B. TECH. PROJECT REPORT
On

Damage Detection in Beams under Static Loading

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DISCIPLINE OF CIVIL ENGINEERING
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A PROJECT REPORT

Submitted in partial fulfillment of the
requirements for the award of the degree

of
BACHELOR OF TECHNOLOGY
in

CIVIL ENGINEERING

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INDIAN INSTITUTE OF TECHNOLOGY INDORE
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CANDIDATE'S DECLARATION

We hereby declare that the project entitled **"Damage Detection in Beams under Static Loading"** submitted in partial fulfillment for the award of the degree of Bachelor of Technology in 'Civil Engineering' completed under the supervision of **Dr. Guru Prakash, Assistant Professor, Discipline of Civil Engineering,** IIT Indore is an authentic work.

Further, we declare that we have not submitted this work for the award of any other degree elsewhere.

Signature and name of the student(s) with date

<u>CERTIFICATE by BTP Guide</u>

It is certified that the above statement made by the students is correct to the best of my knowledge.

Signature of BTP Guide(s) with dates and their designation

Preface

This report on “Damage Detection in Beams under Static Loading" is prepared under the guidance of Dr. Guru Prakash.

The report describes a method to detect damage in beams illustrating the case of a simply supported beam under uniformly distributed load. The introductory part of the report gives the basic layout of the method and explanation of the various concepts used. The next part focusses on the application of the method to a numerical model prepared in Finite Element software.

The latter part deals with the application of the damage detection method on an experimental model set up in the civil department laboratory of IIT Indore.

In the final chapters, conclusions about the practicability and application of the developed damage method are drawn followed by suggestions on the scope of future enhancement of the method.

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Abstract

Beams are among the most critical members of any civil engineering structure. They form the core of most of the modern-day structures. Ranging from girders in bridges to the beams spanning the length and breadth of modern multi-stories there is no doubt on the utility of this invention. However, to keep the structure intact and functioning, maintenance of the structure is of prime importance. This is precisely where structure health monitoring (SHM) comes into the role. SHM uses various techniques to inspect the damages that a structure undergoes and helps to prevent failure of the structure by giving time to carry out repair works.

This project is based on the development and testing of a simplified method that uses SHM based approach to detect and assess damage in a beam in the early stages. Previous researches have used deflection measurements to assess damage in a beam, but most of these methods involved dynamic measurement techniques. In this project, however, static measurement technique has been adopted, which is still an important area of research. An algorithm has been proposed to detect and assess the severity of damage in a beam, and the same is verified using a numerical model. In the final step, a laboratory experiment has been conducted using an aluminium section as the test beam, and the proposed methodology is used in the analysis. The results are validated using numerical modelling. The experimental verification of the method is also unique as it uses a uniformly-distributed load compared to the majority of point-load based tests. The test of the proposed damage detection method is done on an aluminium beam keeping in mind the utility of the material in composite and light-weight structures. Both numerical and experimental analysis results show that the proposed model performs reasonably well for detecting and assessing damage in beams.

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1. Introduction

1.1 Motivation

Damage in a structure is one of the most critical issues that arise for any structural engineer. In any case, it is of extreme importance that the damage in any structural element is detected well in time so that there is a scope of conservation of the structure. Figure 1 shows a damaged section of a concrete bridge. Such failures can be avoided if damages are detected in time and repair work is carried out in advance. The maintenance of a structure is as essential as its construction and aesthetics in the field of civil engineering due to which the concept of Structural Health Monitoring (SHM) increasingly finds importance in the engineering practices for not only the relatively newer structures but also for the renovation and conservation of heritage structures and monuments. As a sub-part of the SHM, damage detection is the first step in checking the fitness of the structure.

Figure 1: A damaged bridge
https://s.hdnux.com/photos/57/36/23/12444870/5/940x0.jpg

There are many damage detection methods for various type of structural elements. Of these, damage detection in beams is a fairly important area of research as beams are one of the most basic yet crucial of the various construction elements used and damaged beams may lead to failure of the entire structure. The detection of damage in the beams especially in the early stages provide enough time to carry out the repair work. Therefore, it becomes important to have a damage detection method that not only gives the location of the damage but is easy to use and can be related to detectable parameters that are simple to monitor.

1.2 Organisation of Report

The whole report is organized into various chapters for the comfort of the reader and a short description about each chapter is provided in Table 1 below.

Table 1: Chapter Summary

Chapter Name	**Chapter Summary**
1.Motivation	This chapter contains the motivation and short description of the scope of the problem along with brief information about all the subsequent chapters.
2.Literature Review	This chapter covers the description of relevant works that have been carried out earlier along with the research gaps and objective of the project.
3.Proposed Methodology	This chapter contains the proposed analytical model and explains various associated concepts.
4.Numerical Modelling	This chapter contains the FEM analysis of the use of the proposed theory on different elements.
5.Experimental Analysis	This chapter contains the experimental analysis of aluminium section using the theory developed.
6.Conclusion and Future Work	This chapter contains a brief summary of the whole project and prospects of future work that can carried out on the topic.

2. Literature Review

This chapter contains brief description about the relevant works that have been done earlier describing SHM, effect of damage on a structure and giving details about the various types of damage detection methods. The following part of the chapter contain identified research gaps and the chapter ends with a description of the precise objective of the project.

2.1 Structural Health Monitoring (SHM)

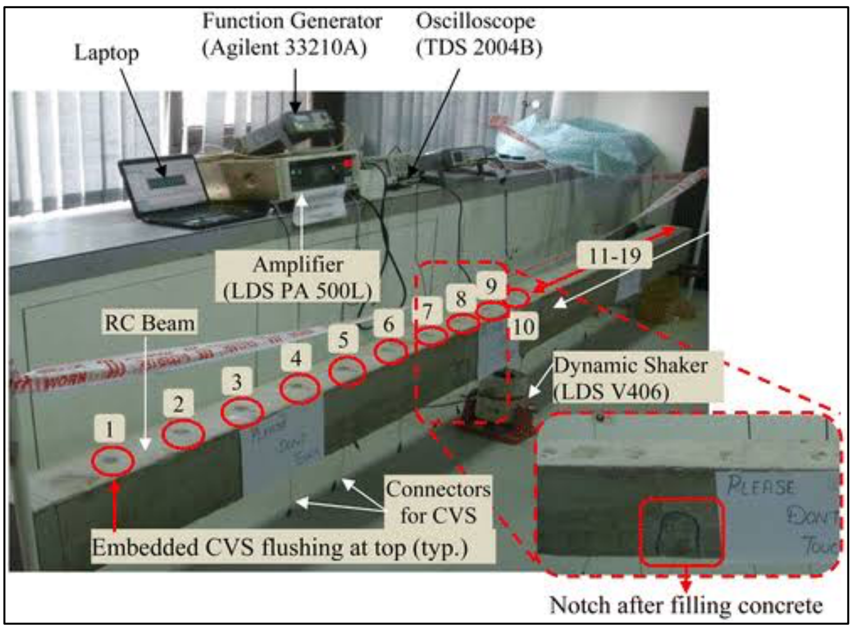

Figure 2: SHM Process: Dynamic Technique with sensors
https://ascelibrary.org/doi/abs/10.1061/%28ASCE%29EY.1943-7897.0000224

Structural Health Monitoring is a term used to denote the practice of putting in effect a damage detection technique for engineering structures and predicting the remaining life or severity of the damage. With the development in the field of civil engineering, the maintenance of structures has emerged as an emerging and challenging area of interest, and hence the research on the techniques of damage detection in a structure has received much interest worldwide. Especially in case of India with such varying topographical challenges, there is a considerable requirement of research in this area.

Damage in a structural element is defined as a change in its geometric or material properties that can lead to diminished strength and can adversely affect the performance of the system in the current scenario or in the future.

The SHM process involves monitoring the system over a period of time and observing changes in damage-sensitive features to predict the overall health of the system. A bridge element (e.g.- beam) is a good example of the use of SHM in practical cases. The remaining usability of a part of the bridge can be assessed using SHM and necessary steps can be taken to repair or replace that part. Typical assessment of damage to a bridge usually involves visual inspection by authorized inspectors. However, this 'judgement' is very subjective and can vary greatly from inspector to inspector (Feng, 2007). On the other hand, the sensor-based inspections are costly and require sophisticated instruments and skilled manpower. A middle course is therefore needed and SHM using static method provides a good alternative especially in case of early damage detection.

2.2 Effect of Damage on a Structure

When a structure is damaged, the effect of damage can be classified into two categories (S. W. Doebling, 1998), linear damage and non-linear damage. In case of linear damage, the initially linear-elastic structure remains linear-elastic after damage while in case of non-linear damage, this situation is not followed. From the available literature it is evident that most of the damage detection methods use the linear damage assumption. This assumption remains fairly accurate when the damage is at early stages. Since the proposed method is also an early detection method, the linear damage assumption is followed.

2.3 Damage Detection Methods

The available damage detection methods can be classified under three categories (N.T. Le, 2019) depending on the nature of the data obtained from experiments.

2.3.1 Dynamic Methods

This method involves the use of dynamic set of data obtained on real time basis and hence is therefore easier since the data set are relatively easier to obtain as compared to the static methods. This is a typical reason for the development of a large number of dynamic SHM methods. Hence, there is abundant availability of literature from the dynamic methods that involves strain energy analysis (A. Dixit, 2011), modal data analysis (M. N. Cerri, 2000), curvature analysis (A.K.Pandey, 1991) etc.

2.3.2 Static Methods

These methods use static data sets like deflection obtained on a long-time basis to assess the damage severity. Compared to the dynamic methods, static methods have lesser number of errors due to change in structural stiffness, mass and damping. Even, the change in structural element like loosening or exclusion can lead to errors in dynamic methods. The static methods on the other hand uses change in structural stiffness as the major criteria for considering damage in a structure (N.T. Le, 2019).

2.3.3 Static-Dynamic Methods

These methods combine the use of both static and dynamic data sets to assess the severity of damage.

2.4 Research Gaps

As evident from the literature, a lot of work has been done on dynamic damage detection methods as explained earlier but there has been less emphasis on the static damage detection methods. Also, with new age construction practices materials like aluminium and composite materials are increasingly finding their usage in structural engineering compared to the traditional steel and concrete materials, on which most of the past researches are based. Another important area which needs mention is the scarce availability of research on uniformly distributed loads (UDL) compared to the abundance of study of point loads. This is partially due to the difficulty in obtaining UDL for experimental analysis in laboratories.

2.5 Objective of the Project

The project has the following objectives:

1. Development of a damage detection method based which uses a static parameter like deflection changes that are easier to identify and monitor.
2. Development of a numerical model based on the proposed method to check its applicability and to further consolidate the analytical model.
3. An experimental analysis of an aluminium beam based on the procedure developed, to check the practical application of the proposed damage detection method.

Aluminium beam was chosen due to the growing use of aluminium in structures specially to make form-works and due to its light weight and durability. The use of aluminium composites is also finding increased uses where a combination of strength and light weight material is required. The study of aluminium and its composites is emerging as a separate research area altogether.

In this chapter, the relevant literature has been briefly described giving a crux of the research gaps which finally leads to the identification of objective of the project. The next chapter is focused on the development of a methodology to give a solution to the identified problem in the objective.

3. Proposed Methodology

The first part of the objective identified in the previous chapter is to develop an analytical base to approach the problem statement. This is done by using concepts of structural analysis and certain assumptions that are described below. The equations so developed are unique and have not been described in other literature.

3.1 Problem Statement

The problem statement of the project is the development of a static damage detection method for a single damage case in a simply supported beam with UDL.

For this, a simply supported Euler-Bernoulli beam under a UDL is considered for study. However, the method can be extended to other loads and support conditions as well as for the indeterminate structures. Another study for a cantilever beam is provided in the appendix at the end of the report. The problem statement can be divided into two parts,

1. Detection of damage.
2. Assessment of damage severity.

The utility of the deflection change to find the location of the damage and assess the has been used earlier in the works of Choi et al. (Yoon Choi, 2004) and N.T. Le et al. (N.T. Le, 2019) and (Yang, 2017), but these methods were not used in case of UDL studies.

