WINDOWED FBSE-EWT METHOD FOR NON-STATIONARY SIGNAL ANALYSIS

M.Tech Thesis

by

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DISCIPLINE OF ELECTRICAL ENGINEERING

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WINDOWED FBSE-EWT METHOD FOR NON-STATIONARY SIGNAL ANALYSIS

A THESIS

Submitted in partial fulfillment of the
requirements for the award of the degree
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RISHITA SHARMA

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **WINDOWED FBSE-EWT METHOD FOR NON-STATIONARY SIGNAL ANALYSIS** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DISCIPLINE OF ELECTRICAL ENGINEERING, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from July, 2019 to June, 2020 under the supervision of Dr. Ram Bilas Pachori, Professor, Discipline of Electrical Engineering, IIT Indore.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

Rishita
22/06/2020

Signature of the student with date
(RISHITA SHARMA)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Signature of the Supervisor

(Dr. RAM BILAS PACHORI)

RISHITA SHARMA has successfully given her M.Tech. Oral Examination held on **22/06/2020**.

Signature of Supervisor of M.Tech. thesis
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Date: 22/06/2020

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ABSTRACT

Time-frequency (TF) analysis is an active research area in the field of signal processing. It has various methodologies to generate TF representation like short time Fourier transform (STFT), Hilbert-Huang transform (HHT), wavelet transform (WT), Wigner-Ville distribution (WVD), empirical wavelet transform (EWT), etc. which help in getting better knowledge about a signal.

In this thesis, we discuss windowed Fourier-Bessel series expansion based empirical wavelet transform (WFBSE- EWT) method for analysis of non-stationary signals which has been developed by enhancing the existing Fourier-Bessel series expansion based empirical wavelet transform (FBSE- EWT) method. It is obtained by segmenting the signal in time domain by using windows such as Gaussian, Hann, Chebyshev and Hamming along with 50% overrun, applying FBSE-EWT on individual segments, adding the resulting intrinsic mode functions (IMFs) and obtaining the TF representation by applying Hilbert transform (HT). This gives us better TF representation as compared to the existing method.

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TF	Time-frequency
STFT	Short-time Fourier transform
FBSE	Fourier-Bessel series expansion
EWT	Empirical wavelet transform
DWT	Discrete wavelet transform
EMD	Empirical mode decomposition
CWT	Continuous wavelet transform
IMF	Intrinsic mode function
FT	Fourier transform
WVD	Wigner-Ville distribution
WT	Wavelet transform
IA	Instantaneous amplitude
IF	Instantaneous frequency
MSE	Mean square error
EEG	Electroencephalogram
SB	Sub-band
HHT	Hilbert-Huang transform
HT	Hilbert transform
HSA	Hilbert spectral analysis
AM	Amplitude modulation
FM	Frequency modulation
TQWT	Tunable Q-wavelet transform

NHT	Normalized Hilbert transform
VMD	Variational mode decomposition
AR	Autoregressive
PPG	Photoplethysmogram
SAE	Sparse autoencoder
WPT	Wavelet packet transform
SVM	Support vector machine
DFT	Discrete Fourier transform
WFBSE-EWT	Windowed Fourier-Bessel series expansion based empirical wavelet transform

Chapter 1

Introduction

The life of a human being is surrounded by signals. A signal is a function that conveys information about a phenomenon. Signal processing is an operation which is applied on a signal in some fashion so that some useful information can be extracted. Signal processing is a subfield of engineering, in particular electrical engineering that focuses on study, alteration, and synthesis of signals such as sound, images, etc.

1.1 Background

1.1.1 The nature of non-stationary signals

Signals can be classified and categorized into various types, given below.

- Continuous and discrete signals
- Periodic and aperiodic signals
- Power and energy signals
- Analog and digital signals
- Stationary and non-stationary signals and many more.

Most real-life signals are essentially non-stationary, such as speech signals, biological signals, civil structure vibration signals, etc. They have their statistics changing with time.

1.1.2 The need for time-frequency (TF) analysis

In Fourier analysis, we assume that signals are infinite in time or of periodic nature, but in reality most of the signals are of small duration and significantly change over the course of time. Fourier analysis analyzes the frequency content of the signal without any details about time. This shortcoming leads to the need for TF analysis.

TF techniques show change in the frequency components of the signal with respect to time, providing a more revealing picture of the time location of the spectral components of the signal.

Ideally, the TF distribution function should have the following properties:

- For both the time domain and frequency domain, it should have high resolution.
- No cross terms.
- It should have lower computational complexity to make sure that it gets appropriate time to process a signal and represent it on the TF plane.

1.1.3 Existing techniques

Various existing techniques for TF analysis of non-stationary signals are: Wigner-Ville distribution (WVD) [8], short-time Fourier transform (STFT) [5,6], wavelet transform (WT) [7], tunable Q-wavelet transform (TQWT) [15], empirical mode decomposition (EMD) and Hilbert-Huang transform (HHT) [9], empirical wavelet transform (EWT) [24], etc.

STFT works by moving window on the signal and obtaining corresponding Fourier transform (FT). But, the STFT also has its drawbacks, like its TF resolution capability is limited due to the uncertainty principle.

STFT has TF localization, which is spaced equally. Another problem with STFT is how to decide the type of window needed and its size too. In order to overcome these drawbacks, WT came into the picture. WT can be both discrete wavelet transform (DWT) and continuous wavelet transform (CWT). WT due to the prefixed nature of the filter banks, they fail to disintegrate the signals correctly. Therefore, the TF representation achieved is not in accordance with the exact information content present in the signal. The wavelet packet transform (WPT) [11, 12], was the enhanced version of the WT introduced so as to increase the signal adaptability. Although, this way was also confined by the fact that it uses a prefixed basis. In [15], TQWT is explained. In TQWT, Q factor can be tuned in conformity to the oscillatory behaviour of the input. The results were found better than that of WT.

All these methods use prefixed basis functions. Hence, they are non-adaptive and rigid in nature. The EMD method decomposes the signal adaptively into amplitude modulation – frequency modulation (AM-FM) components. These components are referred to as intrinsic mode functions (IMFs). In [16], HHT has been expressed as the sum of EMD and Hilbert spectral analysis (HSA). First, EMD decomposes the signal in IMFs, and then Hilbert transform (HT) estimates instantaneous frequency (IF) and instantaneous amplitude (IA) with the help of the IMFs. Further, TF representation is given. Although, EMD faces various drawbacks such as mode mixing, no exact mathematical model, noise sensitivity, etc. The variational mode decomposition (VMD), decomposition of input in a limited number of components is done. When compared to EMD, VMD has proved to be more precise in the separation of any pair of harmonics, irrespective of the closeness of frequencies and relative amplitudes. In [18, 19, 20], the authors proposed the representation by enhanced eigen value decomposition of the Hankel matrix in collaboration with HT. Although, the WVD gives sound localization in both time and frequency, but has its limits due to the presence of cross-terms in components of the signal.

In [23], EWT was suggested for analyzing the non-stationary signals. It is an adaptive decomposition technique. FT spectrum is segregated using scale-space method followed by the designing of the corresponding wavelet filter banks [24]. We will go through EWT in detail in the next chapter.

In [75], Fourier-Bessel series expansion based empirical wavelet transform (FBSE-EWT) gives a better estimate of instantaneous frequency for short-duration signals by replacing the Fourier spectrum to the Fourier Bessel series expansion (FBSE) spectrum.

1.2 Motivation

On comparing FBSE- EWT technique with HHT and EWT for TF representation, FBSE- EWT technique provides better resolution i.e. it can represent the frequency components that are close enough in TF plane, leading to superior TF representation. But, for some non-stationary signals which get easily separated in the time axis but cannot be separated that easily in the frequency axis, FBSE- EWT method fails to give a good TF representation. Therefore, an enhanced method is beneficial in order to meet the aforesaid drawbacks of the existing method. The signal is separated into small time intervals with the help of window function and FBSE- EWT is applied on each segment. By summing all these components, HT based TF representation is taken.

The new suggested method has been applied on bat echo signal as well as on a synthetic linear frequency modulated signal. Using our proposed method, we have got a better TF representation.

1.3 Organization of the thesis

The further portions of this report are organized in the following manner:

1. Chapter 2 provides a detailed description of EWT and FBSE-EWT, along with their properties and drawbacks. HT is also discussed.
2. In Chapter 3, windowed Fourier-Bessel series expansion based empirical wavelet transform (WFBSE-EWT) method has been explained. The performance evaluation based on mean square error (MSE) has also been explained.
3. Chapter 4 presents results, discussion where comparison of our method is done with the existing techniques.
4. Chapter 5 presents the conclusion of whole work. The directions for future research work are also provided in this chapter.

1.4 Summary

In this chapter, we have discussed the nature of real-life signals i.e. non-stationary nature. Further, we have discussed how the need for TF analysis came into the picture and gave us a better view to analyze any signal. The existing techniques for TF representation are also explained. Our goal is to get a better TF representation.

Chapter 2

EWT, FBSE-EWT and HT

Methods such as EMD [16] are used to decompose signals in different modes. Although it finds many applications, still it lacks mathematical theory. EMD faces various drawbacks such as modes getting mixed, boundary effect, stop criteria, etc. This leads to the arising of a new WT called EWT. The idea behind this method is extracting different modes from the signal and designing a suitable wavelet filter bank. Scale-space based approach is used for boundary detection.

Windowing is applied in order to segment the signal in various sub-signals. FBSE spectrum is used for the optimal boundary selection. Further, filter banks based on the selected boundaries have been constructed.

Bessel functions being damped in nature are used as the basis functions in FBSE, this property makes FBSE more competent for non-stationary signal analysis.

HT used to derive the TF representation from the IMFs.

2.1 EWT

TF analysis comprises of techniques through which we are able to study in both the time and frequency domain simultaneously. EWT is one such method.

The method is proposed by Jerome Gilles. Decomposing a signal into wavelet tight frames is the key objective of EWT. These frames are built adaptively. In [23], the process of EWT on one-dimensional signal has been explained. It has been further extended for two-dimensional signal [27]. Fig 2.1 represents the block diagram of EWT.

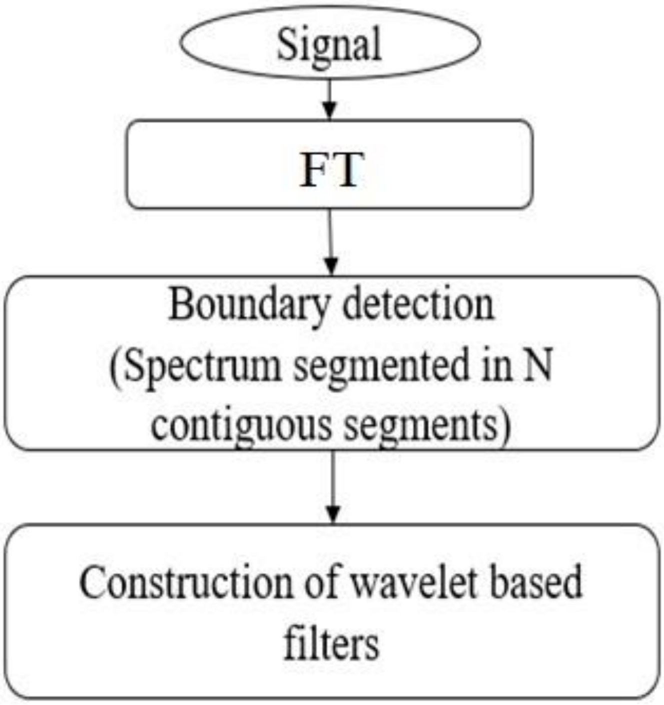


Figure 2.1 Block diagram of EWT

The process involved in EWT is explained below [23]:

- FT of the signal is taken in order to obtain the frequency spectrum.
- With the help of scale-space boundary detection method, segmentation of the obtained spectrum has been done. It has been segregated into N parts. Boundary frequencies are denoted by ω_i. These frequencies are bounded in [0,π].
- The wavelet and scaling functions are specified as the set of band-pass filters for each part. Meyer's wavelets and Littlewood Paley wavelets are used for the construction of filters [30, 57].

