HOLOGRAPHIC DESCRIPTION OF QUANTUM FIELD

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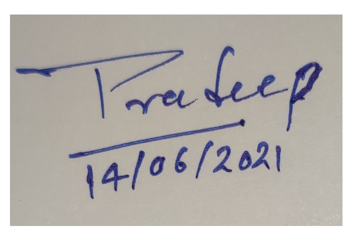
CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Holographic description of Quantum Field"** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF SCIENCE** and submitted in the **DISCIPLINE OF PHYSICS, Indian Institute of Technology Indore,** is an authentic record of my own work carried out during the time period from july 2020 to june, 2021 under the supervision of **Dr. Manavendra N. Mahato,** Associate Professor.The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.



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Abstract

If such AdS/CFT conjecture was valid, the boundary CFT can solve any bulk physics question. However, we still don't know how to convert all of the bulk physics concerns into boundary CFT questions.We build boundary operators to represent local bulk operators inserted behind the Poincare' patch's horizon.Here we write boundary representation of the free field in bulk. We represent the free field in anti-de Sitter space and after that discuss free field reconstruction in mode sum approach.

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Chapter 1

Introduction

Every Conformal Field Theory inside a d-dimensional spacetime is equivalent to a theory of quantum gravity inside a d + 1-dimensional spacetime, as per the AdS/CFT conjecture. As a result, it should also be possible to gain knowledge of bulk physics from the boundaries. If indeed the bulk boundary theories are equal, then the boundary CFT can theoretically answer any issue concerning bulk physics. The bulk quantum theory of gravity is indeed the CFT inside the boundary!

The boundary theory, in principle, understands the solutions to all the issues that remain unsolved, such as:

- What happens when a person passes through the horizon of a black hole?[1]
- How does the black hole information loss paradox handled in general?[13]
- What happens when classical general relativity breaks down at singularities.

We should have solutions to all of these bulk physics concerns if we can just resolve the boundary CFT properly.

In the bulk, semiclassical observables can be translated to the boundary CFT. Quantum gravity concerns, on the other hand, remain a long way off. Even when gravity isn't present.

First, we present a brief recall of the concept of quantum field theory. Then in chapter-3, we discuss the Einstein field equation and maximally symmetric universes. And we derive metrics for de Sitter space and anti-de Sitter space. After that, we draw the conformal diagram for de Sitter space and anti-de Sitter space. After that in chapter-4, we discuss the free field in anti-de Sitter space and calculate the smearing function for the free field.

Chapter 2

Quantum Field Theory

2.1 Noether's Theorem:

Some Lagrangians are well-known in physics. However, a system may not be well known in general, and we must create a Lagrangian that expects to describe it. In such cases, symmetry, i.e. transformations that leave the system invariant, is typically useful. A conserved quantity happens to be connected with each symmetry of a system. Thus, if we know a system's conserved quantity, we can work backwards to identify the system's symmetries and then guess the form of a useful Lagrangian. The conclusion that tied symmetries to conserved quantities is known as Noether's theorem. The conserved quantities are known as Noether's charges and Noether's current.[11]

The equations of motion suggests that each and every continuous symmetry of the Lagrangian leads to a conserved current.

$$\partial_\mu \jmath^\mu = 0 \tag{2.1}$$

where μ is 0,1,2,3 .The equations of motion suggest that each and every continuous symmetry of the Lagrangian leads to a conserved current. Equation (2.1) is called Noether's theorem.

For symmetry transformation

$$\delta\mathcal{L} = \partial_\mu F^\mu \quad \text{and} \quad \delta\phi_a(x) = X_a(\phi)$$

$$\jmath^{\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi_a)} - F^{\mu}(\phi) \tag{2.2}$$

The conservation law can also be stated as follows:

$$Q \equiv \int_{\text{all space}} \jmath^0 d^3x$$

where Q is Noether's charge.Q is constant in time.

2.2 Quntum Field Theory

2.2.1 Propagator:

Quantum field theory aims to explain particle interactions. Calculating cross-sections, transition probabilities, decay rates, and so on will be of interest to us. We need to understand how particles move in space-time to do this. A particle's amplitude when propagating from y to x is $\langle 0|\phi(x)\phi(y)|0\rangle$. This quantity will be referred to as $D(x-y)$.

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3p d^3\acute{p}}{(2\pi)^6\sqrt{4E_{\vec{p}}E_{\vec{\acute{p}}}}} \langle 0|a_{\vec{p}} a^{\dagger}_{\vec{\acute{p}}}|0\rangle e^{-\iota p.x + \iota\acute{p}.y}$$

$$D(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-\iota p.(x-y)}$$

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-\iota p.(x-y)} \tag{2.3}$$

The function D(x-y) is called propagator.

2.2.2 The Feynman propagator:

A time-ordered correlation function for two scalar fields with in vacuum state is the Feynman propagator of such a free real scalar field. according to the definition[14].

$$\triangle_F(x-y) = \langle 0|\mathcal{T}\phi(x)\phi(y)|0\rangle \tag{2.4}$$

$$\triangle_F(x-y) = \begin{cases} D(x-y) & x^0 > y^0 \\ D(y-x) & y^0 > x^0 \end{cases} \tag{2.5}$$

Where T denotes time ordering, all operators analyzed at later times are placed to the left

$$\mathcal{T}\phi(x)\phi(y) = \begin{cases} \phi(x)\phi(y) & x^0 > y^0 \\ \phi(y)\phi(x) & y^0 > x^0 \end{cases} \tag{2.6}$$

The Klein-Gordon equation has a Feynman propagator, as well as a Green's functions.It is Defined as follows:

$$\triangle_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{\iota e^{-\iota p.(x-y)}}{p^2 - m^2 + \iota\varepsilon} \tag{2.7}$$

The Feynman propagator is indeed the expression that we have used to represent the propagation of virtual particles on the interior lines of the Feynman diagram.

2.2.3 Wick's Theorem:

In normal ordering, all annihilation operators are placed on the right and all creation operators on the left. So that it eliminates vacuum. But in time ordering all operators analyzed at later times are placed to the left. So prior times creation operators must've been further to the right than later times annihilation operators. This is not same as of what is required for normal ordering. So we'll need to establish a link between time ordering and normal ordering.

The Wick's theorem explains how to transition from time-ordered to normally ordered products. Wick's theorem establishes a link between time-ordered and normally ordered products.

For any collection of field $\phi_1 = \phi(x_1), \phi_2 = \phi(x_2)$ etc. we have

$$T(\phi_1.............\phi_n) =: \phi_1............\phi_n : + : \text{ all possible contraction } : \tag{2.8}$$

Contraction: The contraction of the pair of a field in a chain of operators ......$\phi(x_1)$......$\phi(x_2)$........ to signify that the Feynman propagator is used to replace those operators, keeping all other operators unaffected. We should use the notation

$$..............\overbrace{\phi(x_1)........\phi(x_2)}.............$$

to denote contraction. So, for example

$$\overbrace{\phi(x)\phi(y)} = \triangle_F(x-y)$$

From Wick's theorem

$$T[\phi(x)\phi(y)] =: \phi(x)\phi(y) : +\triangle_F(x-y) \tag{2.9}$$

2.2.4 Feynman Diagram:

- To every particle within the initial state and for every particle in the final state, draw an exterior line. For mesons, we'll use dotted lines, and for nucleons, we'll use solid lines. Each line should be given a directed momentum p. Add an arrow to solid lines to indicate their charge; inside the initial state for $\psi(\bar{\psi})$, we'll use an incoming (outgoing) arrow. For the final state, we use the contrary convention, with an outgoing arrow denoting ψ .
- Use trivalent vertices to connect the exterior lines (Figure-1).