3.2 Assumptions

The following assumptions have been made to propose the methodology to work out solution to the problem-

1. The damage is a case of single damage scenario, i.e., the beam is damaged at one-point only.
2. The damage follows linear damage situation.
3. The beam chosen is a homogeneous beam.

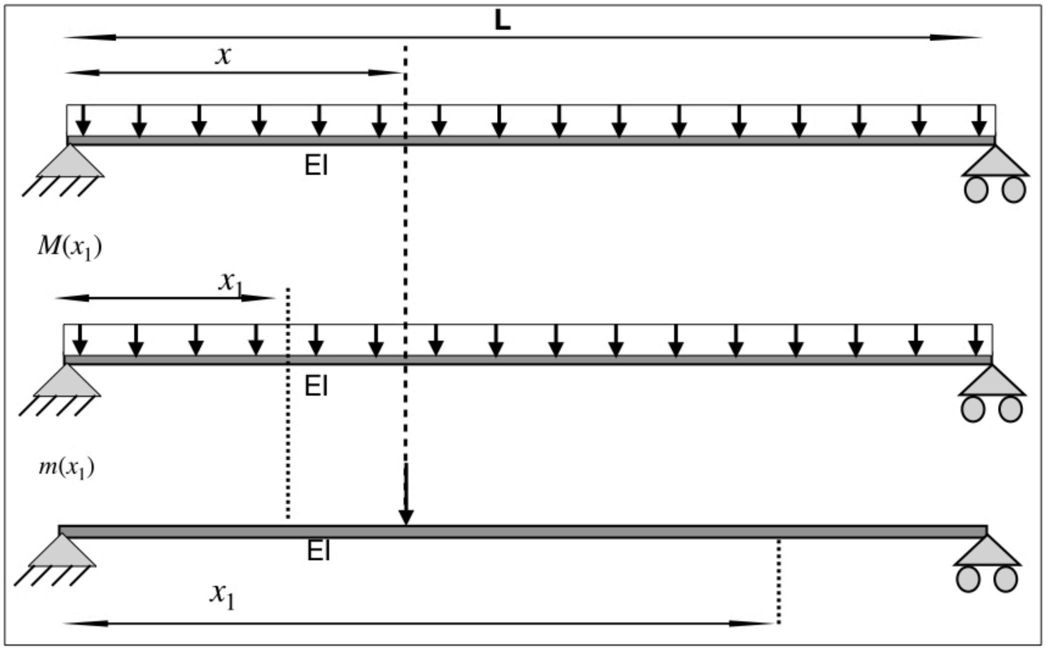

Figure 3: a) Undamaged Simply Supported Beam; b) Calculation of $M(x_1)$

(c) Calculation of $m(x_1)$

3.3 Damage Detection and Localization

Figure 3 shows a simply supported beam of length L, constant bending stiffness EI placed along the x-axis with the left end at $x=0$ and right end at $x=L$, under a UDL of w *kN/m* and Figure 4 shows the same beam, damaged at a single segment $a < x < a + b$ from the hinged end. The damaged section width is b. Since the damage is assumed to be linearly elastic, the stiffness of the damaged section has changed to (1-α)EI, where E denotes the modulus of elasticity of the material and I denotes the second moment of area of the beam.

Since the beam is statically determinate and the moments in statically determinate beams are independent of member stiffness changes, the deflection of the beam under the UDL is formulated using the Virtual Work method. For obtaining deflection at any point x from the left end
The deflection of undamaged beam, using Virtual Work method is,

$$D_h = \frac{\int_0^L M(x_1).m(x_1)dx_1}{EI_x} = \frac{\int_0^L M(x_1).m(x_1)dx_1}{EI}$$

(1)

Where D_h is the deflection of undamaged beam, $M(x_1)$ is the bending moment of beam due to real uniformly distributed load and $m(x_1)$ is the bending moment for virtual unit point load. From structural analysis concepts,

$$M(x_1) = \frac{wLx_1}{2} - \frac{wx_1^2}{2} \qquad for \qquad 0 < x_1 < L$$

(2)

$$m(x_1) = \left(1 - \frac{x}{L}\right).x_1 \qquad for \qquad 0 < x_1 < x$$

(3)

$$m(x_1) = \left(1 - \frac{x_1}{L}\right).x \qquad for \qquad x < x_1 < L$$

(4)

After putting equation (2),(3),(4) in equation (1) and integration,

$$D_h = \frac{1}{EI}\int_0^x \left(\frac{wLx_1}{2} - \frac{wx_1^2}{2}\right).\left(1 - \frac{x}{L}\right).x_1.dx_1 + \int_x^L \frac{wx}{2}\left(1 - \frac{x_1}{L}\right).(Lx_1 - x_1^2).dx_1$$

$$\Rightarrow D_h = \frac{1}{EI}\int_0^x \frac{w}{2}\left(1 - \frac{x}{L}\right).(Lx_1^2 - x_1^3).dx_1 + \int_x^L \frac{wx}{2}\left(Lx_1 - 2x_1^2 + \frac{x_1^3}{L}\right).dx_1$$

$$\Rightarrow D_h = \frac{w}{2EI}\left(1 - \frac{x}{L}\right)\left(\frac{Lx_1^3}{3} - \frac{x_1^4}{4}\right)\Big|_0^x$$

$$\Rightarrow D_h = \frac{w(x^4 - 2Lx^3 + L^3x)}{24EI}$$

(5)

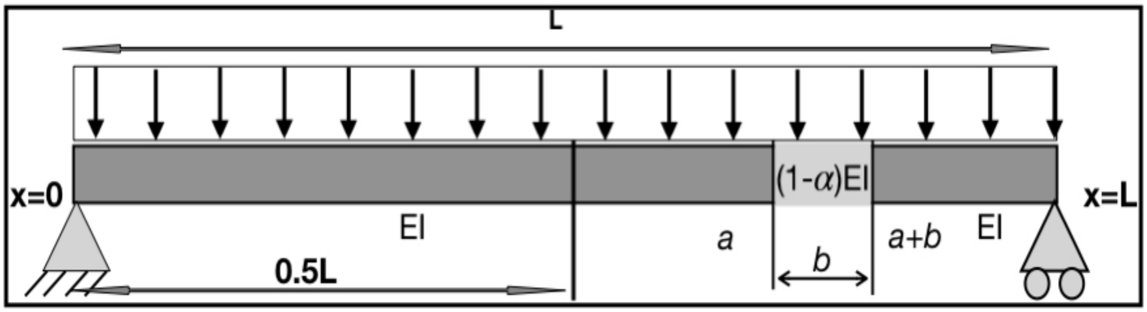

Figure 4: A damaged beam

Similarly, the deflection of the beam as shown in Figure 4 under damaged state (D_d) is also formulated using the Virtual Work method.

$$D_d = \frac{\int_0^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^{a+b} M(x_1).m(x_1)dx_1}{(1-\alpha)EI} + \frac{\int_{a+b}^{L} M(x_1).m(x_1)dx_1}{EI}$$

(6)

The change in deflection (Δ) between the undamaged and damaged state of beam can therefore be calculated from equation (5) and (6) as,

$$\Delta = D_d - D_h = \left(\frac{\alpha}{1-\alpha}\right)\frac{\int_a^{a+b} M(x_1).m(x_1)dx_1}{EI}$$

After integration the results for different positions of damage, i.e., for different values of a and b comes out to be as follows, where, $\beta = \frac{\alpha}{1-\alpha}$

For $x \le a$,

$$D_d = \frac{\int_0^x M(x_1).m(x_1)dx_1}{EI} + \frac{\int_x^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^{a+b} M(x_1).m(x_1)dx_1}{(1-\alpha)EI} + \frac{\int_{a+b}^L M(x_1).m(x_1)dx_1}{EI}$$

D_h can be written as,

$$D_h = \frac{\int_0^x M(x_1).m(x_1)dx_1}{EI} + \frac{\int_x^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^{a+b} M(x_1).m(x_1)dx_1}{EI} + \frac{\int_{a+b}^L M(x_1).m(x_1)dx_1}{EI}$$

Therefore,

$$\Delta = D_d - D_h$$

$$\Delta = \int_a^{a+b} \frac{\beta}{EI}\left(\frac{wLx_1}{2} - \frac{wx_1^2}{2}\right).\left(1 - \frac{x_1}{L}\right).x.dx_1$$

$$\Rightarrow \Delta = \int_a^{a+b} \frac{\beta wx}{2EI}\left(Lx_1 - 2x_1^2 + \frac{x_1^3}{L}\right).dx_1$$

$$\Rightarrow \Delta = \frac{\beta wx}{2EI}\left(\frac{Lx_1^2}{2} - \frac{2x_1^3}{3} + \frac{x_1^4}{L}\right)\Big|_a^{a+b}$$

$$\Delta = \beta C_1 x \tag{7}$$

where,

$$C_1 = \frac{w}{2EI}\left[\frac{L(a+b)^2 - a^2}{2} - \frac{2\{(a+b)^3 - a^3\}}{3} + \frac{\{(a+b)^4 - a^4\}}{4L}\right] \tag{8}$$

And for $x \geq a + b$

$$D_d = \frac{\int_0^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^{a+b} M(x_1).m(x_1)dx_1}{(1-\alpha)EI} + \frac{\int_{a+b}^{x} M(x_1).m(x_1)dx_1}{EI} + \frac{\int_x^L M(x_1).m(x_1)dx_1}{EI}$$

D_h can be written as,

$$D_h = \frac{\int_0^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^{a+b} M(x_1).m(x_1)dx_1}{EI} + \frac{\int_{a+b}^{x} M(x_1).m(x_1)dx_1}{EI} + \frac{\int_x^L M(x_1).m(x_1)dx_1}{EI}$$

$$\Delta = \int_a^{a+b} \frac{\beta}{EI}\left(\frac{wLx_1}{2} - \frac{wx_1^2}{2}\right).\left(1-\frac{x}{L}\right)x_1.\,dx_1$$

$$\Delta = \int_a^{a+b} \frac{\beta w}{2EI}\left(1-\frac{x}{L}\right).(Lx_1^2 - x_1^3).\,dx_1$$

$$\Delta = \frac{\beta w}{2EI}\left(1-\frac{x}{L}\right)\left(\frac{Lx_1^3}{3} - \frac{x_1^4}{4}\right)\Big|_a^{a+b}$$

$$\Delta = \beta C_2(L-x) \tag{9}$$

Where,

$$C_2 = \frac{w}{2LEI}\left[\frac{L\{(a+b)^3 - a^3\}}{3} - \frac{\{(a+b)^4 - a^4\}}{4}\right] \tag{10}$$

also, for $a < x < a + b$,

$$D_d = \frac{\int_0^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^x M(x_1).m(x_1)dx_1}{(1-\alpha)EI} + \frac{\int_x^{a+b} M(x_1).m(x_1)dx_1}{(1-\alpha)EI} + \frac{\int_{a+b}^{L} M(x_1).m(x_1)dx_1}{EI}$$

D_h can be written as,

$$D_h = \frac{\int_0^a M(x_1).m(x_1)dx_1}{EI} + \frac{\int_a^x M(x_1).m(x_1)dx_1}{EI} + \frac{\int_x^{a+b} M(x_1).m(x_1)dx_1}{EI} + \frac{\int_{a+b}^{L} M(x_1).m(x_1)dx_1}{EI}$$

$$\Delta = \int_a^x \frac{\beta w}{2EI}\left(1-\frac{x}{L}\right).(Lx_1 - x_1^2).x_1.dx_1 + \int_a^{a+b} \frac{\beta wd}{2EI}(Lx_1 - x_1^2)\left(1-\frac{x_1}{L}\right).dx_1$$

$$\Delta = \int_a^x \frac{\beta w}{2EI}\left(1-\frac{x}{L}\right).(Lx_1^2 - x_1^3).dx_1 + \int_a^{a+b} \frac{\beta wx}{2EI}\left(Lx_1 - 2x_1^2 + \frac{x_1^3}{L}\right).dx_1$$

$$\Delta = \frac{\beta w}{2EI}\left(1-\frac{x}{L}\right).\left(\frac{Lx_1^3}{3} - \frac{x_1^4}{4}\right)\Big|_a^d + \frac{\beta wd}{2EI}\left(\frac{Lx_1^2}{2} - \frac{2x_1^3}{3} + \frac{x_1^4}{4L}\right)\Big|_d^{a+b}$$

$$\Delta = \beta\frac{w}{2EI}\left[\left(1-\frac{x}{L}\right)\left[\frac{L(x^3-a^3)}{3} - \frac{x^4-a^4}{4}\right] + \left[\frac{L\{(a+b)^2-x^2\}}{2} - \frac{2\{(a+b)^3-x^3\}}{3} + \frac{(a+b)^4-x^4}{4L}\right]\right] \quad (11)$$

The above deflection change (DC) equation when plotted against the length of the beam leads to the graph as shown in Figure 5. It is visible from the graph that as one moves from one support to the other, the deflection change curve has two linear regions and a curved region sandwiched between them. Also, the peak of the curve is in the curved region. Therefore, the location of the damage can be found out using the curve as-

i) the damaged region lies within the linear portions curve, and,

ii) the damaged region is located at the peak area of the deflection change curve.