The scaling function $\Delta_i(\omega)$ is given by,

$$\Delta_i(\omega) = \begin{cases} \cos\left(\frac{\Pi\eta(\mu,\omega_i)}{2}\right) & , \text{if } (1-\mu\omega_i) \le |\omega| \le (1+\mu\omega_i) \\ 1 & , \text{if } |\omega| \le (1-\mu\omega_i) \\ 0 & , \quad \text{otherwise} \end{cases} \quad \text{..............................2.1}$$

The wavelet function $\Omega_i(\omega)$ is given by,

$$\Omega_i(\omega) = \begin{cases} 1 & , \text{if } (1+\mu\omega_i) \le |\omega| \le (1-\mu\omega_{i+1}) \\ \cos\left(\dfrac{\Pi\eta(\mu,\omega_{i+1})}{2}\right) & , \text{if } (1-\mu\omega_{i+1}) \le |\omega| \le (1+\mu\omega_{i+1}) \\ \sin\left(\dfrac{\Pi\eta(\mu,\omega_i)}{2}\right) & , \text{if } (1-\mu\omega_i) \le |\omega| \le (1+\mu\omega_i) \\ 0 & , \text{otherwise} \end{cases},$$

…………………………2.2

where,

$$\eta(\mu,\omega_i) = \emptyset\left(\frac{(|\omega| - (1-\mu)\omega_i)}{2\mu\omega_i}\right)$$

……………………………2.3

and arbitrary function $\emptyset(h)$ is given by,

$$\emptyset(h) = \begin{cases} \emptyset(h) + \emptyset(h+1) = 1, \text{if } h \in [0,1] \\ 1 & , \text{if } h \le 0 \\ 0 & , \text{if } h \ge 1 \end{cases}$$

……………………………2.4

The tight frame condition is given as follows,

$$\mu < \min_i \left(\frac{\omega_{i+1} - \omega_i}{\omega_{i+1} + \omega_i}\right)$$

……………………………2.5

The detail coefficients of i^{th} oscillatory level $X_{z,\Theta}(i,t)$ and the approximation coefficients $X_{z,\Delta}(0,t)$ are,

$$X_{z,\Theta}(i,t) = \int z(\alpha)\overline{\Theta_i(\alpha - t)}\, d\alpha$$

…………………………2.6

$$X_{z,\Delta}(0,t) = \int z(\alpha)\overline{\Delta_1(\alpha - t)}\, d\alpha$$

…………………………..2.7

Approximate sub-band (SB) signal $a_0(t)$ and detail SB signal $a_i(t)$ of i^{th} level is given by,

$$a_0(t) = \Delta_1(t) \star X_{z,\Delta}(0,t)$$

…………………………..2.8

$$a_i(t) = \Theta_i(t) \star X_{z,\Theta}(i,t)$$

…………………………..2.9

In [25], based on EWT method identification of the type of emotion a person is going through using electroencephalogram (EEG) signals has been demonstrated. For extracting the features, it uses two selected channels during a specific time segment. EEG signal is decomposed in various modes using EWT. On the basis of the selected modes, the calculation of autoregressive (AR) coefficients is done. These features form a feature vector, and further, they are input to the classifier for performing emotion identification.

In [26], decomposition of non-stationary bridge vibration signals using the EWT technique has been done.

Hence, we can say that EWT is a new technique that builds adaptive wavelets, and these wavelets have the ability to extract AM-FM components of the data. The primary concept behind this is that these AM-FM components have a close-packed support spectrum derived by FT. Retrieving various modes means segmenting the Fourier spectrum and applying filtering with respect to each support that has been detected. Through these, a set of functions are built that forms an orthonormal basis. It is a fast method too.

2.1.1 Advantages of EWT

Various advantages of EWT are given below:

- Calculations are non-recursive. EMD has highly recursive calculations that are time-consuming. If we compare it to EMD, more consistent decomposition is being provided by EWT.
- Robust against background noise as compared to the non-adaptive techniques such as STFT.
- Modes derived by it are narrow banded functions with fewer mixed modes.

2.1.2 Disadvantages of EWT

The disadvantages of EWT become the base for FBSE-EWT.

- It has a lesser frequency resolution compared to FBSE-EWT.

- In some cases, such as if the signal is formed of two chirps that are overlapping in both the domains, time as well as frequency, then the EWT fails to separate them.

2.2 Scale-space method for boundary detection

By convolving the signal with the Gaussian kernel, scale-space representation is computed. For calculating this for a discrete signal, the formula given in [27] as,

$$g(h, m) = \sum_{n=-P}^{P} f(h-n) \quad x(n; m)$$

…………………………………2.10

Where, $x(n; t) = \frac{1}{\sqrt{2\pi m}} e^{\frac{-n^2}{2m}}$, $P = K\sqrt{m} + 1$ and m is a scale parameter with 1≤ K≤ 3. The scale-space parameter plays the following role:

If the scale-space parameter l is increased, the number of minima in the representation will decrease, and no new minima will appear. Scale-space parameter l, in terms of scale parameter m is given below as [8],

$$l = \sqrt{\frac{m}{m_o}}, l = 1,2, \dots, l_{maximum}$$

…………………………………2.11

Let, m_o represent the overall number of initial minima. Thus, in scale-space plane, each of the initial minima M_i guides to a curve D_i . If Q_i is the lifetime of curve D_i , where i=1, 2, 3, 4, 5, . . . M and this lifetime of curve is expressed as,

$$Q_i = \text{maximum}\left\{\frac{l}{\text{the } i^{th} \text{ minimum exists}}\right\}$$

…………………………..2.12

In the histogram, modes are elucidated by the following way: A significant mode should be limited by two local minima, which will further lead to two long scale-space curves D_i . Their values should be considerably more than the threshold. Hence, the most appropriate threshold (T_h) should be estimated for choosing the scale-space curves of length significantly more than the threshold value [28].

Due to the non-availability of any prior knowledge of which scales to be used in a particular task, the only solution can be the representation of the input data signal at multiple scales. This becomes the base of the idea of scale-space representation.

The motivation to generate a scale-space representation of a signal under analysis has originated from a very basic consideration that the objects of real-world are made up of different structures along with that on different scales. It implies that real-world things, in contradictory to the ideal mathematical body, for example, dots or lines, may look different depending upon their different observation scale. Considering an example, the concept of a tree is precise at the scale of meters, whereas the concepts such as flowers, fruits, buds, etc. are more precise at smaller scales as compared to that of a tree. Similarly, for a computer vision system when a new scene appears in front of it, and it is having no prior information related to it. It has no idea that at which scale it should determine the shape of the unknown. Therefore, the only reasonable approach can be to collect the information on various scales so that it becomes easy to understand the areas where scale variations are in occurrence. This makes it clear that scale-space representation takes into consideration representation at various scales.

Hence, in other words, we can consider this theory of scale-space representation to be a framework for representation on multiple scales. It is highly used in signal and image processing. It represents the image as a single-parameter family of the smoothened images. The suppression of fine-scale structures is done with the help of a parameter, the size of the smoothing kernel.

After this, Otsu's method [29], has been utilized to determine the value of T_h and modes have been derived. Otsu’s method has been explained as the next topic below.

2.3 Otsu's threshold detection method

The Otsu's method takes its name from Nobuyuki Otsu. This method is intended to automated thresholding. This algorithm returns a threshold of a particular intensity level separating the pixels into two different classes: foreground and background, and has a bimodal histogram displaying two peaks. The value of threshold is estimated by either maximizing intra-class intensity variance, or by minimizing inter-class variance [30].

It is effective in low signal to noise and low contrast conditions too.

In a detailed manner, this method probe for the threshold that can minimize the intra-class variance. It is expressed as the weighted sum of variances of both the classes, as follows:

$$\sigma_c^2\,(p) = c_0(p)\sigma_0^2(p) + c_1(p)\sigma_1^2(p)$$

……………………………………2.13

Where, weights c_0 and c_1 are the probabilities of the two classes foreground and background separated by a threshold p, and σ_0^2 andσ_1^2 are the variances of both classes. They are expressed as follows:

$$c_0(p) = \sum_{j=0}^{p-1} f(j)$$

……………………………………2.14

$$c_1(p) = \sum_{j=t}^{K-1} f(j)$$

……………………………………2.15

For these two classes, minimizing the intra-class variance is equivalent to maximizing inter-class variance as,

$$\sigma_b^2(p) = \sigma^2 - \sigma_c^2(p) = c_0(\mu_0 - \mu_T)^2 + c_1(\mu_1 - \mu_T)^2$$
$$= c_0(p)c_1(p)[\mu_0(p) - \mu_1(p)]^2$$

……………………………………2.16

Where, μ is the class mean and c being the class probability. Class means $\mu_0(p)$, $\mu_1(p)$, and μ_T are given as,

$$\mu_0(p) = \frac{\sum_{i=0}^{p-1} i\, f(i)}{c_0(t)}$$

……………………………………2.17

$$\mu_1(p) = \frac{\sum_{i=t}^{K-1} i\, f(i)}{c_1(t)}$$

……………………………………2.18

$$\mu_T = \sum_{i=0}^{K-1} i\, f(i)$$

……………………………………..2.19

These relations can be verified as:

$$c_0\mu_0 + c_1\mu_1 = \mu_T$$

…………………………………….. 2.20

$$c_0 + c_1 = 1$$

………………………………………2.21

These probabilities and means of the class will be evaluated repetitively. In steps, Otsu's method can be given as,

1. Probabilities and histogram of each and every intensity value is computed.
2. Initial values of class probability and mean is set.
3. Traversing through all the thresholds from minimum to the maximum intensity, keep updating the class probability and mean. Simultaneously, keep computing the inter-class variance.
4. The required threshold will be the one which will correspond to the maximum inter-class variance.

2.4 FBSE-EWT

2.4.1 FBSE

FBSE of function f(t) is expressed mathematically as,

$$f(t) = \sum_{i=1}^{N} C_i J_0\left(\frac{\beta_i t}{N}\right), t = 0,1, \dots\dots, N-1$$

……………………………………..2.22

It makes the use of Bessel functions of zero order.

where, C_i are FBSE coefficients of $f(t)$, expressed as follows:

$$C_i = \frac{2}{N^2\left(J_1(\beta_i)\right)^2} \sum_{i=0}^{M-1} t\, x(t) J_0\left(\frac{\beta_i t}{N}\right)$$

……………………………2.23

$J_0(.)$ and $J_1(.)$ denote the Bessel functions of order zero and one, respectively.

The positive roots of the Bessel function are represented by β_i for i = 1, 2, 3, 4, . . . N. Here, order i is corresponding to the continuous-time frequency f_i [4,5], and is related as follows:

$$\beta_i \approx \frac{2\pi f_i N}{f_s}$$

……………………………2.24

$$i \approx \frac{2 f_i N}{f_s}$$

……………………………2.25

where, $\beta_i \approx \beta_{i-1} + \pi \approx i\,\pi$ and f_s is the sampling frequency.

2.4.2 FBSE- EWT method

In [2], a new methodology based on FBSE-EWT came into the picture. It is particularly targeted for the analysis of non- stationary signals.

The non-stationary nature of Bessel functions are of great use to this method. The advantage lies in the point that in Fourier representation, the length of the spectrum is half of the signal length in contrast to FBSE, providing the same length of both. This makes the frequency resolution double for FBSE.

In [3], using FBSE- EWT heart and respiratory rates from photoplethysmogram (PPG) signals have been derived. In [31], with the help of sparse autoencoder with support vector machine (SAE-SVM) network and FBSE-EWT rhythm identification of focal seizure area has been discussed. In this, use of EEG signal has been done. Block diagram of FBSE- EWT is given in fig 2.2.

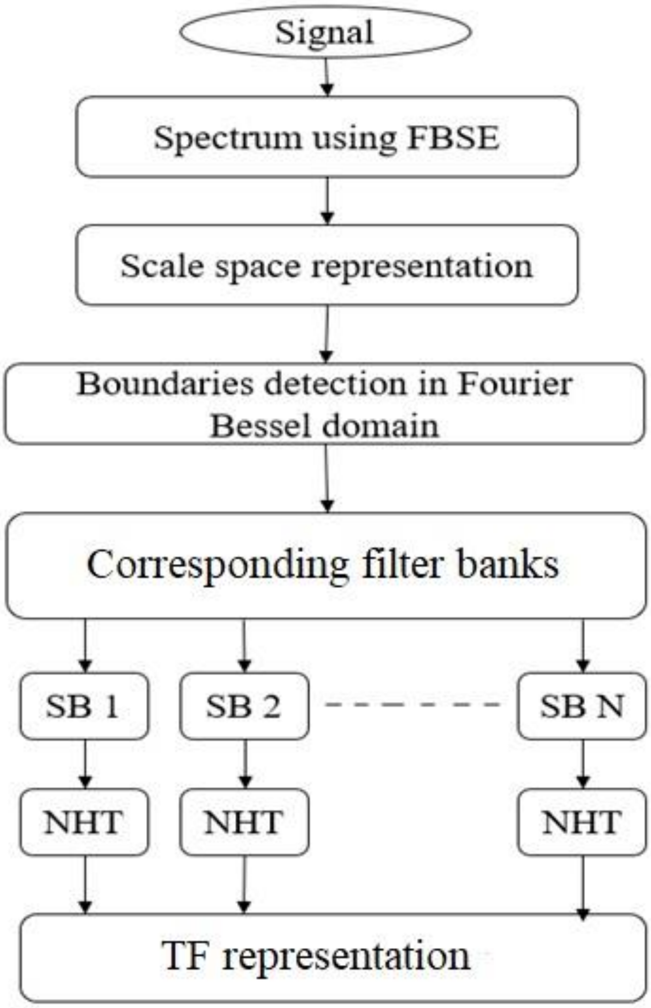


Figure 2.2 Block diagram of FBSE- EWT

For finding meaning modes, scale-space representation is used through which boundaries are detected and the FBSE spectrum is segmented, which results in improved EWT based filter bank and SB signals are obtained. By applying the normalized Hilbert transform (NHT), TF representation is obtained.

