr{B}_{7,3}, \mathscr{B}_{7,4}, \mathscr{B}_{7,5}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, -j]$. We can now start from (3.11) and based on the definition of $f_{\gamma_c}(x)$, divide it into two integrals

$$\bar{P}_e = \underbrace{\int_0^{\gamma_T} P(e|x) f_{\gamma_{FSO}+\gamma_{RF}}(x) dx}_{\mathrm{I}_1} + \underbrace{\int_{\gamma_T}^{\infty} P(e|x)(f_{\gamma_{FSO}}(x) + G(x)) dx}_{\mathrm{I}_2} \tag{3.16}$$

We start solving for $\bar{P}_e$ from I_1. Using the relation $erfc(x) = 1 - erf(x)$, where erf($\cdot$) is the error function, and substituting (3.12) in I_1, we get

$$I_1 = \int_0^{\gamma_T} \frac{A}{2}(1 - erf(\sqrt{x}B)) f_{\gamma_{FSO}+\gamma_{RF}}(x) dx \tag{3.17}$$

We have

$$I_1 = \frac{A}{2}\Bigg(\underbrace{\int_0^{\gamma_T} f_{\gamma_{FSO}+\gamma_{RF}}(x)dx}_{\mathrm{I}_{11}} - \underbrace{\int_0^{\gamma_T} erf(\sqrt{x}B) f_{\gamma_{FSO}+\gamma_{RF}}(x)dx}_{\mathrm{I}_{12}}\Bigg) \tag{3.18}$$

We can easily see that $I_{11} = F_1(\gamma_T)$. To solve for I_{12}, we substitute (3.5) in I_{12}, use the series expansion of the error function and apply [9, Eq. (07.34.21.0084.01)] to obtain

$$I_{12} = \frac{2^{\alpha+\beta-1}(C\gamma_T)^m}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty} \frac{(-1)^n (B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)} \sum_{k=0}^{\infty} \frac{(-1)^k (C\gamma_T)^k}{k!} \Gamma(m+k)$$
$$G_{2,6}^{4,2}\left(\frac{D^2\gamma_T}{16} \;\middle|\; \begin{matrix} \mathscr{B}_8 \\ \mathscr{B}_9 \end{matrix}\right), \tag{3.19}$$

where $\mathscr{B}_8 = [\mathscr{B}_{8,1}, \mathscr{B}_{8,2}] = [\frac{1}{2} - n - m - k, 1]$ and $\mathscr{B}_9 = [\mathscr{B}_{9,1}, \mathscr{B}_{9,2}, \mathscr{B}_{9,3}, \mathscr{B}_{9,4}, \mathscr{B}_{9,5}, \mathscr{B}_{9,6}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 1 - k - m, -n - k - m - \frac{1}{2}]$. From I_{11} and I_{12}, we get I_1.

To solve I_2, we first express it as a sum of two integrals giving us

$$I_2 = \int_{\gamma_T}^{\infty} P(e|x) f_{\gamma_{FSO}}(x)dx + \underbrace{\int_{\gamma_T}^{\infty} P(e|x)G(x)dx}_{\mathrm{I}_{23}} \tag{3.20}$$

Here we can start solving the integral I_{23} directly but the first integral needs to be broken down into two integrals (as shown below) before we start simplifying it. This is because if we solve the first integral directly, we run into convergence problems while extracting the theoretical results of the resultant expression after coding it into softwares like MATLAB. So, we get

$$I_2 = \underbrace{\int_0^{\infty} P(e|x) f_{\gamma_{FSO}}(x)dx}_{\mathrm{H}} - \int_0^{\gamma_T} P(e|x) f_{\gamma_{FSO}}(x)dx + I_{23} \tag{3.21}$$

Here H is the average SER of single-link FSO system which has been determined in [2] and is given by the following expression

$$H = \frac{2^{\alpha+\beta-3}A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} G_{2,5}^{4,2}\left(\left(\frac{D}{4B}\right)^2 \;\middle|\; \begin{matrix} \mathscr{B}_{10} \\ \mathscr{B}_{11} \end{matrix}\right), \tag{3.22}$$

where $\mathscr{B}_{10} = [\mathscr{B}_{10,1}, \mathscr{B}_{10,2}] = [1, \frac{1}{2}]$ and $\mathscr{B}_{11} = [\mathscr{B}_{11,1}, \mathscr{B}_{11,2}, \mathscr{B}_{11,3}, \mathscr{B}_{11,4}, \mathscr{B}_{11,5}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 0]$. The second term in (3.21) can be simplified in the same way as that of I_1. So we first express the complementary error function in terms of the error function and obtain

$$I_2 = \underbrace{H - \frac{A}{2} F_{\gamma_{FSO}}(\gamma_T)}_{\mathrm{I}_{21}} + \underbrace{\frac{A}{2}\int_0^{\gamma_T} erf(\sqrt{x}B) f_{\gamma_{FSO}}(x)dx}_{\mathrm{I}_{22}} + I_{23} \tag{3.23}$$

We can solve I_{22} by using the series expansion of the error function and [9, Eq. (07.34.21.0084.01)]. So, we have

$$I_{22} = \frac{2^{\alpha+\beta-1}}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{(-1)^n (B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)} G_{1,5}^{4,1}\left(\frac{D^2\gamma_T}{16} \mid \begin{array}{c} \mathscr{B}_{12} \\ \mathscr{B}_{13} \end{array} \right), \tag{3.24}$$

where $\mathscr{B}_{12} = [\mathscr{B}_{12,1}] = [\frac{1}{2} - n]$ and $\mathscr{B}_{13} = [\mathscr{B}_{13,1}, \mathscr{B}_{13,2}, \mathscr{B}_{13,3}, \mathscr{B}_{13,4}, \mathscr{B}_{13,5}] = [\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, -n-\frac{1}{2}]$. We can determine I_{23} using [9, Eq. (07.34.21.0085.01)] which gives us

$$I_{23} = \frac{2^{\alpha+\beta-3}(C\gamma_T)^m A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{i=0}^{\infty} \frac{(-C\gamma_T)^i}{i!} \sum_{j=0}^{m+i-1} \binom{m+i-1}{j} (-1)^j G_{1,5}^{4,1}\left(\frac{D^2\gamma_T}{16} \mid \begin{array}{c} \mathscr{B}_6 \\ \mathscr{B}_7 \end{array} \right) G_{2,3}^{3,0}\left(B^2\gamma_T \mid \begin{array}{c} \mathscr{B}_{14} \\ \mathscr{B}_{15} \end{array} \right) \tag{3.25}$$

where $\mathscr{B}_{14} = [\mathscr{B}_{14,1}, \mathscr{B}_{14,2}] = [1, 1-m-i+j]$ and $\mathscr{B}_{15} = [\mathscr{B}_{15,1}, \mathscr{B}_{15,2}, \mathscr{B}_{15,3}] = [-m-i+j, 0, \frac{1}{2}]$. Combining I_{21}, I_{22} and I_{23} we get I_2. Finally, we get $\bar{P}_e$ by adding I_1 and I_2.

3.3 Asymptotic analysis

Asymptotic expressions of outage and average SER can be used to understand their behaviour at high SNRs. To derive asymptotic expressions, we take the limit of outage and average SER expressions when $\bar{\gamma}_{FSO}$ tends to infinity. For the expressions of outage and average SER derived, $\bar{\gamma}_{FSO}$ occurs only in the denominator of the input of Meijer-G functions. So $\bar{\gamma}_{FSO} \to \infty$ implies the input of Meijer-G tending to 0. Hence, we look at expansion of Meijer-G function when its input is zero [9, Eq. (07.34.06.0040.01)]. After expanding all the Meijer-G functions in an expression, we obtain a Taylor series of the form $\sum_{t=0}^{\infty} a_t (\bar{\gamma}_{FSO})^{-t}$. This can be used to determine the diversity order of the system which is the minimum value of t for a non-zero a_t.

3.3.1 Outage probability

For outage probability, when $\gamma_{out} \leq \gamma_T$, the asymptotic expression is

$$P_{AC}^{asy} = \frac{2^{\alpha+\beta-2}C^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty} (-1)^n \frac{C^n}{n!} \Gamma(m+n) \gamma_{out}^{m+n} \mathscr{C}_1, \gamma_{out} \leq \gamma_T \tag{3.26}$$

where

$$\mathscr{C}_1 = \sum_{k=1}^{4} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{4} \Gamma(\mathscr{B}_{3,j} - \mathscr{B}_{3,k}) \prod_{j=1}^{2} \Gamma(1 - \mathscr{B}_{2,j} + \mathscr{B}_{3,k})}{\prod_{j=5}^{6} \Gamma(1 - \mathscr{B}_{3,j} + \mathscr{B}_{3,k})} \left(\frac{D^2 \gamma_{out}}{16} \right)^{\mathscr{B}_{3,k}} \tag{3.27}$$