Both these criteria can be used to locate the damage in the beam. In many cases due to the measurement inaccuracies, linear regions of the curve might be difficult to obtain and, in such cases, the peak area criterion can be used more effectively to ascertain the location of the damage.

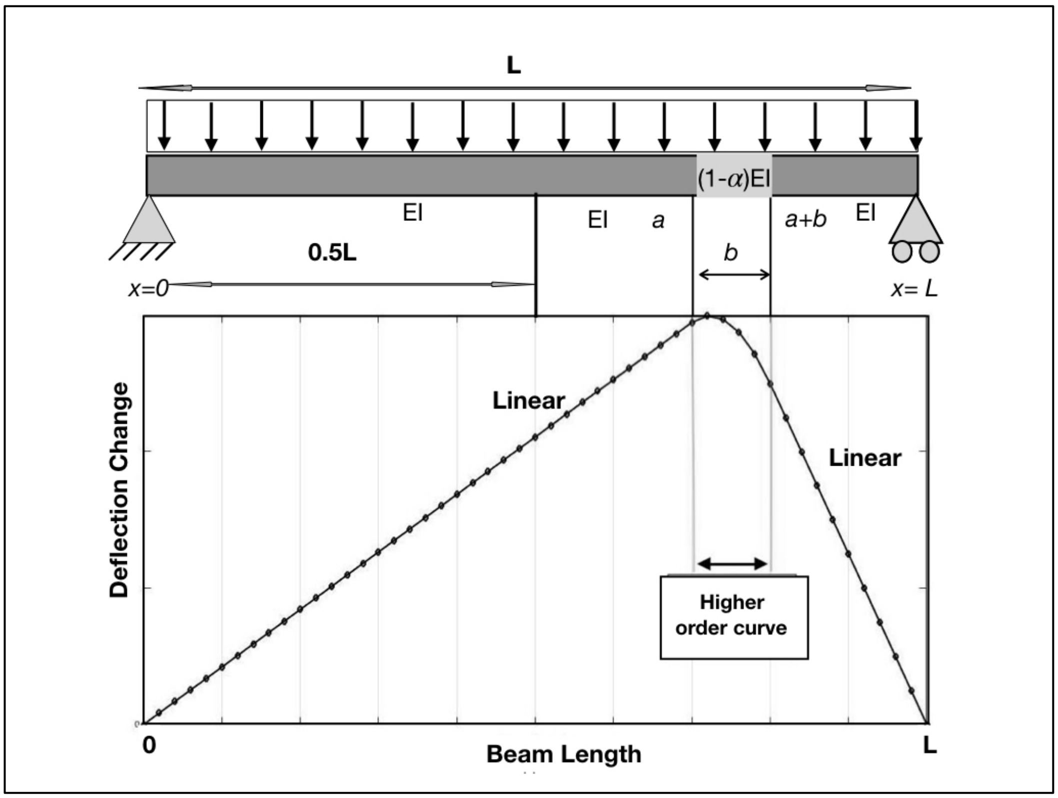

Figure 5: Deflection Change Curve

3.4 Assessment of Damage Severity

Continuing on the previous step, after locating the damage, the next task is to assess the severity of the damage. The deflection change values obtained in the previous step however, also contain stiffness EI in the formula. This quantity in practical applications is not always accurately calculated that can lead to erroneous results. Hence, the deflection change is normalized by taking relative deflection change (RDC) values which eliminates EI from the calculations. RDC is defined as,

$$RDC(x) = \frac{\Delta(x)}{D_h(x)}$$

(12)

From previous equations (5),(7) and (9),

$$RDC(x) = \begin{cases} \beta \dfrac{C_1 x}{D_h}(x), & x < a \\ \beta \dfrac{C_2(L - x)}{D_h}(x), & x > a + b \end{cases}$$

(13)

Another term is defined as RDC^{50} which is the RDC value for a special case when β=1,i.e., when the damage severity is 50% or $\alpha = 0.5$ which can be calculated from the already known parameters a, b length of the beam l and the inspection point coordinate values of x. This provides for a unique RDC^{50} value for each beam element which can be calculated analytically without the requirement to know the damage severity at this stage.

$$RDC^{50}(x) = \begin{cases} \dfrac{C_1 x}{D_h}(x) \quad , & x \leq a \\ \dfrac{C_2(L - x)}{D_h}(x) \ , & x > a + b \end{cases}$$

(14)

Hence,

$$RDC(x) = \beta \, RDC(x)^{50}$$

(15)

Another important observation from equations (13), (14) and (15) is that the measured value of RDC^{50} at each value of x differs from the RDC value at that point by a factor of β. Hence, we can use the RDC^{50} value as the comparison value to calculate β.

This gives us a fairly simple approach to calculate the severity of damage. For further improvement in calculation another function called consistency function c(x) is defined which is used to calculate the value of β for different measurement points. Hence,

$$c(x) = \frac{RDC(x)}{RDC^{50}(x)} \;\; ; for\ x \notin (a, a+b) \tag{16}$$

It is clear that a good consistency and minimal error in measurements will give a nearly constant consistency function while a large variation in values of consistency function will imply measurement noise. The average value of the consistency function is then used to calculate the value of β which in turn gives the damage severity α. Therefore,

$$\beta = \overline{c(x)} \;\; ; for\ x \notin (a, a+b) \tag{17}$$

$$\alpha = \frac{\beta}{1+\beta} \tag{18}$$

Hence, the whole process to detect and assess the damage can be broken down into steps as shown in the flowchart in Figure 6. The suspected beam is first divided into smaller elements and the beams deflection is measured at various measurement points. After this, calculation of DC and RDC values is done followed by calculation of consistency function c(f). The average of c(f) is then used to calculate β which in turn gives the value of damage severity α.

3.5 Algorithm Flowchart

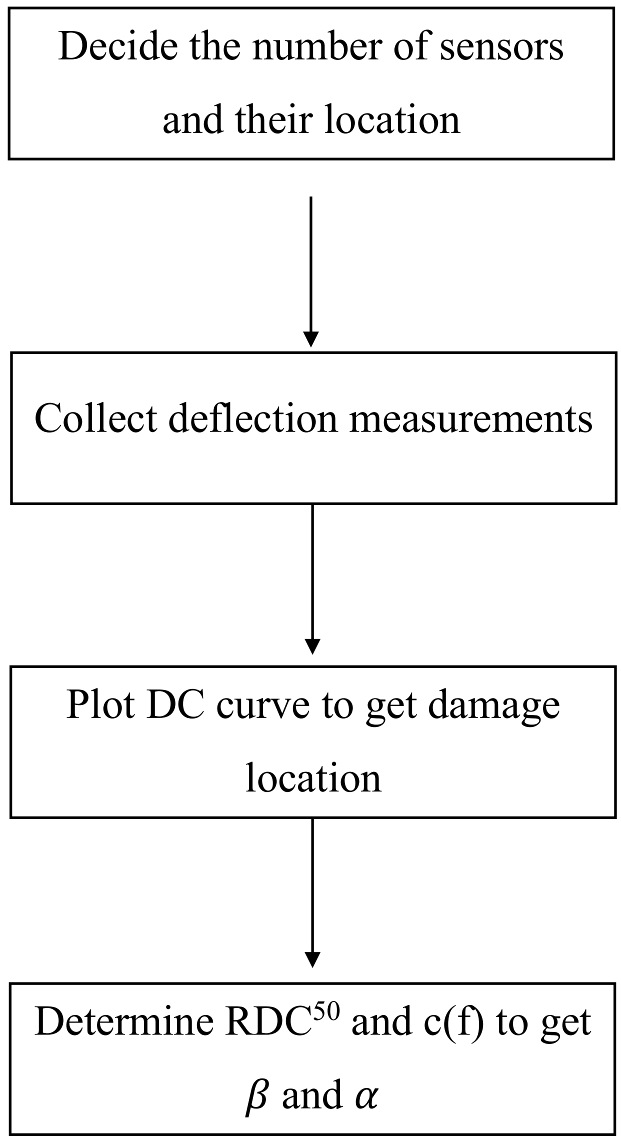

Figure 6 : A flowchart showing key steps in proposed algorithm

4. Numerical Modelling

The previously developed equations were tested on practical beams using a Finite Element Method software package, ABAQUS. The numerical analysis was done on a number of rectangular beams of different materials and with varying degree of damage. After this, using the previously developed theory, the results of the numerical modelling are analysed. The results of the analysis are then shown using figures and graphs.

4.1 Numerical Model

A 12m long simply-supported aluminum beam is modelled using the Finite Element (FE) software. The properties of the material and dimensions of the beam are shown in Table 2. For the purpose of measurements, the beam is divided into 12 equal elements of length 1 m each.

Table 2: Details of the beam

Young's Modulus	71GPa
Poisson's Ratio	0.32
Density	2770 kg/m^3
Beam Length	12 m
Beam Width	1.5 m
Beam Depth	1 m

A UDL of 1000 kN/m is applied on the beam and the boundary conditions of the beam are that of a simply-supported beam i.e., one hinged support and other roller support. This is shown in Figure 7.

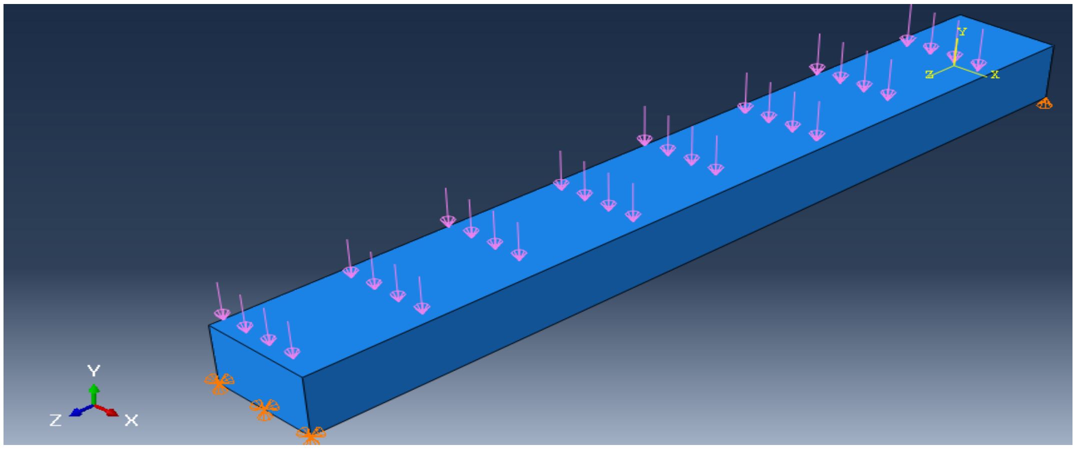

Figure 7: Loading and Boundary Conditions

4.2 Damage Simulation

Damage is simulated in one of the elements by reducing the stiffness of that element by reducing its second moment of area. Damage is given to a selected part (at element number 9) by introducing a cut along the width, perpendicular to the length of the beam. The intensity of the damage can be changed with the depth of the cut.

