HIGHER ORDER STATISTICS FOR UNDERLAY COGNITIVE RADIO SYSTEM

M.Tech. Thesis

By

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DISCIPLINE OF ELECTRICAL ENGINEERING

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HIGHER ORDER STATISTICS FOR UNDERLAY COGNITIVE RADIO SYSTEM

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **HIGHER ORDER STATISTICS FOR UNDERLAY COGNITIVE RADIO SYSTEM** in the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** and submitted in the **DISCIPLINE OF ELECTRICAL ENGINNERING,, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from JULY 2014 to JUNE 2016 under the supervision of Dr.Prabhat Kumar Upadhyay, Assistant Professor, Discipline of Electrical Engineering, IIT Indore.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

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This is to certify that the above statement made by the candidate is correct to the best of my/our knowledge.

Signature of the Supervisor of
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Abstract

In last few decades, the wireless communication technology observes a rapid advancement because of its escalating demand and day-to-day importance. Subsequently, the field has attracted recent interest from researchers to find new approaches which can be more promising for throughput enhancement over the restrained spectrum. These new arising techniques must be able to utilize scarce spectrum resources efficiently and reliably. One of such emerging technologies is cognitive radio which can be seen as a potential solution to mitigate the problem of spectrum scarcity.

In this thesis, we investigate the outage probability and second order statistics viz., level crossing rate (LCR) and average fade duration (AFD) of an underlay cognitive radio system in the presence of multiple primary receivers. We consider an underlay scenario where the communication between secondary source and secondary destination takes place over the same spectrum licensed to primary system. This cognitive radio communication is subject to the constraint that interference temperature at primary receiver should lie below a predefined level. With this setup, we investigate first order statistics such as outage probability of the considered system. The first order statistics alone are not sufficient to address wireless communication system design issues. Since the pragmatic wireless channel is slowly time-varying, various design issues can be addressed by combining first order statistics with the second order statistics of wireless communication systems. In the considered system, the receivers are assumed to be mobile and hence the fixed-to-mobile channels are adopted. To the best of our knowledge, little attention has been directed for such systems in terms of the temporal characteristics such as LCR and AFD without which overall system may not be designed and rigorously tested. In this thesis, we investigate second order statistics viz., LCR and AFD of the cognitive radio system in a Rayleigh fading environment. First, we obtain closed-form expressions of LCR and AFD by using appropriate mathematical tools. Then, we confirm our analysis through numerical and simulation investigation. Finally, the impact of system parameters on LCR and AFD are highlighted. Results demonstrated that by using efficient power and bandwidth allocation method, the throughput of the cognitive radio system can be increased. The results presented in the thesis can thus provide vital information for proper deployment of a cognitive radio system.

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Chapter 1

Introduction

1.1 Overview of Cognitive Radio Systems

In the communication industry, wireless communication is the fastest growing segment, and it has completely revolutionized our lives. To meet the demand for high quality and high-speed services like wi-fi, video calling, etc., continuous efforts are made to develop new wireless devices capable of achieving higher data rate. However, these devices require larger bandwidth to operate which results in an increase in demand for radio spectrum. One of most popular and successful wireless services is mobile communication which is totally dependent on small usable part of very expensive radio-frequency spectrum. If we follow the technological advancement in mobile communication as suggested by Table 1.1, we can easily observe that in near future, we will run out of electromagnetic spectrum to accommodate emerging wireless products.

Technology	1G	2G	3G	4G
Deployment	1970/1984	1980/1999	1990/2002	2000/2010
Bandwidth	2Kbps	14-64Kbps	2Mbps	200Mbps
Multiplexing	FDMA	TDMA/FDMA	CDMA	OFDMA/SC-FDMA

Table 1.1: Technology advancement in wireless communication [1].

Mostly, radio spectrum between 30 MHz to 30 GHz is used for wireless applications as earth curvature does not affect these frequencies, and moderately sized antennas are required for communication at these frequencies. Table 1.2 shows the spectrum allocated to various wireless technology.

AM Radio	534-1605 KHz
FM Radio	88-108 MHz
Broadcast TV	54-88 MHz, 174-216 MHz, 470-806 MHz
3G Broadband Wireless	746-764 MHz, 776-794 MHz
3G Broadband Wireless	1.7-1.85 MHz ,2.5-2.69 MHz
2G Digital Cellular Phones	806-902 MHz
Personal Communication Services(2G Cell Phones)	1.85-1.99 GHz
Wireless Communication Service	2.305-2.32 GHz, 2.345-2.36 GHz
Satellite Digital Radio	2.32-2.325 GHz
Multichannel Multipoint Distribution Service	2.15-2.68 GHz
Digital Broadcast Satellite	12.2-12.7 GHz
Digital Electronic Message Service	24.25-24.45 GHz, 25.05-25.25 GHz
Local Multipoint Distribution Service	27.5-29.5 GHz, 31-31.3 GHz
Fixed Wireless Services	38.6-40 GHz

Table 1.2: Radio spectrum allocation [2].

Various government regulating authorities of conducted a survey on spectrum occupancy measurement [3]. Their reports suggest that the spectrum is poorly utilized. In fact, Federal Communications Commission (FCC) clearly states that “rather than the physical scarcity of spectrum, spectrum access is a more significant problem. The ability to obtain such access of any potential spectrum user is limited by currently used legacy command and control regulation”. This inflexible spectrum allocation procedure followed worldwide by government agencies has resulted in spectrum holes, i.e., the unused band at the particular time or geographical location. A technology named ‘Cognitive Radio’ was proposed by Mitola [4] and have been suggested as a solution to this spectrum scarcity. The problem of an insufficient spectrum can be combated by exploiting spectrum holes and is the basis of cognitive radio.

Cognitive radio is designed in such a way that it can use the underutilized spectrum licensed to the primary user. Thus, by using this technology, additional spectrum user could be accommodated which results in spectrum efficiency improvement. Cognitive radio is supposed to work in *spectrum agile* mode so that it can adapt its system parameters according to the environment [5]. For cognitive radio deployment, there exists a wide choice of RF spectrum which can be seen as a potential candidate. For example, due to low propagation losses, all band below 3.5 GHz are a potential candidates for cognitive radio deployment [5]. FCC considers ultra high frequency (UHF) band to allow dynamic spectrum access to cognitive radio devices. Feasible option for deployment of spectrum agile cognitive radio devices is provided by cellular and fixed wireless access band. Cognitive radio is defined as the radio system that can change its transmitter parameters based on the environment characteristics in which it operates. Cognitive radio is allowed to exist simultaneously with the primary user, but ideally, it is not allowed to interfere with the operation of the primary user. Spectrum access paradigms utilized by cognitive radio are interweave, underlay, overlay paradigm [5-7]. These spectrum access paradigms are briefly described below and schematically represented in Fig. 1.1.

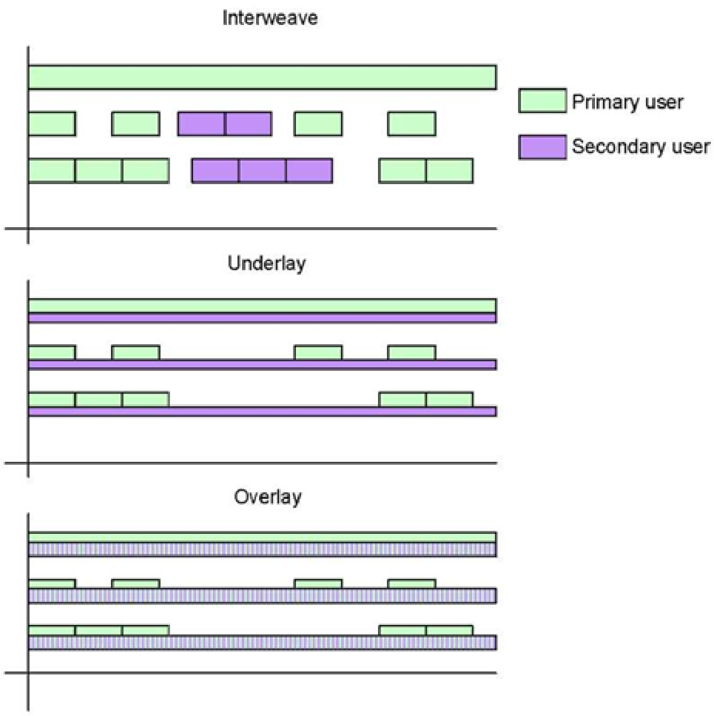

Figure 1.1: Spectrum Access Paradigm

In interweave cognitive radio model paradigm, cognitive users use the spectrum licensed to primary user only when the primary or legacy user is inactive. In this, cognitive user exploits the spectrum holes in licensed spectrum, i.e., such user uses spectrum which is not used by the primary user. For this, the cognitive radio must have an awareness of the precise location of spectrum holes and must be equipped with a proper spectrum sensing signal processing module. The task of detecting spectrum holes become very difficult if primary user spectral activity changes rapidly. As the time gap between the idle and used states of the spectrum would be very small and therefore, the cognitive user must be very agile to switch between these states. In addition to this, the range of secondary transmission also plays a vital role since the primary user activity sensed by both cognitive transmitter and receiver may vary due to varying primary signal strength. This is arguably the simplest spectrum access paradigm. It is an interference avoidance strategy. In this paradigm, the performance of cognitive radio depends on the accuracy of detecting spectrum holes, so that primary user can be protected from harmful interference.