2.5 HT

For getting the TF representation, the idea of HT has been used in this work. It works well on signals with non-stationary and non- linear nature. It is like an algorithm. It is used to decompose a signal into IMFs. Each and every component will have its HT defined as follows:

$$h_j(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{c_i(\alpha)}{t-\alpha} d\alpha$$

…………………………….2.26

The analytic signal z(t) is given by,

$$z(t) = c(t) + ih(t) = b(t)e^{i\Omega(t)}$$

……………………………2.27

Where, IA b(t), phase function Ω and IF ω are expressed as,

$$b(t) = \sqrt{c^2 + h^2}$$

…………………………….2.28

$$\Omega(t) = \arctan(h/c)$$

…………………………….2.29

$$\omega = d\Omega \,/\, d\,t$$

…………………………….2.30

After the application of HT on every component, the actual signal can be expressed as follows:

$$c(t) = R\left\{\sum_{j=1}^{n} b_i(t) \exp\left[j \int \omega_i(t) d\,t\right]\right\}$$

………………………………2.31

where, R denotes the real part.

We will apply the HT to each filter output in order to get the TF representation of the WFBSE- EWT.

HT finds usage in various other fields such as medical imaging, system response analysis, sampling of narrowband signals in telecommunication, array processing for direction of arrival, etc.

2.6 Window function

A window is a function that has zero value outside a particular time interval. The window function is also known as tampering function and apodization function. Mostly, it is symmetric around the middle of the interval. It can be multiplied to any signal to get the view of a particular time interval that comes inside the window. The window can be of different shapes such as bell-shaped, rectangular, triangular, or any other function depending upon the need of the problem.

In signal processing, it finds multiple applications in the fields such as spectral analysis, designing of finite impulse filters, and other applications such as antenna design, etc.

Various windows [82] that can be used for the purpose of analysis can be:

2.6.1 Gaussian window

The Gaussian window is bell-shaped with equal probabilities both sides about the mean. It extends till infinity. Its FT is also Gaussian in nature. With standard deviation σ and sampling periods M/2, it can be expressed as,

$$f[n] = \exp\left(-\frac{1}{2}\left(\frac{n - M/2}{\sigma M/2}\right)^2\right), \quad 0 \leq n \leq M$$

………………………….2.32

The plot of Gaussian window is given in fig 2.3.

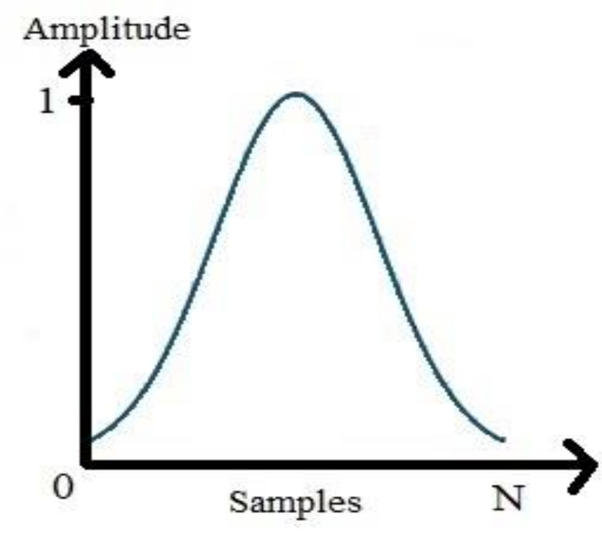


Figure 2.3 Gaussian window

2.6.2 Rectangular window

Also known as Dirichlet or Boxcar window, it is one of the simplest windows. The rectangular window can be viewed as a series of M values equal to 1 and except that all the values being 0. In the form of an equation, it is expressed as follows:

$$f[n]=1$$

…………………………2.33

The plot of rectangular window is given in fig 2.4.

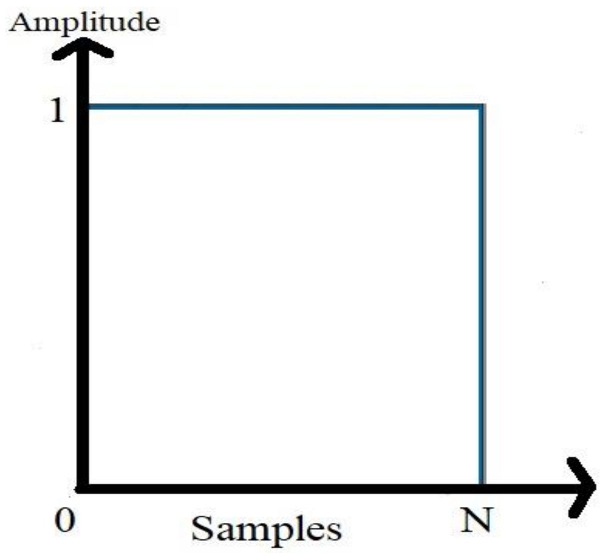


Figure 2.4 Rectangular window

2.6.3 Hamming window

It was named behind R. W. Hamming, an associate of J. W. Tukey. It uses raised cosine. It has non-zero ends. In the time domain, it smoothens the autocovariance function. It is also known as an apodization or tapering function.

$$f(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{W-1}\right) \quad 0 \le n \le W-1$$

……………………………..2.34

where, a_0= 0.54 and a_1=0.46.

The plot of Hamming window is given in fig 2.5.

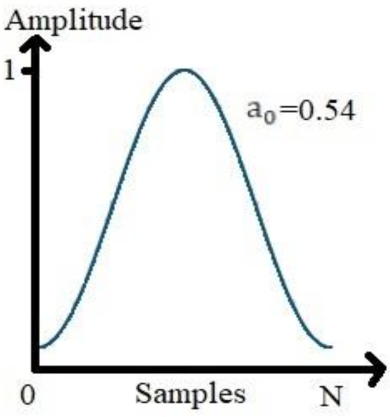


Figure 2.5 Hamming window

2.6.4 Chebyshev window

Chebyshev window is also known as Dolph-Chebyshev and Dolph window. The work of this window is that for a provided width of the main lobe, it will reduce side lobe's Chebyshev norm. Discrete Fourier transform (DFT), $F_0(k)$ of the function $f_0(n)$ is expressed as,

$$F_0(k) = \frac{X_N\left(\beta \cos\left(\frac{\pi k}{N+1}\right)\right)}{X_N(\beta)}.$$

$$= \frac{X_N\left(\beta \cos\left(\frac{\pi k}{N+1}\right)\right)}{10^{\alpha}}, 0 \leq k \leq N$$

……………………………..2.35

$F_0(k)$ is real valued.

l-th polynomial $X_l(p)$ is given by,

$$X_l(p) = \begin{cases} \cos(l\cos^{-1}(p)) & , \text{if } -1 \leq p \leq 1 \\ \cosh(l\cosh^{-1}(p)) & , \text{if } p \geq 1 \\ (-1)^l \cosh(l\cosh^{-1}(p)) & , \text{if } p \leq -1 \end{cases}$$

………………………………2.36

and

$$\beta = \cosh\left(\frac{1}{N}\cosh^{-1}(10^{\alpha})\right)$$

………………………………2.37

The plot of Chebyshev window is shown in fig 2.6.

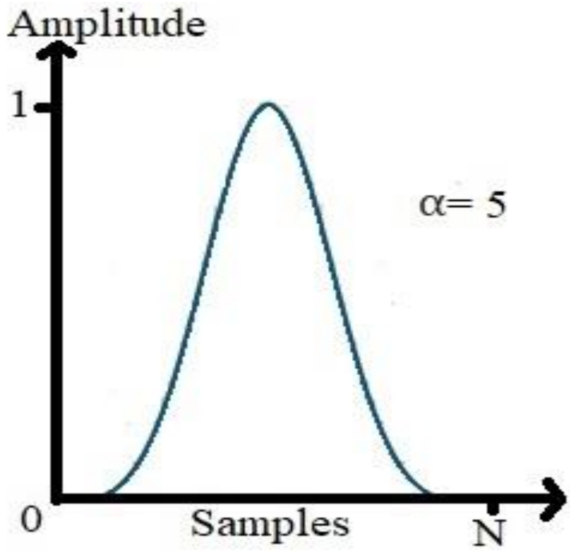


Figure 2.6 Chebyshev window

2.6.5 Hann window

It is also known as Hann filter, von Hann window, etc. The Hann function of length L used to perform Hann smoothing, the name of this window has been given behind Julius von Hann, an Austrian meteorologist. For performing Hann smoothing, we use a function of L length. The function of the window is given by,

$$f(g) = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi g}{L}\right)\right) \quad , |g| \leq (L/2)$$

$$= 0 \quad , \text{otherwise}$$

………………………………..2.38

$$f_0(n) = f\left(\frac{L}{N}\left(n - \frac{N}{2}\right)\right)$$

$$f_0(n) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N}\right)\right] \quad , 0 \leq |n| \leq N$$

…………………………….2.39

N can take either odd or even value. The plot of Hann window is shown in fig 2.7.

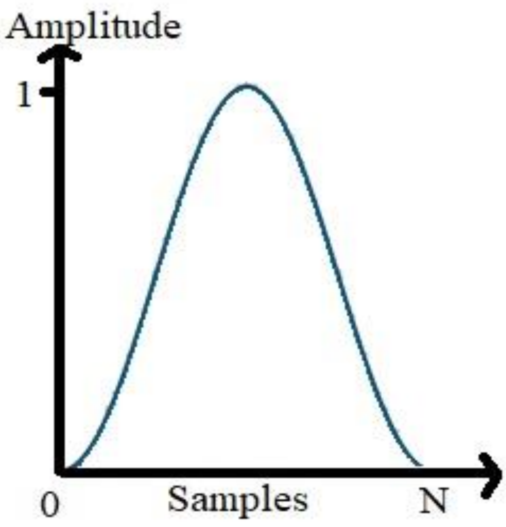


Figure 2.7 Hann window

Hann window can also be defined as a continuous combination of rectangular windows. The difference between the Hann window and the Hamming window is that the Hann window touches zero on both the sides, whereas the Hamming window doesn’t touch zero exactly. It ends slightly before zero.

Hann removes the discontinuity, whereas Hamming has a slight amount of discontinuity left.

In this way, we have used various windows for segmenting the signal and for its analysis in the further steps.

2.7 Summary

In this chapter, we have discussed EWT with its advantages and disadvantages, boundary detection using scale-space representation, Otsu’s thresholding method. In EWT, the empirical scaling functions and wavelet functions have been designed as band-pass filters, and Meyer's wavelets and Littlewood Paley wavelets have been used to design these filters.

After this, we have discussed the FBSE- EWT method, HT, and various types of window functions.

Chapter 3

Proposed Method

3.1 Proposed method WFBSE-EWT

We have proposed a method for analyzing non-stationary signals and named it as WFBSE-EWT. We have considered a bat echo signal and a synthetic linear frequency modulated signal for that purpose. Fig 3.1 depicts the block diagram of the proposed method.