Three damage scenarios are simulated by reducing the stiffness by 20%, 30% and 40%. This is done by introduction of cuts of width 3 mm and overall reduction in breadth of the beam by 20%, 30% and 40%. For instance, to reduce the stiffness by 20%, the breadth is decreased by 0.3m.

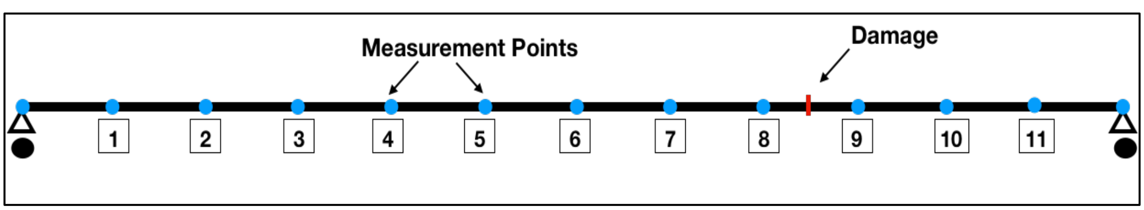

Figure 8: Representation of the beam

4.3 Deflection Curves

The deflection curve of the beam is plotted in Figure 9 for undamaged and three damage scenarios under the same UDL as applied on the beam. The increasing trend in the deflection shows the increase in the damage. But, for locating the damage and for assessing the severity, further information is required.

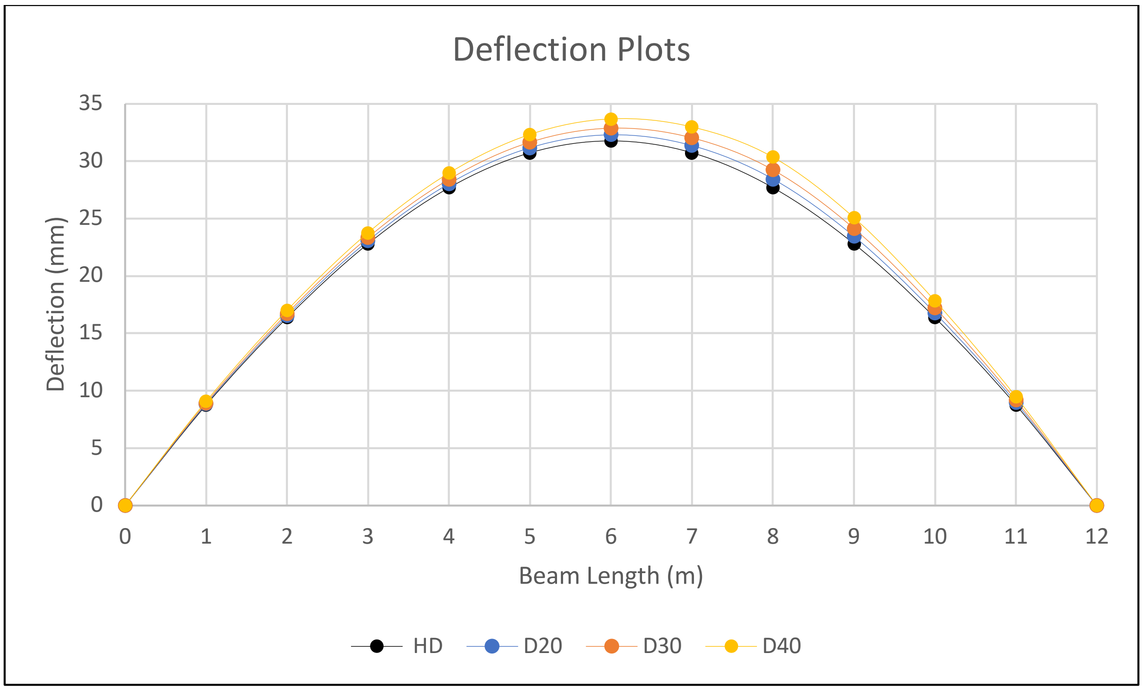

Figure 9: Deflection Curves for beam

As is evident from Figure 9 that with increase in damage, the deflection of the beam increases as compared to the undamaged beam. As per theory explained earlier highest curve in plots will be for highest damage i.e., 40%.

4.4 Damage Localization

To get the damage location, deflection change (DC) curves are plotted by taking difference between the damaged deflection and undamaged deflection for each point. As expected, there is a peak at element 9 in the DC plots. As is evident from Figure 10, there are two linear regions surrounding the damaged element of the beam which is similar to the analytical concept explained earlier. Hence, the location of the damage is fixed and element 9 is the suspected element for damage.

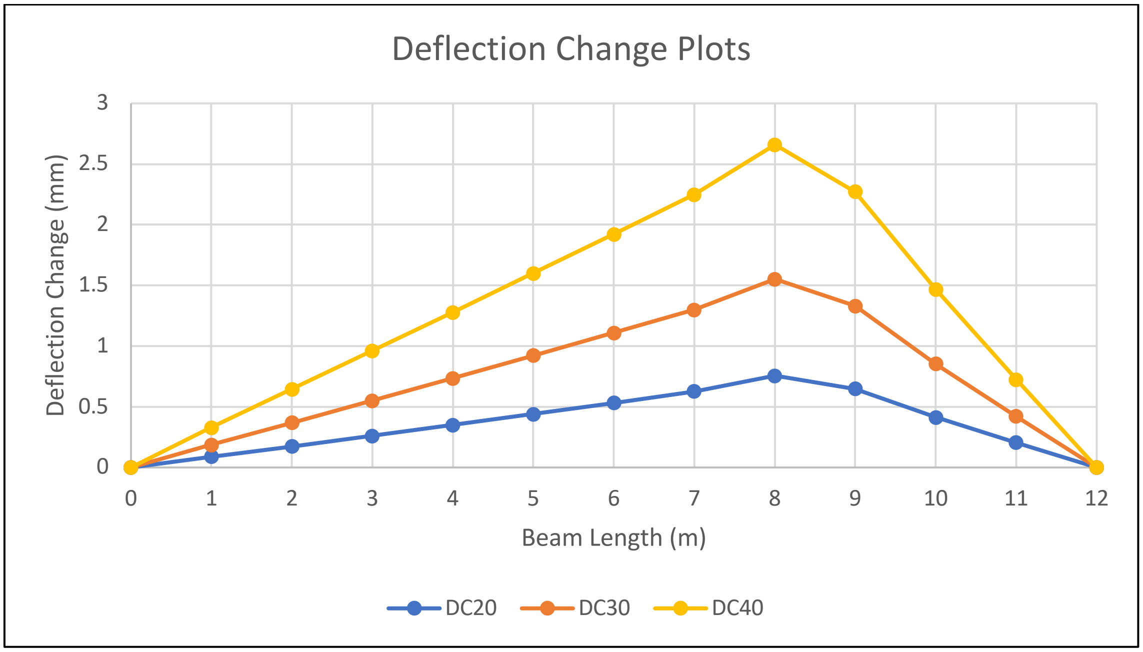

Figure 10: Deflection Change Plots for Beam

4.5 Assessment of Damage Severity

After the damaged element is identified, the damage severity is assessed by first calculating RDC and RDC50 for various inspection points. After which, the value of consistency function is also plotted in the following Figure 11. Finally, Table 3 shows the comparative values of damage severity and error between actual and model values.

Table 3: Comparison between analytical and model values

Node Number	**RDC**				**c(f)**		
Damage	**20%**	**30%**	**40%**		**20%**	**30%**	**40%**
1	0.010	0.021	0.038		0.165	0.350	0.614
2	0.010	0.023	0.039		0.167	0.354	0.619
3	0.011	0.024	0.042		0.169	0.357	0.621
4	0.013	0.027	0.046		0.170	0.358	0.623
5	0.014	0.030	0.052		0.171	0.360	0.625
6	0.017	0.035	0.060		0.173	0.361	0.626
7	0.020	0.042	0.073		0.174	0.363	0.627
8	0.027	0.056	0.096		0.184	0.378	0.648
9	0.028	0.058	0.099		0.173	0.356	0.608
10	0.025	0.052	0.090		0.164	0.339	0.584
11	0.023	0.048	0.083		0.157	0.326	0.561

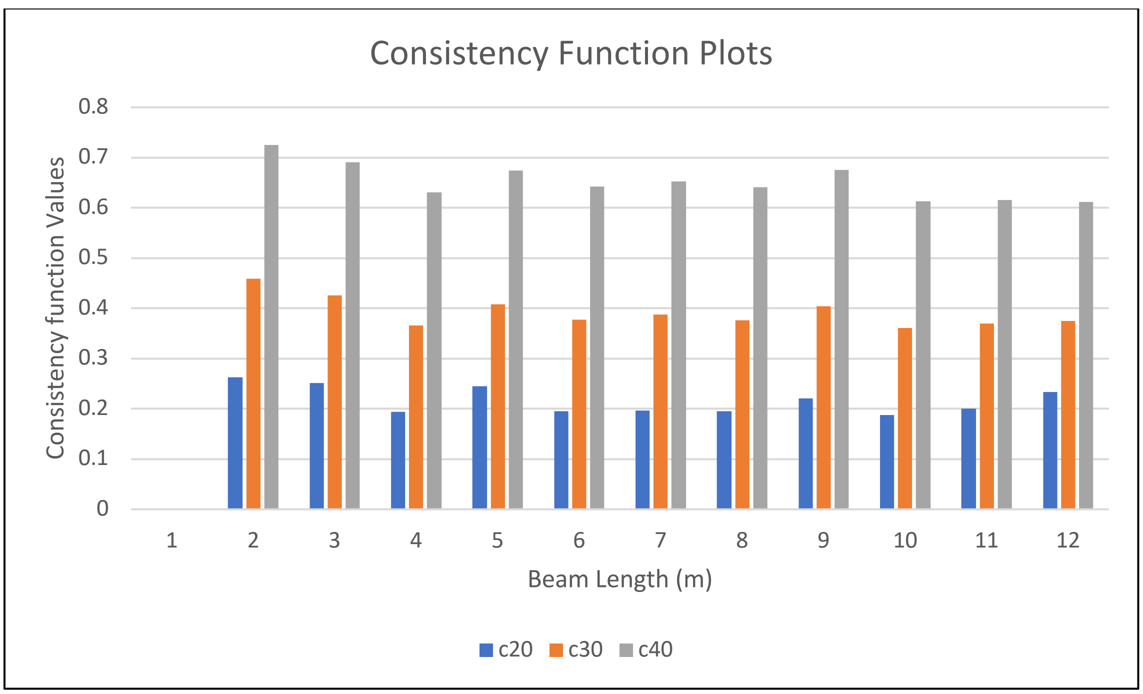

Figure 11: Consistency Function Plot

Table 4: Results

Case	$\boldsymbol{\beta = \overline{c(f)}}$	$\boldsymbol{\alpha}$	**Damage Severity Percentage**	**Error**
20%	0.217	0.178	17.8%	11%
30%	0.392	0.281	28.1%	6.34%
40%	0.652	0.394	39.4%	1.5%

4.6 Conclusion

As evident from the Figure 11, the consistency function values are nearly constant which shows acceptable measurement prudence. Also, the damage severity as detected is within acceptable error limits. Hence the numerical model so developed follows the theory proposed earlier.

5. Experimental Studies

The final part of the project involved experimental studies of the proposed method of damage detection. The experiment was set up in the laboratory of IIT Indore. The details of the equipment used are given in the subsequent sections. An FEM model of the aluminium section used in experiment is first validated and finally the results of the experiment are analysed and verified using the validating model.