To find the scattering amplitude $\iota\mathcal{M}$, Feynman rule

- Draw all possible diagrams as you can with proper external legs and at each vertex, apply 4-momentum conservation.
- At each vertex, jot down a $(-\iota g)$ factor.
- Take note of the propagator to every internal line.

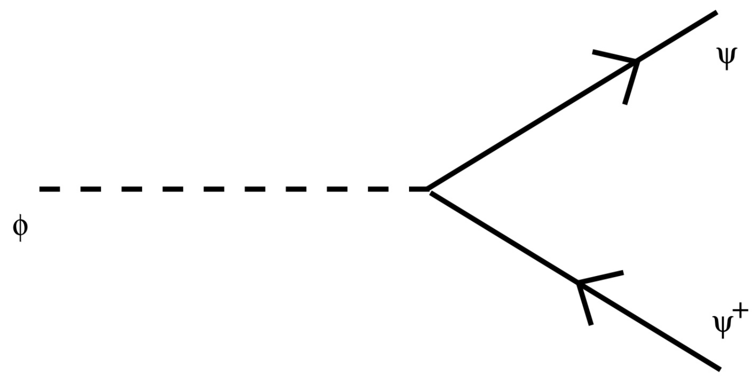


Figure 2.1: Allow vertex

- Integrate across the flow of momentum k via each loop $\int \frac{d^4k}{(2\pi)^4}$

2.2.5 Clifford Algebra or Dirac Algebra:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} I \tag{2.10}$$

Where γ^μ is a group of four matrices, with $\mu = 0, 1, 2, 3$.

$\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu$ When $\mu \neq \nu$

$$(\gamma^0)^2 = 1, \quad (\gamma^i)^2 = -1 \tag{2.11}$$

where $i = 1, 2, 3$.

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

where each component is a 2×2 matrix on its own, with σ^i is the Pauli matrices. This representation is referred to as the Weyl or chiral representation.

Chiral spinors:

In chiral representation, the spinor rotation matrix is $S[\Lambda_{rot}]$ and boost matrix is $S[\Lambda_{boost}]$.Both are block diagonal

$$S[\Lambda_{rot}] = \begin{pmatrix} e^{\iota\vec{\varphi}.\vec{\sigma}/2} & 0 \\ 0 & e^{-\iota\vec{\varphi}.\vec{\sigma}/2} \end{pmatrix}$$

and

$$S[\Lambda_{boost}] = \begin{pmatrix} e^{\overrightarrow{\chi}.\overrightarrow{\sigma}/2} & 0 \\ 0 & e^{\overrightarrow{\chi}.\overrightarrow{\sigma}/2} \end{pmatrix}$$

This implies that the Lorentz group's Dirac spinor representation is reducible. It breaks down into two irreducible representations that exclusively act on two-component spinors $u_{\pm}$ which are specified by in the chiral representation.

$$\psi = \begin{pmatrix} u_+ \\ u_- \end{pmatrix}$$

The two-component objects $u_{\pm}$ are called Weyl spinors or chiral spinors. When rotated, they transform in about the same way

$$u_{\pm} \rightarrow e^{\iota\overrightarrow{\varphi}.\overrightarrow{\sigma}/2} u_{\pm}$$

2.2.6 Fermionic propagator:

The fermionic propagator can be defined as the following:

$$\iota S(x-y) = \iota S_{\alpha\beta} = \{\psi_\alpha(x), \overline{\psi}_\beta(y)\}$$

$$\iota S(x-y) = \int \frac{d^3p d^3q}{(2\pi)^6} \frac{1}{\sqrt{4E_{\overrightarrow{p}} E_{\overrightarrow{q}}}} [\{b^s_{\overrightarrow{p}}, b^{r\dagger}_{\overrightarrow{q}}\} u^s(\overrightarrow{p}) \overline{u}^r(\overrightarrow{q}) e^{-\iota(p.x-q.y)} + \{c^{s\dagger}_{\overrightarrow{p}}, c^r_{\overrightarrow{q}}\} v^s(\overrightarrow{p}) \overline{v}^r(\overrightarrow{q}) e^{+\iota(p.x-q.y)}]$$

$$\iota S(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\overrightarrow{p}}} [u^s(\overrightarrow{p}) \overline{u}^s(\overrightarrow{p}) e^{-\iota p.(x-y)} + v^s(\overrightarrow{p}) \overline{v}^s(\overrightarrow{p}) e^{+\iota p.(x-y)}]$$

$$\iota S(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\overrightarrow{p}}} [(\gamma^\mu p_\mu + m) e^{-\iota p.(x-y)} + (\gamma^\mu p_\mu - m) e^{+\iota p.(x-y)}]$$

We can then write

$$\iota S(x-y) = (\iota\gamma^\mu \partial + m)(D(x-y) - D(y-x)) \tag{2.12}$$

2.2.7 The Feynman propagator for Fermions:

We can figure out what the vacuum expectation value is

$$\langle 0|\psi_\alpha(x)\overline{\psi}_\beta(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\overrightarrow{p}}}(\gamma^\mu p_\mu + m)_{\alpha\beta}e^{-\iota p.(x-y)}$$

$$\langle 0|\overline{\psi}_\beta(y)\psi_\alpha(x)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\overrightarrow{p}}}(\gamma^\mu p_\mu - m)_{\alpha\beta}e^{+\iota p.(x-y)}$$

As such a time-ordered product, we establish the Feynman propagator $S_F(x-y)$, which is still another 4×4 matrix.

$$S_F(x-y) = \langle 0|T\psi(x)\overline{\psi}(y)|0\rangle = \begin{cases}\langle 0|\psi(x)\overline{\psi}(y)|0\rangle & x^0 > y^0\\ \langle 0|-\overline{\psi}(y)\psi(x)|0\rangle & y^0 > x^0\end{cases}$$

For such Feynman propagator, we have had the 4-momentum essential representation.

$$S_F(x-y) = \iota\int\frac{d^4p}{(2\pi)^4}\frac{1}{2E_{\overrightarrow{p}}}e^{-\iota p.(x-y)}\frac{(\gamma^\mu p_\mu + m)}{p^2 - m^2 + \iota\varepsilon} \tag{2.13}$$

Feynman Rules for Fermions:

The below are now the rules for evaluating amplitudes:

- We assign a spinor $u^r(\overrightarrow{p})$ to each entering fermion having momentum p and spin r (Figure-2).$\bar{u}^r(\overrightarrow{p})$ is assigned to outgoing fermions (Figure-3).

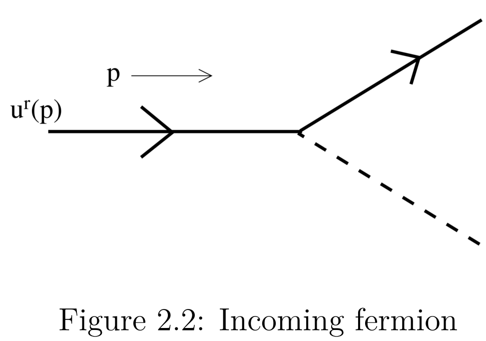


Figure 2.2: Incoming fermion

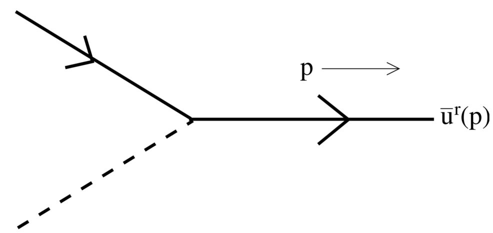


Figure 2.3: Outgoing fermion

- We assign a spinor $\bar{v}^r(\overrightarrow{p})$ to each entering fermion having momentum p and spin r (Figure-4). $v^r(\overrightarrow{p})$ is assigned to outgoing anti-fermions(Figure-5).