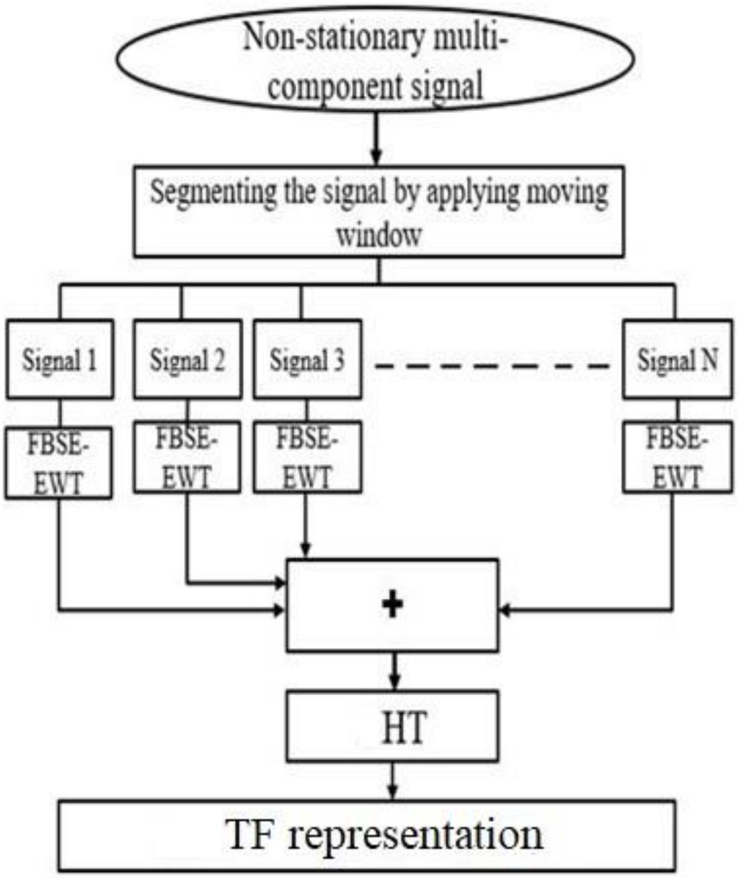


Figure 3.1 Block diagram of proposed method

In fig 3.1, + indicates the concatenation process of the obtained components.

The process proceeds in the following manner:

The signal is segmented in equal parts using a moving window concept. A window is a function that has zero value outside a particular time interval. A window function is also known as tampering function and apodization function. Various windows used are Gaussian window, Hann window, Chebyshev window and Hamming window. As this process of windowing causes disturbances and discontinuities on both ends, we have done an overlapping of 50% while performing the segmentation of the signal in the time domain. For the sub-signals obtained, FBSE-EWT is applied for each sub-signal. It refers to obtaining the FBSE spectrum of individual sub-signal. Scale-space based boundary detection approach applied to find meaningful boundaries, which results in design of improved EWT based filter bank and finally, SB signals are obtained using filter bank.

The IMFs are obtained for each segment separately. Further, the IMFs are concatenated, and then HT is applied to get the TF representation of the signal.

3.2 Performance evaluation

For evaluating the performance of the TF representation, we compute MSE between the expected and the proposed method's TF representation. The MSE is expressed by,

$$\mathrm{MSE} = \frac{1}{\mathrm{RS}} \sum_{f=1}^{R} \sum_{t=1}^{S} \left(\mathrm{TF}_2(f,t) - \mathrm{TF}_1(f,t) \right)^2$$

……………………………..3.1

where, R- total frequency points.

S - total time instants.

and TF_2 and TF_1 represent the expected and the proposed method's TF representations, respectively.

3.3 Summary

In this chapter, we have discussed the proposed method to analyze non-stationary signal using WFBSE-EWT method. The aim is to get a better TF representation. The experimental results of these methods are shown in the next chapter. This can be observed/viewed that the proposed method gives preferably better results than the other approaches discussed in Chapter 2.

Chapter 4

Simulation results

The performance of the proposed method has been evaluated by applying it to two non-stationary signals. One signal is a real signal, and the other one is a synthetic signal. For the real signal bat echo signal has been used whereas, on the other hand a synthetic frequency modulated signal has also been used.

The results are obtained by applying different types of windows. Further, the results are compared by the existing FBSE- EWT method.

4.1 Bat echo signal

This bat chirp has been taken from a large brown bat called Eptesicus fuscus from [81]. The duration of the signal is 2.5 milliseconds, with the sampling period of 7 microseconds. Fig 4.1 shows the plot of bat-echo signal.

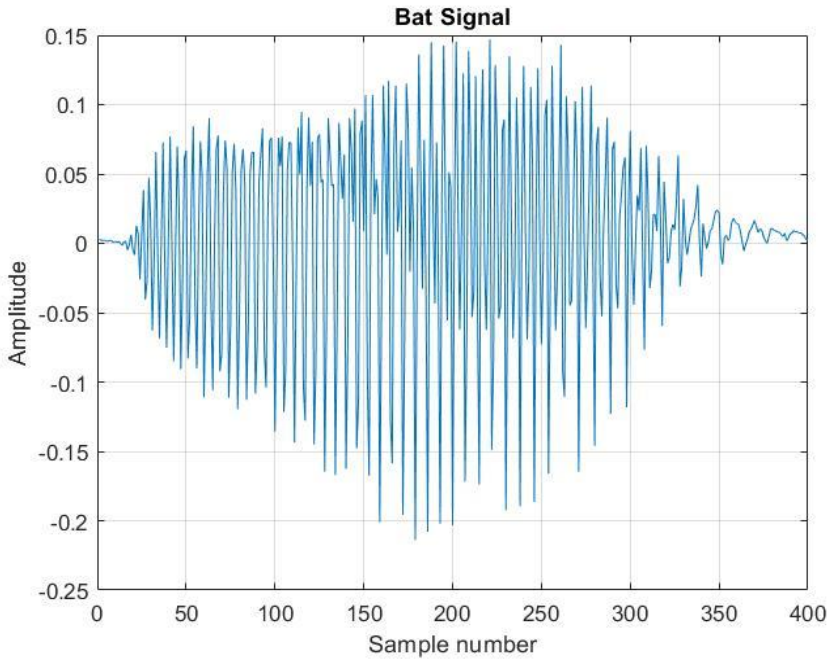


Figure 4.1 Bat-echo signal

The boundaries detected and the corresponding filter banks can be given as in fig 4.2 (a) and 4.2 (b).

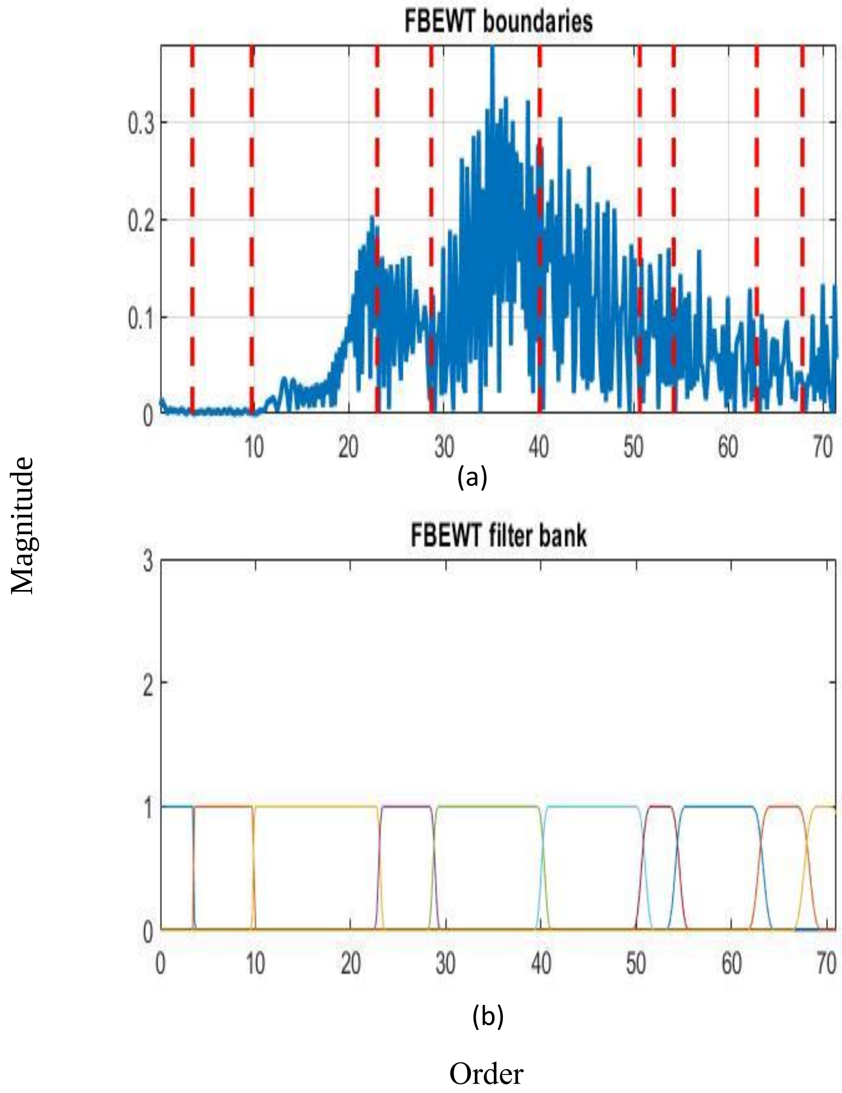


Figure 4.2 (a) Boundaries obtained by FBSE-EWT for bat signal and (b) corresponding filter banks.

The IMFs for the FBSE-EWT of bat signal are shown in fig 4.3.

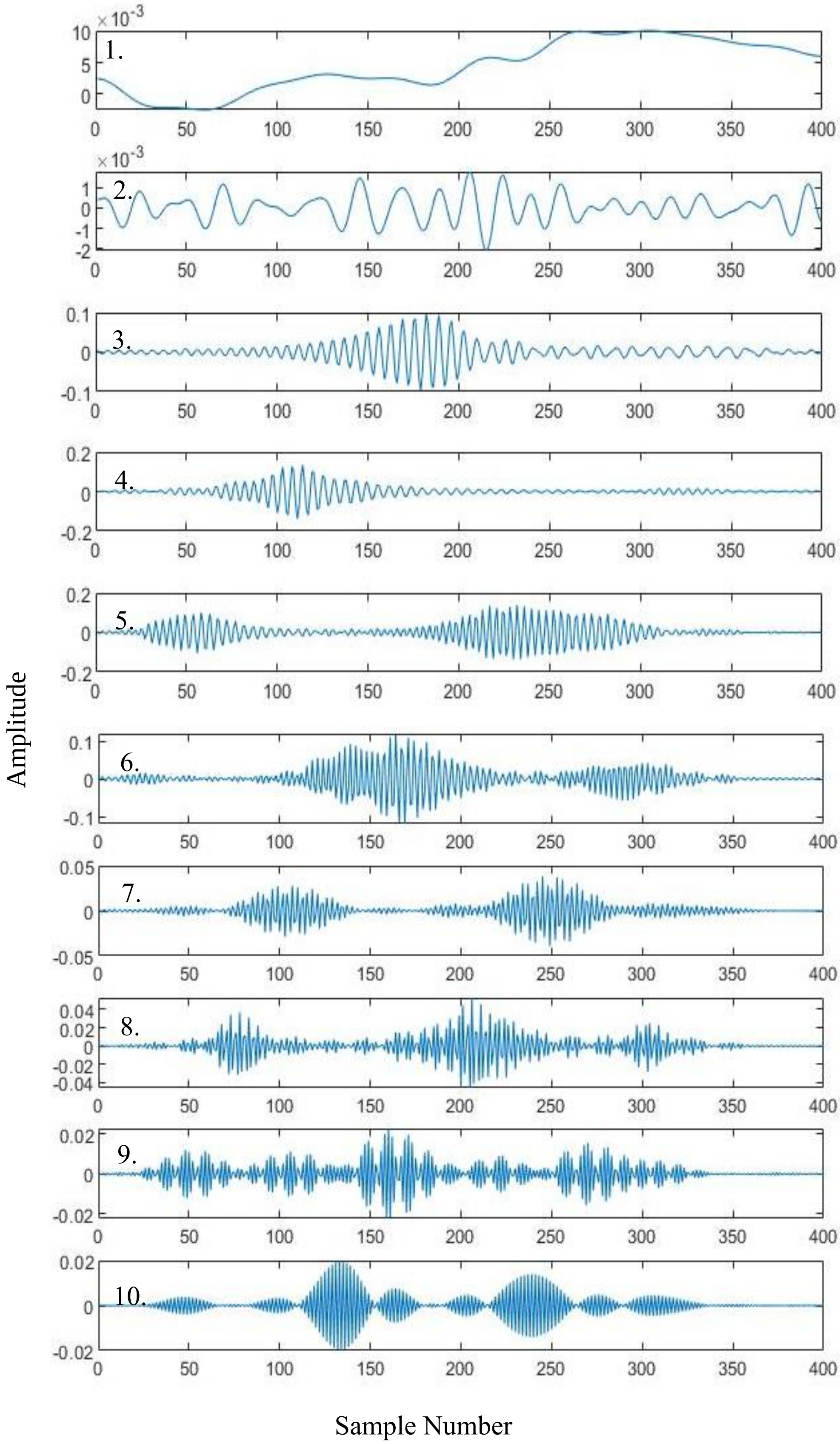


Figure 4.3 IMFs of bat signal using FBSE-EWT, where (1.) IMF-1, (2.) IMF-2, (3.) IMF-3, (4.) IMF-4, (5.) IMF-5, (6.) IMF-6, (7.) IMF-7, (8.) IMF-8, (9.) IMF-9, (10.) IMF-10.