Figure 12: Experimental set-up

5.1 Instrumentation

The experimental setup involved the use of the following equipment.

5.1.1 Aluminium Beam

A hollow, rectangular aluminium section element of length 2 m and of the cross section as depicted in Figure 13 is used as a beam element.

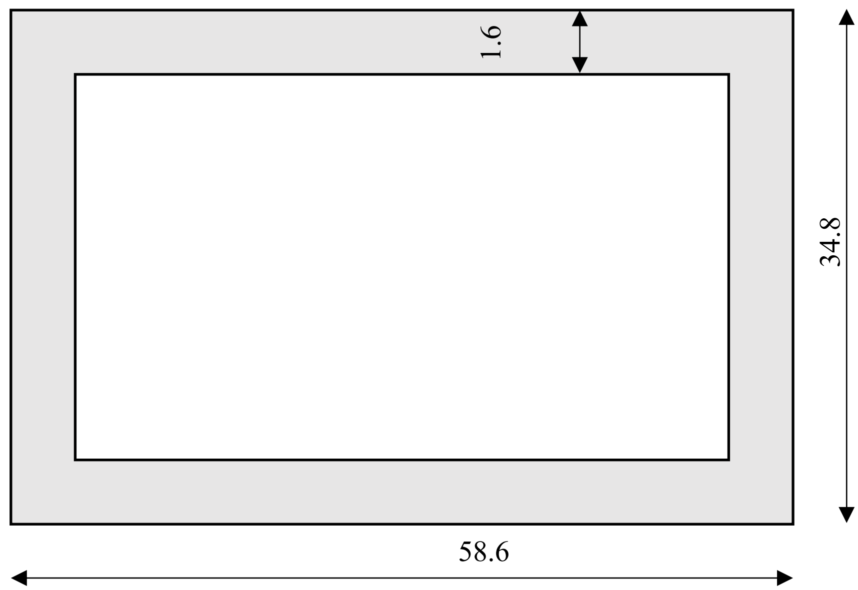

Figure 13: Aluminium Beam Section

5.1.2 UDL Set-up

Due to the difficulty in obtaining an exact UDL in this scenario, an innovative approach to obtain a near UDL is used in the experiment. Ten equal sized fly ash bricks with approximately equal weights are used to obtain the UDL as shown in Figure 14. Also, the weight of the bricks is chosen such that it is well below the failure load of the beam. The details of the UDL are mentioned in Table 5.

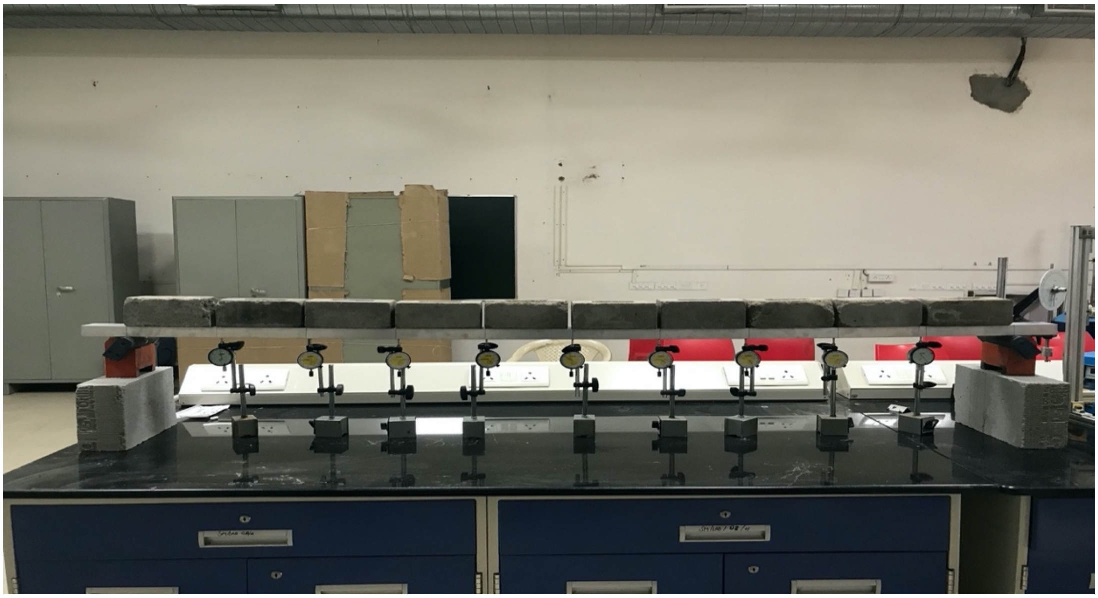

Figure 14: UDL set-up

Table 5: Details of UDL

Dimensions of each brick	19X10X10 cm^3
Gap between bricks	1 cm
Average weight of each brick	2748.5 g $\pm$ 0.8%
Average value of UDL on beam	134.814 N/m

5.1.3 Damage Initiation

Damage to the beam was introduced at a distance of 1400 mm from left end by introducing 1mm width cuts as shown in Figure 15 using Hacksaw of 24 TPI. The depth of the cuts was increased subsequently for assigning various degree of damage.

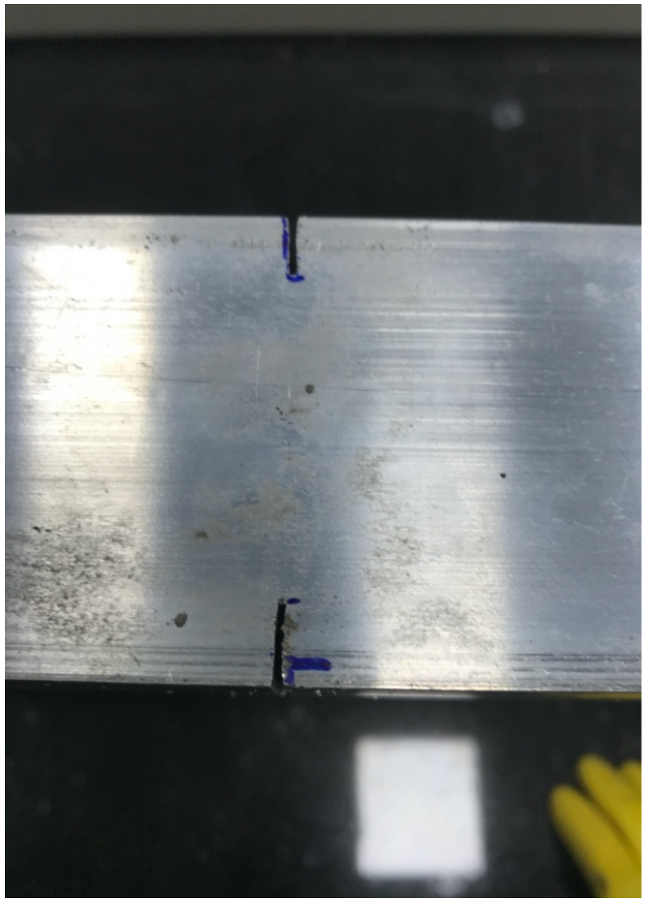

Figure 15: Damage Introduction

5.1.4 Supports and Dial Gauges

In total, 9 dial gauges were placed at a distance of 200 mm from each other. The 1st and the 9th were of 10 mm range and 0.01 mm resolution while the other 7 were of 25 mm range and 0.01 resolution. The supports used were simple roller supports and two concrete bricks were used to raise the supports.

5.2 Data Acquisition

Each reading was taken 3 times loading and unloading the beam using the same set of weights. As shown in Figure 16, in the first reading, the left end of the brick was placed at the left end of the element, similarly for second reading right end of the brick was placed at right end of the element.

Finally for the third set of data the brick was placed at the middle of the element (giving a margin of 0.5 cm on each side). The average values of the three set of readings were taken for calculation. This was done to accommodate errors arising when some part of the beam is not under loading due to the gaps between the bricks. Also, after loading, 120 seconds were given for the beam to deflect, similarly a time of 120 seconds was given between unloading the beam and taking next set of measurements. This was done to make sure that the errors due to measurement were minimised.

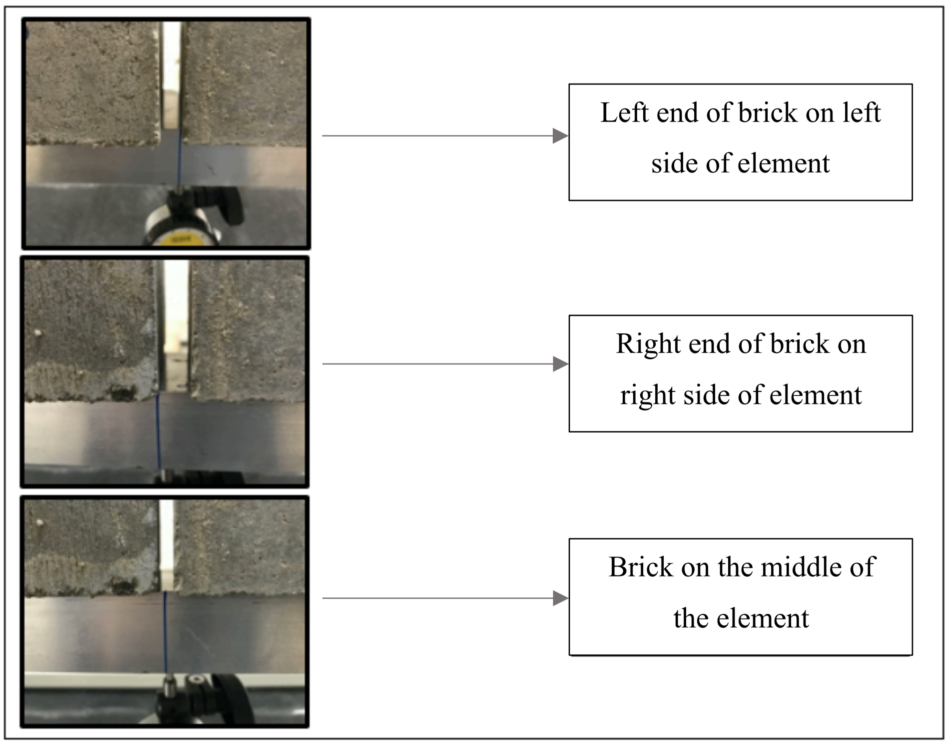

Figure 16: Load Placement

5.3 Experimental Analysis

Three damage scenarios (D1, D2 and D3) are initiated by introducing 1 mm width cuts of total depth (from both sides) 12 mm, 18 mm and 24 mm respectively. The severity of the damage is not known beforehand. The results are shown for each of the cases in the following sub-heads.