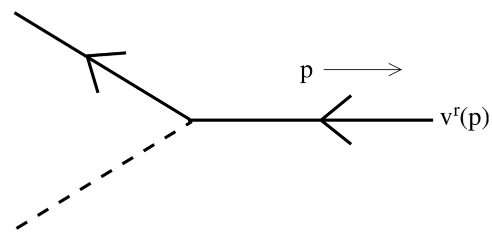


Figure 2.4: Incoming anti-fermion

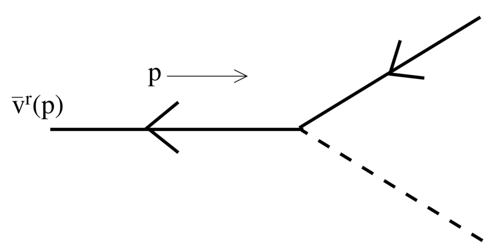


Figure 2.5: Outgoing anti-fermion

- Each vertex is multiplied by a factor of $-\iota\lambda$.
- The relevant propagator's factor is given for each internal line.

$$- - -^{p\rightarrow} - - - \qquad \frac{\iota}{p^2 - \mu^2 + \iota\varepsilon} \qquad \text{for scalars} \tag{2.14}$$

$$\xrightarrow{p\rightarrow} \qquad \frac{\iota(\gamma^\mu p_\mu + m)}{p^2 - m^2 + \iota\varepsilon} \qquad \text{for fermions} \tag{2.15}$$

The fermion lines' arrows must follow a constant path throughout the illustration (this ensures fermion number conservation). The fermionic propagator is indeed a 4×4 matrix, as you can see. At each vertex, the matrix indices are constricted, either with further propagators or through external spinors $u, \bar{u}, v$ or $\bar{v}$.

- Integrate over unknown loop momenta while imposing momentum conservation on every vertex.
- For statistics, add additional minus signs.

Chapter 3

General Relativity

3.1 Einstein Field equation:

The Einstein field equations describe how the existence of matter causes space-time curvature[6].

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} \tag{3.1}$$

or

$$G_{\mu\nu} = 8\pi GT_{\mu} \tag{3.2}$$

or

$$R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}) \tag{3.3}$$

where $R_{\mu\nu}$ is Ricci tesor,

$T_{\mu\nu}$ is stress-energy tensor,

G is Newton's constant of gravitation,

$G_{\mu\nu}$ is Einstein tensor, and $g_{\mu\nu}$ is metric.

$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$ and $R^{\lambda}_{\mu\lambda\nu}$ is Ricci curvature tensor[6].

And for vacuum $T_{\mu\nu}$ is zero. So, Einstein field equation for vacuum is

$$R_{\mu\nu} = 0 \tag{3.4}$$

The Einstein field equations have been used to define the spacetime geometry that results from the existence of mass-energy and linear momentum, or the metric of spacetime for a specific arrangement of stress-energy within spacetime.

The Schwarzschild Solution: The Schwarzschild geometry is the geometry of the vacuum spacetime outside a spherical star. In spherical coordinates $\{t, r, \theta, \phi\}$, the metric is given by

$$ds^2 = -(1 - \frac{2GM}{r})dr^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2 \tag{3.5}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$

$$ds^2 = -(1 - \frac{2GM}{r})dr^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2 d\theta^2 + sin^2\theta d\phi^2 \tag{3.6}$$

The constant M is interpreted as the mass of the gravitating object.

$$g_{\mu\nu} = diag(-(1 - \frac{2GM}{r}), (1 - \frac{2GM}{r})^{-1}, r^2, sin^2\theta) \tag{3.7}$$

in equation (3.6) metric called Schwarzschild metric.

Schwarzschild radius R_S is

$$R_S = 2GM \tag{3.8}$$

3.2 Maximally Symmetric Universes

Homogeneity and isotropy are useful because they suggest that space is maximally symmetric. Consider isotropy to be invariance under rotations, and homogeneity to be invariance under translations. The combination of homogeneity and isotropy implies that space has the largest number of Killing vectors imaginable. The Copernican principle could be taken to its logical conclusion by claiming that spacetime is maximally symmetric. In reality, this will not be the case. We know from observation that the cosmos is homogenous and isotropic in space, but not in all of spacetime. However, it's worth thinking about spacetimes that are maximally symmetric first (which are, after all, special cases of the more general situation in which only space is maximally symmetric). As we will show, such universes are ground states of general relativity in several ways[6].

For a maximally symmetric n-dimensional manifold with metric $g_{\mu\nu}$, the Riemann tensor is represented as

$$R_{\rho\sigma\mu\nu} = \Lambda\left(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\nu}g_{\sigma\mu}\right) \tag{3.9}$$

where Λ is a Ricci curvature measure that has been normalized, and

$$\Lambda = \frac{R}{d(d-1)} \tag{3.10}$$

and the Ricci scalar R will be a constant over the manifold.

For vanishing curvature $\Lambda = 0$ the maximally symmetric spacetime is well known; it is simply Minkowski space with metric

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{3.11}$$

The maximally symmetric spacetime with positive curvature $\Lambda > 0$ is called **de Sitter space**. Consider a Minkowski space with five dimensions and metric $ds_5^2 = -du^2 + dx^2 + dy^2 + dz^2 + dw^2$, Then insert a hyperboloid provided by

$$-u^2 + x^2 + y^2 + z^2 + w^2 = \ell^2 \tag{3.12}$$

Now induce coordinates t, χ, θ, ϕ on the hyperboloid via

$$\begin{aligned}
u &= \ell\sinh(t/\ell)\\
w &= \ell\cosh(t/\ell)\cos(\chi)\\
x &= \ell\cosh(t/\ell)\sin(\chi)\cos(\theta)\\
y &= \ell\cosh(t/\ell)\sin(\chi)\sin(\theta)\cos(\phi)\\
z &= \ell\cosh(t/\ell)\sin(\chi)\sin(\theta)\sin(\phi)
\end{aligned} \tag{3.13}$$

The metric on the hyperboloid is then

$$ds^2 = -dt^2 + \ell^2\cosh(\tau/\ell)\left[d\chi^2 + \sin^2(\chi)\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)\right] \tag{3.14}$$