When $\gamma_{out} > \gamma_T$, we have

$$P_{AC}^{asy} = \frac{2^{\alpha+\beta-2}}{\pi\Gamma(\alpha)\Gamma(\beta)}\left[\mathscr{C}_2 - e^{-C\gamma_{out}}\sum_{n=0}^{\infty}\frac{(C\gamma_T)^n}{n!}\sum_{k=0}^{m-1}\frac{(C\gamma_{out})^k}{k!}\sum_{j=0}^{k}\binom{k}{j}\left(\frac{-\gamma_T}{\gamma_{out}}\right)^j\mathscr{C}_3\right], \tag{3.28}$$

where

$$\mathscr{C}_2 = \sum_{k=1}^{4}\frac{\prod_{\substack{j=1\\j\neq k}}^{4}\Gamma(b_j - b_k)}{b_k}\left(\frac{D^2\gamma_{out}}{16}\right)^{b_k} \tag{3.29}$$

$$\mathscr{C}_3 = \sum_{k=1}^{4}\frac{\prod_{\substack{j=1\\j\neq k}}^{4}\Gamma(\mathscr{B}_{5,j} - \mathscr{B}_{5,k})}{n+j+\mathscr{B}_{5,k}}\left(\frac{D^2\gamma_T}{16}\right)^{\mathscr{B}_{5,k}} \tag{3.30}$$

3.3.2 Average SER

We start deriving the asymptotic expression of average SER with the asymptotic expression of I_{11}

$$I_{11}^{asy} = \frac{2^{\alpha+\beta-2}C^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)}\sum_{n=0}^{\infty}(-1)^n\frac{C^n}{n!}\Gamma(m+n)\gamma_T^{m+n}\mathscr{C}_4, \tag{3.31}$$

where

$$\mathscr{C}_4 = \sum_{k=1}^{4}\frac{\prod_{\substack{j=1\\j\neq k}}^{4}\Gamma(\mathscr{B}_{3,j} - \mathscr{B}_{3,k})\prod_{j=1}^{2}\Gamma(1-\mathscr{B}_{2,j}+\mathscr{B}_{3,k})}{\prod_{j=5}^{6}\Gamma(1-\mathscr{B}_{3,j}+\mathscr{B}_{3,k})}\left(\frac{D^2\gamma_T}{16}\right)^{\mathscr{B}_{3,k}} \tag{3.32}$$

Now, we have asymptotic expression of I_{12}

$$I_{12}^{asy} = \frac{2^{\alpha+\beta-1}(C\gamma_T)^m}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)}\sum_{n=0}^{\infty}\frac{(-1)^n(B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)}\sum_{k=0}^{\infty}\frac{(-1)^k(C\gamma_T)^k}{k!}\Gamma(m+k)\mathscr{C}_5, \tag{3.33}$$

where

$$\mathscr{C}_5 = \sum_{k=1}^{4}\frac{\prod_{\substack{j=1\\j\neq k}}^{4}\Gamma(\mathscr{B}_{9,j} - \mathscr{B}_{9,k})\prod_{j=1}^{2}\Gamma(1-\mathscr{B}_{8,j}+\mathscr{B}_{9,k})}{\prod_{j=5}^{6}\Gamma(1-\mathscr{B}_{9,j}+\mathscr{B}_{9,k})}\left(\frac{D^2\gamma_T}{16}\right)^{\mathscr{B}_{9,k}} \tag{3.34}$$

Next we have asymptotic expression of I_{21}

$$I_{21}^{asy} = \frac{2^{\alpha+\beta-3}A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)}\mathscr{C}_6 - \frac{2^{\alpha+\beta-3}A}{\pi\Gamma(\alpha)\Gamma(\beta)}\mathscr{C}_7, \tag{3.35}$$

where

$$\mathscr{C}_6 = \sum_{k=1}^{4}\frac{\prod_{\substack{j=1\\j\neq k}}^{4}\Gamma(\mathscr{B}_{11,j} - \mathscr{B}_{11,k})\Gamma(\frac{1}{2}+\mathscr{B}_{11,k})}{\mathscr{B}_{11,k}}\left(\frac{D^2}{16B^2}\right)^{\mathscr{B}_{11,k}} \tag{3.36}$$

$$\mathscr{C}_7 = \sum_{k=1}^{4}\frac{\prod_{\substack{j=1\\j\neq k}}^{4}\Gamma(b_j - b_k)}{b_k}\left(\frac{D^2\gamma_T}{16}\right)^{b_k} \tag{3.37}$$

We have asymptotic expression of I_{22}

$$I_{22}^{asy} = \frac{2^{\alpha+\beta-1}}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{(-1)^n (B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)} \mathscr{C}_8, \tag{3.38}$$

where

$$\mathscr{C}_8 = \sum_{k=1}^{4} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{4} \Gamma(\mathscr{B}_{13,j} - \mathscr{B}_{13,k})}{n + \frac{1}{2} + \mathscr{B}_{13,k}} \left(\frac{D^2\gamma_T}{16}\right)^{\mathscr{B}_{13,k}} \tag{3.39}$$

Finally, we have asymptotic expression of I_{23}

$$I_{23}^{asy} = \frac{2^{\alpha+\beta-3}(C\gamma_T)^m A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{i=0}^{\infty} \frac{(-C\gamma_T)^i}{i!} \sum_{j=0}^{m+i-1} \binom{m+i-1}{j} (-1)^j \mathscr{C}_9 G_{2,3}^{3,0}\left(B^2\gamma_T \;\middle|\; \begin{matrix} \mathscr{B}_{14} \\ \mathscr{B}_{15} \end{matrix}\right) \tag{3.40}$$

where

$$\mathscr{C}_9 = \sum_{k=1}^{4} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{4} \Gamma(\mathscr{B}_{7,j} - \mathscr{B}_{7,k})}{j + \mathscr{B}_{7,k}} \left(\frac{D^2\gamma_T}{16}\right)^{\mathscr{B}_{7,k}} \tag{3.41}$$

We get the asymptotic expression of average SER by combining all the asymptotic expressions in this sub-section.

From the asymptotic expressions of both outage and average SER, we can observe that the diversity gain is given by $\min\{\frac{\alpha}{2}, \frac{\beta}{2}\}$.

Chapter 4

Performance analysis of hybrid FSO/RF system with pointing errors

This chapter deals with outage and average SER analysis of hybrid FSO/RF system using adaptive combining taking pointing errors into account. Also included is the asymptotic analysis of the outage and average SER expressions derived.

4.1 FSO channel model with pointing errors

Pointing errors effect the quality of data transmission due to misalignment between the transmitting and receiving apertures. These errors primarily occur due to beam wander and building sway [11]. The PDF of γ_{FSO} taking into account pointing errors is given by [12][13]

$$f_{\gamma_{FSO}}(x) = \frac{\xi^2}{2x\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(pD\sqrt{x} \mid \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, \beta \end{matrix} \right) \tag{4.1}$$

where ξ is the pointing errors parameter [14] and $p = \frac{\xi^2}{\xi^2+1}$. We can derive its CDF by integrating the PDF [12], which gives us

$$F_{\gamma_{FSO}}(x) = \frac{2^{\alpha+\beta-3}\xi^2}{\pi\Gamma(\alpha)\Gamma(\beta)} G_{3,7}^{6,1}\left(\frac{(pD)^2 x}{16} \mid \begin{matrix} \mathscr{B}_{16} \\ \mathscr{B}_{17} \end{matrix} \right) \tag{4.2}$$

where $\mathscr{B}_{16} = [\mathscr{B}_{16,1}, \mathscr{B}_{16,2}, \mathscr{B}_{16,3}] = [1, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{17}$=$[\mathscr{B}_{17,1}, \mathscr{B}_{17,2}, \mathscr{B}_{17,3}, \mathscr{B}_{17,4}, \mathscr{B}_{17,5}, \mathscr{B}_{17,6}, \mathscr{B}_{17,7}]$ = $[\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 0]$. In the derivations presented in the subsequent sections only the PDF and CDF of γ_{FSO} change while the PDF and CDF of γ_{RF} remain the same.

4.2 Outage probability

We derive the outage probability for hybrid FSO/RF system taking pointing errors into account in the same way as done for the case without pointing errors in the previous chapter. For $f_{\gamma_{FSO}+\gamma_{RF}}(x)$, we substitute (4.1) and (2.15) in (2.9)

$$f_{\gamma_{FSO}+\gamma_{RF}}(x) = \frac{2^{\alpha+\beta-3}\xi^2 C^m x^{m-1}}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty}(-1)^n \frac{C^n}{n!}\Gamma(m+n)x^n G_{3,7}^{6,1}\left(\frac{(pD)^2 x}{16} \mid \begin{matrix}\mathscr{B}_{18}\\ \mathscr{B}_{19}\end{matrix}\right), \tag{4.3}$$

where $\mathscr{B}_{18} = [\mathscr{B}_{18,1}, \mathscr{B}_{18,2}, \mathscr{B}_{18,3}] = [1, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{19}$=$[\mathscr{B}_{19,1}, \mathscr{B}_{19,2}, \mathscr{B}_{19,3}, \mathscr{B}_{19,4}, \mathscr{B}_{19,5}, \mathscr{B}_{19,6}, \mathscr{B}_{19,7}]$ = $[\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 1-m-n]$. Integrating $f_{\gamma_{FSO}+\gamma_{RF}}(x)$, we can obtain $F_1(x)$ using [9, Eq. (07.34.21.0084.01)]

$$F_1(x) = \frac{2^{\alpha+\beta-3}\xi^2 C^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty}(-1)^n \frac{C^n}{n!}\Gamma(m+n)x^{m+n} G_{4,8}^{6,2}\left(\frac{(pD)^2 x}{16} \mid \begin{matrix}\mathscr{B}_{20}\\ \mathscr{B}_{21}\end{matrix}\right), \tag{4.4}$$