Damage Level 1

Damage Localization

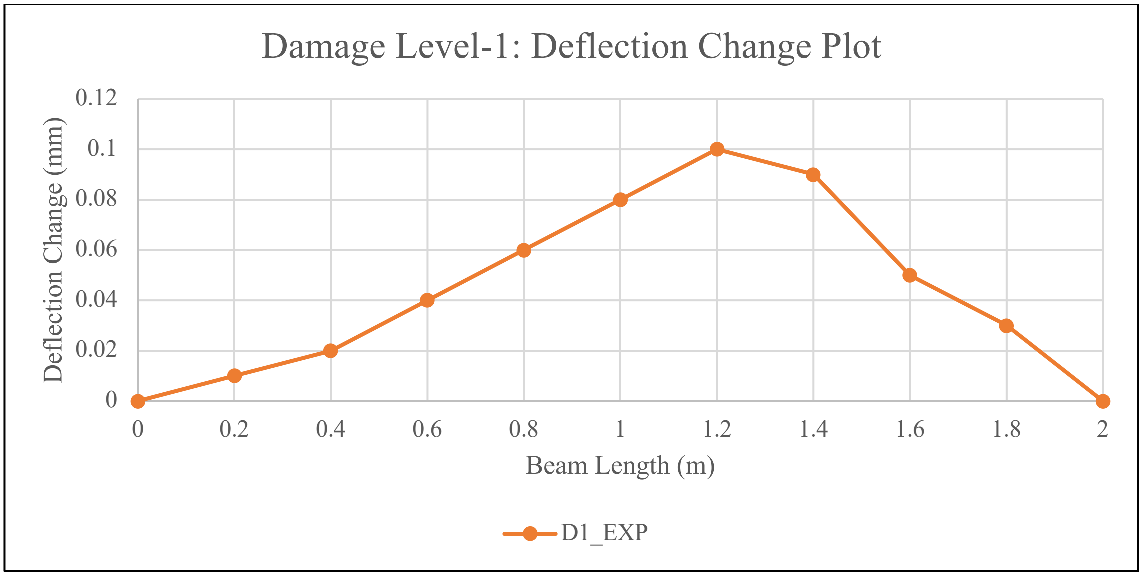

Figure 17: Case-1-Deflection Change Plot

Damage Severity

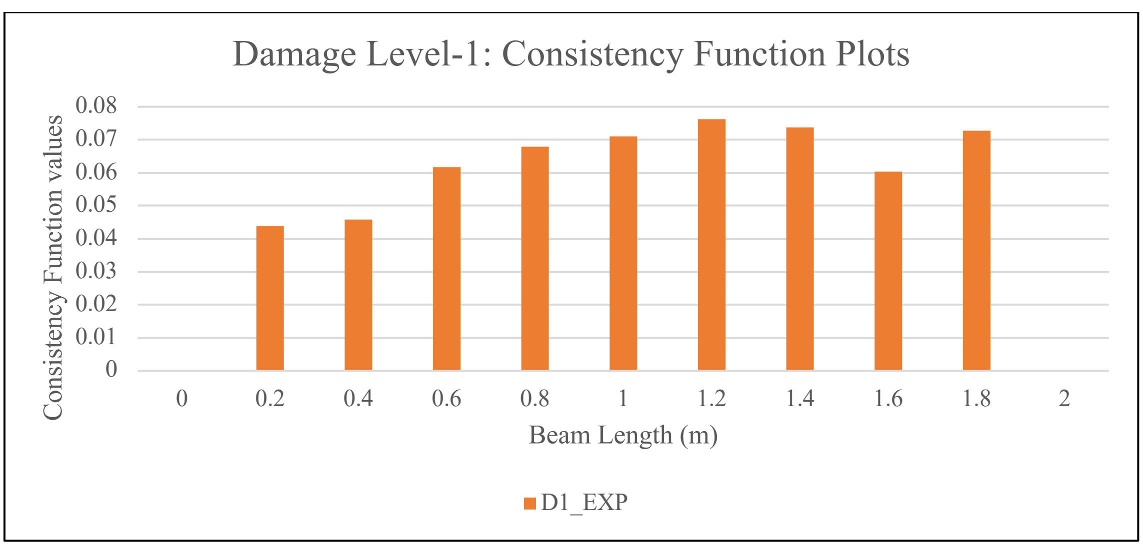

Figure 18: : Case-1-Consistency Function Plot

Damage Level 2

Damage Localization

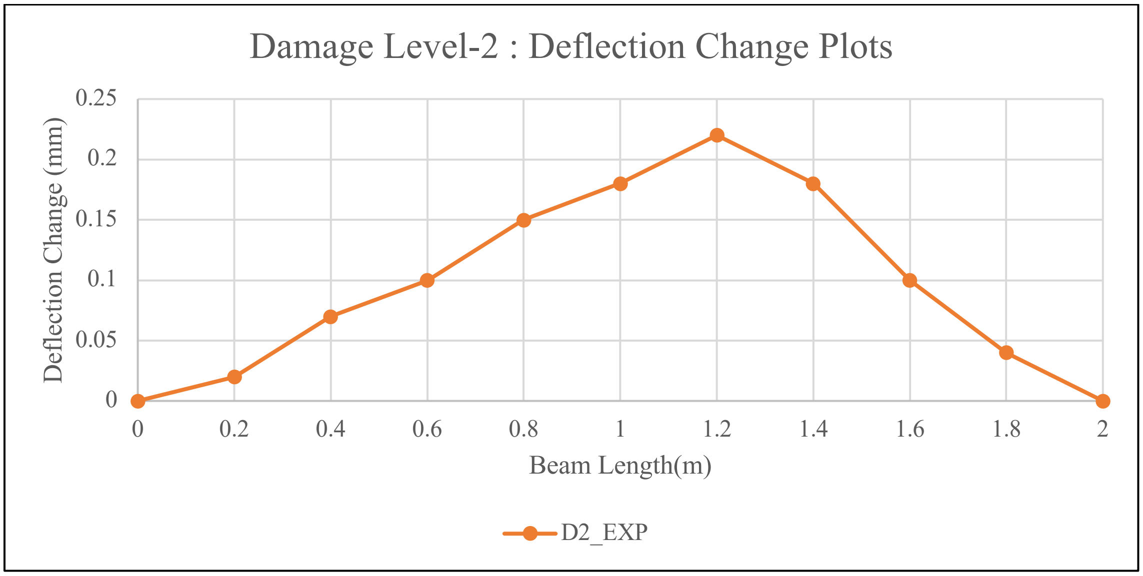

Figure 19: Case-2-Deflection Change Plot

Damage Severity

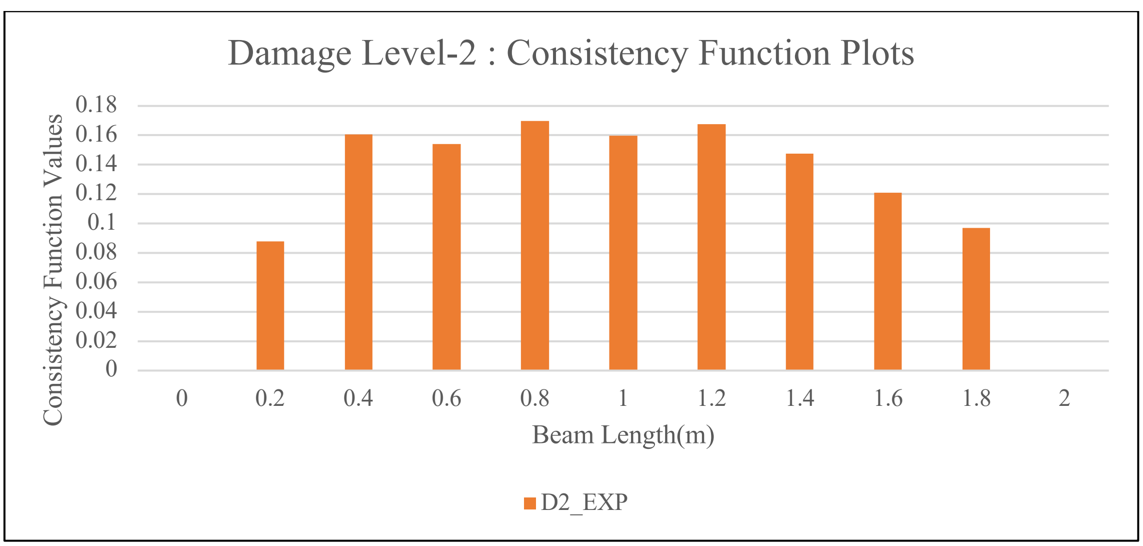

Figure 20: Case-2-Consistency Function Plot

Damage Level 3

Damage Localization

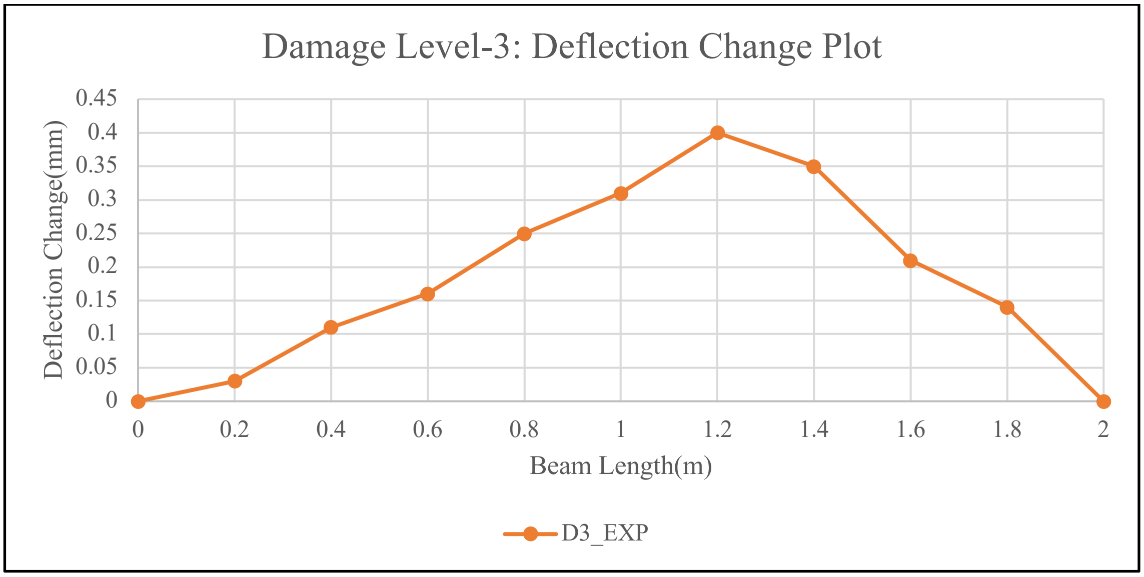

Figure 21: Case-3-Deflection Change Plot

Damage Severity

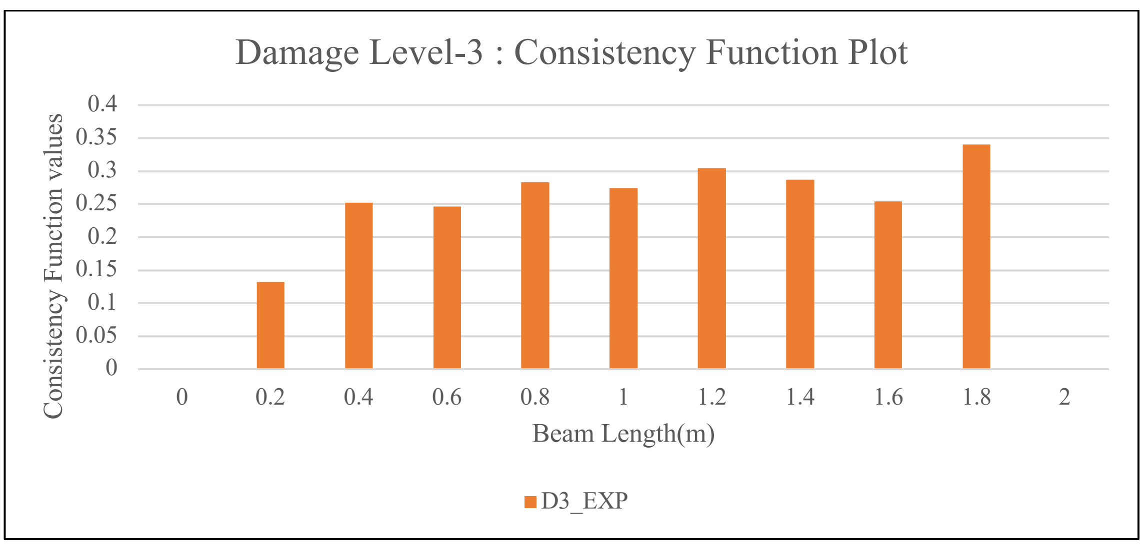

Figure 22: Case-3-Consistency Function Plot

5.4. Numerical Model Validation

The experimental beam was modelled on FE software using the average value of UDL obtained earlier to get a working model that can be used as a baseline for experimental verification. The results for deflection in undamaged case for the numerical model and experimental beam are plotted in Figure 23. The model and experiment results are in agreement to each other with error ranging between 7% to 13%. Hence, the model can be used to verify the experimental data.