Consider coordinate transformation from $t to \tau$ via

$$\cosh(t/\ell) = \frac{1}{\cos(\tau)} \tag{3.15}$$

Tme metric equation(3.14) now becomes

$$ds^2 = \frac{\ell^2}{\cos^2(\tau)} d\overline{s}^2 \tag{3.16}$$

where $d\overline{s}^2$ represents the metric on the Einstein static universe,

$$d\overline{s}^2 = -(\tau)^2 + d\chi^2 + \sin^2(\chi) d\Omega_2^2 \tag{3.17}$$

The range of the new time coordinate is

$$-\frac{\pi}{2} < \tau < \frac{\pi}{2} \tag{3.18}$$

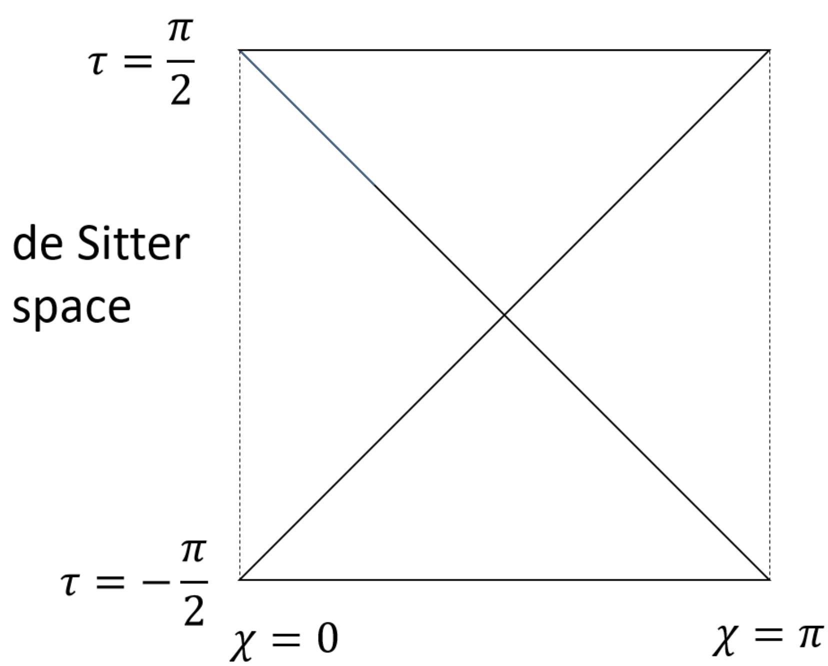


Figure 3.1: Conformal diagram for de Sitter spacetime. Spacelike slices are three spheres, so that points on the diagram represent two-spheres except for those at left and right edges, which are points.

A similar hyperboloid construction reveals the $\Lambda < 0$ spacetime of maximal symmetry, known as anti-de Sitter space. "Begin with a fictitious five-dimensional flat manifold with metric $ds_5^2 = -du^2 - dv^2 + dx^2 + dy^2 + dz^2$, and embed a hyperbolid given by

$$-u^2 - v^2 + x^2 + y^2 + z^2 = -\ell^2 \tag{3.19}$$

Then we can induce coordinate $\{\tau, \rho, \theta, \phi\}$ on the hyperboloid via

$$\begin{aligned} u &= \ell \sin(\tau)\cosh(\rho) \\ w &= \ell \cos(\tau)\cosh(\rho) \\ x &= \ell \sinh(\rho)\cos(\theta) \\ y &= \ell \sinh(\rho)\sin(\theta)\cos(\phi) \\ z &= \ell \sinh(\rho)\sin(\theta)\sin(\phi) \end{aligned} \tag{3.20}$$

yielding a metric on this hyperboloid of the form

$$ds^2 = \ell^2 \left(-\cosh^2(\rho)d\tau^2 + d\rho^2 + \sinh^2(\rho)d\Omega_2^2\right) \tag{3.21}$$

To derive the conformal diagram, perform a coordinate transformation analogous to that used for de Sitter, but now on the radial coordinate:

$$\cosh(\rho) = \frac{1}{\cos(\chi)} \tag{3.22}$$

so that

$$ds^2 = \frac{\ell^2}{\cos^2(\chi)} d\bar{s}^2 \tag{3.23}$$

where $d\bar{s}^2$ represents the metric on the Einstein static universe equation (3.17). Unlike in de Sitter, the radial coordinate now appears in the conformal factor. In addition, for anti-de Sitter, the τ coordinate goes from minus infinity to plus infinity, while the range of the radial coordinate is

$$0 \leq \chi < \frac{\pi}{2} \tag{3.24}$$

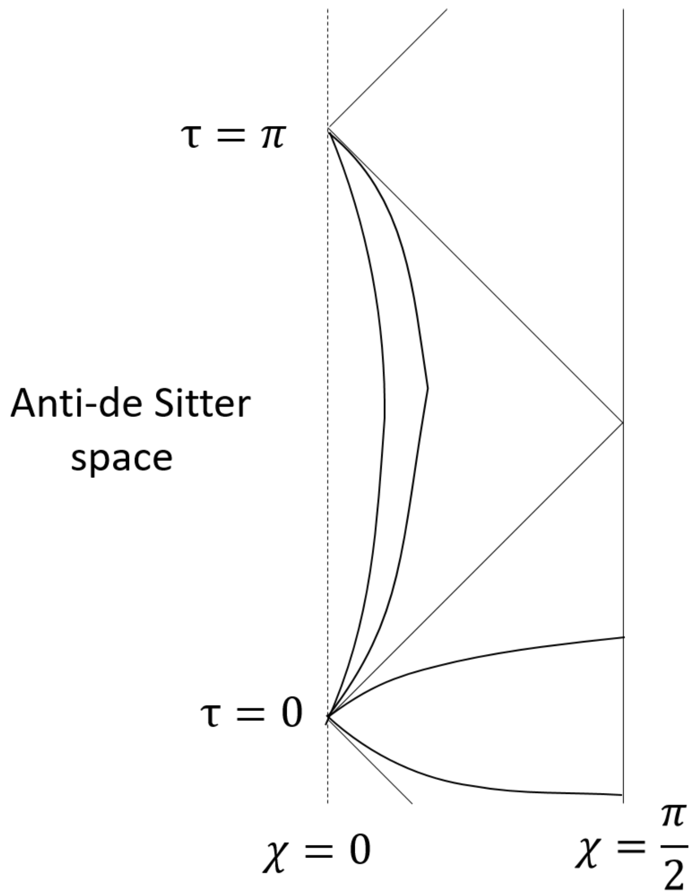


Figure 3.2: Conformal diagram for anti-de Sitter spacetime. Spacelike slices have the topology of R^3, which we have represented in polar coordinates, so that points on the diagram stand for two-spheres except those at the left side, which stand for single points at the spatial origin. Infinity is a timelike surface at the right side.

Chapter 4

Bulk Reconstruction

4.1 Introduction

The AdS/CFT correspondence is commonly expressed as the equality of the bulk and boundary theories partition functions[12][7][15].
A different formulation of the correspondence, which is expected to be equivalent to the statement above, is the extrapolation dictionary[2][3][4], which we state here for scalar fields :

$$\lim_{r\to\infty} r^{n\Delta}\langle\phi(r,t_1,\Omega_1)\phi(r,t_2,\Omega_2)......\phi(r,t_n,\Omega_n)\rangle_{\text{pure AdS}}$$
$$= \langle 0|\mathcal{O}(t_1,\Omega_1)\mathcal{O}(t_2,\Omega_2)........\mathcal{O}(t_n,\Omega_n)|0\rangle \quad (4.1)$$

Here $\mathcal{O}$ is the scalar primary dual to the bulk scalar ϕ. It has dimension Δ which is related to the mass M of the scalar field as $\Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4M^2}$ where d is the number of space dimensions. A similar dictionary can be written down for other fields.
This was for pure AdS. More generally, for any semi-classical asymptotically AdS geometry g we expect that there will be a dual state $|\psi_g\rangle$

$$g \longleftrightarrow |\psi_g\rangle \quad (4.2)$$

such that

$$\lim_{r\to\infty} r^{n\Delta}\langle\phi(r,t_1,\Omega_1)\phi(r,t_2,\Omega_2)......\phi(r,t_n,\Omega_n)\rangle_g$$
$$= \langle\psi_g|\mathcal{O}(t_1,\Omega_1)\mathcal{O}(t_2,\Omega_2)........\mathcal{O}(t_n,\Omega_n)|\psi_g\rangle \quad (4.3)$$