The underlay approach is the more conservative approach. In this approach, rather than tracking the spectrum holes, the cognitive radio is allowed to use licensed spectrum simultaneously with the primary user with a constraint that the cognitive user transmits at limited power to ensure the interference at primary user due secondary transmission is below a certain predefined threshold. To satisfy the interference constraint, cognitive radio must be aware of the channel gains of the legacy user as side information. By using the channel gain knowledge, the cognitive radio can use efficient power allocation algorithm or use multiple antenna techniques to transmit its signal in such a way that legacy user is most possibly unaware of its presence. From the point of view of the deployment of cognitive radio in near future, the underlay cognitive radio model seems to be the promising technique because of the simplicity of its approach. For example, a technique commonly used in an ultra wide band in which cognitive signal is spread below the noise level than at cognitive receiver end nullifying the spreading effect to recover the cognitive signal. However, underlay spectrum access technique have its unique challenges. For instance, transmitter has to acquire perfect channel state information of primary receiver which is a very daunting task for which cognitive radio system must be equipped with complex and tedious signal processing system. Similarly, maintaining the certain interference level at primary user leaves cognitive radio to communicate over a short range.

In overlay paradigm, similar to underlay approach, the secondary user is allowed to utilize the licensed spectrum simultaneously with the primary user, but the part of the transmission power of the secondary user is devoted to strengthening the primary signal and assists its detection at the primary receiver. In exchange for this cooperation, the interference level at primary receiver due to secondary transmission is higher than underlay approach. Overlay cognitive radio has higher acceptance from the primary user because of the fact that it help to improve detection of the primary user. Overlay paradigm is an advancement in underlay approach, where the degree of cooperation between the primary user and the secondary user must be high so that higher performance can be achieved. The basic principle of this approach is that the secondary users which are in closer proximity of primary transmitter can successfully decode the primary signal. Because they have access to high-quality primary signal. Then, by using the knowledge of primary message, the secondary user produces a signal that aids the primary signal and thus, improves the probability of detection of the primary signal. At the same time, low power communication takes place between secondary transmitter and receiver. In overlay approach, either secondary receiver or transmitter or both have the knowledge of primary signal. The main advantage of this approach is that the interference level which can be tolerated is higher, and it facilitates higher probability of detection of the primary user. However, this approach also has its challenges such as it requires secondary transceiver which has a higher degree of complexity and able to produce reconcilable signal format when complementing primary signal.

From the above overview, interweave model can be easily categorized as an example of opportunistic spectrum access philosophy and remaining two techniques are considered as the examples of spectrum sharing models.

1.2 Motivation

As introduction suggests, the cognitive radio has been shown to be the promising wireless communication technology for efficient spectrum utilization. Extensive research has been carried out worldwide to address the problem and gaps in knowledge such as outage probability, outage capacity, the fundamental limit on data rate, etc., which need to be addressed regarding the performance limits of the cognitive radio. The first order statistics such as outage probability, bit error rate, etc., of cognitive radio system is widely investigated in the literature. Authors in [8], have analyzed the outage probability of cognitive multiple relays network with a direct link, but in the presence of a single primary user. In [9], outage probability of cognitive relay networks with interference constraints without direct link has been investigated. In [10], authors have investigated the outage performance in case of cognitive relay network with an approach of underlay spectrum sharing.

To address all design issues of the cognitive radio communication technology, first order statistics alone are not sufficient since the channel is time varying in practical scenario. Fortunately, the design issues which are not dealt by first-order statistics can be addressed by second order statistics such as level crossing rate (LCR) and average fade duration (AFD). These second order statistics provide a dynamic representation of channel and complements the first order statistics [11]. LCR and AFD of the relaying network are widely addressed in the literature. Authors in [12] have investigated the statistical properties of amplify and forward relay channel which experiences Rayleigh fading. In [13], authors have evaluated the outage rates and outage duration of an opportunistic

relaying system. Recently authors in [14] have investigated LCR and AFD of decode and forward relay network with N^{th} relay selection scheme in Rayleigh fading channel.
The work towards the investigation of second order statistics in cognitive radio is still open. Recently, authors in [11] investigated the LCR and AFD of the cognitive opportunistic relaying system with mobile nodes. As the field of second order statistics of cognitive radio is still raw, in our thesis, we investigate the second-order statistics of basic cognitive radio system in the presence of multiple primary receivers. References [13,15] are two important works, which are considered for our research work in this thesis.

1.3 Organization of The Thesis

The rest of the thesis is organized as follows.
In Chapter 2, we discuss wireless communication fundamental theory, in which we first discuss about the performance metrics like bit error rate, outage probability. Then in the later part, we give a brief overview of higher order statistics like LCR and AFD.
In Chapter 3, we discuss about the system model and channel model which is used for further investigation.
In Chapter 4, the considered system is analyzed by deriving its closed-form expression of outage probability, LCR and AFD.
Results and conclusion are presented in Chapter 5. In Chapter 6, we conclude our work with the discussion on scope of future work in the areas.

Chapter 2

Fundamental of Wireless Communication Theory

The principal characteristics of the wireless channel and Rayleigh fading model are over-viewed briefly in initial part of chapter. In the latter part, the performance metrics used to evaluate the performance of wireless communication system are discussed.

2.1 Fading in Wireless Channel

For reliable high-speed communication, a severe challenge is posed by the medium which is a wireless radio channel, as it is not only affected by noise and other channel restriction but this restriction changes in an uncertain way over time due to movement of the user.
In wireless communication, the transmitted signal received by the receiver through more than two paths due to the atmospheric ducting reflection from objects like the mountain, water bodies, buildings and ionospheric reflection and refraction. this propagation phenomenon is called multipath. Due to multipath propagation, multipath interference arises which results in fading. Also, due to movements of transmitter and receiver over the small duration of time, the received signal overall amplitude and phase changes. This is the main reason of fading. Fading or **small scale fading** is defined as the rapid swing in the amplitude and phase of received signal.

2.1.1 Factors Influencing Fading

In radio propagation, the small-scale fading is influenced by following physical factors: [16]

(1) Multipath Propagation - It is the propagation phenomenon due to which multiple copies of a radio signal is received by the receiver via two or more paths. Its effects include phase shifting of signal, and constructive and destructive interference.

(2) Speed of Mobile - The relative motion between receiver and transmitter results in Doppler shift in each multipath which results in random frequency modulation.

(3) Transmission Bandwidth of Signal - When transmitted signal bandwidth is greater than multipath channel bandwidth (*coherence bandwidth*), then distorted signal is received at the receiver.

2.1.2 Types of Small Scale Fading

There are four types of fading which can be categorized into two categories:

Flat Fading and Frequency Selective Fading

Flat fading occurs when coherence bandwidth is greater than signal bandwidth. In this case, channel exhibits constant gain and linear phase response over its bandwidth.

$$B_s << B_c$$

Where B_s and B_c are signal and coherence bandwidth respectively.
When this inequality is reversed, frequency selective fading is observed. This type of channel introduces inter-symbol interference.

Fast Fading and Slow Fading

The channel in which channel impulse response changes are much less than the transmitted signal is called slowly faded channel. In such a channel, for one symbol duration, the channel remains constant.

$$T_s << T_c$$

where T_s and T_c are signal and coherence time respectively.
In fast fading, within the symbol duration, the channel response changes rapidly. The received signal is distorted because signal undergoes frequency dispersion due to Doppler spreading. Signal undergoes fast fading if above inequality is reversed.