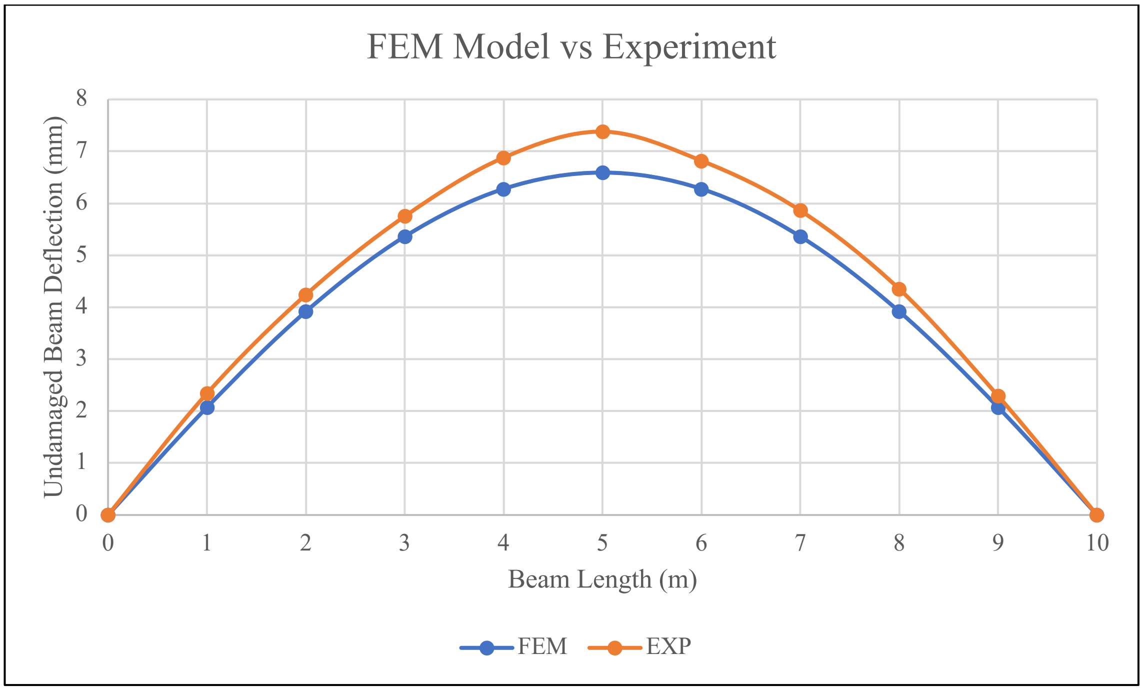

Figure 23: Deflection plots for validation model and experiment

5.5. Results

The following section shows three cases of damage given to the experimental beam and its comparison with validation model. Table 6 shows comparison between the damage severity values found experimentally and from validation model. The damage is clearly identified between 1.2 m and 1.4 m of the beam which is in accordance with the given damage at 1.30 m from left end of the beam.