where $\mathscr{B}_{20} = [\mathscr{B}_{20,1}, \mathscr{B}_{20,2}, \mathscr{B}_{20,3}, \mathscr{B}_{20,4}] = [1-n-m, 1, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{21} = [\mathscr{B}_{21,1}, \mathscr{B}_{21,2}, \mathscr{B}_{21,3}, \mathscr{B}_{21,4}, \mathscr{B}_{21,5}, \mathscr{B}_{21,6}, \mathscr{B}_{21,7}, \mathscr{B}_{21,8}]$ = $[\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 1-n-m, -n-m]$. For $F_2(x)$ we substitute (4.1) and (2.16) in (2.8) to obtain

$$F_2(x) = F_{\gamma_{FSO}}(\gamma_T) - \left[\frac{2^{\alpha+\beta-3}\xi^2 e^{-Cx}}{\pi\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty}\frac{(C\gamma_T)^n}{n!}\sum_{k=0}^{m-1}\frac{(Cx)^k}{k!}\sum_{j=0}^{k}\binom{k}{j}\left(\frac{-\gamma_T}{x}\right)^j G_{3,7}^{6,1}\left(\frac{(pD)^2\gamma_T}{16} \mid \begin{matrix}\mathscr{B}_{22}\\ \mathscr{B}_{23}\end{matrix}\right)\right], \tag{4.5}$$

where $\mathscr{B}_{22} = [\mathscr{B}_{22,1}, \mathscr{B}_{22,2}, \mathscr{B}_{22,3}] = [1-n-j, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{23} = [\mathscr{B}_{23,1}, \mathscr{B}_{23,2}, \mathscr{B}_{23,3}, \mathscr{B}_{23,4}, \mathscr{B}_{23,5}, \mathscr{B}_{23,6}, \mathscr{B}_{23,7}]$ = $[\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, -n-j]$. $F_{\gamma_c}(x)$ can be determined by substituting (4.4), (4.5) and (2.22) in (2.6), which gives us the outage probability of the system by plugging in γ_{out} in F_{γ_c}.

4.3 Average SER

Similar to outage, all the expressions of average SER for this case can be derived in the same manner as done in the case without pointing errors. We start by obtaining G(x) by substituting (4.1) and (2.15) in (2.11) and using the series expansion of the exponential and binomial expansion, which gives us

$$G(x) = \frac{2^{\alpha+\beta-3}(Cx)^m\xi^2}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{i=0}^{\infty}\frac{(-Cx)^i}{i!}\sum_{j=0}^{m+i-1}\frac{(-\gamma_T)^j}{x^{j+1}}\binom{m+i-1}{j} G_{3,7}^{6,1}\left(\frac{(pD)^2\gamma_T}{16} \mid \begin{matrix}\mathscr{B}_{24}\\ \mathscr{B}_{25}\end{matrix}\right) \tag{4.6}$$

where $\mathscr{B}_{24} = [\mathscr{B}_{24,1}, \mathscr{B}_{24,2}, \mathscr{B}_{24,3}] = [1-j, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{25} = [\mathscr{B}_{25,1}, \mathscr{B}_{25,2}, \mathscr{B}_{25,3}, \mathscr{B}_{25,4}, \mathscr{B}_{25,5}, \mathscr{B}_{25,6}, \mathscr{B}_{25,7}] = [\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, -j]$. Now, we start solving for average SER, $\bar{P}_e$, by solving I_1. Similar to the previous case, we have $I_{11} = F_1(\gamma_T)$ and we can evaluate I_{12} using the series expansion of the error function, (4.3) and [9, Eq.(07.34.21.0084.01)]

$$I_{12} = \frac{2^{\alpha+\beta-2}\xi^2 (C\gamma_T)^m}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty} \frac{(-1)^n (B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)} \sum_{k=0}^{\infty} \frac{(-1)^k (C\gamma_T)^k}{k!} \Gamma(m+k) G_{4,8}^{6,2}\left(\frac{(pD)^2\gamma_T}{16} \;\middle|\; \begin{matrix} \mathscr{B}_{26} \\ \mathscr{B}_{27} \end{matrix} \right), \tag{4.7}$$

where $\mathscr{B}_{26} = [\mathscr{B}_{26,1}, \mathscr{B}_{26,2}, \mathscr{B}_{26,3}, \mathscr{B}_{26,4}] = [\frac{1}{2}-n-m-k, 1, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{27} = [\mathscr{B}_{27,1}, \mathscr{B}_{27,2}, \mathscr{B}_{27,3}, \mathscr{B}_{27,4}, \mathscr{B}_{27,5}, \mathscr{B}_{27,6}, \mathscr{B}_{27,7}, \mathscr{B}_{27,8}] = [\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 1-k-m, -n-k-m-\frac{1}{2}]$. In order to derive H in this scenario, we use the Meijer-G representation of complementary error function and the result of integrating two Meijer-G functions as given in [15] which gives us

$$H = \frac{2^{\alpha+\beta-4}\xi^2 A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} G_{4,7}^{6,2}\left(\left(\frac{pD}{4B}\right)^2 \;\middle|\; \begin{matrix} \mathscr{B}_{28} \\ \mathscr{B}_{29} \end{matrix} \right), \tag{4.8}$$

where $\mathscr{B}_{28} = [\mathscr{B}_{28,1}, \mathscr{B}_{28,2}, \mathscr{B}_{28,3}, \mathscr{B}_{28,4}] = [1, \frac{1}{2}, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{29} = [\mathscr{B}_{29,1}, \mathscr{B}_{29,2}, \mathscr{B}_{29,3}, \mathscr{B}_{29,4}, \mathscr{B}_{29,5}, \mathscr{B}_{29,6}, \mathscr{B}_{29,7}] = [\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, 0]$. Next, we derive I_{22} using series expansion of error function, (4.1) and [9, Eq.(07.34.21.0084.01)]

$$I_{22} = \frac{2^{\alpha+\beta-2}\xi^2}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{(-1)^n (B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)} G_{3,7}^{6,1}\left(\frac{(pD)^2\gamma_T}{16} \;\middle|\; \begin{matrix} \mathscr{B}_{30} \\ \mathscr{B}_{31} \end{matrix} \right), \tag{4.9}$$

where $\mathscr{B}_{30} = [\mathscr{B}_{30,1}, \mathscr{B}_{30,2}, \mathscr{B}_{30,3}] = [\frac{1}{2}-n, \frac{\xi^2+1}{2}, \frac{\xi^2+2}{2}]$ and $\mathscr{B}_{31} = [\mathscr{B}_{31,1}, \mathscr{B}_{31,2}, \mathscr{B}_{31,3}, \mathscr{B}_{31,4}, \mathscr{B}_{31,5}, \mathscr{B}_{31,6}, \mathscr{B}_{31,7}] = [\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\beta+1}{2}, -n-\frac{1}{2}]$. Finally, we obtain I_{23} using [9, Eq. (07.34.21.0085.01)]

$$I_{23} = \frac{2^{\alpha+\beta-4}\xi^2 (C\gamma_T)^m A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{i=0}^{\infty} \frac{(-C\gamma_T)^i}{i!} \sum_{j=0}^{m+i-1} \binom{m+i-1}{j} (-1)^j G_{3,7}^{6,1}\left(\frac{(pD)^2\gamma_T}{16} \;\middle|\; \begin{matrix} \mathscr{B}_{24} \\ \mathscr{B}_{25} \end{matrix} \right) G_{2,3}^{3,0}\left(B^2\gamma_T \;\middle|\; \begin{matrix} \mathscr{B}_{14} \\ \mathscr{B}_{15} \end{matrix} \right) \tag{4.10}$$

Combining the above expressions we obtain I_1 and I_2, which in turn give us the average SER of the hybrid FSO/RF system taking pointing errors into account.

4.4 Asymptotic analysis

We derive asymptotic expressions for outage and average SER in the same manner as done in the previous chapter.

4.4.1 Outage probability

The asymptotic expression of outage probability when $\gamma_{out} \leq \gamma_T$ is given by

$$P_{AC}^{asy} = \frac{2^{\alpha+\beta-3}\xi^2 C^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{n=0}^{\infty}(-1)^n \frac{C^n}{n!}\Gamma(m+n)\gamma_{out}^{m+n}\mathscr{C}_{10}, \tag{4.11}$$

where

$$\mathscr{C}_{10} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1\\j\neq k}}^{6}\Gamma(\mathscr{B}_{21,j}-\mathscr{B}_{21,k})\Gamma(\mathscr{B}_{21,k})}{\prod_{j=3}^{4}\Gamma(\mathscr{B}_{20,j}-\mathscr{B}_{21,k})\Gamma(1+m+n+\mathscr{B}_{21,k})}\left(\frac{(pD)^2\gamma_{out}}{16}\right)^{\mathscr{B}_{21,k}} \tag{4.12}$$

We have asymptotic expression of outage probability when $\gamma_{out} > \gamma_T$

$$P_{AC}^{asy} = \frac{2^{\alpha+\beta-3}\xi^2}{\pi\Gamma(\alpha)\Gamma(\beta)}\left[\mathscr{C}_{11} - e^{-C\gamma_{out}}\sum_{n=0}^{\infty}\frac{(C\gamma_T)^n}{n!}\sum_{k=0}^{m-1}\frac{(C\gamma_{out})^k}{k!}\sum_{j=0}^{k}\binom{k}{j}\left(\frac{-\gamma_T}{\gamma_{out}}\right)^j\mathscr{C}_{12}\right], \tag{4.13}$$

where

$$\mathscr{C}_{11} = \sum_{k=1}^{6}\frac{\prod_{\substack{j=1\\j\neq k}}^{6}\Gamma(\mathscr{B}_{17,j}-\mathscr{B}_{17,k})}{\prod_{j=2}^{3}\Gamma(\mathscr{B}_{16,j}-\mathscr{B}_{17,k})\mathscr{B}_{17,k}}\left(\frac{(pD)^2\gamma_{out}}{16}\right)^{\mathscr{B}_{17,k}} \tag{4.14}$$

$$\mathscr{C}_{12} = \sum_{k=1}^{6}\frac{\prod_{\substack{j=1\\j\neq k}}^{6}\Gamma(\mathscr{B}_{23,j}-\mathscr{B}_{23,k})}{\prod_{j=2}^{3}\Gamma(\mathscr{B}_{22,j}-\mathscr{B}_{23,k})(n+j+\mathscr{B}_{23,k})}\left(\frac{(pD)^2\gamma_T}{16}\right)^{\mathscr{B}_{23,k}} \tag{4.15}$$