2.2 Time Varying Fading Channel Impulse Response

In this thesis, we assume that the channel is time-varying and follows flat narrowband fading model. The input-output relation for such a channel in SISO scenario is given as

$$y(t) = h(t)x(t) + n(t) \tag{2.1}$$

where $x(t)$, $y(t)$, $h(t)$ and $n(t)$ are transmitted signal, received signal, channel impulse response and additive white Gaussian noise (AWGN) respectively. We can write transmitted signal as

$$x(t) = \mathcal{R}\left[\tilde{x}(t)exp(2j\pi f_c t)\right] \tag{2.2}$$

where $\mathcal{R}$ specifies the real part of complex variable, $\tilde{x}(t)$ is complex lowpass equivalent signal and signal $x(t)$ is considered to be bandpass signal whose carrier frequency is f_c.
In our work, we consider that the receiver is moving with constant velocity v with respect to the transmitter. Due to the relative motion of receiver with the transmitter, the frequency of received signal shifted by an amount of f_d, and this is called Doppler shift. In wireless communication, if electromagnetic signal which is arriving at the receiver makes an angle θ with the direction of motion, then Doppler shift is given as

$$f_d = f_c \frac{v}{c} cos(\theta) = \frac{v}{\lambda} cos(\theta) \tag{2.3}$$

where $c = 3 \times 10^8 m/s$, is the speed of light, $\theta = \frac{c}{f_c}$.
In wireless communication, the transmitted signal arrives at the receiver through N paths due to the presence of multipath.The received bandpass signal is given as

$$y(t) = \mathcal{R}\left[\sum_{i=1}^{N} A_i exp\left[2j\pi[f_c + f_{d,i}](t-\tau_i)\right]\tilde{x}(t-\tau_i)\right] \tag{2.4}$$

Here, A_i, θ_i, τ_i are amplitude, angle of arrival and time delay respectively experienced by ith multipath component. Here index i starts from 1 as in our work, as we do not consider a line

of sight (LOS) between the transmitter and the receiver.
Let $\phi_i(t) = 2\pi[(f_c + f_{d,i})\tau_i - f_{D,i}t]$ be the phase of ith component, then

$$y(t) = \mathcal{R}\left[\sum_{i=1}^{N} A_i exp\left[-j\phi_i(t)\right] exp\left[2j\pi f_c t\right] \tilde{x}(t-\tau_i)\right] \tag{2.5}$$

From the above expression, we can easily write the expression of complex envelope of received signal as

$$\tilde{y}(t) = \sum_{i=1}^{N} A_i exp\left[-j\phi_i(t)\right] \tilde{x}(t-\tau_i) \tag{2.6}$$

Thus, the time varying channel response is given as

$$h(t,\tau) = \sum_{i=1}^{N} A_i exp\left[-j\phi_i(t)\right] \delta(t-\tau_i) \tag{2.7}$$

2.3 Statistical Properties of Recevied Envelope

For characterization of the received envelope, we have to address a large number of random variable. To overcome this issue, we use central limit theorem.
Under the narrowband condition, we can consider $x(t)$ to be an unmodulated signal i.e.,

$$\tilde{x}(t) = \mathcal{R}\left[e^{2j\pi f_c t}\right] \tag{2.8}$$

Thus, the received signal in this assumption is given as

$$y(t) = \mathcal{R}\left[\sum_{i=1}^{N} A_i e^{-j\phi_i(t)} e^{2j\pi f_c t}\right] = R_I(t) cos 2\pi f_c t + R_Q(t) sin 2\pi f_c t \tag{2.9}$$

where $R_I(t)$ and $R_Q(t)$ are in-phase and quadrature components which are given by

$$R_I(t) = \sum_{1}^{N} A_i cos\phi_i(t) \tag{2.10}$$

and

$$R_Q(t) = \sum_{1}^{N} A_i sin\phi_i(t) \tag{2.11}$$

If N is very large and $A_i(t)$, $\phi_i(t)$ are stationary and ergodic, then we can use central limit theorem to approximate $R_I(t)$ and $R_Q(t)$ as jointly Gaussian random process with zero mean [2]. If $A_i(t)$ and $\phi_i(t)$ are Rayleigh distributed and uniformly distributed on $[-\pi, \pi]$ then even if N is small, above approximation holds true. We assume that both in-phase and quadrature component has equal variance σ^2, then the amplitude of received signal envelope is

$$z(t) = |y(t)| = \sqrt{R_I(t)^2 + R_Q(t)^2} \tag{2.12}$$

By using the standard transformation [18], it can be easily proven that received signal envelope follows Rayleigh distribution, thus the probability distribution function of Z is given as

$$f_Z(z) = \frac{z}{\sigma^2} exp\left[-\frac{z^2}{2\sigma^2}\right] \tag{2.13}$$

The received signal envelope average power is given as

$$\mathbb{E}[R^2] = 2\sigma^2 \tag{2.14}$$

The distribution of received signal power is characterized by an exponential distribution which can be easily obtained by making change of variable $z^2(t)$ in (2.13) as

$$f_{Z^2}(z) = \frac{1}{2\sigma^2} exp\left[-\frac{z}{2\sigma^2}\right] \tag{2.15}$$

2.4 Performance Metrics

In this section, we provide a brief overview of the performance metrics used in this thesis. These are outage probability and second order statistics which are Level crossing rate (LCR), average fade duration (AFD) to evaluate the performance of our system.

2.4.1 Outage Probability

In wireless communication, if the value of received SNR goes below the predefined minimum value, then the performance of wireless communication system becomes unacceptable. This condition is called outage. The probability of occurrence of outage is called outage probability. Mathematically, it is given as

$$P_{out} = P_r(\gamma \leq \gamma_{th}) \tag{2.16}$$

2.4.2 Second Order Statistics

In practical mobile communication, the channel gain are slowly time-varying. In this case, first order statistics such as outage probability, bit error rate are not able to address all system design issues such as packet length selection, power and bandwidth allocation, maximum delay requirement [11]. Thus, we can say that when the channel is time varying, the first order statistics are not able to address all the design issues. But fortunately, by investigating second order statistics such as LCR and AFD of the system, these design issues can be addressed.

Level Crossing Rate (LCR)

Level crossing rate L_z, is defined as the expected rate at which the received signal envelope crosses the level Z in the downward direction [2]. To obtain the expression of LCR, we require the joint PDF of signal envelope z and its time derivative $\dot{z}$, $f_{Z\dot{Z}}(z\dot{z})$. We now see how to derive L_z based on the joint PDF $f_{Z\dot{Z}}(z\dot{z})$.
Let us consider a random process shown in Fig 2.1 below.

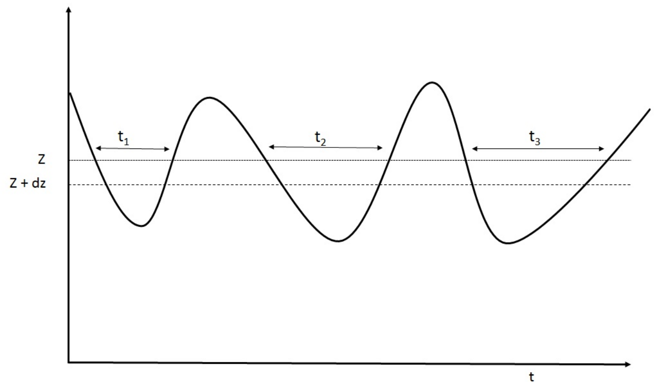

Figure 2.1: Random Fading Process

The received signal envelope in the interval of $(Z, Z+dz)$ with range of envelope slope $(\dot{z}, \dot{z}+d\dot{z})$ spends expected amount of time over the time duration dt is given as [2]

$$A = f(Z, \dot{z})dz d\dot{z}dt \tag{2.17}$$

The required time to cross once for the given slope $d\dot{z}$ from Z to $Z + dz$ is given as

$$B = \frac{dz}{\dot{z}} \tag{2.18}$$

The ratio of (2.17) and (2.18) gives the expected number of crossing for the given slope $\dot{z}$ over time duration of dt within the interval of $(Z, Z + \dot{z})$. Thus, the expected number of crossing of envelope level Z with negative slope over time interval $[0.T]$ is

$$N_Z = T \int_0^{\infty} \dot{z} f(Z, \dot{z}) d\dot{z} \tag{2.19}$$

Finally, the general expression of LCR for any random process is given as

$$L_Z = \int_0^{\infty} \dot{z} f(Z, \dot{z}) d\dot{z} \tag{2.20}$$

The closed-form expression of LCR for the simplest case of Rayleigh faded signal is given by [19].

$$L_Z = \sqrt{\frac{\beta}{2\pi}} \frac{Z}{\sigma_p^2} e^{-\frac{Z^2}{2\sigma^2}} \tag{2.21}$$

where σ_p^2 denotes the mean power of underlying Gaussian process. The parameter β represents negative curvature of autocorrelation funtion.
The closed-form expression of LCR for Rician process is also presented in [19].