Equation (4.1) is a specific situation in which geometry g is pure AdS and the dual state is the CFT vacuum $|0\rangle$:

$$\text{Pure AdS} \longleftrightarrow |0\rangle$$

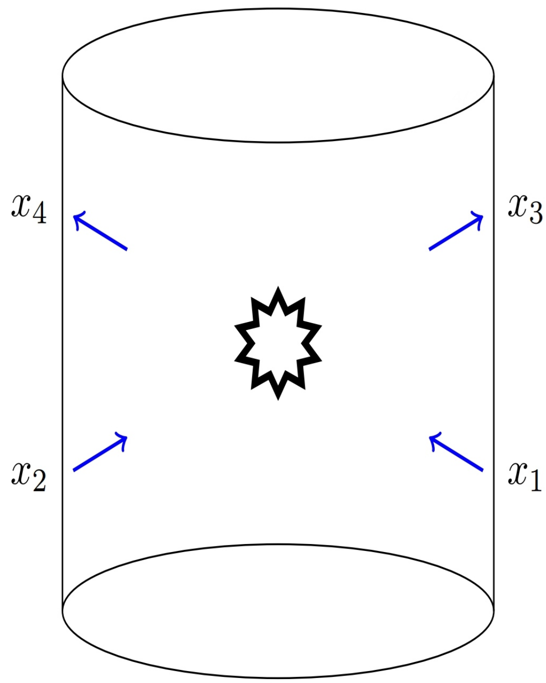
The two-sided eternal black hole is another example of semiclassical asymptotically AdS spacetime. Two asymptotic boundaries exist for the eternal black hole. As an outcome, the tensor product of the Hilbert Spaces of the two CFTs on the two boundaries must contain the state dual to the eternal black hole. The theromofield double state is the correct dual state:

$$\text{Eternal Black Hole} \longleftrightarrow \frac{1}{\sqrt{Z(\beta)}}\sum_E e^{-\beta E}|E\rangle\langle E| \quad (4.4)$$

where β is the inverse temperature of the black hole and $Z(\beta)$ is the partition function at inverse temperature β, and the sum is over energy eigenstates.

In general, we don't know what a boundary state's bulk dual is.

The extrapolation dictionary already contains some bulk physics information. We can do scattering experiments, in which we send in wave packets from close to the boundary, scatter them, and then collect them close to the boundary later. The CFT correlator $\langle\mathcal{O}(X_1)\mathcal{O}(X_2)\mathcal{O}(X_3)\mathcal{O}(X_4)\rangle$ will have the results of such scattering experiments.



$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

Figure 4.1: A bulk scattering experiment.

However, this does not cover all bulk information. For example, the extrapolate dictionary cannot directly consider the problem of the correlator between bulk fields for finite values of r, which may be relevant for the description of a local bulk experiment. A bulk-boundary dictionary would need to be further developed.

4.1.1 Statement of the program

We'll work with semi-classical bulk geometry all the time. The gravitational constant $G << \ell^{d-1}$, where ℓ is the AdS radius, is required for semiclassicality.

A set of conditions must be satisfied for a CFT to have a semi-classical bulk dual[10]. It is critical to have a parameter $N >> 1$ that governs the factorization of the CFT correlators and can be dual to the perturbative parameter in the bulk theory. In the CFTs known to have a bulk dual, the role of N is played by the central charge of the CFT. In the CFT, N is the expansion parameter, and it is related to the gravitational constant as follows:

$$N^2 = \frac{l^{d-1}}{G} \tag{4.5}$$

In the CFT, we've only introduced one expansion parameter thus far. As a consequence, the most general dual bulk theory with Einstein gravity and scalar fields will have a similar action(in units where a radius of AdS is one):

$$\begin{aligned} S = \frac{1}{G}\int d^{d+1}y\sqrt{-g}R + \int d^{d+1}y\sqrt{-g}\left(\partial_\mu\phi\partial^\mu\phi + M^2\phi^2\right) \\ + \lambda\sqrt{G}\int d^{d+1}y\sqrt{-g}\left(\frac{\phi^3}{3!} + \text{all possible cubic coupling}\right) \\ + \lambda^{'}G\int d^{d+1}y\sqrt{-g}\left(\frac{\phi}{4!} + \text{all possible quartic coupling}\right) + ... \end{aligned} \tag{4.6}$$

In this case, λ and $\lambda^{'}$ are $\mathcal{O}(1)$ numbers. The couplings' strengths are constrained. A general bulk field theory can contain couplings of varying strengths (e.g., the standard model), but a theory with a holographic CFT dual cannot unless more expansion parameters are included.

Our goal is to find CFT operators for all bulk points that represent bulk fields. That is, ϕ_{CFT} of them fulfill the following criteria:

$$\langle\phi(r_1,t_1,\Omega_1)\phi(r_2,t_2,\Omega_2)\rangle_g = \langle\psi_g|\phi_{CFT}(r_1,t_1,\Omega_1)\phi_{CFT}(r_2,t_2,\Omega_2)|\psi_g\rangle \tag{4.7}$$

4.2 Boundary representation of free fields in the bulk

4.2.1 Free scalar fields in AdS :-

The AdS metric is given by:

$$ds^2 = \ell^2 \left(-\cosh^2(\rho)d\tau^2 + d\rho^2 + sinh^2(\rho)d\Omega_{d-1}^2\right)$$

With the transformations $r = \ell sinh(\rho)$ and $t = \ell\tau$ we can have the usual AdS_{d+1} metric in global coordinates:

$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{1 + \frac{r^2}{\ell^2}} + r^2 d\Omega_{d-1}^2 \tag{4.8}$$

where ℓ is AdS radius.

We'll take $\ell = 1$ from now on.

$$ds^2 = -\left(1 + r^2\right)dt^2 + \frac{dr^2}{(1 + r^2)} + r^2 d\Omega_{d-1}^2$$

We shall operate in a semi-classical framework in which bulk action is provided by (4.6).We find the free field equation in pure AdS by setting $G \to 0(N \to \infty$ limit in the CFT) .

$$\left(\Box - M^2\right)\phi = 0 \tag{4.9}$$

where $\Box$ is the D'Alembartian in anti-de Sitter space-time. And M is mass parameter for the field ϕ.

The D'Alembartion operator in d+1 dimensions is written as

$$\Box = \frac{1}{\ell^2}\left(-\frac{\ell^2\partial_t^2}{\left(1 + \frac{r^2}{\ell^2}\right)} + \ell^2\left(1 + \frac{r^2}{\ell^2}\right)\partial_r^2 + \left(\frac{d-1}{r}\ell^2\left(1 + \frac{r^2}{\ell^2}\right) + 2r\right)\partial_r + \frac{\ell^2}{r^2}\Box_{\Omega_{d-1}}\right)$$

We'll take $\ell = 1$ and we get

$$\Box = \left(-\frac{\partial_t^2}{(1 + r^2)} + \left(1 + r^2\right)\partial_r^2 + \left(\frac{d-1}{r}\left(1 + r^2\right) + 2r\right)\partial_r + \frac{1}{r^2}\Box_{\Omega_{d-1}}\right)$$

Gravity is turned off at this limit. As a result, we may ignore gravity and consider the scalar field in a fixed background. Let's look up the quantum theory that applies to this field.

From rotational and time translation symmetry of the metric (4.8) we know that the solution to equation (4.9) will be of the form

$$f_{\omega l\overrightarrow{m}}(r,t,\Omega) = \psi_{\omega l}(r)e^{-\iota\omega t}Y_{l\overrightarrow{m}}(\Omega)$$

where $Y_{l\overrightarrow{m}}(\Omega)$ are the usual spherical harmonics.