4.4.2 Average SER

We have the asymptotic expression for I_{11}

$$I_{11}^{asy} = \frac{2^{\alpha+\beta-3}\xi^2 C^m}{\pi\Gamma(\alpha)\Gamma(\beta)\Gamma(m)}\sum_{n=0}^{\infty}(-1)^n\frac{C^n}{n!}\Gamma(m+n)\gamma_T^{m+n}\mathscr{C}_{13}, \tag{4.16}$$

where

$$\mathscr{C}_{13} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1\\j\neq k}}^{6}\Gamma(\mathscr{B}_{21,j}-\mathscr{B}_{21,k})\Gamma(\mathscr{B}_{21,k})}{\prod_{j=3}^{4}\Gamma(\mathscr{B}_{20,j}-\mathscr{B}_{21,k})\Gamma(1+m+n+\mathscr{B}_{21,k})}\left(\frac{(pD)^2\gamma_T}{16}\right)^{\mathscr{B}_{21,k}} \tag{4.17}$$

Next, we look at the asymptotic expression of I_{12}

$$I_{12}^{asy} = \frac{2^{\alpha+\beta-2}\xi^2(C\gamma_T)^m}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)}\sum_{n=0}^{\infty}\frac{(-1)^n(B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)}\sum_{k=0}^{\infty}\frac{(-1)^k(C\gamma_T)^k}{k!}\Gamma(m+k)\mathscr{C}_{14}, \tag{4.18}$$

where

$$\mathscr{C}_{14} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{6} \Gamma(\mathscr{B}_{27,j} - \mathscr{B}_{27,k})\Gamma(\mathscr{B}_{27,k})}{\prod_{j=3}^{4} \Gamma(\mathscr{B}_{26,j} - \mathscr{B}_{27,k})\Gamma(k+m+\mathscr{B}_{27,k})} \left(\frac{(pD)^2\gamma_T}{16}\right)^{\mathscr{B}_{27,k}} \tag{4.19}$$

Now, we have the asymptotic expression of I_{21}

$$I_{21}^{asy} = \frac{2^{\alpha+\beta-4}\xi^2 A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)}\mathscr{C}_{15} - \frac{2^{\alpha+\beta-4}\xi^2 A}{\pi\Gamma(\alpha)\Gamma(\beta)}\mathscr{C}_{16} \tag{4.20}$$

where

$$\mathscr{C}_{15} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{6} \Gamma(\mathscr{B}_{29,j} - \mathscr{B}_{29,k})\Gamma(\frac{1}{2}+\mathscr{B}_{29,k})}{\prod_{j=3}^{4} \Gamma(\mathscr{B}_{28,j} - \mathscr{B}_{29,k})\mathscr{B}_{29,k}} \left(\frac{(pD)^2}{16B^2}\right)^{\mathscr{B}_{29,k}} \tag{4.21}$$

$$\mathscr{C}_{16} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{6} \Gamma(\mathscr{B}_{17,j} - \mathscr{B}_{17,k})}{\prod_{j=2}^{3} \Gamma(\mathscr{B}_{16,j} - \mathscr{B}_{17,k})\mathscr{B}_{17,k}} \left(\frac{(pD)^2\gamma_T}{16}\right)^{\mathscr{B}_{17,k}} \tag{4.22}$$

We have the asymptotic expression of I_{22}

$$I_{22}^{asy} = \frac{2^{\alpha+\beta-2}\xi^2}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{(-1)^n (B\sqrt{\gamma_T})^{2n+1}}{n!(2n+1)}\mathscr{C}_{17}, \tag{4.23}$$

where

$$\mathscr{C}_{17} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{6} \Gamma(\mathscr{B}_{31,j} - \mathscr{B}_{31,k})}{\prod_{j=2}^{3} \Gamma(\mathscr{B}_{30,j} - \mathscr{B}_{31,k})(n+\frac{1}{2}+\mathscr{B}_{31,k})} \left(\frac{(pD)^2\gamma_T}{16}\right)^{\mathscr{B}_{31,k}} \tag{4.24}$$

Finally, we have the asymptotic expression of I_{23}

$$I_{23}^{asy} = \frac{2^{\alpha+\beta-4}\xi^2 (C\gamma_T)^m A}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\Gamma(m)} \sum_{i=0}^{\infty} \frac{(-C\gamma_T)^i}{i!} \sum_{j=0}^{m+i-1} \binom{m+i-1}{j} (-1)^j \mathscr{C}_{18} G_{2,3}^{3,0}\left(B^2\gamma_T \;\middle|\; \begin{matrix} \mathscr{B}_{14} \\ \mathscr{B}_{15} \end{matrix}\right) \tag{4.25}$$

where

$$\mathscr{C}_{18} = \sum_{k=1}^{6} \frac{\prod_{\substack{j=1 \\ j\neq k}}^{6} \Gamma(\mathscr{B}_{25,j} - \mathscr{B}_{25,k})}{\prod_{j=2}^{3} \Gamma(\mathscr{B}_{24,j} - \mathscr{B}_{25,k})(j+\mathscr{B}_{25,k})} \left(\frac{(pD)^2\gamma_T}{16}\right)^{\mathscr{B}_{25,k}} \tag{4.26}$$

We can obtain the asymptotic expression for average SER by substituting the derived asymptotic expressions in I_1 and I_2. In this scenario, we can see that the diversity order of the system is given by $\min\{\frac{\xi^2}{2}, \frac{\alpha}{2}, \frac{\beta}{2}\}$.

Chapter 5

Numerical results and discussions

This chapter looks at the results obtained by plotting the derived expressions of outage and average SER. The system parameters used to obtain the plots are specified in the captions of the figures. The list of summation limits (for infinite summations) used to evaluate the expressions are as follows:

1. For $F_1(x)$, in (3.6) and (4.4), we set the upper limit of n to 50.

2. For $F_2(x)$, in (3.10) and (4.5), we set the upper limit of n to 30.

3. For I_{12}, in (3.19) and (4.7), we set the upper limit of n to 10 and that of k to 10.

4. For I_{22}, in (3.24) and (4.9), we set the upper limit of n to 50.

5. For I_{23}, in (3.25) and (4.10), we set the upper limit of i to 20.

All these upper limits are for both the cases of adaptive combining with and without pointing errors. If we take higher values for upper limits than the ones listed, it would have no effect on the fifth decimal of the final value.

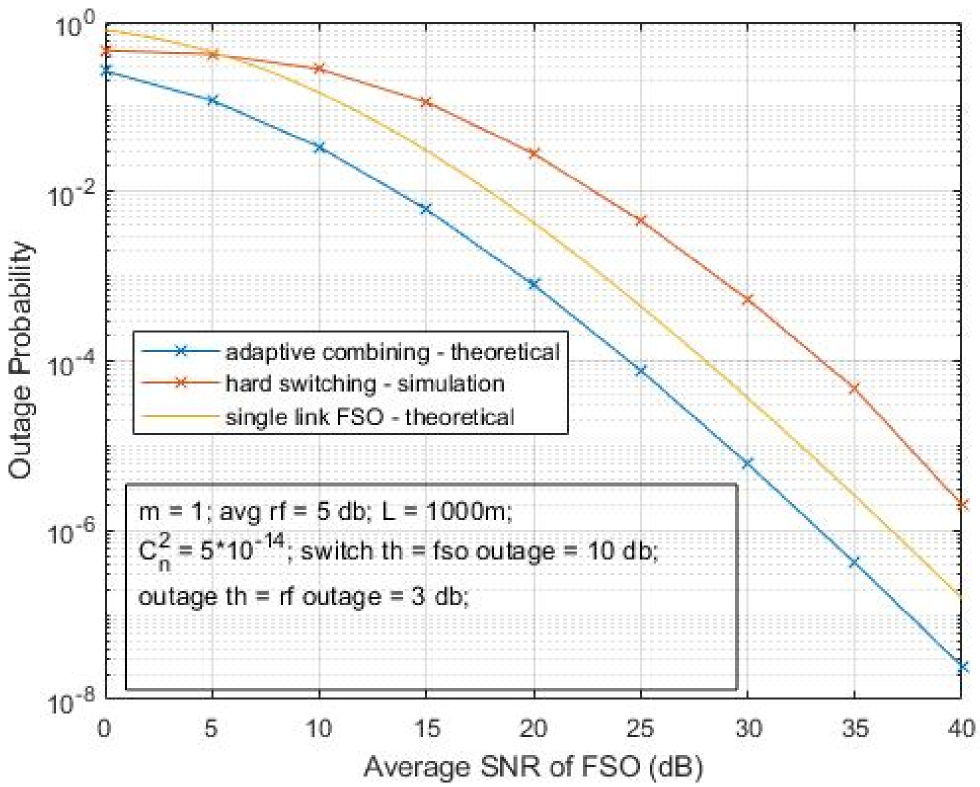

Figure 5.1: Outage performance comparison (without pointing errors) of adaptive combining, hard switching and single-link FSO system

Fig. 5.1 shows the outage performance (without pointing errors) comparison of adaptive combining, hard switching [5] and single-link FSO system. We can observe that adaptive combining provides better performance over hard-switching and single-link FSO.