IMFs of bat signal using WFBSE-EWT is given below in fig 4.4.

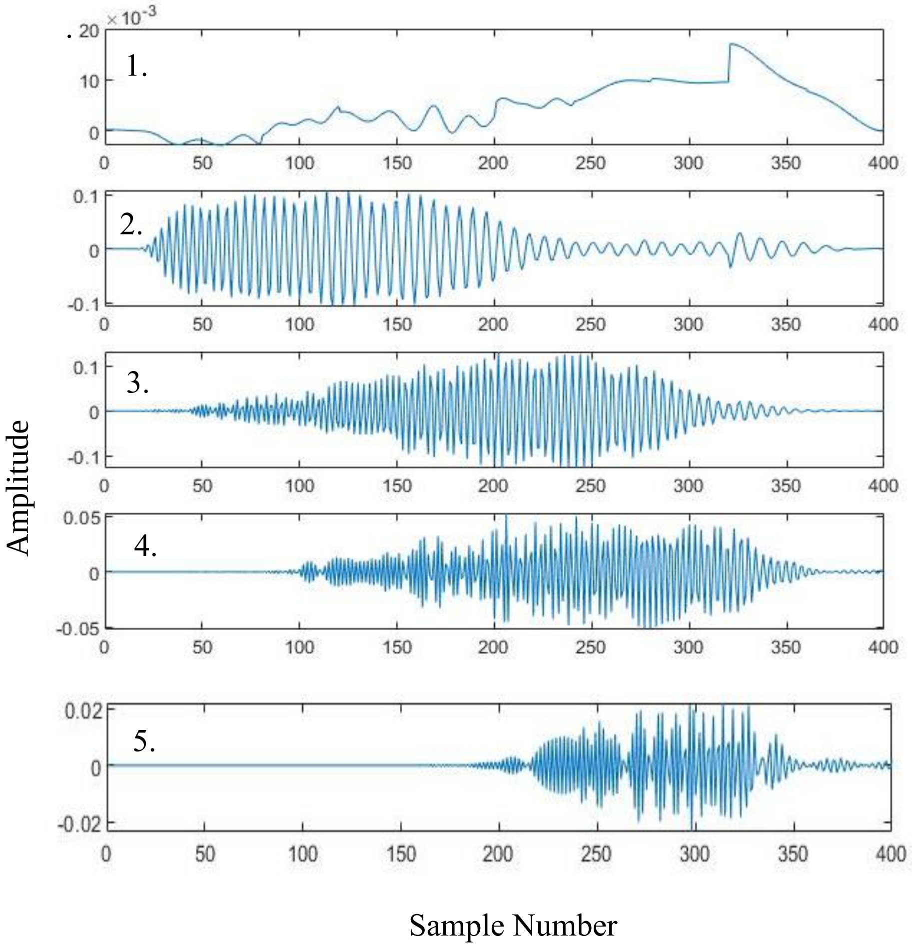


Figure 4.4 IMFs of bat signal using WFBSE-EWT, where (1.) IMF-1, (2.) IMF-2, (3.) IMF-3, (4.) IMF-4, (5.) IMF-5.

Fig 4.5 depicts TF representation of bat signal using FBSE-EWT.

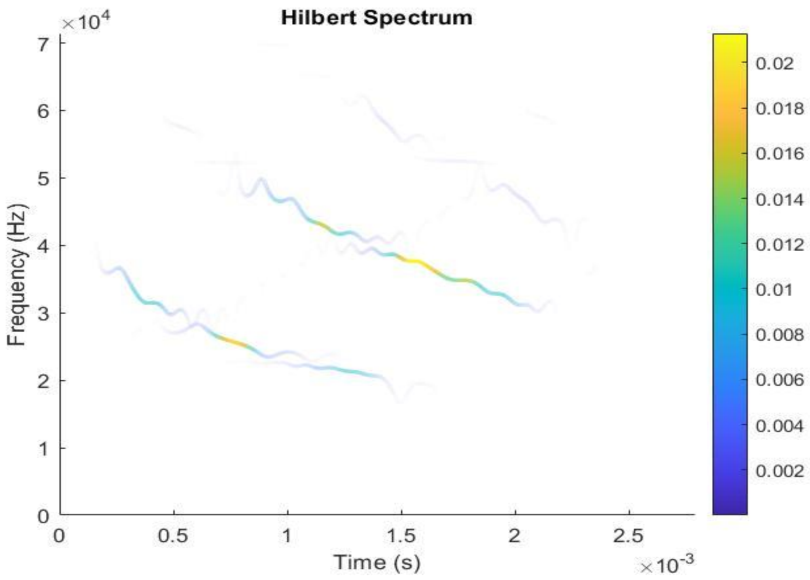


Figure 4.5 TF representation of bat signal using FBSE- EWT

TF representation using WFBSE-EWT for bat signal with Gaussian, Hamming, Hann and Chebyshev window are shown in fig 4.6(a), 4.6(b), 4.6(c) and 4.6(d), respectively.

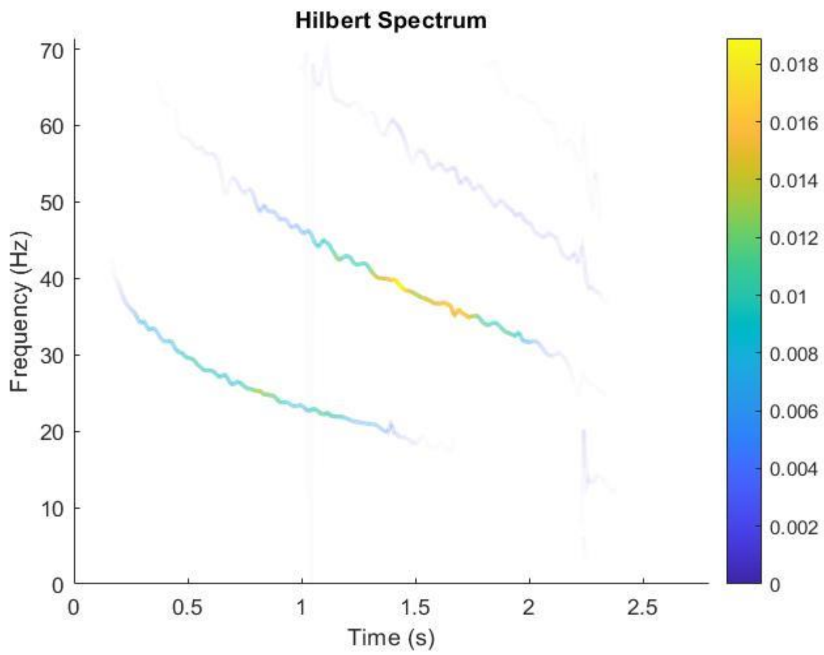


Figure 4.6 (a) TF representation using Gaussian window in WFBSE-EWT (bat signal)

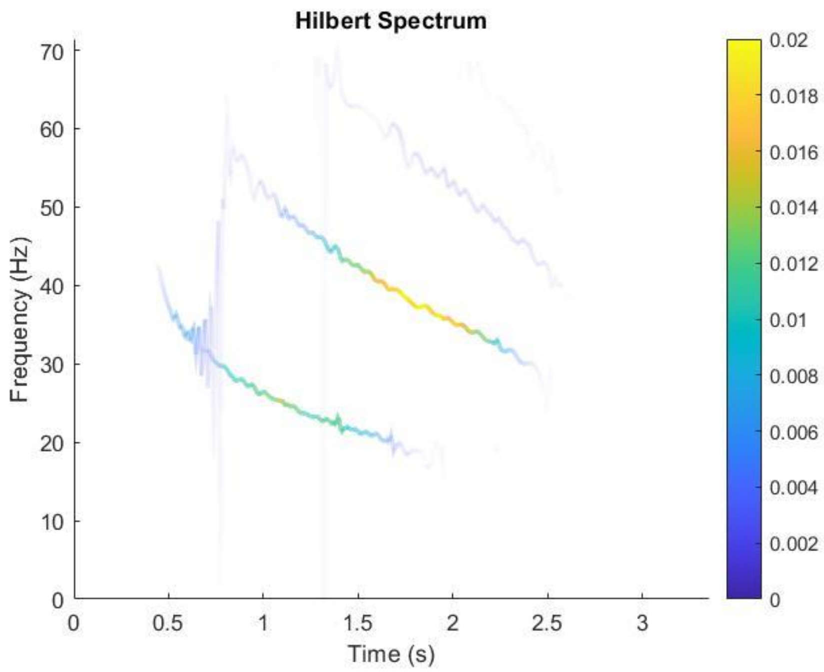


Figure 4.6 (b) TF representation using Hamming window in WFBSE-EWT (bat signal)

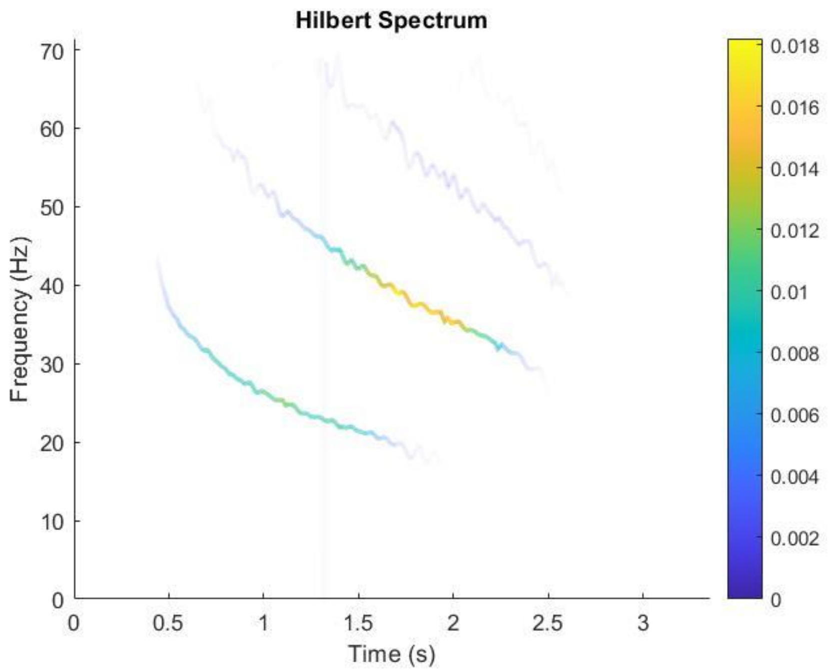


Figure 4.6 (c) TF representation using Hann window in WFBSE-EWT (bat signal)

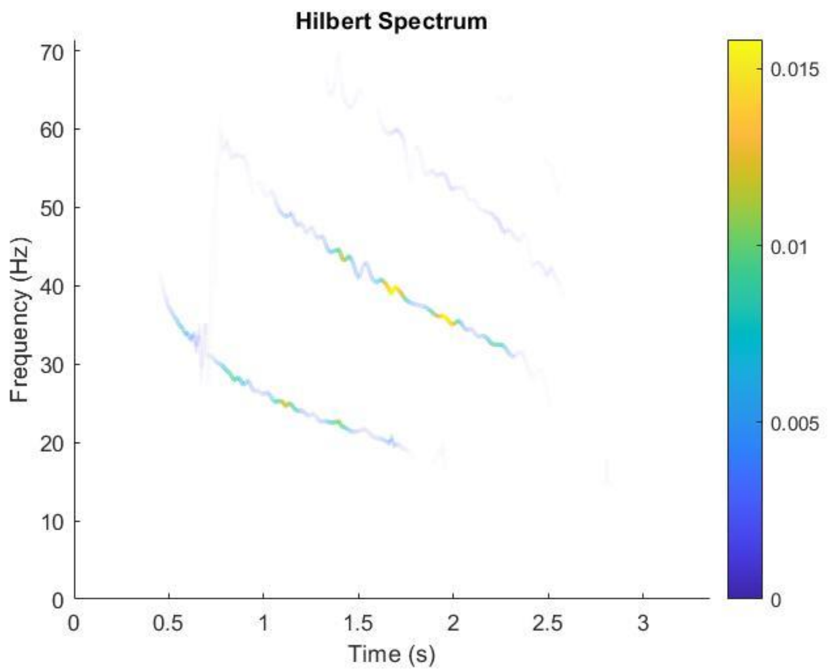


Figure 4.6 (d) TF representation using Chebyshev window in WFBSE-EWT (bat signal)

4.2 Synthetic linear frequency modulated signal

For n varying from 1 to 500. The signal taken is:

$$S= \left(\frac{1}{99}\right)\left(\cos\left(\frac{1}{200}\pi n \; + 1\right)\frac{3}{50}n\right) + \left(\frac{1}{99}\right)\left(\cos\left(\frac{3}{100}\pi n \; + 188\right)\frac{1}{100}n\right)$$

The time domain representation of the signal is given below in fig 4.7 along with boundaries and filter banks obtained using FBSE-EWT in fig 4.8(a) and 4.8(b), respectively.