Substituting this in equation (4.9) gives:(For derivation see Appendix A.)

$$(1+r^2)\psi'' + \left(\frac{d-1}{r}(1+r^2)+2r\right)\psi' + \left(\frac{\omega^2}{(1+r^2)} - \frac{l(l+d-2)}{r^2} - M^2\right)\psi = 0 \tag{4.10}$$

At large r this becomes

$$r^2\psi'' + \left(\frac{d-1}{r}r^2+2r\right)\psi' - M^2\psi = 0$$

$$r^2\psi'' + ((d-1)r+2r)\psi' - M^2\psi = 0$$

$$r^2\psi'' + (rd-r+2r)\psi' - M^2\psi = 0$$

$$r^2\psi'' + (d+1)\,r\psi' - M^2\psi = 0 \tag{4.11}$$

There are certainly polynomial solutions of the kind $r^{-\alpha}$. By substituting $\psi(r) = r^{-\alpha}$ in the previous equation, we have two independent solutions, $\alpha = \Delta, d-\Delta$.

where

$$\Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4M^2} \tag{4.12}$$

As a conclusion, the asymptotic solution to equation (4.9) will take the following form:

$$\phi(r,t,\Omega) = r^{\Delta-d}K(t,\Omega) + r^{-\Delta}L(t,\Omega) \tag{4.13}$$

Normalizable modes are the ones with $r^{-\Delta}$ fall off. These are the ones we need to define a unitary field theory in AdS.

We further impose smoothness at $r = 0$ which quantizes ω :

$$\omega_{nl} = \Delta + l + 2n \tag{4.14}$$

where $n = 0, 1, 2, ...$

The full solution of $f_{\omega l \vec{m}}(r, t, \Omega)$ is given by

$$f_{\omega l \vec{m}}(r, t, \Omega) = \frac{1}{N_{\Delta nl}} e^{-\iota \omega_{nl} t} Y_{l \vec{m}}(\Omega) \left(\frac{r}{\sqrt{1+r^2}} \right)^l \left(\frac{1}{\sqrt{1+r^2}} \right)^\Delta F_1 \left(-n, \Delta + l + n, l + \frac{d}{2}, \frac{r^2}{1+r^2} \right) \tag{4.15}$$

where N is the normalization constant.

Now that we have the modes we can quantize the fields.

$$\phi(r, t, \Omega) = \phi^- + \phi^+$$

$$\phi(r, t, \Omega) = \sum_{nl\vec{m}} \left(f_{\omega l \vec{m}}(r, t, \Omega) a_{\omega l \vec{m}} + f^*_{\omega l \vec{m}}(r, t, \Omega) a^\dagger_{\omega l \vec{m}} \right) \tag{4.16}$$

Here $a, a^\dagger$are the annihilation and creation operator. They create normalizable particle excitations in the bulk.

4.2.2 Free field reconstruction in mode sum approach:-

As a CFT operator, we want to reproduce the free scalar field from the previous section. That is, we seek a CFT operator that satisfies the following criteria:

$$\langle \phi(r_1, t_1, \Omega_1) \phi(r_2, t_2, \Omega_2) \rangle_{pureAdS} = \langle 0 | \phi_{CFT}(r_1, t_1, \Omega_1) \phi_{CFT}(r_2, t_2, \Omega_2) | 0 \rangle \tag{4.17}$$

Because higher-order correlators factorize to products of two-point functions in free field theory, it is sufficient to study only two-point functions. Large N factorization is a dual phenomenon in the CFT.

How do we obtain such a ϕ_{CFT} ? This problem was first solved in [5][9][8]. The HKLL construction is named after Hamilton, Kabat, Lifschytz, and Lowe, who pioneered some of the techniques used in this field.

To get this representation, we would first have to know that the bulk field fulfills the free field equation.

$$\left(\Box - M^2\right)\phi = 0 \tag{4.18}$$

We also notice that the extrapolate dictionary resembles a bulk field boundary condition:

$$\lim_{r\to\infty} r^{\Delta}\phi(r,t,\Omega) = \mathcal{O}(t,\Omega) \tag{4.19}$$

In conformal field theory, this equation connects the field's boundary value to a primary operator. If we solve equation (4.9) with equation (4.19) as boundary conditions, we have an expression for ϕ in terms of conformal field theory operators $\mathcal{O}$.

Of course, because it maps fields between two separate spaces, equation (4.19)ef isn't truly a boundary condition. The right-hand side is a CFT operator acting on the CFT Hilbert space, whereas the left-hand side is a bulk field's boundary value. So what we'll truly want to do is find a CFT operator $\phi_{CFT}(r,t,\Omega)$ that fulfills the necessary criteria:

$$\left(\Box - M^2\right)\phi_{CFT}(r,t,\Omega) = 0 \tag{4.20}$$

This conformal field theory operator is dependent on r, which may be thought of as a parameter. Then we demand that in the limit where this parameter becomes large, ϕ_{CFT} is given by equation (4.19). Then we figure out how to solve this conformal field theory operator.

This is the correct way of thinking about bulk reconstruction, but in terms of problem-solving logistics, it's the same as solving equation (4.9) as a bulk equation of motion as a boundary value problem using equation (4.19) as the boundary value. This distinction is not made in the literature. The bulk field ϕ and its CFT representation ϕ_{CFT} is usually denoted by the same notation. Now that we understand what's going on, we'll drop the distinction and refer to ϕ_{CFT} as simply ϕ from now on, unless there's a risk of confusion between the bulk field and its CFT representation.

It's important to realize that equation (4.19) isn't a typical boundary value problem. In most cases, initial conditions are specified on a space-like Cauchy surface in field theory. We're defining boundary conditions on a time-like surface in this case. In mathematics, this is not a well-studied problem. As we'll see, the solution isn't unique.

In this case, though, solving the boundary value problem is quite simple. For the sake of simplicity, we'll assume Δ is an integer. The solution then becomes time-periodic, and we can restrict the range of t to $-\pi$ to π.

We start from the expansion equation (4.16) and plug it in (4.19):

$$\lim_{r\to\infty} r^{\Delta}\phi(r,t,\Omega) = \lim_{r\to\infty} r^{\Delta}\left(\sum_{nl\vec{m}}\left(f_{\omega l\vec{m}}(r,t,\Omega)a_{\omega l\vec{m}} + f^*_{\omega l\vec{m}}(r,t,\Omega)a^{\dagger}_{\omega l\vec{m}}\right)\right) = \mathcal{O}(t,\Omega) \tag{4.21}$$

Now

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} r^{\Delta}\frac{1}{N_{\Delta nl}}e^{-\iota\omega_{nl}t}Y_{l\vec{m}}(\Omega)\left(\frac{r}{\sqrt{1+r^2}}\right)^{l}\left(\frac{1}{\sqrt{1+r^2}}\right)^{\Delta} F_1\left(-n,\Delta+l+n,l+\frac{d}{2},\frac{r^2}{1+r^2}\right)$$

For derivation see Appendix B.

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \frac{1}{N_{\Delta nl}}e^{-\iota\omega_{nl}t}Y_{l\vec{m}}(\Omega)F_1\left(-n,\Delta+l+n,l+\frac{d}{2},1\right) \tag{4.22}$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = g_{\omega l\vec{m}}(t,\Omega) \tag{4.23}$$

where

$$g_{\omega l\vec{m}}(t,\Omega) = \frac{1}{N_{\Delta nl}}e^{-\iota\omega_{nl}t}Y_{l\vec{m}}(\Omega)F_1\left(-n,\Delta+l+n,l+\frac{d}{2},1\right)$$

and similarly

$$\lim_{r\to\infty} r^{\Delta} f^*_{\omega l\vec{m}}(r,t,\Omega) = g^*_{\omega l\vec{m}}(t,\Omega)$$

$$\sum_{nl\vec{m}}\left(g_{\omega l\vec{m}}(t,\Omega)a_{\omega l\vec{m}} + g^*_{\omega l\vec{m}}(t,\Omega)a^{\dagger}_{\omega l\vec{m}}\right) = \mathcal{O}(t,\Omega) \tag{4.24}$$