Damage Level-1

Damage Localization

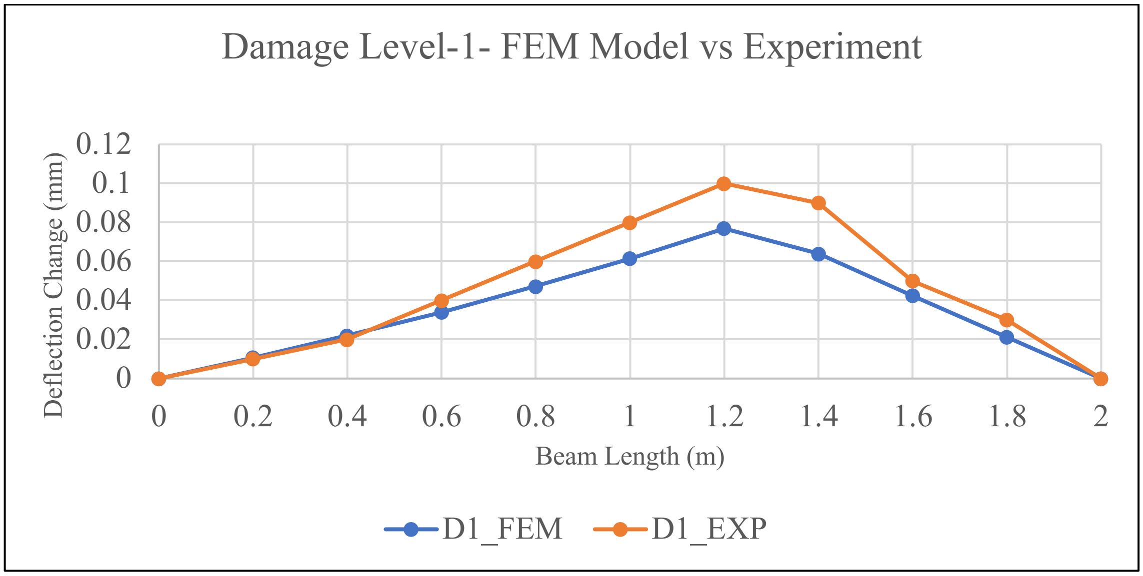

Figure 24: Case 1- Deflection Change comparison

Damage Severity

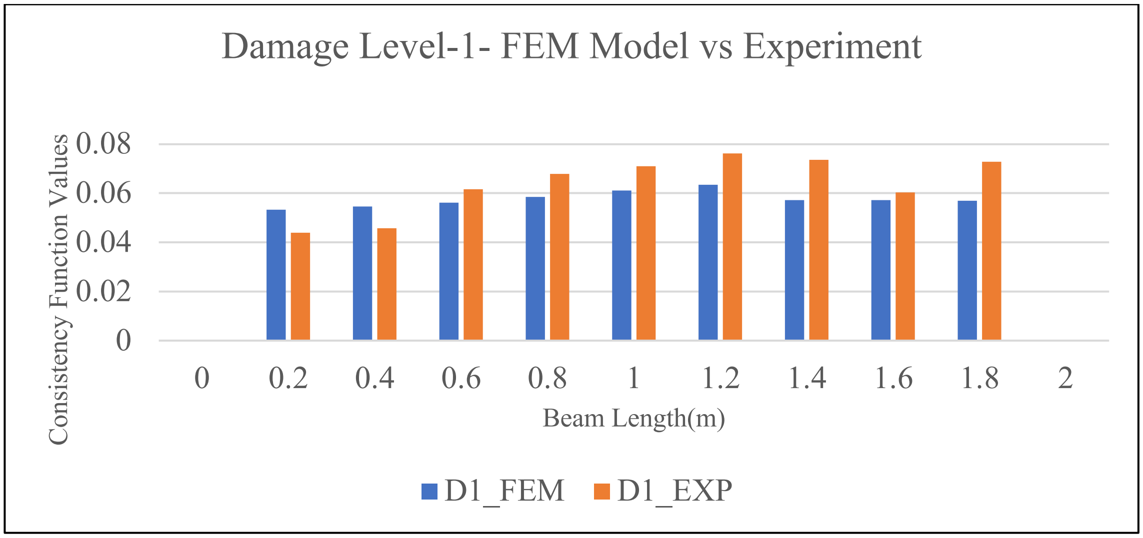

Figure 25: Damage Level-1: Damage Severity comparison

Damage Level-2

Damage Localization

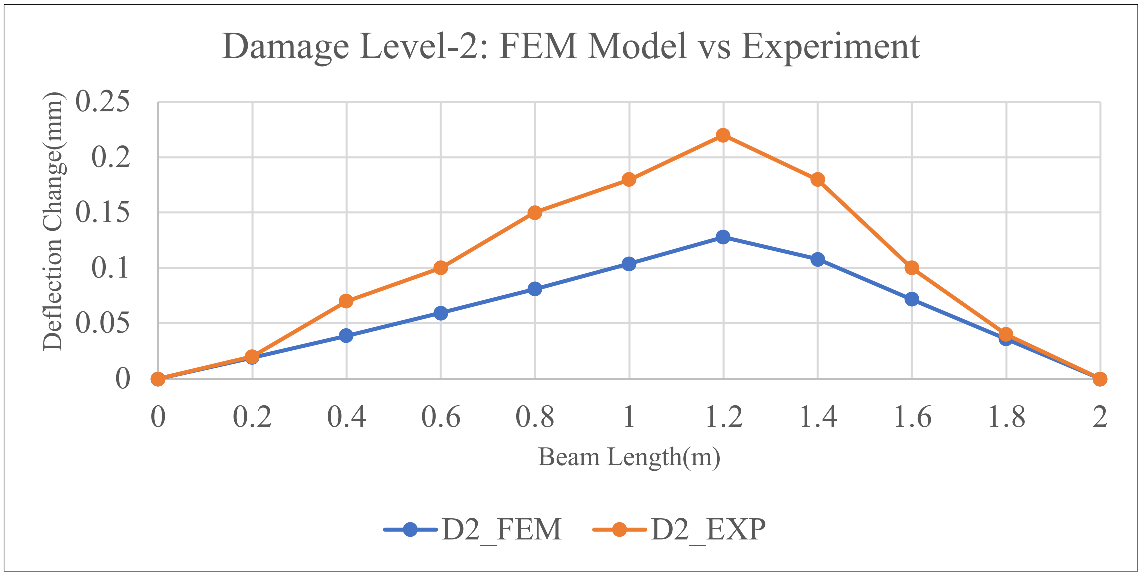

Figure 26: Case 2- Deflection Change comparison

Damage Severity

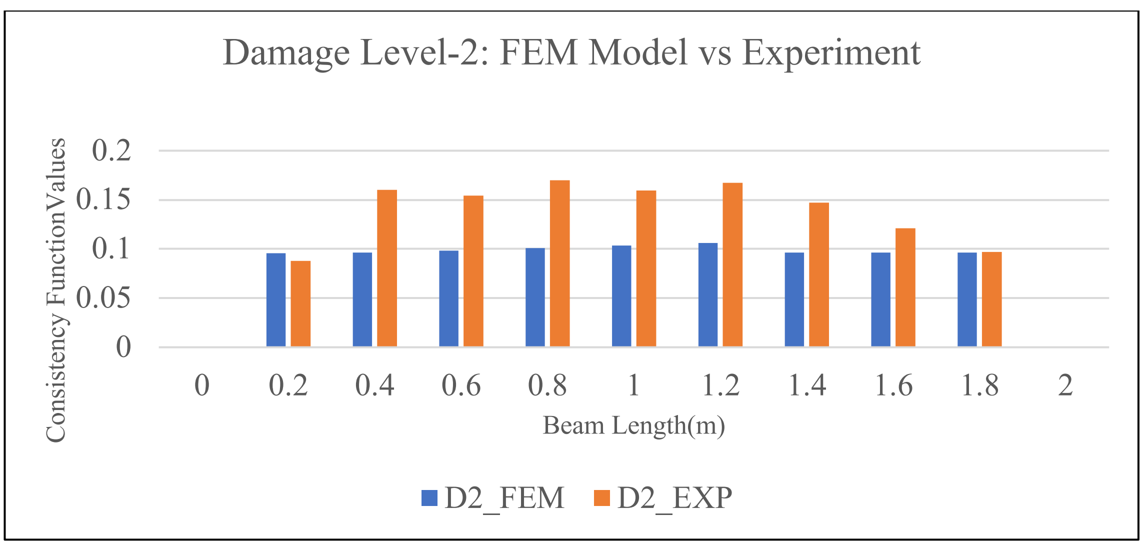

Figure 27: Case 2- Damage Severity comparison

Damage Level-3

Damage Localization

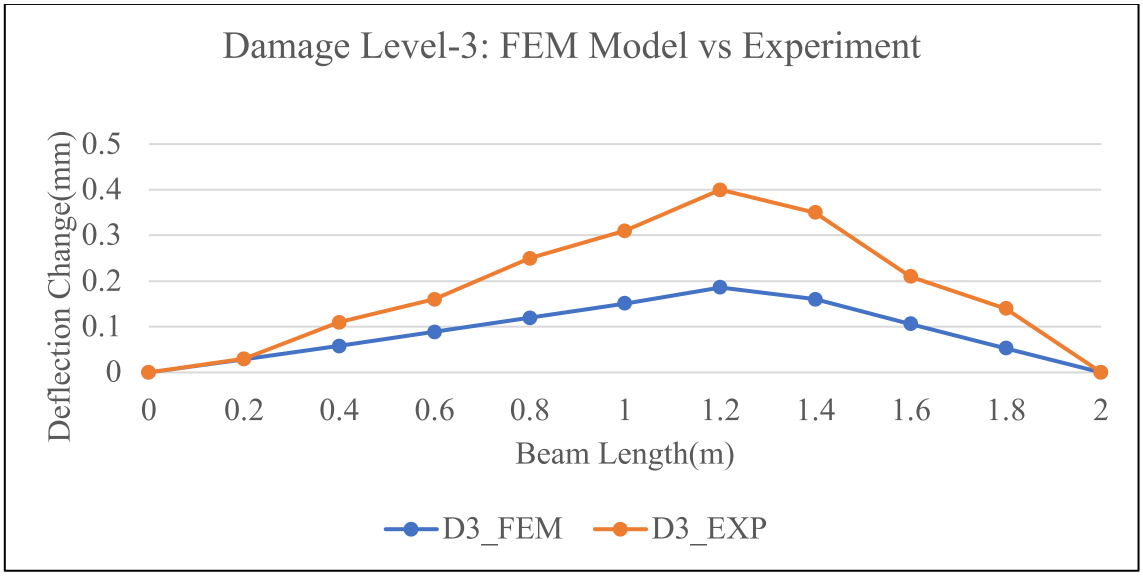

Figure 28: Case 3- Deflection Change comparison

Damage Severity

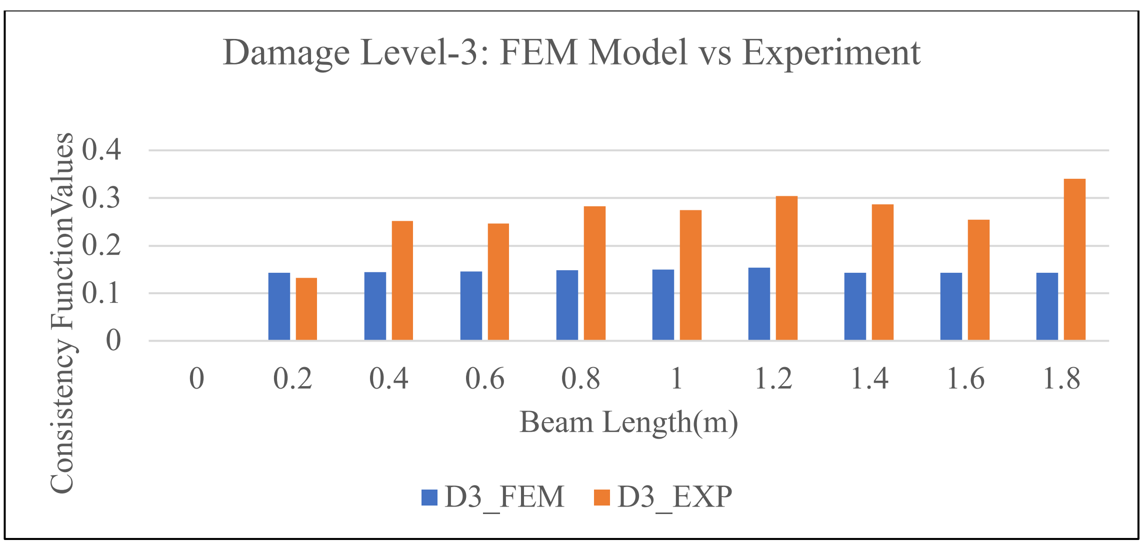

Figure 29: Case 3- Damage Severity comparison

Table 6: Comparison of experimental results with validation model

Case	**α from numerical model**	**α from experiment**	**Error**
1	**0.05988**	**0.05449**	**9.0%**
2	**0.0891**	**0.1231**	**38.1%**
3	**0.1270**	**0.2086**	**38.9%**

5.6. Conclusion

The experimental analysis and validated model comparison of the aluminium beam chosen for study demonstrates the proposed methodology and follows the developed equations. The damage is clearly identified between $x = 1.2\ m\ and\ x = 1.4\ m$. Also, the damage severity so assessed is good in case of early damage detection, i.e., when the damage given is less. But, for higher damage cases, the beam is assessed as being more damaged than what is assessed by the validating model. Overall, the method shows acceptable results in case of early damage detection. The errors so obtained may have arisen due to inaccuracies in the method of giving damage for which better approaches may be adopted.

6. Summary and Future Work

The project aimed to propose a simple method of early damage detection that can be used without much nuances to give an early estimate of the part of a beam that is damaged. As demonstrated using both numerical studies and laboratory experiment, the proposed method gives acceptable results in practical scenarios. The simplicity of the method in no way compromises on its utility to be developed into a more robust and accommodating structural health monitoring method. The case of single damaged as is proposed in the project can be extended to accommodate damage at multiple positions while using the same approach. Also, the proposed method is supported strongly by analytical model and equations which makes it free from using only specific set of parameters and hence the proposed method is a generalized one and not specific to one set of inputs. Therefore, the work can be extended for other types of beams like cantilever beams and even for indeterminate beams.

The proposed method in future developed form can find its use in damage analysis and structural health monitoring of real bridges and daily use beams. Also, due to its simplicity in usage, it does not require a very high set of skilled manpower and equipment which is one of the main plus points of the method. The whole proposal can also be developed into a software form that can process various combinations of input.

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Image Sources

Figure 1 https://s.hdnux.com/photos/57/36/23/12444870/5/940x0.jpg

Figure 2 https://ascelibrary.org/doi/abs/10.1061/%28ASCE%29EY.1943-7897.0000224

Appendix: Cantilever Beam under UDL

As the equation developed for Undamaged deflection, deflection change and RDC in the theory chapter for simply supported beam with UDL similar equations are developed for the cantilever beam under UDL.

The Undamaged beam deflection using Virtual Work method,

$$D_h = \frac{\int_0^L M(x_1).m(x_1)dx_1}{EI_x} = \frac{\int_0^L M(x_1).m(x_1)dx_1}{EI}$$

$$M(x_1) = wLx_1 - \frac{wx_1^2}{2} - \frac{wL^2}{2} \qquad 0 \leq x_1 \leq L$$

$$m(x_1) = x_1 - x \qquad 0 \leq x_1 \leq x$$

$$m(x_1) = 0 \qquad x < x_1 \leq L$$

Where *M(x)* is the bending moment of beam due to real uniformly distributed load and *m(x)* is the bending moment for virtual unit point load. After putting the values of different parameters in the equation and integration, we have,

$$D_h = \frac{1}{EI}\int_0^x \left(wLx_1 - \frac{wx_1^2}{2} - \frac{wL^2}{2}\right).(x_1 - x).dx_1 + \int_x^L (wLx_1 - \frac{wx_1^2}{2} - \frac{wL^2}{2}).(0).dx_1$$

$$D_h = \frac{1}{EI}\int_0^x \left(wLx_1^2 - \frac{wx_1^3}{2} - \frac{wL^2x_1}{2} - wLx_1x + \frac{wx_1^2x}{2} + \frac{wL^2x}{2}\right).dx_1$$

$$D_h = \frac{1}{EI}\left(\frac{wLx_1^3}{3} - \frac{wx_1^4}{8} - \frac{wL^2x_1^2}{4} - \frac{wLx.x_1^2}{2} + \frac{wx.x_1^3}{6} + \frac{wL^2x_1x}{2}\right)\Big|_0^x$$

$$D_h = \frac{wx^2}{24EI}(x^2 - 4xL + 6L^2)$$

Similarly, Deflection change between undamaged and damaged state is calculated for different damage position,

For $0 \leq x \leq a$

$$\Delta = 0 \tag{19}$$

For $a \leq x \leq a + b$

$$\Delta = \frac{\beta w}{24EI}\left((x^4) - (4x^3L) - (6x^2L^2) + x(12La^2 - 12L^2a - 4a^3) + (8L^2a^2 - 6La^3 + 3a^4)\right) \tag{20}$$

For $x \geq a + b$

$$\Delta = \frac{\beta w}{24EI}(\{(a+b)^3 - a^3\} - 12L\{(a+b)^2 - a^2\} + 12L^2b\,).x + (8L\{(a+b)^3 - a^3\} - 3\{(a+b)^4 - a^4\} - 6\{(a+b)^2 - a^2\}L^2) \tag{21}$$

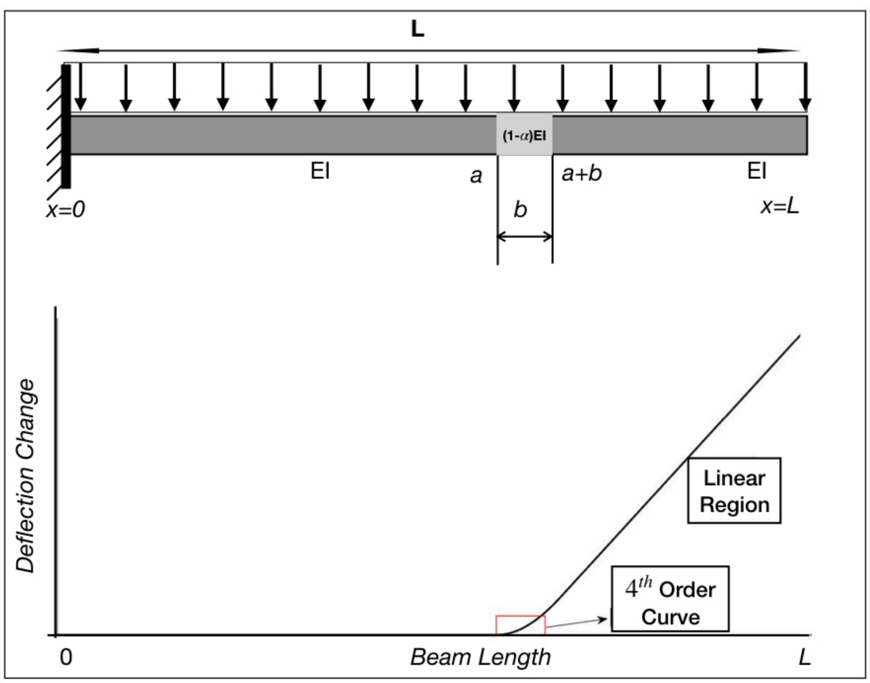

Figure 30 : Deflection Change Plot in Cantilever Beam under UDL

The deflection change (DC) equations , (19), (20), (21), when plotted against the length of the beam leads to the graph, as shown in Figure 30. The plot indicates that there will be no change in the deflection for $x \leq a$. Deflection change increases by a fourth order degree function with respect x in the damaged region $a < x \leq a + b$ and there is a linear portion for region $x > a + b$. From equation (12) it is known that,

$$RDC(x) = \frac{\Delta(x)}{D_h(x)}$$

Therefore,

For $0 \leq x \leq a$

$RDC = 0$

For $a \leq x \leq a + b$

$$RDC = \frac{\beta\big((x^4) - (4x^3L) - (6x^2L^2) + x(12La^2 - 12L^2a - 4a^3) + (8L^2a^2 - 6La^3 + 3a^4)\big)}{(x^4 - 4x^3L + 6L^2x^2)}$$

For $x \geq a + b$

$$RDC = \beta \frac{\begin{matrix}[(\{(a+b)^3 - a^3\} - 12L\{(a+b)^2 - a^2\} + 12L^2b\ ).x \\ +8L\{(a+b)^3 - a^3\} - 3\{(a+b)^4 - a^4\} - 6\{(a+b)^2 - a^2\}L^2]\end{matrix}}{x^4 - 4x^3L + 6L^2x^2}$$

C(f) function can be calculated from equation (16) and then damage severity is estimated from equation (18).