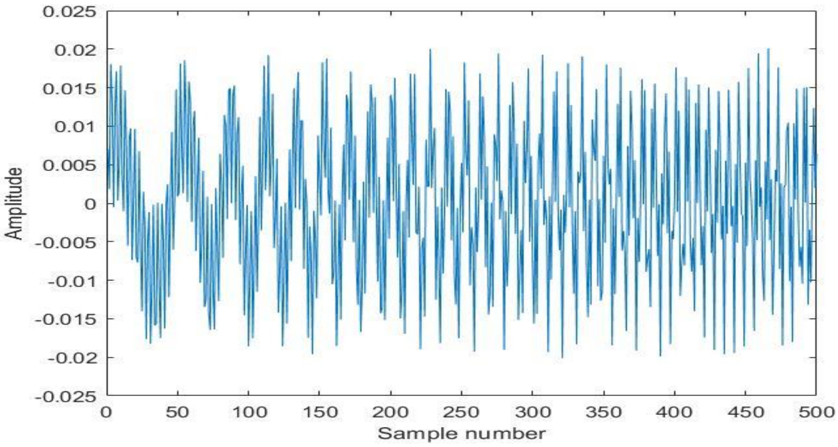
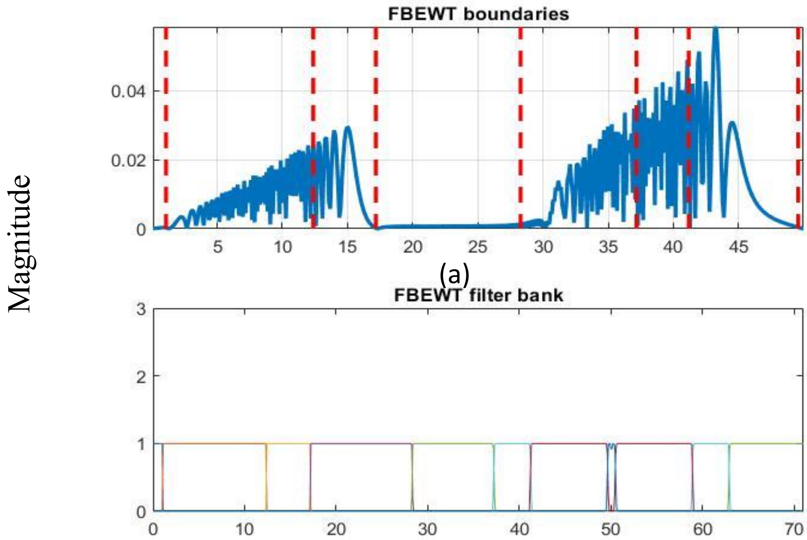


Figure 4.7 Linear frequency modulated signal



(b)
Order

Figure 4.8 (a) Boundaries obtained by FBSE-EWT for linear frequency modulated signal and (b) corresponding filter banks.

The IMFs for the FBSE-EWT of linear frequency modulated signal are shown below in fig 4.9.

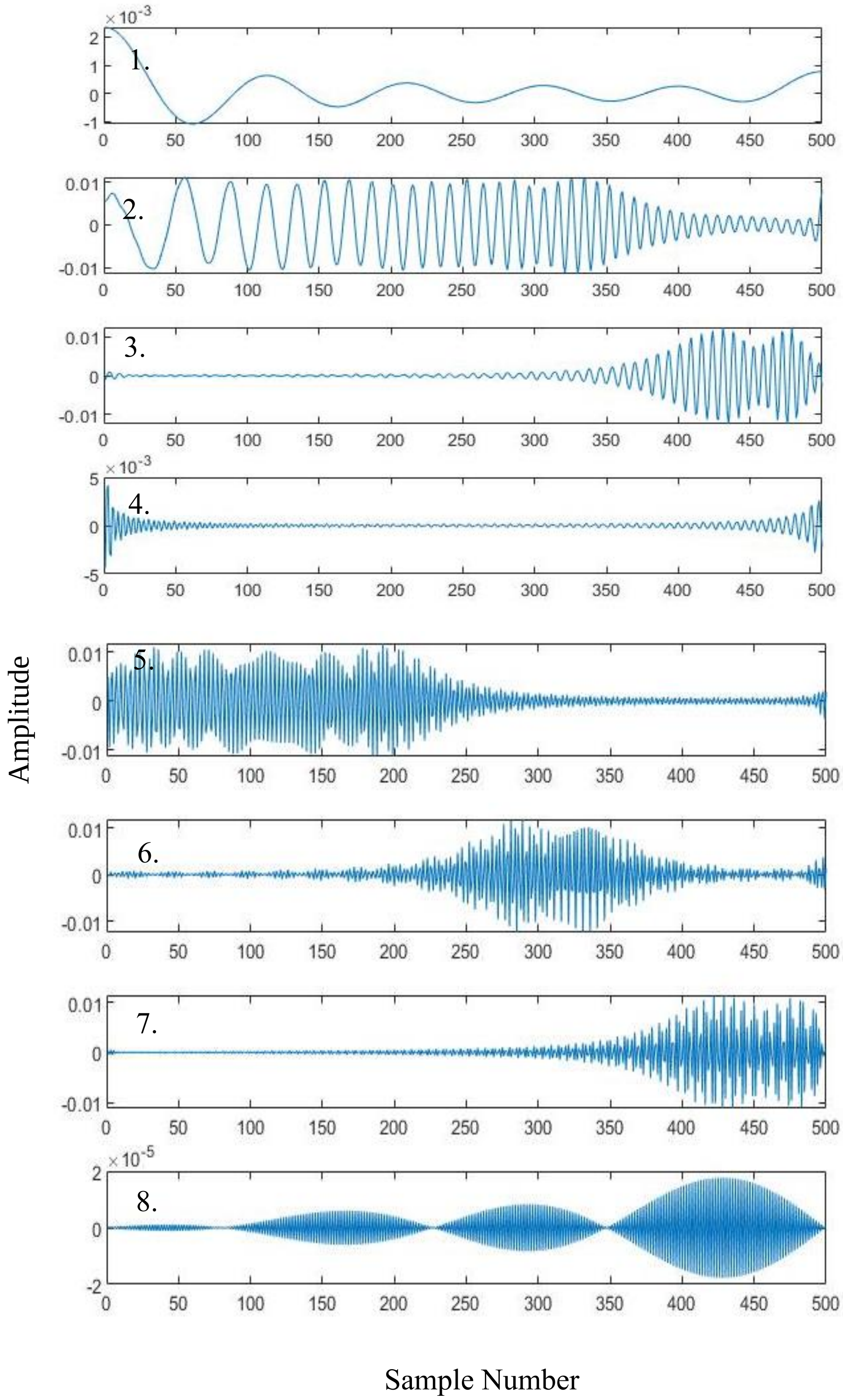


Figure 4.9 IMFs of linear frequency modulated signal using FBSE-EWT, where (1.) IMF-1, (2.) IMF-2, (3.) IMF-3, (4.) IMF-4, (5.) IMF-5, (6.) IMF-6, (7.) IMF-7, (8.) IMF-8.

IMFs of linear frequency modulated signal using WFBSE-EWT is given below in fig 4.10.

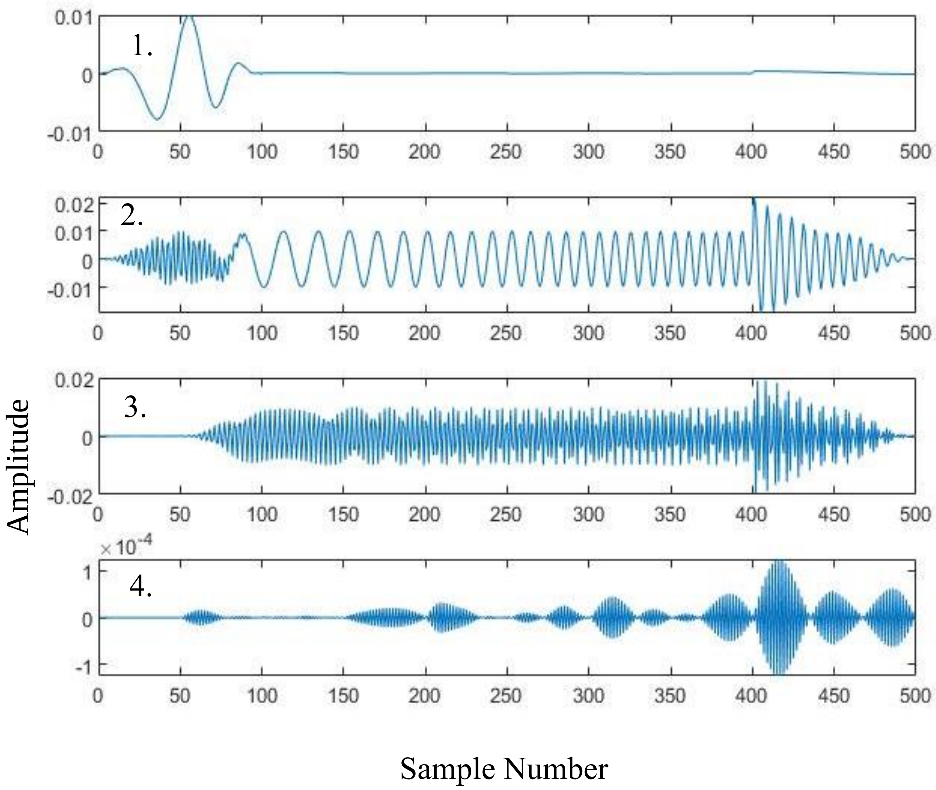


Figure 4.10 IMFs of linear frequency modulated signal using WFBSE-EWT, where (1.) IMF-1, (2.) IMF-2, (3.) IMF-3, (4.) IMF-4.

Fig 4.11 depicts TF representation of linear frequency modulated signal using FBSE-EWT.

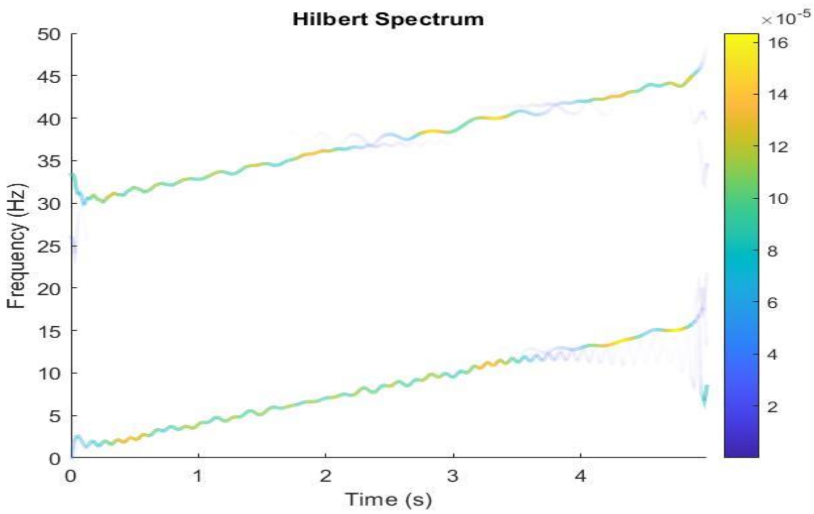


Figure 4.11 TF representation of linear frequency modulated signal using FBSE-EWT

TF representation using WFBSE-EWT for linear frequency modulated signal with Gaussian, Hamming, Hann and Chebyshev window are shown in fig 4.12 (a), 4.12 (b), 4.12 (c) and 4.12 (d), respectively.