To achieve an outage of 10^{-2}, adaptive combining scheme requires $\bar{\gamma}_{FSO}$ to be set to 14 dB, whereas hard-switching and single-link FSO require 23 dB and 17 dB respectively. Here, adaptive combining provides a coding gain of 9 dB on hard-switching and 3 dB on single-link FSO system.

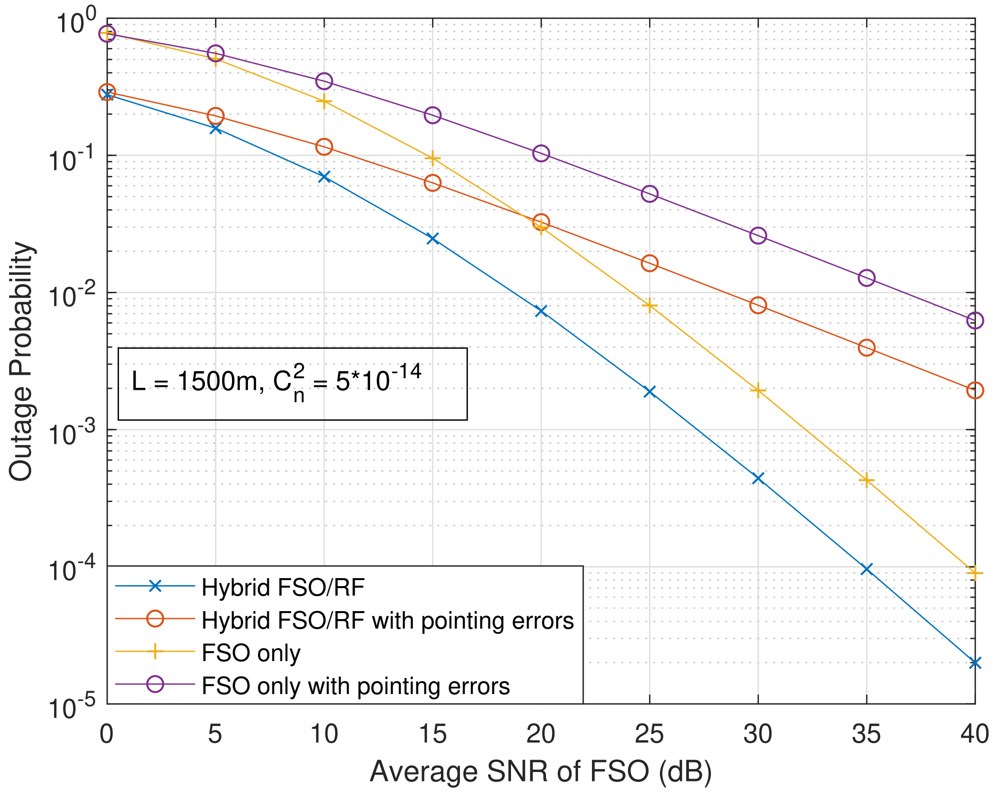

Figure 5.2: Outage probability coding gain comparison of adaptive combining with and without pointing errors. $\bar{\gamma}_{RF}$ = 5 dB, γ_T = 10 dB, γ_{out} = 3 dB, m = 1 and ξ = 1.1170

Fig. 5.2 is to illustrate the coding gains offered by adaptive combining with and without pointing errors over single-link FSO systems with and without pointing errors respectively.

For the case without pointing errors, in the given plot, to achieve an outage of 10^{-2}, we need $\bar{\gamma}_{FSO}$ = 18 dB for a hybrid FSO/RF system and 24 dB for a single-link FSO system. So, in this case, the hybrid system offers a coding gain of 6 dB on the single-link FSO system.

In the case involving pointing errors, an outage of 10^{-2} is obtained with $\bar{\gamma}_{FSO}$ = 28 dB for hybrid FSO/RF system and 36 dB for single-link FSO system. Thus, the hybrid system provides a coding gain of 8 dB on single-link FSO.

Hence, we can conclude that the back-up RF link provided better performance enhancement in worst-case scenarios.

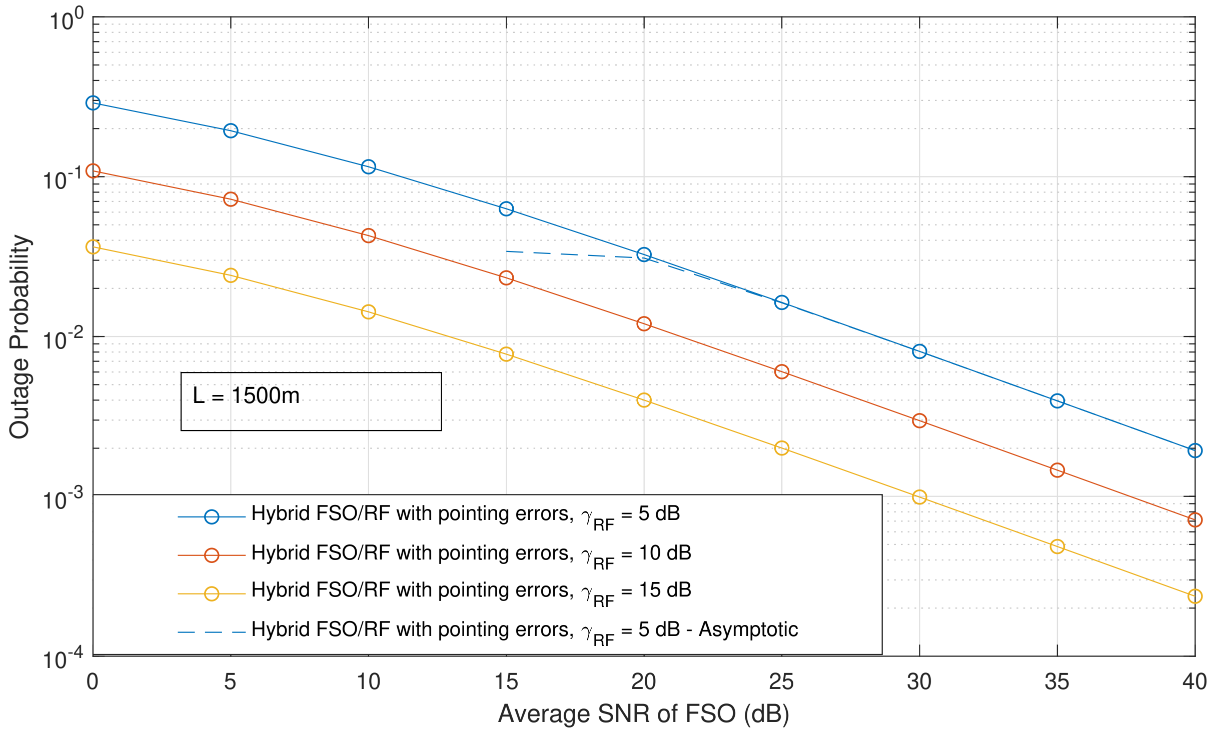

Figure 5.3: Outage performance (with pointing errors) at different values of $\bar{\gamma}_{RF}$. $C_n^2 = 5\times10^{-14}$, γ_T = 10 dB, γ_{out} = 3 dB, $m = 1$ and $\xi = 1.1170$

Fig. 5.3 shows the variation in outage performance (with pointing errors) at different $\bar{\gamma}_{RF}$. We also have an asymptotic curve plotted for $\bar{\gamma}_{RF}$ = 5 dB. We can see that as $\bar{\gamma}_{RF}$ increases the outage performance also increases.

At $\bar{\gamma}_{RF}$ = 5 dB, we need $\bar{\gamma}_{FSO}$ = 28 dB to get an outage of 10^{-2}. For $\bar{\gamma}_{RF}$ = 10 dB, we require $\bar{\gamma}_{FSO}$ to be 21 dB for an outage of 10^{-2}. Finally, for $\bar{\gamma}_{RF}$ = 15 dB, we need average SNR of 13 dB to get an outage of 10^{-2}. So, for $\bar{\gamma}_{RF}$ = 15 dB, we have SNR gains of 8 dB and 15 dB on $\bar{\gamma}_{RF}$ = 10 dB and 5 dB respectively.

As $\bar{\gamma}_{RF}$ increases, the quality of the RF link increases, due to which the RF link acts as a better back-up link to the FSO link which increases the overall performance of the system.