Average Fade Duration (AFD)

Average fade duration (AFD) is another second order statistics which is used as a performance metric in wireless communication. The concept of AFD is related to LCR, and it is defined as the average duration of time for which signal envelope stays below the certain predefined target rate Z. The target level is often obtained from signal amplitude or power level required for given performance metric [2].
Let the duration of the ith fade be t_i as illustrated in Fig 2.1. Therefore, the time duration t_i is equal to the time duration for which received signal envelope stays below target level on its ith crossing. Since, the considered random process is stationary and ergodic, for large T, we have

$$P_{out} = \frac{1}{T} \sum_i t_i \tag{2.22}$$

Thus, AFD is given as

$$T_Z(z) = \frac{1}{T L_Z(z)} \sum_{i=1}^{L_Z(z)} t_i \approx \frac{P_r(z(t) \leq Z)}{L_Z(z)} \tag{2.23}$$

Thus, mathematically, the AFD is equal to the ratio of outage probability and LCR.

Chapter 3

Underlay Cognitive Radio System and Channel Model

In this chapter, first we discuss the typical underlay cognitive radio system model, and then, we explain the mathematical modeling of the system model considered in this thesis work.

3.1 Channel Model

In wireless communication, channels are characterized by the fading. In our system, we consider that the channel experiences Rayleigh fading. Thus, our channel gains are Rayleigh distributed. Let X be a random variable which follows Rayleigh distribution, then its probability density function (PDF) is given as follows

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} exp\left[-\frac{x^2}{2\sigma^2}\right] & if\ x > 0 \\ 0 & if\ x < 0 \end{cases} \tag{3.1}$$

where σ is the scale parameter.
The cumulative distribution function (CDF) is another statistics used for characterization of a random variable which is obtained by integration of PDF over defined limits. The CDF of Rayleigh distribution is given as

$$F_X(x) = \begin{cases} 1 - exp\left[-\frac{x^2}{2\sigma^2}\right] & if\ x > 0 \\ 0 & if\ x < 0 \end{cases} \tag{3.2}$$

Let us consider a random variable Y depends on Rayleigh distributed random variable X as

$$Y = |X|^2 \tag{3.3}$$

In this case, the random variable Y follows exponential distribution and its PDF and CDF are given by

$$f_X(x) = \begin{cases} \lambda exp\left[-\lambda y\right] & if\ y > 0 \\ 0 & if\ y < 0 \end{cases} \tag{3.4}$$

$$F_X(x) = \begin{cases} 1 - exp\left[-\lambda y\right] & if\ y > 0 \\ 0 & if\ y < 0 \end{cases} \tag{3.5}$$

where $\lambda = 2\sigma^2$.

3.2 Underlay Cognitive Radio System Model

A typical underlay cognitive radio network consists of N primary transmitter-receiver pairs and a cognitive (secondary) transmitter-receiver pair. The cognitive transmitter-receiver pair tries to access the licensed spectrum of primary radio in underlay spectrum access paradigm. The transmission power of the transmitter of the cognitive radio system is constraint in such a way that interference at primary receiver caused by the cognitive radio transmission lies below the certain predefined threshold.

The system model of typical underlay cognitive radio system is shown in Fig. 3.1, which consist of N primary radio systems and a cognitive radio system. Interference caused at cognitive receiver due to primary communication is shown by blue dotted lines and the interference caused by cognitive communication at primary receiver is shown by the black dotted lines. h_{ji} and h_{TR} are the channel gains of the communication links between the primary transmitter and receiver, and cognitive transmitter and receiver respectively. h_{jR} and h_{Ti} are the channel gains of the interference links between primary transmitter and cognitive receiver, and cognitive transmitter and primary receiver respectively. Where $i, j \in 1, 2, ..N$.

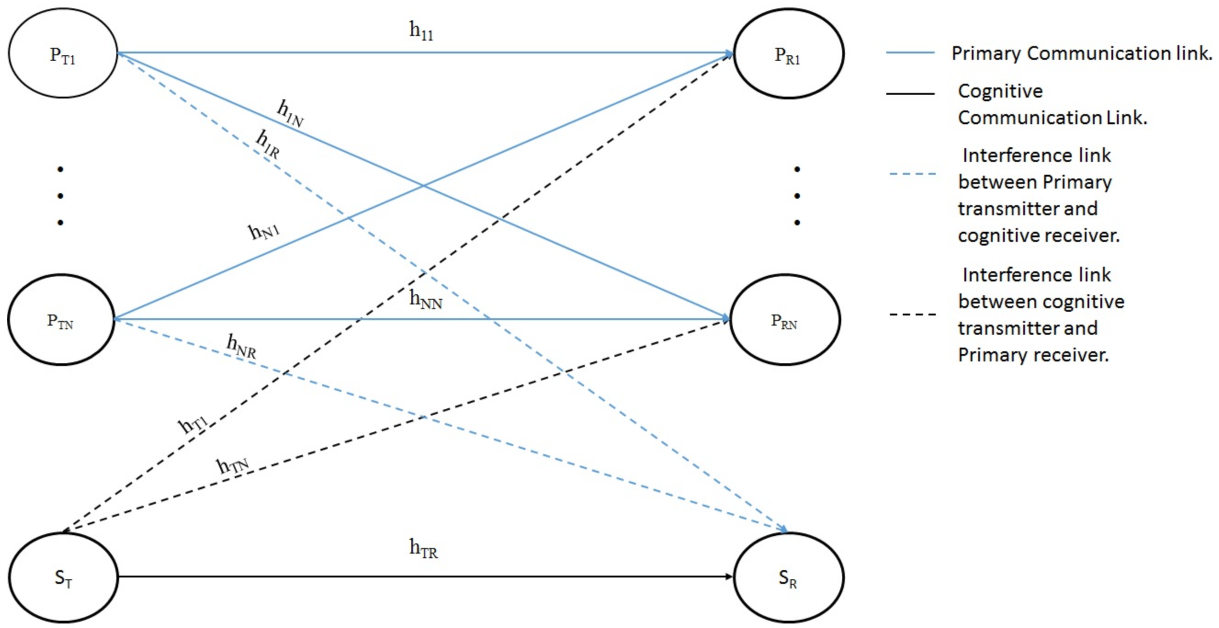

Figure 3.1: Typical Underlay Cognitive Radio System

3.3 System Model Under Consideration

In this thesis, we consider that the primary transmitters are at very large distance from cognitive radio system geographically so that it will not cause any interference at cognitive receiver. Thus, our considered system model consists of N primary receivers P_{Rj} where$j \in 1, 2, ..N$, one cognitive source S_T and one cognitive destination S_R. Cognitive receiver and primary receivers are moving with a certain velocities which cause maximum Doppler frequency shifts of f_1 and f_2 respectively in received signal. The considered system model is shown in below Fig. 3.2.

The message signal transmitted through the channel is prone to fading due to various factors. Depending on the various elements of communication environment, the fading can be modeled using Rayleigh distribution, Rician distribution, Nakagami distribution, etc. In our system, we consider that all channels experience Rayleigh fading. The channel gain between cognitive source S_T and N primary receivers is denoted as h_{Ti} where $i \in 1, 2, ...N$. The channel gain between cognitive source S_T and the cognitive destination S_R is denoted as h_0.

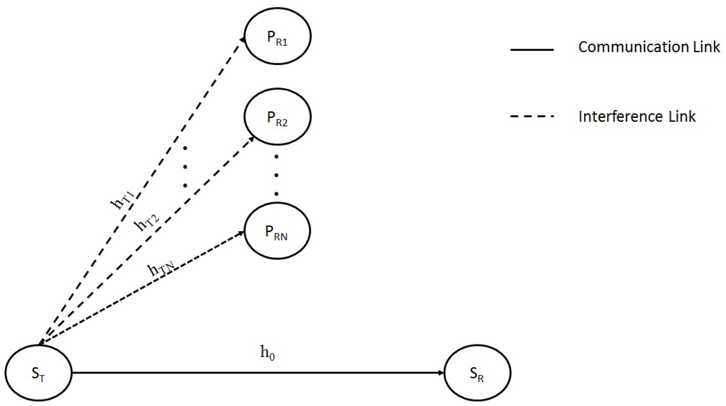

Figure 3.2: System Model

The cognitive user communicates over spectrum licensed to primary user, but the cognitive user transmission power is restricted so that the interference caused by cognitive transmission at primary receivers is below the predefined threshold.