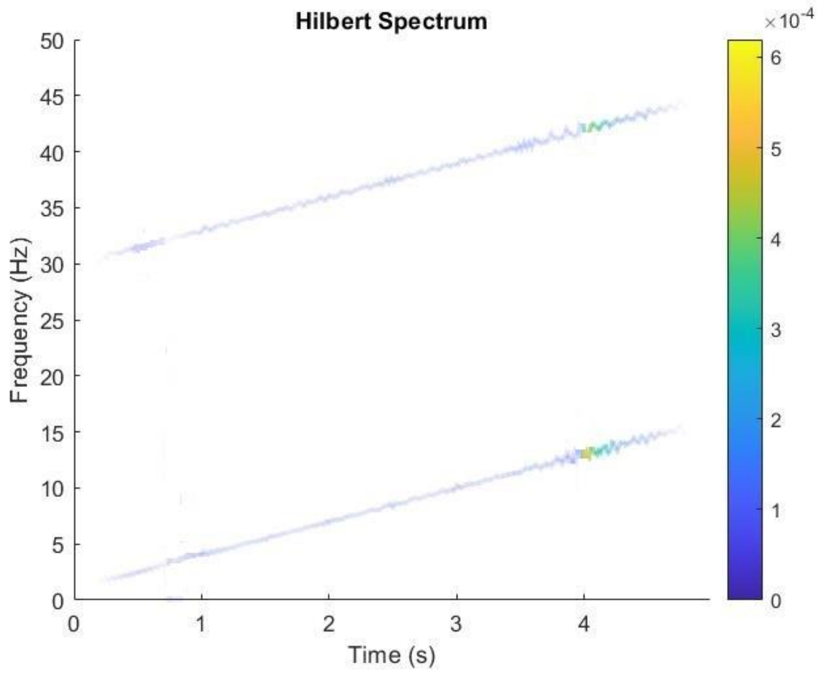


Figure 4.12 (a) TF representation using Gaussian window in WFBSE-EWT (linear frequency modulated signal)

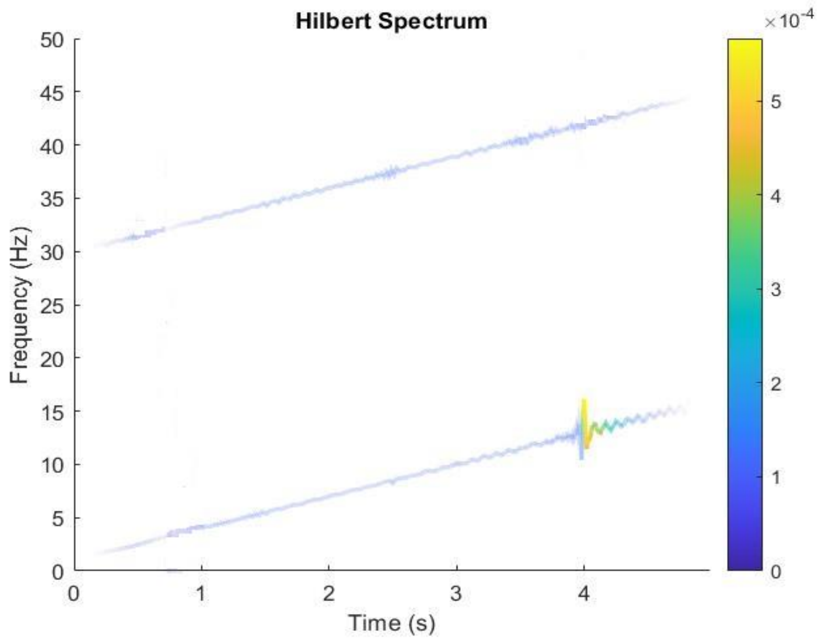


Figure 4.12 (b) TF representation using Hamming window in WFBSE-EWT (linear frequency modulated signal)

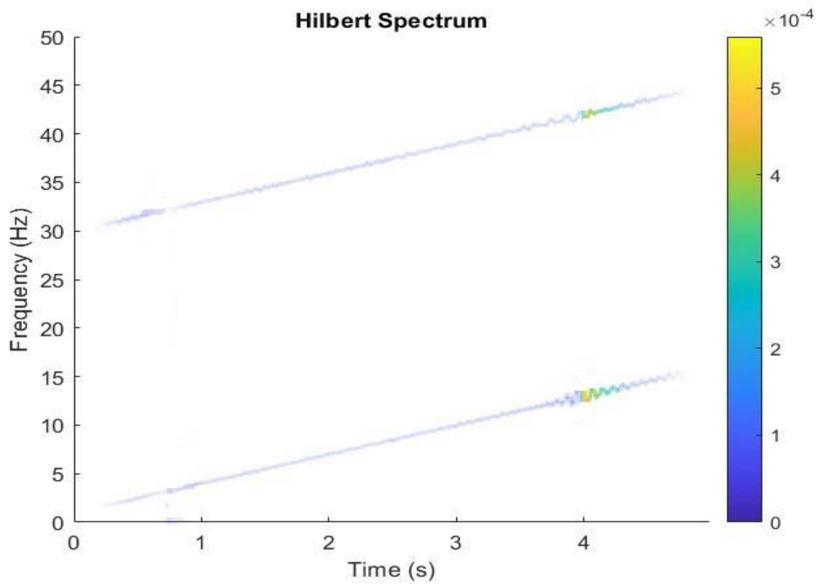


Figure 4.12 (c) TF representation using Hann window in WFBSE-EWT (linear frequency modulated signal)

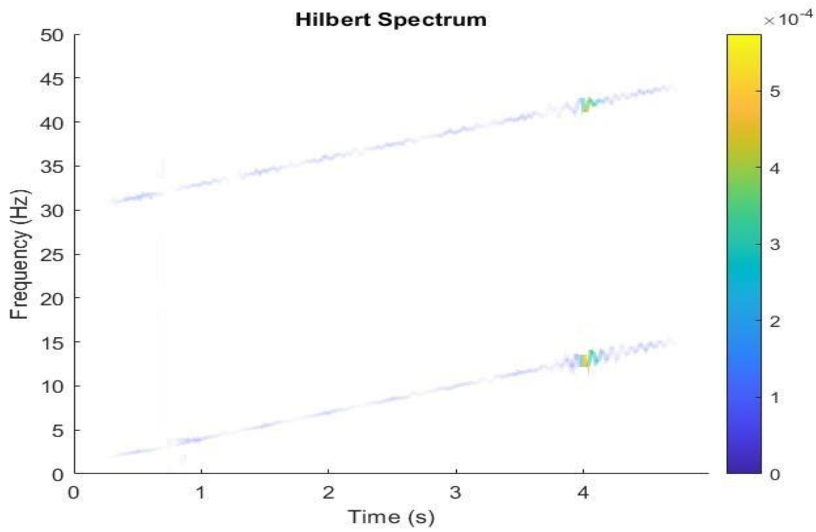


Figure 4.12 (d) TF representation using Chebyshev window in WFBSE-EWT (linear frequency modulated signal)

In both the cases fig 4.5 and fig 4.11, we have obtained discontinuous TF representation in the case of FBSE-EWT, whereas by using WFBSE-EWT, there are fewer discontinuities in the representation.

Further, the results can be verified by the MSE comparison made between the existing method and the proposed method.

The comparison table of MSE is given below in table 4.1.

Table 4.1 Comparison table of MSE

Signals	**FBSE -EWT**	**WFBSE -EWT (Gauss -ian)**	**WFBSE - EWT (Hamm -ing)**	**WFBSE -EWT (Hann)**	**WFBSE -EWT (Cheby -shev)**
Bat echo signal	9.5124 e-07	7.8473 e-07	8.0523 e-06	8.1028 e-07	3.5441 e-07
Linear frequency modulated signal	2.0244 e-10	2.0033 e-10	2.6695 e-10	2.1237 e-10	1.4262 e-10

From the above table, we draw the following conclusions:

- In both cases, we can observe that Gaussian, Hann and Chebyshev window, gives better performance as compared to the existing method.
- But, the MSE for Hamming window is more as compared to the existing method.
- MSE for Chebyshev window is less as compared to all other windows.

4.3 Summary

In this chapter, we have presented the results based on TF representation of the existing FBSE-EWT method and WFBSE-EWT method (Gaussian window, Hamming window, Hann window and Chebyshev window). Further, the discussions have been made by comparing the

MSE of the existing and proposed method. The discussion shows that the proposed method has been found highly effective.

Chapter 5

Conclusion and future work

5.1 Conclusion

We have proposed WFBSE-EWT method to get a better TF representation of non-stationary signals. After comparing the results with the existing FBSE-EWT technique, we come to the conclusion that using Gaussian, Hann and Chebyshev window, we get better TF representation.

We obtain lesser MSE in case of these three windows, best in the case of Chebyshev window.

5.2 Future work

We can extend this method for two-dimension. Also, the method can be improved so that it becomes immune to noise. The proposed method can be used to determine the features for the analysis and classification of biomedical signals. The method can be studied for the analysis of wide class of non-stationary signals and can be compared with other existing methods.

REFERENCES

[1] R.B. Pachori and P. Sircar, “Speech analysis using Fourier-Bessel expansion and discrete energy separation algorithm,” *In 12th Digital Signal Processing Workshop, 4th-Signal Processing Education Workshop* pp. 423-428, 2006.

[2] P. Noll, “MPEG digital audio coding,” *IEEE Signal Processing Magazine*, Vol.14(5), pp. 59-81, 1997.

[3] A. Skodras and C. Christopoulos, “The JPEG 2000 still image compression standard,” *IEEE Signal Processing Magazine*, Vol. 18(5), pp. 36-58, 2001.

[4] G.K. Wallace, “The JPEG still picture compression standard,” *IEEE Transactions in Consumer Electronics*, Vol. 38(1), pp. 30-44, 1992.

[5] M. Richards, “Fundamental of radar signal processing,” *McGraw-Hill Professional*, New York, NY, 2005.

[6] S. Haykin, “Adaptive radar signal processing,” *Hoboken*, NJ, Wiley, 2007.

[7] G.B. Giannakis, Y. Hua, P. Stoica, and L. Tong, “Signal processing advances in wireless and mobile communications, volume 1,” *Trends in Channel Estimation and Equalization*, Upper Saddle River, NJ: Prentice-Hall, 2000.

[8] R. McAulay and T. Quatieri, “Speech analysis/synthesis based on a sinusoidal representation," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol 34(4), pp.744-754.

[9] M. Krumpholz and L.P. Katehi, “New time-domain schemes based on multiresolution analysis,” *IEEE Transactions on Microwave Theory and Techniques*, Vol. 44(4), pp.555-571.

[10] K. Rolf and D. Seltzer, “Signal energy distribution in time and frequency,” *IEEE Transactions on Information Theory*, Vol. 14(3), pp.369-374. 1968

[11] K. Rolf and D. Seltzer, “Frequency analysis method,” *U.S. Patent 5,583,784*, issued December 10, 1996.

[12] L. Cohen, “Time-frequency analysis,” *Prentice hall*, Vol. 778.

[13] L. Cohen, “Time-frequency distributions a review,” *Proceedings of the IEEE*, 77(7), pp. 941-981, 1989.

[14] L.B. Almeida, “The fractional Fourier transform and time-frequency representations," *IEEE Transactions on Signal Processing*, Vol. 42(11), pp. 3084-3091, 1994.

[15] H.M. Ozaktas, Z. Zalevsky, and M.A. Kutay, “The fractional Fourier Transform with applications in optics and signal processing," *New York*, Wiley, 2001.

[16] A. Bhattacharyya, R.B. Pachori, A. Upadhyay, and U.R. Acharya, “Tunable-Q wavelet transform based multiscale entropy measure for automated classification of epileptic EEG signals," *Applied Sciences*, Vol. 7(4), 385, 2017.

[17] A. Bhattacharyya, R.B. Pachori, and U.R. Acharya, “Tunable-Q wavelet transform based multivariate sub-band fuzzy entropy with application to focal EEG signal analysis,” *Entropy*, Vol. 19(3), 99, 2017.

[18] R.B. Pachori, “Discrimination between ictal and seizure-free EEG signals using empirical mode decomposition,” Research Letters in Signal Processing, Vol. 2008, 2008.

[19] S. Patidar and R.B. Pachori, “Classification of cardiac sound signals using constrained tunable-Q wavelet transform,” *Expert System Application*, Vol. 41(16), pp. 7161-7170, 2014.

[20] A. Bhattacharyya, M. Sharma, R.B. Pachori, P. Sircar, and U.R. Acharya, “A novel approach for automated detection of focal EEG signals using empirical wavelet transform,” *Neural Computing and Applications*, Vol. 28(8), pp. 47-57, 2018.

[21] S. Lahmiri, “A variational mode decomposition approach for analysis and forecasting of economic and financial time series,” *Expert System Application*, Vol. 55(C), pp. 268-273, 2016.

[22] F. Amini, N.K. Hazaveh, and A.A. Rad, “Wavelet PSO-based LQR algorithm for optimal structural control using active tuned mass dampers,” *Computer Aided Civil and Infrastructure Engineering*, Vol. 28(7), pp. 542-557, 2013.

[23] M.O. Byrne, F. Schoefs, B. Ghosh, and V. Pakrashi, “Texture analysis based damage detection of ageing infrastructural elements,” *Computer Aided Civil and Infrastructure Engineering*, Vol. 28(3), pp. 162-177, 2013.