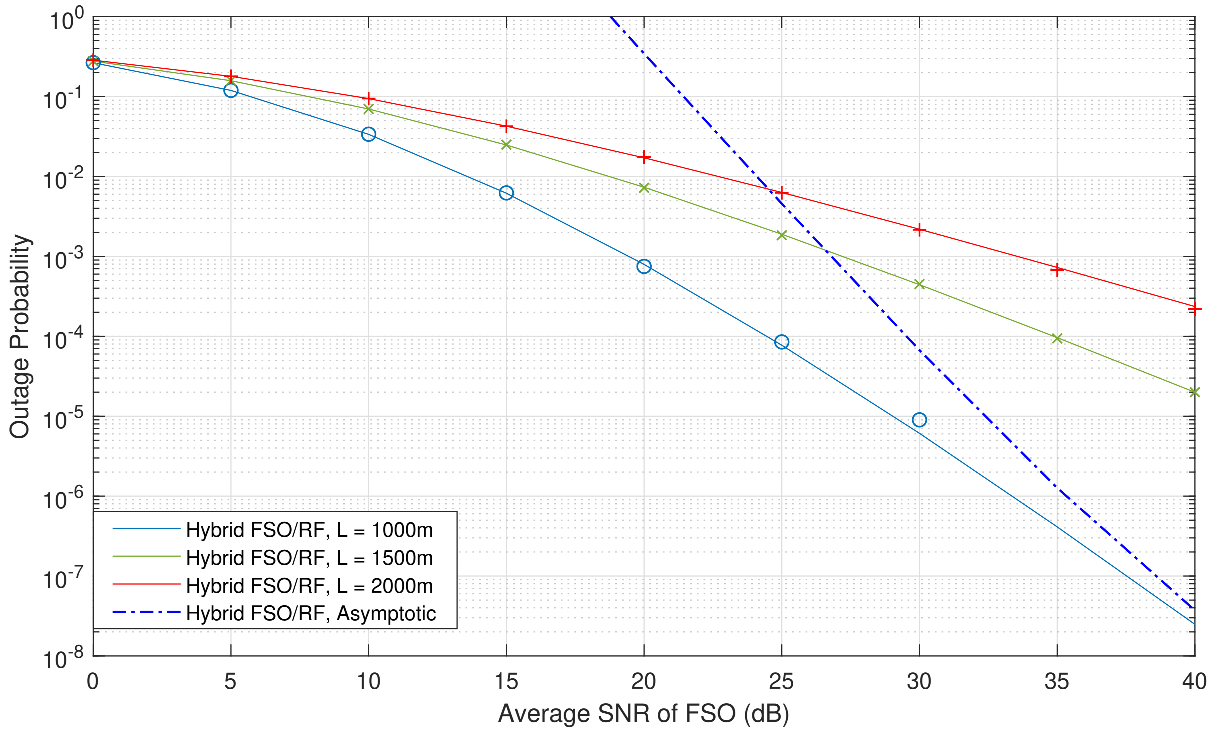

Figure 5.4: Outage performance (without pointing errors) at different values of link distance. $\bar{\gamma}_{RF} = 5$ dB, $C_n^2 = 5\times10^{-14}$, $\gamma_T = 10$ dB, $\gamma_{out} = 3$ dB and $m = 1$

Fig. 5.4 illustrates outage performance (without pointing errors) at various link distances along with an asymptotic curve at L = 1000 m. We can observe that as link distance increases, the outage performance of the system decreases.

For a target outage probability of 10^{-3}, we need $\bar{\gamma}_{FSO}$ to be 20 dB for L = 1000 m. Similarly for L = 1500 m, we need $\bar{\gamma}_{FSO} = 26$ dB. For L = 2000 m, we set $\bar{\gamma}_{FSO}$ to 34 dB to achieve an outage probability of 10^{-3}. So, for L = 1000 m, we have SNR gains of 6 dB and 14 dB on L = 1500 m and 2000 m respectively.

This behaviour can be explained by the fact that as link distance increases, the small and large scale scattering parameters decrease, thereby decreasing the diversity gain of the system. This leads to deterioration of outage performance of the system.

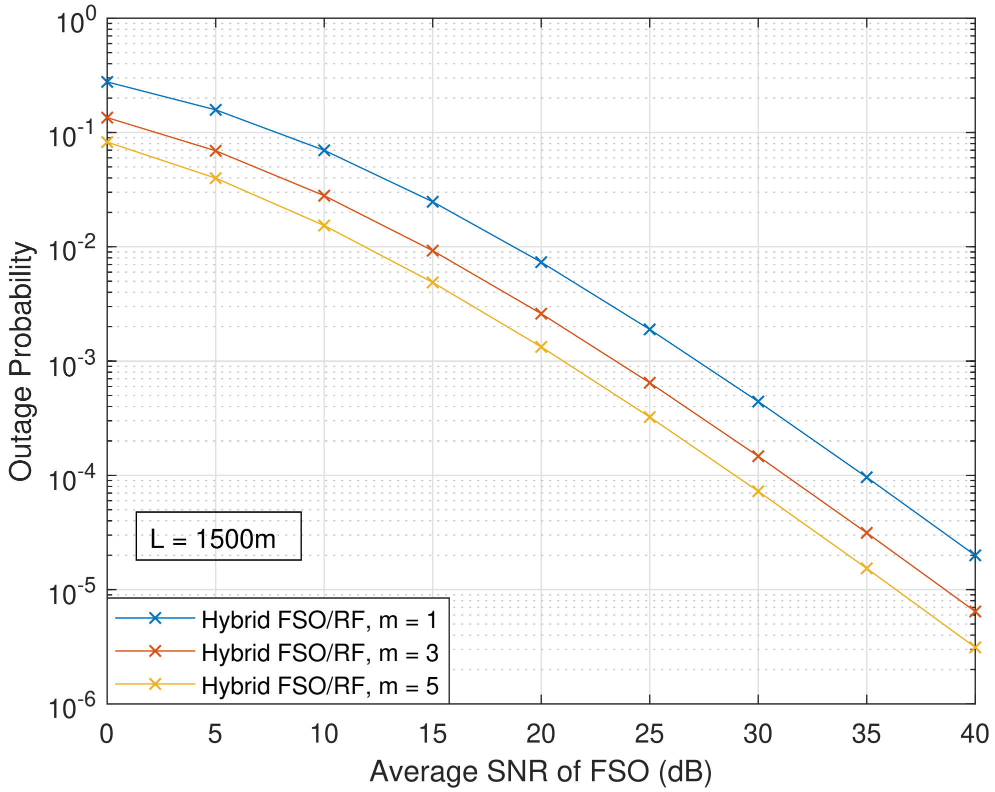

Figure 5.5: Outage performance (without pointing errors) at different values of m. $\bar{\gamma}_{RF}$ = 5 dB, C_n^2 = 5×10^{-14}, γ_T = 10 dB and γ_{out} = 3 dB

Fig. 5.5 shows outage probability plots (without pointing errors) at various fading severity parameters (m) of the RF link. Clearly, as m is increasing, we can see that outage performance is also increasing.

To achieve an outage probability of 10^{-3}, at m = 1, we need $\bar{\gamma}_{FSO}$ to be 26 dB. Similarly, for m = 3, $\bar{\gamma}_{FSO}$ has to be 24 dB. For m = 5, we have $\bar{\gamma}_{FSO}$ = 21 dB to obtain an outage probability of 10^{-3}. Here, for m = 5, we have coding gains of 5 dB and 3 dB on m = 1 and m = 3 respectively.

As m increases, the quality of the back-up RF link increases, as seen in the case of increasing $\bar{\gamma}_{RF}$. Hence, the outage performance of the system increases.

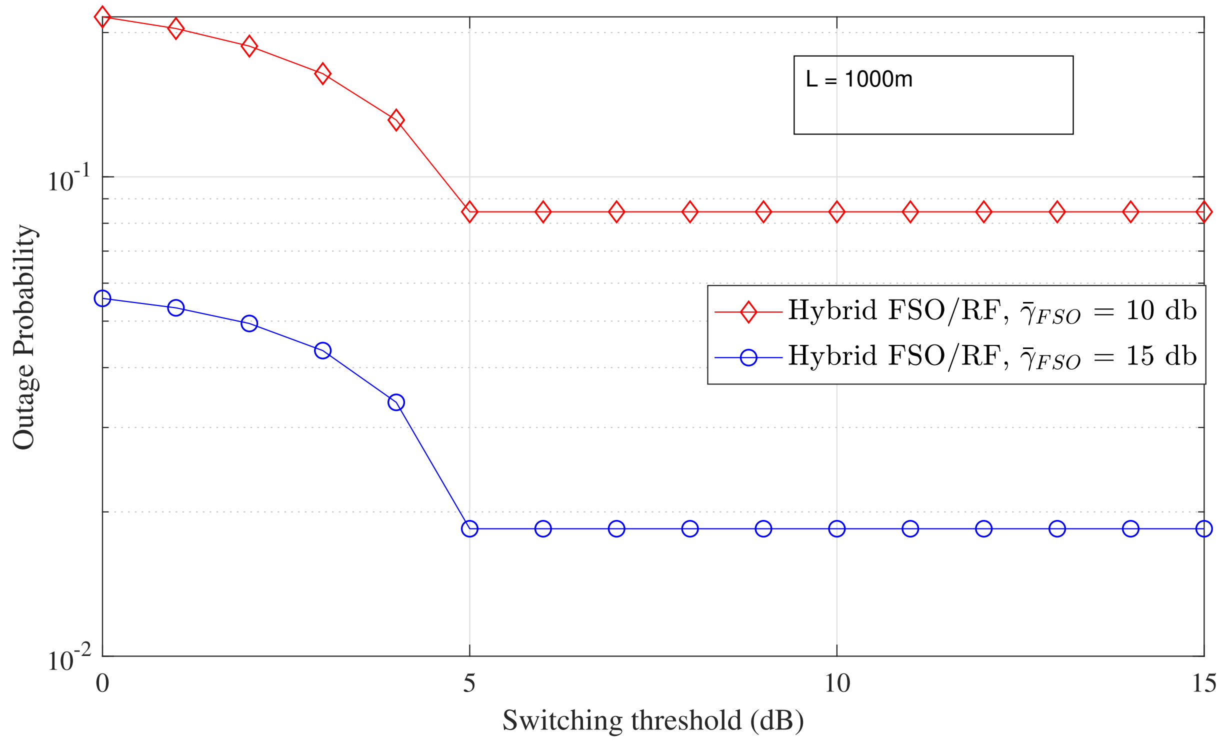

Figure 5.6: Outage probability vs γ_T at different $\bar{\gamma}_{FSO}$. $\bar{\gamma}_{RF}$ = 5 dB, $C_n^2 = 5\times10^{-14}$, γ_T = 10 dB, γ_{out} = 3 dB and m = 1

Fig. 5.6 shows outage probability vs switching threshold plots (without pointing errors) for different $\bar{\gamma}_{FSO}$ values. Here, instead of $\bar{\gamma}_{FSO}$, the outage probability is varied w.r.t γ_T.