Let the maximum transmission power of cognitive user be P_m and the maximum allowable interference at primary receivers be Q, then the transmission power P_s of cognitive user is constraint as

$$P_s = min\left\{P_m, \frac{Q}{max\left\{|h_{T1}(t)|^2, |h_{T2}(t)|^2, ..., |h_{TN}(t)|^2\right\}}\right\} = min\left\{P_m, Q.|g(t)|^2\right\} \quad (3.6)$$

where $g(t)$ is a new random variable which depends on channel gain as

$$g(t) = min\left[h_{T1}(t), h_{T2}(t), ..., h_{TN}(t)\right] \quad (3.7)$$

As random variable $g(t)$ is minimum of the N Rayleigh distributed random variable, its CDF is given as

$$\begin{aligned} F_G(g) &= 1 - \{(1 - F_{H_{T1}}(g))(1 - F_{H_{T2}}(g)).....(1 - F_{H_{TN}}(g))\} \\ &= 1 - \left\{(1 - (1 - e^{-\frac{g^2}{2\sigma_1^2}}))(1 - (1 - e^{-\frac{g^2}{2\sigma_2^2}})).....(1 - (1 - e^{-\frac{g^2}{2\sigma_N^2}}))\right\} \\ &= 1 - \left\{(e^{-\frac{g^2}{2\sigma_1^2}})(e^{-\frac{g^2}{2\sigma_2^2}}).....(e^{-\frac{g^2}{2\sigma_N^2}})\right\} \\ &= 1 - exp\left[\sum_{i=1}^{N} -\frac{g^2}{2\sigma_i^2}\right] \end{aligned} \quad (3.8)$$

PDF is obtained by differentiation of CDF. Thus, the PDF of random variable g is given as

$$f_G(g) = \sum_{i=1}^{N} \frac{g}{\sigma_i^2} exp\left[-\sum_{i=1}^{N} \frac{g^2}{2\sigma_i^2}\right] \tag{3.9}$$

In our system, we consider fixed to mobile channel [20,21] and it is found that time derivative of Rayleigh distributed channel is independent from channel itself and follows Gaussian distribution [20-22]. Thus, PDF of the time derivatives of h and g is given by

$$f_{\dot{H}}(\dot{h}) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{H}}} exp\left[-\frac{\dot{h}^2}{2\sigma_{\dot{H}}^2}\right] \tag{3.10}$$

where

$$\sigma_{\dot{H}}^2 = \pi^2 \sigma_0^2 f_1^2 \tag{3.11}$$

and

$$f_{\dot{G}}(\dot{g}) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{G}}} exp\left[-\frac{\dot{g}^2}{2\sigma_{\dot{G}}^2}\right] \tag{3.12}$$

where

$$\sigma_{\dot{G}}^2 = \pi^2 \sigma_g^2 f_2^2 \tag{3.13}$$

In (3.13), $\sigma_g^2 = \left(\sum_{i=0}^{N} \frac{1}{\sigma_i^2}\right)^{-1}$ and f_2 is the maximum Doppler frequency shift induced due to the motion of primary receivers.

In (3.11), f_1 is the maximum Doppler frequency shift induced due to the motion of cognitive receiver.

Let the cognitive transmitter transmits message signal $x(t)$. The noise present in wireless channel severely affects the transmitted signal. In our system, we modeled the noise as additive white Gaussian noise (AWGN) with zero mean and variance N_0. Thus, the signal received at cognitive receiver is mathematically modeled as

$$y = \sqrt{P_s} h_0(t) x(t) + n(t) \tag{3.14}$$

where y is the received signal at the cognitive receiver, n is AWGN noise at the cognitive receiver.

3.4 Signal-to Noise (SNR) Expression

For performance analysis of considered system, first we need to evaluate the expression of the instantaneous SNR which is defined as the ratio of signal power and noise power. The signal power is given as

$$P_{signal} = P_s \left|h_0(t)\right|^2 \tag{3.15}$$

The noise power is equal to the variance of additive white Gaussian noise as it has zero mean. Thus, we have

$$P_{noise} = N_0 \tag{3.16}$$

From (3.15) and (3.16), the end-to-end instantaneous SNR is given as

$$\gamma = \frac{P_s \left|h_0(t)\right|^2}{N_0} \tag{3.17}$$

From Eq. (3.6), the transmission power P_s is P_m when $g > \sqrt{\frac{P_m}{Q}}$ and when $g < \sqrt{\frac{P_m(t)}{Q}}$, P_s is $Q\left|g\right|^2$. Thus the expression of end-to-end instantaneous SNR can be written as

$$\gamma = \begin{cases} \frac{Q|g(t)|^2|h(t)|^2}{N_0} & \text{if } g(t) < \sqrt{\frac{P_m}{Q}} \\ \frac{P_m|h(t)|^2}{N_0} & \text{if } g(t) > \sqrt{\frac{P_m}{Q}} \end{cases} \tag{3.18}$$

Further on, for simplicity, we will omit the index t.

Chapter 4

Statistics of Underlay Cognitive Radio

4.1 Overview

In this chapter, we first derive the closed-form expression for first order statistics (Outage Probability). To address all the communication system design issues, LCR and AFD which are defined as the average rate at which received signal envelope cross a predefined threshold and duration for which received signal lies below the predefined threshold respectively are of prime importance. These two metrics are second order statistics whose closed-form expression is evaluated in the later part of this chapter.

4.2 Outage Probability

As discussed in Chapter 2, outage probability is a very important metric used to evaluate the performance of wireless communication system, and it is defined as the probability of end-to-end instantaneous SNR that is below a predefined threshold. Thus, we have

$$P_{out} = P_r(\gamma \leq \gamma_{th}) \tag{4.1}$$

By introducing (3.18) into (4.1), we get,

$$P_{out} = P_r\left(\frac{|h|^2 \min(P_m, Q|g|^2)}{N_0} \leq \gamma_{th}\right) \tag{4.2}$$

To evaluate closed-form expression of end-to-end SNR, we fix the random variable $|g|^2$, so (4.2) becomes

$$P_{out} = P_r\left(|h|^2 \leq \frac{\gamma_{th} N_0}{\min\left(P_m, Q|g|^2\right)}\right) \tag{4.3}$$

which in turn is equal to

$$P_{out} = \int_{|g|^2=0}^{\infty} F_{|H|^2}\left(\frac{\gamma_{th}N_0}{\min\left(P_m, Q|g|^2\right)}\right) f_{|G|^2}(|g|^2)d|g|^2 \tag{4.4}$$

By using (3.2), the CDF of $|h|^2$ and $|g|^2$ is given by (4.5) and (4.6) respectively.

$$F_{|H|^2}(|h|^2) = 1 - exp\left[-\lambda_0|h|^2\right] \tag{4.5}$$

$$F_{|G|^2}(|g|^2) = 1 - exp\left[-\sum_{i=1}^{N}\lambda_i|g|^2\right] \tag{4.6}$$

The PDF of $|g|^2$ can be obtained by taking differentiation of (4.6) which is given as

$$f_{|G|^2}(|g|^2) = \sum_{i=1}^{N}\lambda_i exp\left[-\sum_{i=1}^{N}\lambda_i|g|^2\right] \tag{4.7}$$

In (4.5), (4.6), and (4.7), $\lambda_k = 2\sigma_k^2; k = 0, 1, 2, ...N.$

Now, (4.4) can be written as

$$P_{out} = \int_{|g|^2=0}^{\frac{P_m}{Q}} F_{|H|^2}\left(\frac{\gamma_{th}N_0}{Q|g|^2}\right) f_{|G|^2}(|g|^2)d|g|^2 + \int_{|g|^2=\frac{P_m}{Q}}^{\infty} F_{|H|^2}\left(\frac{\gamma_{th}N_0}{P_m}\right) f_{|G|^2}(|g|^2)d|g|^2 \tag{4.8}$$

By introducing (4.5) into (4.8), it reduces to

$$\begin{aligned} P_{out} = &\int_{|g|^2=0}^{\frac{P_m}{Q}} f_{|G|^2}(|g|^2)d|g|^2 - \int_{|g|^2=0}^{\frac{P_m}{Q}} exp\left[-\frac{\lambda_0\gamma_{th}N_0}{Q|g|^2}\right] f_{|G|^2}(|g|^2)d|g|^2 \\ &+ \int_{|g|^2=\frac{P_m}{Q}}^{\infty} f_{|G|^2}(|g|^2)d|g|^2 - \int_{|g|^2=\frac{P_m}{Q}}^{\infty} exp\left[-\frac{\lambda_0\gamma_{th}N_0}{P_m}\right] f_{|G|^2}(|g|^2)d|g|^2 \end{aligned} \tag{4.9}$$

By using (4.7) into above expression, we get

$$P_{out} = 1 - \int_{|g|^2=0}^{\frac{P_m}{Q}} \sum_{i=1}^{N}\lambda_i exp\left[-\zeta_1(|g|^2)\right] d|g|^2 - exp\left[-\frac{\lambda_0\gamma_{th}N_0}{P_m}\right] \int_{|g|^2=\frac{P_m}{Q}}^{\infty} f_{|g|^2}(|g|^2)d|g|^2 \tag{4.10}$$

where $\zeta_1(|g|^2) = \dfrac{\lambda_0\gamma_{th}N_0}{Q|g|^2} + \sum_{i=1}^{N}\lambda_i|g|^2$.