[24] F. Hlawatsch and G.F. Boudreaux-Bartels, “Linear and Quadratic Time- Frequency Signal Representations,” *IEEE Signal Processing Magazine*, Vol. 9(2), pp. 21-67, 1992.

[25] M. Portnoff, “Time frequency representation of digital signals and systems based on short-time Fourier analysis,” *IEEE Transactions on Acoustic Speech and Signal Processing*, Vol. 28(1), pp. 55-69, 1980.

[26] I. Daubechies, “Ten Lectures on Wavelets,” *CBMSNSF Regional Conference Series Applied Mathematics*, Philadelphia, PA, USA, 1992.

[27] B. Boashash, “Time Frequency Signal Analysis and Processing: A Comprehensive Reference,” *Academic Press*, Oxford, UK, 2003.

[28] N.E. Huang, Z. Shen, S.R. Long, M.C. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung, and H.H. Liu, “The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis,” *Proceedings of the Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 454(1971), pp. 903-995, 1998.

[29] E. Sejdic, I. Djurovic, and J. Jiang, “Time-frequency feature representation using energy concentration: an overview of recent advances,” *Digital Signal Processing*, Vol. 19(1), pp. 153-183, 2009.

[30] H. Tao, J.M. Zain, M.M. Ahmed, A.N. Abdalla, and W. Jing, “A wavelet-based particle swarm optimization algorithm for digital image watermarking,” *Integrated Computer Aided Engineering*, Vol.19(1), pp. 81-91, 2012.

[31] G. Prez, A. Conci, A.B. Moreno, and J.A. Hernandez-Tamames, “Rician noise attenuation in the wavelet packet transformed domain for brain MRI,” *Integrated Computer Aided Engineering*, Vol. 21(2), pp. 163-175, 2014.

[32] X. Jiang and H. Adeli, "Wavelet packet autocorrelation function method for traffic flow pattern analysis," *Computer Aided Civil Infrastructure Engineering*, Vol. 19(5), pp. 324-337, 2014.

[33] I. W. Selesnicki, "Wavelet transform with tunable Q-factor," *IEEE Transactions in Signal Processing*, Vol. 59(8), pp. 3560-3575, 2011.

[34] A. Bhattacharyya, L. Singh, and R.B. Pachori, "Identification of epileptic seizures from scalp EEG signals based on TQWT," *InInternational Conference on Machine Intelligence and Signal Processing*., 22-24 Dec. 2017.

[35] I. Daubechies, J. Lu, and H.T. Wu, "Synchrosqueezed wavelet transforms: an empirical mode decomposition like tool," *Applied and Computational Harmonic Analysis*, Vol. 30(2), pp. 243-261, 2011.

[36] M.Sharma, P.V. Achuth, R.B. Pachori, and V.M. Gadre, "A parametrization technique to design joint time-frequency optimized discrete-time biorthogonal wavelet bases," *Signal Processing*, Vol. 135, pp. 107-120, 2017.

[37] K. Dragomiretskiy and D. Zosso, "Variational mode decomposition," *IEEE Transactions on Signal Processing*, Vol. 62(3), pp. 531-544, 2014.

[38] A. Upadhyay and R.B. Pachori, "Instantaneous voiced/non-voiced detection in speech signals based on variational mode decomposition," *Journal of Franklin Institute*, Vol. 352(7), pp. 2679-2707, 2015.

[39] A. Upadhyay and R.B. Pachori, "Speech enhancement based on mEMD-VMD method," *Electronics Letters*, Vol. 53(7), pp. 502-504, 2017. 2679-2707.

[40] N.E. Huang, Z. Wu, S.R. Long, K.C. Arnold, X. Chen, and K. Blank, "On instantaneous frequency," *Advance in Adaptive Data Analysis*, Vol. 1(02), pp. 177-229, 2009.

[41] Z. Yang, L. Yang, C. Qing, and D. Huang, "A method to eliminate riding waves appearing in the empirical AM/FM

demodulation," *Digital Signal Processing*, Vol. 18(4) pp. 488-504, 2008.

[42] P. Singh, S.D. Joshi, R.K. Patney, and K. Saha, "The Fourier decomposition method for nonlinear and non-stationary time series analysis," *Proceedings of the Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 473(2199), 2017.

[43] G.K. Apostolidis and L. J. Hadjileontiadis, "Swarm decomposition: a novel signal analysis using swarm intelligence," *Signal Processing*, Vol. 132, pp. 40-50, 2017.

[44] R.R. Sharma and R.B. Pachori, "Time-frequency representation using IEVDHM-HT with application to classification of epileptic EEG signals," *IET Science, Measurement and Technology*., Vol. 12(1), pp. 72-82, 2018.

[45] R.R. Sharma and R.B. Pachori, "A new method for non-stationary signal analysis using eigenvalue decomposition of the Hankel matrix and Hilbert transform," *In Fourth International Conference on Signal Processing and Integrated Networks (SPIN 2017)*, Noida, India, pp. 484-488, 2017.

[46] R.R. Sharma and R.B. Pachori, "Eigenvalue decomposition of Hankel matrix based time-frequency representation for complex signals," *Circuits Systems and Signal Processing*, 2018.

[47] R.R. Sharma and R.B. Pachori, "Baseline wander and power line interference removal from ECG signals using eigenvalue decomposition," *Biomedical Signal Processing and Control*, Vol. 45, pp.33-49.

[48] R.R. Sharma and R.B. Pachori, "Improved eigenvalue decomposition-based approach for reducing cross-terms in Wigner-Ville distribution," *Circuits, Systems, and Signal Processing*, pp.1-2, 2018.

[49] R.R. Sharma, P. Chandra, and R.B. Pachori, "Electromyogram signal analysis using eigenvalue decomposition of the Hankel matrix," *International Conference on Machine Intelligence and Signal Processing*, December 22-24, 2017, Indore, India.

[50] R.R. Sharma, M. Kumar, and R.B. Pachori, “Automated CAD identification system using time-frequency representation based on eigenvalue decomposition of ECG signals,” *International Conference on Machine Intelligence and Signal Processing*, December 22-24, 2017, Indore, India.

[51] R.B. Pachori and P. Sircar, “A new technique to reduce cross terms in the Wigner distribution,” *Digital Signal Processing*, Vol. 17(2), pp. 466-474, 2007.

[52] R.B. Pachori and A. Nishad, “Cross-terms reduction in the Wigner Ville distribution using tunable-Q wavelet transform,” *Signal Processing*, Vol. 120, pp.288-304, 2016.

[53] J. Gilles, “Empirical wavelet transform,” *IEEE Transactions on Signal Processing*, Vol. 61(16), pp. 3999-4010, 2013.

[54] J. Gilles, G. Tran, and S. Osher, “2D empirical transforms. Wavelets, ridgelets, and curvelets revisited,” *SIAM Journal on Imaging Sciences*, Vol. 7(1), pp. 157-186, 2014.

[55] A. Bhattacharyya and R.B. Pachori, “A multivariate approach for patient-specific EEG seizure detection using empirical wavelet transform,” *IEEE Transactions on Biomedical Engineering*, Vol. 64(9), pp. 2003-2015, 2017.

[56] J. P. Amezquita-Sanchez and H. Adeli, “A new music-empirical wavelet transform methodology for time-frequency analysis of noisy nonlinear and non-stationary signals,” *Digital Signal Processing*, Vol. 45, pp. 55-68, 2015.

[57] E.O. Brigham, “The fast Fourier transform and its applications,” *prentice Hall Englewood Cliffs*, NJ, Vol. 448, 1998.

[58] K. Katoh, K. Misawa, K. Kumar, and T. Miyata, “A novel method for rapid multiple sequence alignment based on fast Fourier transform," Nucleic Acids Research, Vol. 30(14), pp. 3059-3066, 2002.

[59] K. Gopalan, T.R. Anderson, and E.J. Cupples, “A comparison of speaker identification results using features based on cepstrum and Fourier-Bessel expansion,” *IEEE Transactions on Speech and Audio Processing*, Vol. 7(3), pp. 289-294, 1999.

[60] R.B. Pachori and P. Sircar, “Analysis of multicomponent AM-FM signals using FB-DESA method,” *Digital Signal Processing*, Vol. 20(1), pp. 42-62, 2010.

[61] P. Jain and R.B. Pachori, “Time-order representation based method for epoch detection from speech signals,” *Journal of Intelligent Systems*, Vol. 21(1), pp.79-95, 2012.

[62] J. Gilles and K. Heal, “A parameterless scale-space approach to find meaningful modes in histograms- Application to image and spectrum segmentation,” *International Journal of Wavelets, Multiresolution and Information Processing*, Vol. 12(06), 2014.

[63] F. Mokhtarian and B. Miroslaw, “Curvature scale space representation: theory, applications, and MPEG-7 standardization,” *Springer Science and Business Media*, Vol. 25, 2003.

[64] N. Otsu, “A threshold selection method from gray-level histograms,” *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 9(1), pp. 62-66, 1979.

[65] J. Schroeder, “Signal processing via Fourier-Bessel series expansion,” *Digital Signal Processing*, Vol. 3(2), pp.112-124, 1993.

[66] R.B. Pachori and P. Sircar, “EEG signal analysis using FB expansion and second-order linear TVAR process,” *Signal Processing*, Vol. 88(2), pp. 415-420, 2008.

[67] P. Jain and R.B. Pachori, “Event-based method for instantaneous fundamental frequency estimation from voiced speech based on eigenvalue decomposition of the Hankel matrix,” *IEEE Transactions on Audio, Speech and Language Processing*, Vol. 22(10), pp. 1467-1482, 2014.

[68] R.B. Pachori and P. Sircar, “Non-stationary signal analysis: methods based on Fourier-Bessel representation,” *LAP LAMBERT Academic Publishing*, Germany, 2010.

[69] C. Chen, K. Gopalan, and P. Mitra, “Speech signal analysis and synthesis via Fourier-Bessel representation,” *In Acoustics, Speech, and Signal Processing, IEEE International Conference on ICASSP*, Vol. 10, pp. 497-500, 1985.

[70] J. Bird and D. George, “The use of the Fourier-Bessel series in calculating error probabilities for digital communication systems,” *IEEE Transactions on Communications*, Vol. 29(9), pp.1357-1365, 1981.

[71] K. Gopalan and C. Chen, “Numerical evaluation of Fourier-Bessel series expansion,” *IEEE International Conference on Acoustics, Speech, and Signal Processing*, ICASSP'83., Vol. 8, pp. 151-154, 1983.

[72] J. Schroeder, Signal processing via Fourier–Bessel series expansion, *Digital Signal Process*. 3 (1993), 112–124.

[73] L. Cohen and C. Lee, “Instantaneous frequency, its standard deviation and multicomponent signals,” *Proceedings of SPIE,* Vol. 975, pp. l86-208, 1988.

[74] P. Maragos, J.F. Kaiser, and T.F. Quatieri, “Energy separation in signal modulation with application to speech analysis,” *IEEE Transactions on Signal Processing,* Vol. 41(10), pp. 3024-3051, 1993.

[75] A. Bhattacharyya, L. Singh, and R.B. Pachori, “Fourier-Bessel series expansion based empirical wavelet transform for analysis of nonstationary signals”, *Digital Signal Processing*, Vol. 78, pp. 185-196, July 2018.

[76] R. Katiyar, V. Gupta, and R.B. Pachori, “FBSE-EWT-based approach for the determination of respiratory rate from PPG signals,” *IEEE Sensors Letters*, Vol. 03, no. 07, article sequence no. 7001604, July 2019.

[77] Y. Ephraim and H.L. Van Trees, “A signal subspace approach for speech enhancement,” *IEEE Transaction on Speech and Signal Processing*, Vol. 3(4), pp. 251-266, 1995.

[78] T.S. Kumar, M.A. Hussain, and V. Kanhangad, “Classification of voiced and non-voiced speech signals using empirical wavelet transform and multi-level local patterns,” *IEEE International Conference on Digital Signal Processing (DSP)*, Singapore, pp. 163-167, 2015.

[79] S.G. Chang, B. Yu, and M. Vetterli, “Adaptive wavelet thresholding for image denoising and compression," *IEEE Transactions on Image Processing*, Vol.9(9), pp. 1532-1546, 2000.

[80] J.L. Starck, E.J. Candes, and D.L. Donoho, “The curvlet transform for image denoising,” *IEEE Transactions on Image Processing*, Vol. 11(6), pp. 670-684, 2002.

[81] https://www.ece.rice.edu/dsp/software/bat.shtml

[82] Alan V. Oppenheim, Ronald W. Schafer, John R. Buck, “ Discrete-time signal processing (2nd ed.),” Upper Saddle River, N.J.: Prentice Hall. pp. *465*–478.