We can see that for every $\bar{\gamma}_{FSO}$, the outage probability is minimum whenever the switching threshold is greater than or equal to the outage threshold. In fact, the outage becomes constant when $\gamma_T \geq \gamma_{out}$. This is because from (3.6), we can see that outage probability is independent of γ_T under this condition.

Hence, for optimum outage performance, we need to have $\gamma_T \geq \gamma_{out}$.

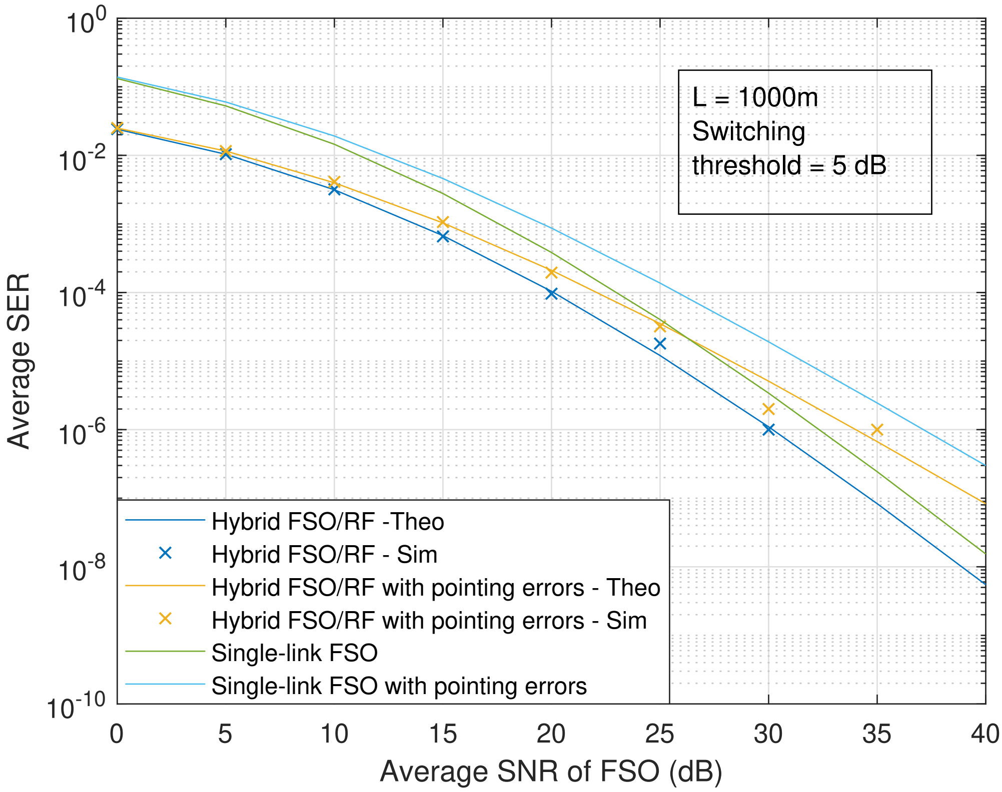

Figure 5.7: Average SER coding gain comparison of adaptive combining with and without pointing errors. $\bar{\gamma}_{RF}$ = 5 dB, $C_n^2 = 5\times10^{-14}$, m = 1 and ξ = 1.9335

Fig. 5.7, like Fig. 5.2, shows the coding gains (w.r.t average SER) of adaptive combining with and without pointing errors on single-link FSO systems with and without pointing errors.

Starting with the case of adaptive combining without pointing errors, we can see that to achieve a target average SER of 10^{-4}, the hybrid FSO/RF system needs $\bar{\gamma}_{FSO}$ to be 20 dB, while the single-link FSO system needs it to be around 23 dB. For the case of adaptive combining with pointing errors, the hybrid system requires $\bar{\gamma}_{FSO}$ = 22 dB while the single-link FSO system requires $\bar{\gamma}_{FSO}$ = 26 dB to obtain an average SER of 10^{-4}.

Hence we get a coding gain of 3 dB in the scenario without pointing errors and 4 dB in the scenario with pointing error over the single-link FSO counterparts. So, similar to outage probability, we can infer that the RF link provides better improvement of performance under worst-case conditions.

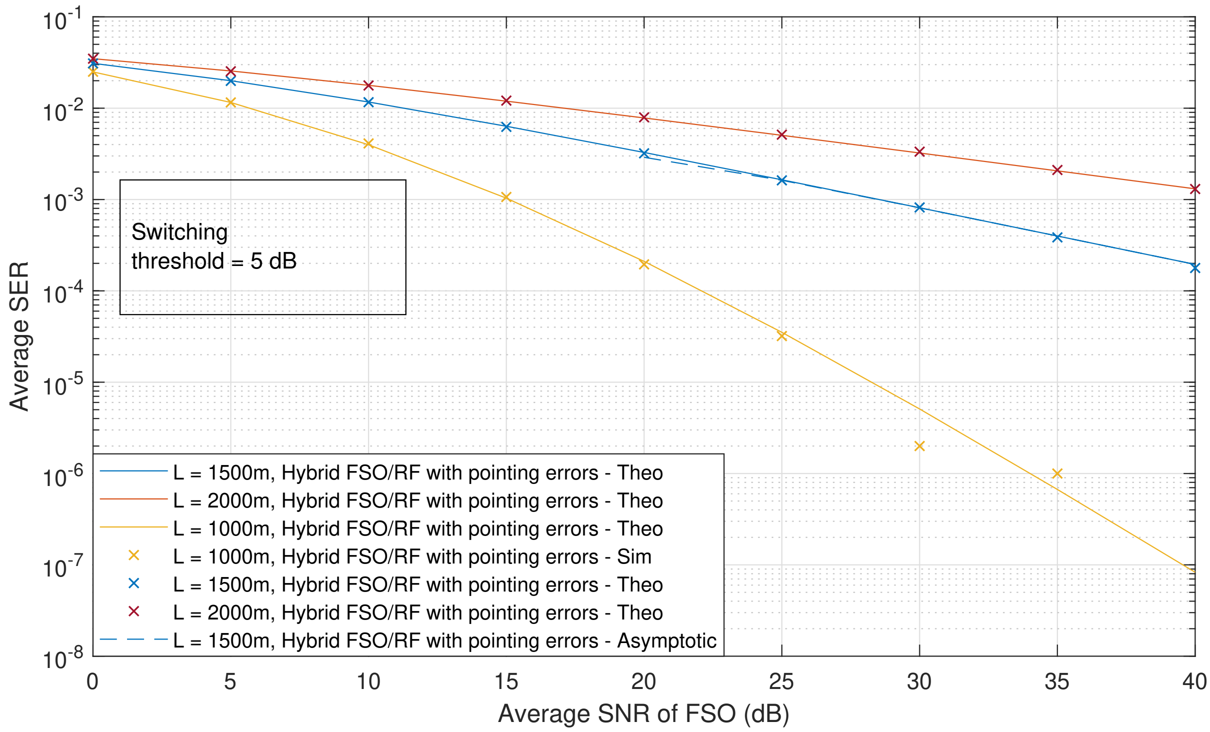

Figure 5.8: Average SER (with pointing errors) at different values of link distance. $\bar{\gamma}_{RF}$ = 5 dB, C_n^2 = 5×10^{-14}, γ_T = 5 dB and m = 1

Fig. 5.8 shows the average SER performance of hybrid FSO/RF system (with pointing errors) at varying link distance along with an asymptotic plot of average SER at L = 1500 m. We can see that as link distance increases, the performance of the system decreases.

To achieve an average SER of 10^{-2}, we need $\bar{\gamma}_{FSO}$ = 5 dB for L = 1000 m, $\bar{\gamma}_{FSO}$ = 11 dB for L = 1500 m, and $\bar{\gamma}_{FSO}$ = 16 dB for L = 2000 m. This implies that at L = 1000 m, we get coding gains of 6 dB and 11 dB on L = 1500 m and 2000 m respectively. We can also observe that the asymptotic plot perfectly coincides with the average SER plot in the high SNR region.

This behaviour can be attributed to the decrease in diversity gain due to increase in the link distance, which lowers the performance of the system.