On solving above expression, we get the final closed-form expression of end-to-end instantaneous SNR as

$$P_{out} = 1 - exp\left[-\left(\frac{\lambda_0\gamma_{th}N_0}{P_m} + \sum_{i=1}^{N}\lambda_i|g|^2\right)\right] - \int_{|g|^2=0}^{\frac{P_m}{Q}} \sum_{i=1}^{N}\lambda_i exp\left[-\zeta_1(|g|^2)\right] d|g|^2 \tag{4.11}$$

4.3 Second Order Statistics

In this section, we first derive the closed-form expression of LCR. As the AFD is the ratio of outage probability and LCR, thus by using the expression of LCR and outage probability, we find the expression of AFD.

4.3.1 Level Crossing Rate (LCR)

As we discussed in Chapter 2, to evaluate closed-form expression of LCR of considered system, we need to determine joint PDF of received signal envelope z and its time derivative $\dot{z}$.
The received signal envelope is equal to the square root of SNR and is given as

$$z = \sqrt{\gamma} = \begin{cases} \frac{\sqrt{Q}|g||h|}{\sqrt{N_0}} & if\ g < \sqrt{\frac{P_m}{Q}} \\ \frac{\sqrt{P_m}|h|}{\sqrt{N_0}} & if\ g > \sqrt{\frac{P_m}{Q}} \end{cases} \tag{4.12}$$

The time derivative of z is given as

$$\dot{z} = \begin{cases} z\left(\frac{\dot{h}}{h} + \frac{\dot{g}}{g}\right) & if\ g < \sqrt{\frac{P_m}{Q}} \\ \dot{h}\sqrt{\frac{P_m}{N_0}} & if\ g > \sqrt{\frac{P_m}{Q}} \end{cases} \tag{4.13}$$

It is known from [21,22] that time derivative of Rayleigh faded envelope follows Guassian disrtribution with zero mean and variance $\sigma_{\dot{z}}^2 = \pi^2\sigma^2 f^2$ where σ^2 is the variance of received signal envelope and f is maximum Doppler shift induced due to motion of receiver.

$$f_{\dot{z}} = \frac{1}{\sqrt{2\pi}\sigma_{\dot{z}}} exp\left[-\frac{\dot{z}^2}{2\pi\sigma_{\dot{z}}^2}\right] \tag{4.14}$$

The LCR of considered system is evaluated by using Rice formula [21] as

$$L_Z(z) = \int_0^{\infty} \dot{z} f_{Z\dot{Z}}(z\dot{z}) d\dot{z} \tag{4.15}$$

As it is mathematically very tedious to determine LCR of considered system directly using Rice formula, so we first find the conditional LCR $L_{Z|G}(z|g)$ then unconditioned it to find LCR as

$$\begin{aligned} L_Z(z) &= \int_{g=0}^{\infty}\int_{\dot{z}=0}^{\infty} \dot{z} f_{Z\dot{Z}|G}(z\dot{z}|g) f_G(g) dg d\dot{z} \\ &= \int_{g=0}^{\sqrt{\frac{P_m}{Q}}}\int_{\dot{z}=0}^{\infty} \dot{z} f_{Z\dot{Z}|G}(z\dot{z}|g) f_G(g) dg d\dot{z} + \int_{g=\sqrt{\frac{P_m}{Q}}}^{\infty}\int_{\dot{z}=0}^{\infty} \dot{z} f_{Z\dot{Z}|G}(z\dot{z}|g) f_G(g) dg d\dot{z} \\ &= L_1 + L_2 \end{aligned} \tag{4.16}$$

where

$$L_1 = \int_{g=0}^{\sqrt{\frac{P_m}{Q}}}\int_{\dot{z}=0}^{\infty} \dot{z} f_{Z\dot{Z}|G}(z\dot{z}|g) f_G(g) dg d\dot{z} \tag{4.17}$$

$$L_2 = \int_{g=\sqrt{\frac{P_m}{Q}}}^{\infty}\int_{\dot{z}=0}^{\infty} \dot{z} f_{Z\dot{Z}|G}(z\dot{z}|g) f_G(g) dg d\dot{z} \tag{4.18}$$

As we can see from (4.17) and (4.18), we need to determine the joint PDF of z and $\dot{z}$ conditioned over g, based on the value of g, two cases arises which are mentioned below.

CASE 1: When $g < \sqrt{\frac{P_m}{Q}}$ the received signal envelope and its time derivative is given as $z = \frac{\sqrt{Q}|g||h|}{\sqrt{N_0}}$ and $\dot{z} = z\left(\frac{\dot{h}}{h} + \frac{\dot{g}}{g}\right)$ respectively.
The LCR for this case is equal to L_1. The conditional PDF $f_{Z\dot{Z}|G}(z\dot{z}|g)$ can be further simplified by using Total probability by fixing Z = z,

$$f_{Z\dot{Z}|G}(z\dot{z}|g) = f_{\dot{Z}|ZG}(\dot{z}|zg) \times f_{Z|G}(z|g) \tag{4.19}$$

Thus, to evaluate L_1, we need to determine conditional PDF $f_{Z|G}(z|g)$ for which we first determine Conditional CDF $F_{Z|G}(z|g)$ which is given as

$$\begin{aligned} F_{Z|G}(z|g) &= P_r(Z \leq z) \\ &= P_r\left(\frac{\sqrt{Q}.h.g}{\sqrt{N_0}} \leq z\right) \\ &= P_r\left(h \leq \frac{z}{g}\sqrt{\frac{N_0}{Q}}\right) \\ &= F_{H_0}\left(\frac{z}{g}\sqrt{\frac{N_0}{Q}}\right) \end{aligned}$$

therefore, we have

$$F_{Z|G}(z|g) = 1 - exp\left[-\frac{z^2 N_0}{2g^2 Q\sigma_0^2}\right] \tag{4.20}$$

After evaluating conditional CDF, the conditional PDF is evaluated by taking the derivative of conditional CDF which is given as

$$f_{Z|G}(z|g) = \frac{zN_0}{g^2 Q\sigma_0^2} exp\left[-\frac{z^2 N_0}{2g^2 Q\sigma_0^2}\right] \tag{4.21}$$

As we already discussed that the time derivative of signal envelope follows Gaussian distribution, so the conditional PDF $f_{\dot{Z}|ZG}(\dot{z}|zg)$ follows the Gaussian distribution with zero mean and variance.

$$\sigma^2_{\dot{Z}|ZG} = \frac{Qg^2\sigma^2_{\dot{h}}}{N_0} + \frac{z^2\sigma^2_{\dot{g}}}{g^2} \tag{4.22}$$

By introducing (3.9), (4.19), (4.21) into (4.17), we obtain following expression

$$L_1 = \int_{g=0}^{\sqrt{\frac{P_m}{Q}}} \left(\int_{\dot{z}=0}^{\infty} \dot{z} f_{\dot{Z}|ZG}(\dot{z}|zg) d\dot{z}\right) \frac{zN_0}{g^2 Q\sigma_0} exp\left[-\frac{z^2 N_0}{2g^2 Q\sigma_0^2}\right] \sum_{i=1}^{N} \frac{g}{\sigma_i^2} exp\left[-\sum_{i=1}^{N}\frac{g^2}{2\sigma_i^2}\right] dg \tag{4.23}$$

The integral in bracket in (4.23) is found by using (4.22) as

$$L_1 = \int_{g=0}^{\sqrt{\frac{P_m}{Q}}} \frac{\sigma_{\dot{Z}|ZG}}{\sqrt{2\pi}} \frac{zN_0}{gQ\sigma_0^2} \left(\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\right) exp\left[-\left(\frac{z^2 N_0}{2g^2 Q\sigma_0^2} + \sum_{i=1}^{N}\frac{g^2}{2\sigma_i^2}\right)\right] dg \tag{4.24}$$