When Δ is an integer, $g_{\omega l \overrightarrow{m}}(t, \Omega)$ and $g^*_{\omega l \overrightarrow{m}}(t, \Omega)$ are orthogonal. Using the orthonormality and completeness of the functions $e^{-\iota \omega_{nl} t}$ and $Y_{l\overrightarrow{m}}(\Omega)$, we can now invert this relationship. As a result, we define:

$$\tilde{g}_{\omega l \overrightarrow{m}}(t, \Omega) = \frac{N_{\Delta nl}}{{}_2F_1\left(-n, \Delta + l + n, l + \frac{d}{2}, 1\right)} e^{-\iota \omega_{nl} t} Y_{l\overrightarrow{m}}(\Omega)$$

Then we can come up with a solution for a :

$$a_{\omega l \overrightarrow{m}} = \int_{-\pi}^{\pi} dt \int d\Omega \tilde{g}^*_{\omega l \overrightarrow{m}}(t, \Omega) \mathcal{O}(t, \Omega) \tag{4.25}$$

and similarly, we can solve for $a^\dagger$:

$$a_{\omega l \overrightarrow{m}} = \int_{-\pi}^{\pi} dt \int d\Omega \tilde{g}^*_{\omega l \overrightarrow{m}}(t, \Omega) \mathcal{O}(t, \Omega)$$

When we put it back in, we get

$$\begin{aligned}\phi(r, t, \Omega) = \sum_{nl\overrightarrow{m}} \Bigg(& f_{\omega l \overrightarrow{m}}(r, t, \Omega) \int_{-\pi}^{\pi} dt' \int d\Omega' \tilde{g}^*_{\omega l \overrightarrow{m}}(t', \Omega') \mathcal{O}(t', \Omega') \\ & + f^*_{\omega l \overrightarrow{m}}(r, t, \Omega) \int_{-\pi}^{\pi} dt'' \int d\Omega'' \tilde{g}_{\omega l \overrightarrow{m}}(t'', \Omega'') \mathcal{O}(t'', \Omega'') \Bigg)\end{aligned}$$

$$\phi(r, t, \Omega) = \int_{-\pi}^{\pi} dt' \int d\Omega' \left(\sum_{nl\overrightarrow{m}} f_{\omega l \overrightarrow{m}}(r, t, \Omega) \tilde{g}^*_{\omega l \overrightarrow{m}}(t', \Omega') + \text{complex conjugate} \right) \mathcal{O}(t', \Omega') \tag{4.26}$$

It turns out that $\sum\limits_{nl\overrightarrow{m}} f_{\omega l \overrightarrow{m}}(r, t, \Omega) \tilde{g}^*_{\omega l \overrightarrow{m}}(t', \Omega')$ is real. So this is equal to its complex conjugate.

$$\phi(r, t, \Omega) = \int_{-\pi}^{\pi} dt' \int d\Omega' 2 \left(\sum_{nl\overrightarrow{m}} f_{\omega l \overrightarrow{m}}(r, t, \Omega) \tilde{g}^*_{\omega l \overrightarrow{m}}(t', \Omega') \right) \mathcal{O}(t', \Omega')$$

$$\phi(r, t, \Omega) = \int_{-\pi}^{\pi} dt' \int d\Omega' K(r, t, \Omega, t', \Omega') \mathcal{O}(t', \Omega') \tag{4.27}$$

where

$$K(r, t, \Omega, t', \Omega') = 2 \left(\sum_{nl\overrightarrow{m}} f_{\omega l \overrightarrow{m}}(r, t, \Omega) \tilde{g}^*_{\omega l \overrightarrow{m}}(t', \Omega') \right) \tag{4.28}$$

This is known as the smearing function.

Using equation (4.22) we have that

$$K(r,t,\Omega,t',\Omega') \propto \sum_{nl\vec{m}} f_{\omega l\vec{m}}(r,t,\Omega)e^{\iota\omega_{nl}t}Y^*_{l\vec{m}}(\Omega) \tag{4.29}$$

As a consequence, the smearing function is proportional to the mode function's Fourier transform.

It's worth noting that the smearing feature isn't unique. We can see from equation (4.14) that only modes between $-\Delta$ and Δ appear in the solution for $\mathcal{O}(t,\Omega)$. Therefore if we add a term $e^{\iota kt}$ to the smearing function where k is any integer between $-\Delta+1$ and $\Delta-1$, the integration $\int dte^{\iota kt}\mathcal{O}(r,t,\Omega)$ vanishes. So we can add any term of the form $\sum_k c_k e^{\iota kt}$ to the smearing function without changing equation (4.27).

This freedom allows us to put the smearing function in a convenient form. In particular, we can arrange for the smearing function to be non-zero only at boundary points space-like separated from the bulk point (r, t). This is the minimal support that it can have[9].

We now have an expression for the bulk field's boundary representation. We can represent it simply by writing the bulk coordinate as y and the boundary coordinate as X:

$$\phi(y) = \int dXK(y;X)\mathcal{O}(X) \tag{4.30}$$

Where the range of integration is over all points X in the boundary which are space-like separated from the bulk point y. Note that this is a non local operator in the CFT.

Now that we have the CFT representation $\phi(r,t,\Omega)$ we can check whether it indeed satisfies the condition equation (4.17). Let's sketch down the check's steps. To begin, we need to point out that

$$\langle 0|\phi(y)\phi(y')|0\rangle = \int dXdX'K(y,X)K(y',X')\langle 0|\mathcal{O}(X)\mathcal{O}(X')|0\rangle \tag{4.31}$$

Where we've used equation (4.30). Now $\langle 0|\mathcal{O}(X)\mathcal{O}(X')|0\rangle$ is fixed completely by symmetry. We can easily evaluate the equation above. It gives the correct bulk two-point function, as one may expect. The information about the bulk has been encoded in the boundary operator through the smearing function.

We worked in global coordinates here, but we could have used Poincare coordinates as well. This produces a smearing function with support on the Poincare patch's boundary. This corresponds to the global smearing function in Poincare patch coordinates, up to the ambiguities in the definition of the smearing function mentioned above[8].

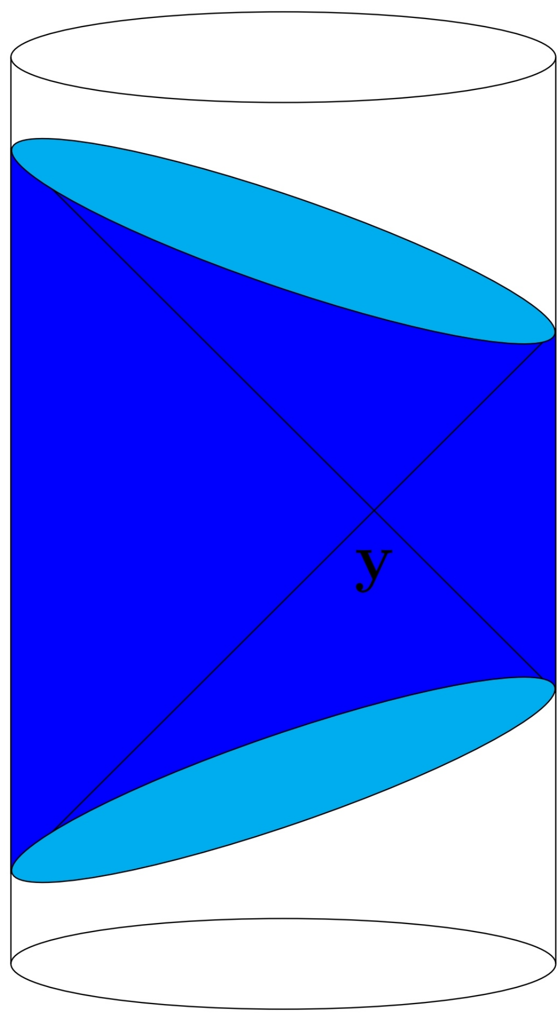


Figure 4.2: The boundary representation of a bulk scalar field at a point y has support on all boundary points space-like separated from y.