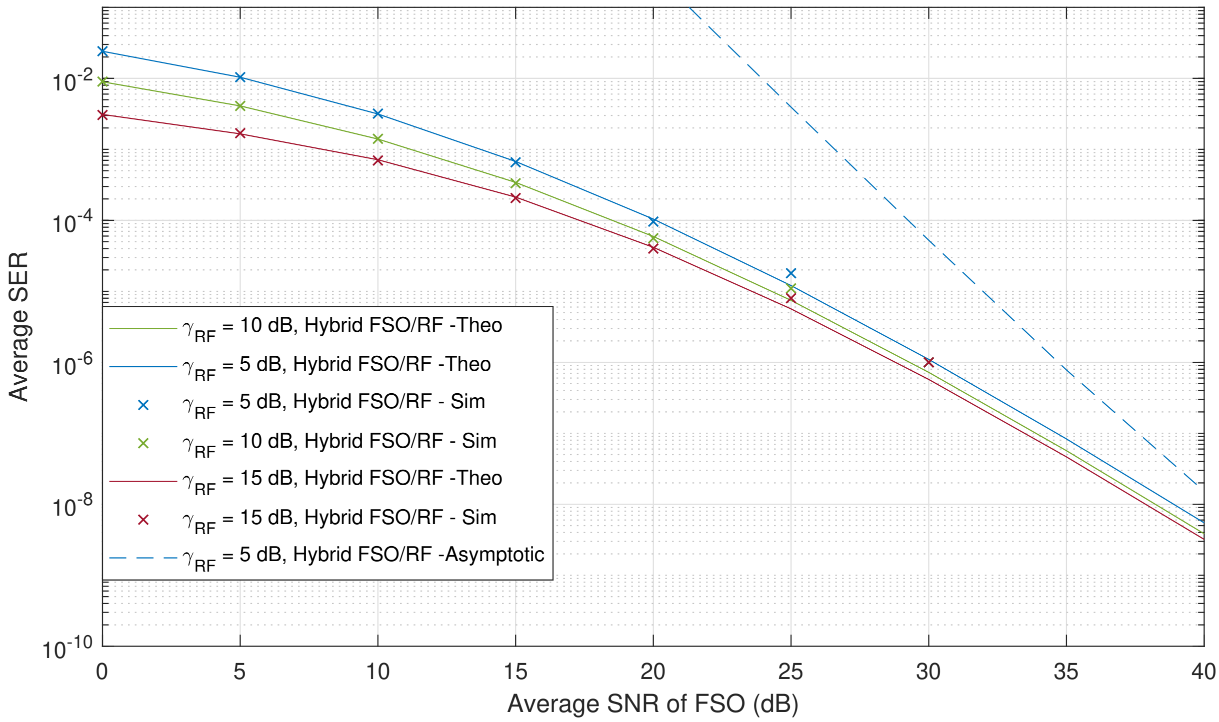

Figure 5.9: Average SER (without pointing errors) at different values of $\bar{\gamma}_{RF}$. L = 1000m, C_n^2 = 5×10^{-14}, γ_T = 5 dB and m = 1

Fig. 5.9 illustrates the variation of average SER performance (without pointing errors) at varying $\bar{\gamma}_{RF}$ as well as the asymptotic average SER plot for $\bar{\gamma}_{RF}$ = 5 dB. The performance of the system increases upon increasing $\bar{\gamma}_{RF}$.

For an average SER of 10^{-3}, at $\bar{\gamma}_{RF}$ = 5 dB, we need $\bar{\gamma}_{FSO}$ to be around 14 dB. Similarly for $\bar{\gamma}_{RF}$ = 10 dB, we need to have $\bar{\gamma}_{FSO}$ = 11 dB. Finally, for $\bar{\gamma}_{FSO}$ = 15 dB, we need $\bar{\gamma}_{FSO}$ = 7 dB to obtain an average SER of 10^{-3}. So, $\bar{\gamma}_{RF}$ = 15 dB offers SNR gains of 7 dB and 4 dB on $\bar{\gamma}_{RF}$ = 5 dB and 10 dB respectively. Also, the asymptotic curve matches with the exact average SER plot in high SNR region.

As $\bar{\gamma}_{RF}$ increases, the quality of the back-up RF link increases, due to which the average SER performance, like the outage performance, increases.

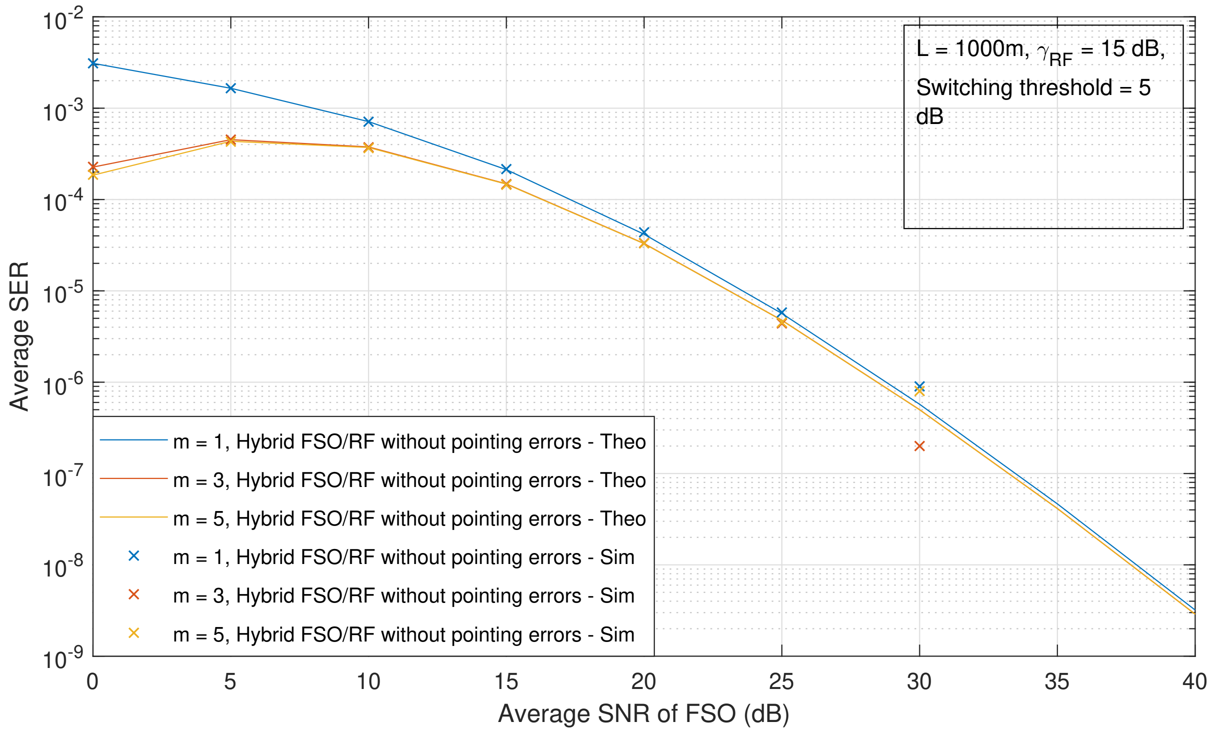

Figure 5.10: Average SER (without pointing errors) at different values of *m*. $C_n^2 = 5\times10^{-14}$

In Fig. 5.10, we have average SER plots (without pointing errors) at varying fading severity parameter (m). We can see that there is only a marginal improvement in the average SER performance of the system.

For a target average SER of 10^{-4}, for m = 1, we need $\bar{\gamma}_{FSO}$ = 17 dB. For m = 3 and m = 5, the $\bar{\gamma}_{FSO}$ needed is practically the same, which is around 16 dB. So the coding gains offered by m = 5 on m = 1 and m = 3 are 1 dB and 0 dB respectively.

So, the SNR gain is meagre when RF link is subjected to varying fading scenarios. The low switching threshold contributes to this, as its low value ensures that only the FSO link is active most of the time.

Chapter 6

Conclusion & future works

In this project, we derived the outage and average SER expressions of adaptive combining with and without pointing errors. We established the fact that adding a back-up RF link significantly increases the performance of an FSO system. We have seen that the back-up RF link offers better performance enhancement in worst-case scenarios. Also, it was illustrated that adaptive combining provides better performance in both outage as well as average SER than hard-switching and single-link FSO system. Several plots illustrating the effects of various parameters on the performance of the system have been shown in the previous chapter. From those plots, we can infer that outage and average performances increase upon increasing $\bar{\gamma}_{RF}$, increasing m and decreasing the link distance. For optimum outage performance, we need to set the switching threshold greater than or equal to the outage threshold as the outage probability function is constant and minimum in the range of switching threshold. Asymptotic expressions have also been derived from which the diversity gains of the hybrid systems with and without pointing errors have been determined. Also, the asymptotic plots of outage and average SER coincide with the plots of their exact expressions.

The analysis carried out in this project evaluates only outage probability and average SER as performance metrics. But, further work can be done on the analysis of adaptive combining scheme using ergodic capacity analysis. Also, the analysis can be extended to non-line of sight scenarios such as relay-based cooperative systems and also MIMO scenarios. In order to obtain a more general mathematical model for the hybrid FSO/RF system discussed in this report, we can explore and formulate the expressions of performance metrics using the more general distributions like Malaga and α-η-κ-μ distributions.

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1. Siddharth Mokkapati, Suyash Shah and Swaminathan R., “Outage Analysis of Adaptive Combining Scheme for Hybrid FSO/RF Communication,” submitted to *2020 National Conference on Communications (NCC)*, Kharagpur, India, 2020.

2. Siddharth Mokkapati, Suyash Shah, and Swaminathan R, “Performance analysis of adaptive combining based hybrid FSO/RF terrestrial communication,” to be submitted to IEEE journals/transactions.

3. Suyash Shah, Siddharth Mokkapati, and Swaminathan R, “Performance analysis of SAGIN with hybrid optical/RF communication,” to be submitted to IEEE journals/transactions.