By substituting (4.22) in (4.24), we obtain exact expression of L_1 as

$$L_1 = \frac{zN_0}{\sqrt{2\pi}Q\sigma_0^2}\left(\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\right) \int_{g=0}^{\sqrt{\frac{P_m}{Q}}} \left(\frac{Q\sigma^2_{\dot{h}}}{N_0} + \frac{z^2\sigma^2_{\dot{g}}}{g^4}\right)^{\frac{1}{2}} exp\left[-\left(\frac{z^2 N_0}{2g^2 Q\sigma_0^2} + \sum_{i=1}^{N}\frac{g^2}{2\sigma_i^2}\right)\right] dg \tag{4.25}$$

CASE 2: When $g > \sqrt{\frac{P_m}{Q}}$ the received signal envelope and its time derivative is $\frac{\sqrt{P_m}|h|}{\sqrt{N_0}}$ and $\dot{h}\sqrt{\frac{P_m}{N_0}}$ respectively.
The LCR for this case is equal to L_2. As we can se that both z and $\dot{z}$ are independent of g thus, conditional PDF $f_{Z\dot{Z}|G}(z\dot{z}|g)$ can be simplified as

$$f_{Z\dot{Z}|G}(z\dot{z}|g) = f_{Z\dot{Z}}(z\dot{z}) \tag{4.26}$$

We already discussed that if a random variable is Rayleigh distributed then its time derivative is independent of the random variable and follows Gaussian distribution, thus

$$f_{Z\dot{Z}}(z\dot{z}) = f_Z(z) f_{\dot{Z}}(\dot{z}) \tag{4.27}$$

To determine PDF $f_Z(z)$, we first calculate CDF $F_Z(z)$ which is given as

$$\begin{aligned} F_z(z) &= P_r\left(\sqrt{\frac{P_m}{N_0}}.h \le z\right) \\ &= P_r\left(h \le z\sqrt{\frac{N_0}{P_m}}\right) \\ &= F_H\left(z\sqrt{\frac{N_0}{P_m}}\right) \end{aligned}$$

therefore, we have

$$F_z(z) = 1 - exp\left[-\frac{z^2 N_0}{2P_m\sigma_0^2}\right] \tag{4.28}$$

Now PDF $f_Z(z)$ is obtained by taking derivative of CDF $F_Z(z)$ which is given as

$$f_Z(z) = \left(\frac{zN_0}{P_m\sigma_0^2}\right) exp\left[-\frac{z^2 N_0}{2P_m\sigma_0^2}\right] \tag{4.29}$$

As we already discussed that $\dot{z}$ is zero mean Gaussian distributed with variance

$$\sigma_{\dot{Z}}^2 = \frac{P_m\sigma_{\dot{h}}^2}{N_0} \tag{4.30}$$

By introducing (4.26) and (4.27) in (4.18), we obtain

$$L_2 = \int_{g=\sqrt{\frac{P_m}{Q}}}^{\infty}\left(\int_{\dot{z}=0}^{\infty}\dot{z} f_{\dot{Z}}(\dot{z})d\dot{z}\right) f_Z(z) f_G(g) dg \tag{4.31}$$

The bracketed integral is equal to $\frac{\sigma_z}{\sqrt{2\pi}}$ thus introducing this and (4.29), we get

$$L_2 = \int_{g=\sqrt{\frac{P_m}{Q}}}^{\infty} \frac{\sigma_z}{\sqrt{2\pi}} \left(\frac{zN_0}{P_m\sigma_0^2}\right) exp\left[-\frac{z^2N_0}{2P_m\sigma_0^2}\right] f_G(g)dg \tag{4.32}$$

Using (4.30), the above expression becomes

$$L_2 = \int_{g=\sqrt{\frac{P_m}{Q}}}^{\infty} \sqrt{\frac{P_m}{2\pi N_0}}\sigma_{\dot{h}} \left(\frac{zN_0}{P_m\sigma_0^2}\right) exp\left[-\frac{z^2N_0}{2P_m\sigma_0^2}\right] f_G(g)dg$$

$$L_2 = \frac{\sigma_{\dot{h}} z}{\sigma_0^2}\sqrt{\frac{N_0}{2\pi P_m}} exp\left[-\frac{z^2N_0}{2P_m\sigma_0^2}\right] \int_{g=\sqrt{\frac{P_m}{Q}}}^{\infty} f_G(g)dg$$

$$L_2 = \frac{\sigma_{\dot{h}} z}{\sigma_0^2}\sqrt{\frac{N_0}{2\pi P_m}} exp\left[-\frac{z^2N_0}{2P_m\sigma_0^2}\right] \left[1 - F_G\left(\sqrt{\frac{P_m}{Q}}\right)\right] \tag{4.33}$$

From (3.8), we use CDF of $F_G(g)$ in above expression

$$L_2 = \frac{\sigma_{\dot{h}} z}{\sigma_0^2}\sqrt{\frac{N_0}{2\pi P_m}} exp\left[-\left(\frac{z^2N_0}{2P_m\sigma_0^2} + \sum_{i=1}^{N}\frac{P_m}{2Q\sigma_i^2}\right)\right] \tag{4.34}$$

From (4.17), (4.25) and (4.34), we get the expression of LCR of considered system as

$$\begin{aligned} L_Z(z) = {} & \frac{zN_0}{\sqrt{2\pi}Q\sigma_0^2}\left(\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\right)\int_{g=0}^{\sqrt{\frac{P_m}{Q}}}\left(\frac{Q\sigma_{\dot{h}}^2}{N_0} + \frac{z^2\sigma_{\dot{g}}^2}{g^4}\right)^{\frac{1}{2}} exp\left[-\left(\frac{z^2N_0}{2g^2Q\sigma_0^2} + \sum_{i=1}^{N}\frac{g^2}{2\sigma_i^2}\right)\right] dg \\ & + \frac{\sigma_{\dot{h}} z}{\sigma_0^2}\sqrt{\frac{N_0}{2\pi P_m}} exp\left[-\left(\frac{z^2N_0}{2P_m\sigma_0^2} + \sum_{i=1}^{N}\frac{P_m}{2Q\sigma_i^2}\right)\right] \end{aligned} \tag{4.35}$$

Introducing (3.11) and (3.13) into (4.35), we get

$$\begin{aligned} L_Z(z) = {} & \frac{\sqrt{\pi}zN_0}{\sqrt{2}Q\sigma_0^2}\left(\sum_{i=0}^{N}\frac{1}{\sigma_i^2}\right)\int_{g=0}^{\sqrt{\frac{P_m}{Q}}}\left(\frac{Qf_1^2\sigma_h^2}{N_0} + \frac{z^2f_2^2\sigma_g^2}{g^4}\right)^{\frac{1}{2}} exp\left[-\left(\frac{z^2N_0}{2g^2Q\sigma_0^2} + \sum_{i=1}^{N}\frac{g^2}{2\sigma_i^2}\right)\right] dg \\ & + \frac{f_1 z}{\sigma_0}\sqrt{\frac{N_0\pi}{2P_m}} exp\left[-\left(\frac{z^2N_0}{2P_m\sigma_0^2} + \sum_{i=1}^{N}\frac{P_m}{2Q\sigma_i^2}\right)\right] \end{aligned} \tag{4.36}$$

where $\sigma_g^2 = \left(\sum_{i=0}^{N}\frac{1}{\sigma_i^2}\right)^{-1}$.

The closed-form expression of LCR for considered system is given by (4.36).