Chapter 5

Conclusions

The HKLL construction in Anti-de Sitter spacetime is studied, and the smearing function for free field theories was derived. We discovered AdS-covariant smearing functions with support simply at spacelike separation through global coordinates.We worked in global coordinates here, but we could have used Poincare coordinates as well. This produces a smearing function with support on the Poincare patch's boundary.We only have a good understanding of bulk reconstruction for large N; finite N (i.e. quantum gravity in the bulk) is still a mystery.$1/N$ corrections are known in many examples.

Chapter 6

Appendix

6.1 Appendix A

From rotational and time translation symmetry of the metric (4.8) we know that the solution to (4.9) will be of the form

$$f_{\omega l \overrightarrow{m}}(r, t, \Omega) = \psi_{\omega l}(r) e^{-\iota \omega t} Y_{l \overrightarrow{m}}(\Omega)$$

where $Y_{l\overrightarrow{m}}(\Omega)$ are the usual spherical harmonics.

Substituting this in (4.9) gives:

$$\left(\Box - M^2\right) \psi_{\omega l}(r) e^{-\iota \omega t} Y_{l \overrightarrow{m}}(\Omega) = 0 \tag{6.1}$$

$$\left(-\frac{\partial_t^2}{(1+r^2)} + \left(1+r^2\right)\partial_r^2 + \left(\frac{d-1}{r}\left(1+r^2\right) + 2r\right)\partial_r + \frac{1}{r^2}\Box_{\Omega_{d-1}} - M^2\right) \psi_{\omega l}(r) e^{-\iota \omega t} Y_{l \overrightarrow{m}}(\Omega) = 0$$

$$\begin{aligned} &-\psi_{\omega l}(r) Y_{l \overrightarrow{m}}(\Omega) \frac{\partial_t^2 e^{-\iota \omega t}}{(1+r^2)} + e^{-\iota \omega t} Y_{l \overrightarrow{m}}(\Omega)\left(1+r^2\right)\partial_r^2 \psi_{\omega l}(r) \\ &\quad + e^{-\iota \omega t} Y_{l \overrightarrow{m}}(\Omega)\left(\frac{d-1}{r}\left(1+r^2\right) + 2r\right)\partial_r \psi_{\omega l}(r) \\ &\quad + \psi_{\omega l}(r) e^{-\iota \omega t}\frac{1}{r^2}\Box_{\Omega_{d-1}} Y_{l \overrightarrow{m}}(\Omega) - M^2 \psi_{\omega l}(r) e^{-\iota \omega t} Y_{l \overrightarrow{m}}(\Omega) = 0 \end{aligned}$$

above equation divided by $\psi_{\omega l}(r)e^{-\iota\omega t}Y_{l\overrightarrow{m}}(\Omega)$

$$-\frac{1}{e^{-\iota\omega t}}\frac{\partial_t^2 e^{-\iota\omega t}}{(1+r^2)}+\frac{(1+r^2)}{\psi_{\omega l}(r)}\partial_r^2\psi_{\omega l}(r)+\left(\frac{d-1}{r}\left(1+r^2\right)+2r\right)\frac{1}{\psi_{\omega l}(r)}\partial_r\psi_{\omega l}(r) \\ +\frac{1}{r^2Y_{l\overrightarrow{m}}(\Omega)}\Box_{\Omega_{d-1}}Y_{l\overrightarrow{m}}(\Omega)-M^2=0$$

$$-\frac{1}{e^{-\iota\omega t}}\frac{(-\omega^2 e^{-\iota\omega t})}{(1+r^2)}+\frac{(1+r^2)}{\psi}\psi''+\left(\frac{d-1}{r}\left(1+r^2\right)+2r\right)\frac{1}{\psi}\psi' \\ +\frac{1}{r^2Y_{l\overrightarrow{m}}(\Omega)}(-l(l+d-2)Y_{l\overrightarrow{m}}(\Omega))-M^2=0$$

where we have used

$$\Box_{\Omega_{d-1}}Y_{l\overrightarrow{m}}(\Omega)=-l(l+d-2)Y_{l\overrightarrow{m}}(\Omega)$$

$$\frac{\omega^2}{(1+r^2)}+\frac{(1+r^2)}{\psi}\psi''+\left(\frac{d-1}{r}\left(1+r^2\right)+2r\right)\frac{1}{\psi}\psi'-\frac{l(l+d-2)}{r^2}-M^2=0$$

$$\frac{(1+r^2)}{\psi}\psi''+\left(\frac{d-1}{r}\left(1+r^2\right)+2r\right)\frac{1}{\psi}\psi'\left(\frac{\omega^2}{(1+r^2)}-\frac{l(l+d-2)}{r^2}-M^2\right)=0$$

$$(1+r^2)\psi''+\left(\frac{d-1}{r}(1+r^2)+2r\right)\psi'+\left(\frac{\omega^2}{(1+r^2)}-\frac{l(l+d-2)}{r^2}-M^2\right)\psi=0 \tag{6.2}$$

6.2 Appendix B

We start from the expansion equation (4.16) and plug it in equation (4.19):"

$$\lim_{r\to\infty}r^{\Delta}\phi(r,t,\Omega)=\lim_{r\to\infty}r^{\Delta}\left(\sum_{nl\overrightarrow{m}}\left(f_{\omega l\overrightarrow{m}}(r,t,\Omega)a_{\omega l\overrightarrow{m}}+f^*_{\omega l\overrightarrow{m}}(r,t,\Omega)a^\dagger_{\omega l\overrightarrow{m}}\right)\right)=\mathcal{O}(t,\Omega) \tag{6.3}$$

Now

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} r^{\Delta}\frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega)\left(\frac{r}{\sqrt{1+r^2}}\right)^{l}\left(\frac{1}{\sqrt{1+r^2}}\right)^{\Delta}$$
$$F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{r^2}{1+r^2}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} r^{\Delta}\frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega)(r^{l})\left(\frac{1}{\sqrt{1+r^2}}\right)^{l}\left(\frac{1}{\sqrt{1+r^2}}\right)^{\Delta}$$
$$F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{r^2}{r^2\left(\frac{1}{r^2}+1\right)}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) r^{\Delta+l}\left(\frac{1}{\sqrt{1+r^2}}\right)^{\Delta+l}$$
$$F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{1}{\left(\frac{1}{r^2}+1\right)}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) r^{\Delta+l}\left(\frac{1}{r\sqrt{(\frac{1}{r^2}+1}}\right)^{\Delta+l}$$
$$F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{1}{\left(\frac{1}{r^2}+1\right)}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) r^{\Delta+l}\frac{1}{r^{\Delta+l}}\left(\frac{1}{\sqrt{(\frac{1}{r^2}+1}}\right)^{\Delta+l}$$
$$F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{1}{\left(\frac{1}{r^2}+1\right)}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \lim_{r\to\infty} \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) \left(\frac{1}{\sqrt{(\frac{1}{r^2}+1}}\right)^{\Delta+l} F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{1}{\left(\frac{1}{r^2}+1\right)}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) \left(\frac{1}{1}\right)^{\Delta+l} F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, \frac{1}{1}\right)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, 1\right) \quad (6.4)$$

$$\lim_{r\to\infty} r^{\Delta} f_{\omega l\vec{m}}(r,t,\Omega) = g_{\omega l\vec{m}}(t,\Omega) \quad (6.5)$$

where

$$g_{\omega l\vec{m}}(t,\Omega) = \frac{1}{N_{\Delta nl}} e^{-\iota\omega_{nl}t} Y_{l\vec{m}}(\Omega) F_1\left(-n, \Delta+l+n, l+\frac{d}{2}, 1\right)$$

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