4.3.2 Average Fade Duration (AFD)

As discussed earlier in Chapter 2, the AFD is given as the ratio of outage probability and LCR. Thus, from (4.11), and (4.36) closed-form expression of AFD is given as

$$T_Z(z) = \frac{1 - exp\left[-\left(\frac{\lambda_0 \gamma_{th} N_0}{P_m} + \sum_{i=1}^{N} \lambda_i |g|^2\right)\right] - \int_{|g|^2=0}^{\frac{P_m}{Q}} \left(\sum_{i=1}^{N} \lambda_i\right) exp[-\zeta_1(|g|^2)] d|g|^2}{\frac{\sqrt{\pi} z N_0}{\sqrt{2} Q \sigma_0^2} \left(\sum_{i=0}^{N} \frac{1}{\sigma_i^2}\right) \int_{g=0}^{\sqrt{\frac{P_m}{Q}}} \left(\frac{Q f_1^2 \sigma_h^2}{N_0} + \frac{z^2 f_2^2 \sigma_g^2}{g^4}\right)^{\frac{1}{2}} exp[-\zeta_2(g)] dg + \frac{f_1 z}{\sigma_0} \sqrt{\frac{N_0 \pi}{2 P_m}} exp\left[-c\right]} \quad (4.37)$$

where $\zeta_1(g) = \left(\frac{\lambda_0 \gamma_{th} N_0}{Q|g|^2} + \sum_{i=1}^{N} \lambda_i |g|^2\right)$, $\zeta_2(g) = \left(\frac{z^2 N_0}{2 g^2 Q \sigma_0^2} + \sum_{i=1}^{N} \frac{g^2}{2\sigma_i^2}\right)$
and $c = \left(\frac{z^2 N_0}{2 P_m \sigma_0^2} + \sum_{i=1}^{N} \frac{P_m}{2 Q \sigma_i^2}\right)$

Chapter 5

Simulation Results

In this chapter, the numerical results based on the analysis presented in the previous chapter are provided and are verified using Monte Carlo simulation. Finally using these results, we comment on the system performance.

5.1 Numerical Plots and Discussion

In this section, numerical and simulation results of outage probability, LCR and AFD is provided. During simulation, we use MATLAB to build Monte Carlo simulation. The wireless communication channel is considered as Rayleigh fading channel and hence, methods that are widely available in the literature can be used [19,20,23]. We apply Zheng and Xiao's statistical simulation model [23] for channel modeling.

The maximum Doppler shift introduced by the motion of primary receivers and secondary receivers are assumed to be same i.e., $f_1 = f_2 = f_0$ Hz. We verify our derivation with Monte Carlo simulation for two cases viz., different number of primary receivers and variable maximum Doppler shift.

In Table 5.1, we summarize all the parameters and their values used for the numerical investigations.

Table 5.1: Values of parameters used in simulation

Parameters	Values
Maximum transmission power (P_m)	100
Threshold level	2 dB
Variance (σ_i^2) where $i = 0, 1, ...N$	1
Noise Variance N_0	1

Now, we analyze the performance of system, based on the following two cases under Rayleigh fading channels.

CASE 1: For different number of primary receiver.

In this case, we assume fixed maximum Doppler shift $f_0 = 50$ dB and the number of primary receiver are varied as $N = [2, 5, 7]$. The figures below shows the numerical and analytical plot of outage probability, normalized LCR, and AFD.

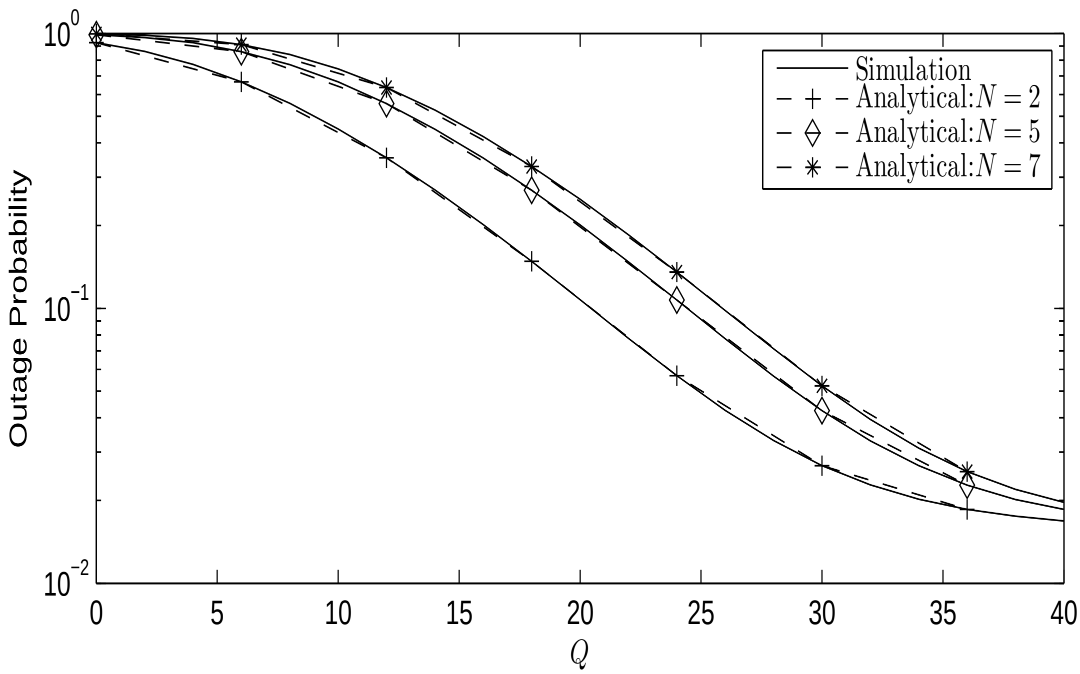

Figure 5.1: Outage Probability for Fixed Maximum Doppler Shift.

Fig. 5.1 shows the outage probability of considered system as a function of interference temperature. From these plots, we can say that as interference temperature increases, outage probability of the system decreases because the primary receivers can tolerate more interference due to secondary transmission with increase of interference temperature i.e., the system performance improves. Also from Fig. 5.1, we observe that as N increases from 2 to 7, the outage probability curve is shifting in upward direction i.e., as the number of primary user increases the outage probability of considered system gets degraded.

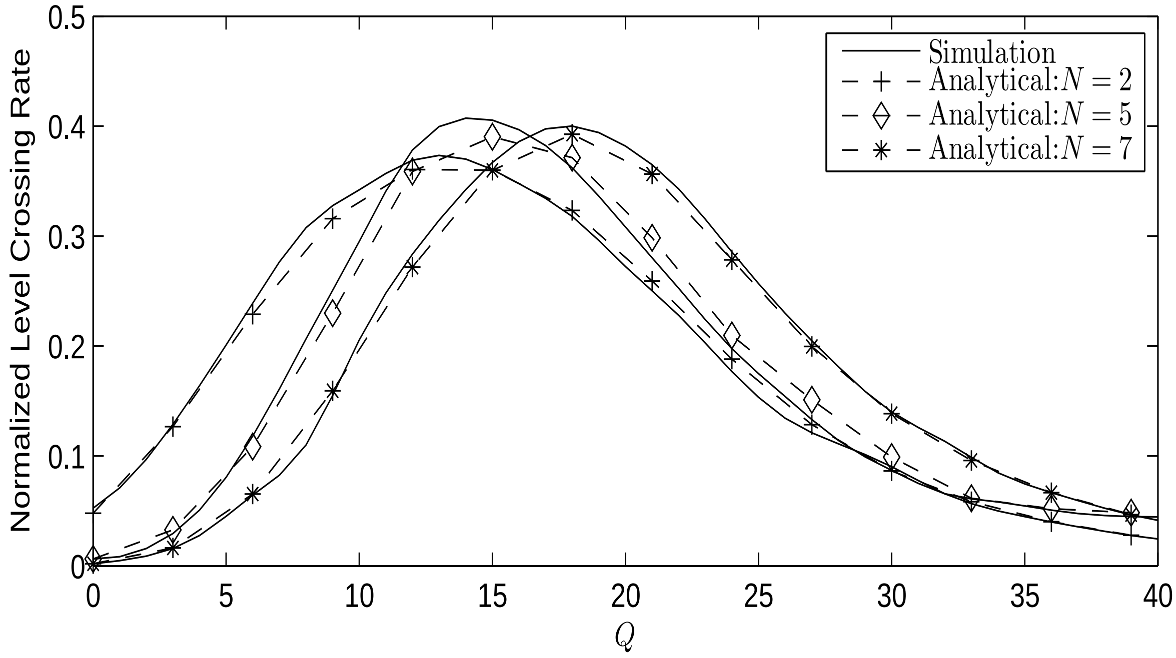

Figure 5.2: Normalized Level Crossing Rate (LCR) for Fixed Maximum Doppler Shift.

Fig. 5.2 shows the normalized LCR of considered system. The LCR is normalized by f_0. From Fig. 5.1 and 5.2, we can see that when the interference temperature is in low SNR regime, the number of times signal envelope crosses the threshold level is less, as the most of time signal envelope lies below the threshold level.

In mid SNR regime, the number of times signal envelope crosses the threshold is high as the signal envelope vary frequently in this range.

In high SNR regime, again the number of times the signal envelope crosses the threshold level is less because in high SNR regime the received signal envelope lies above threshold level.

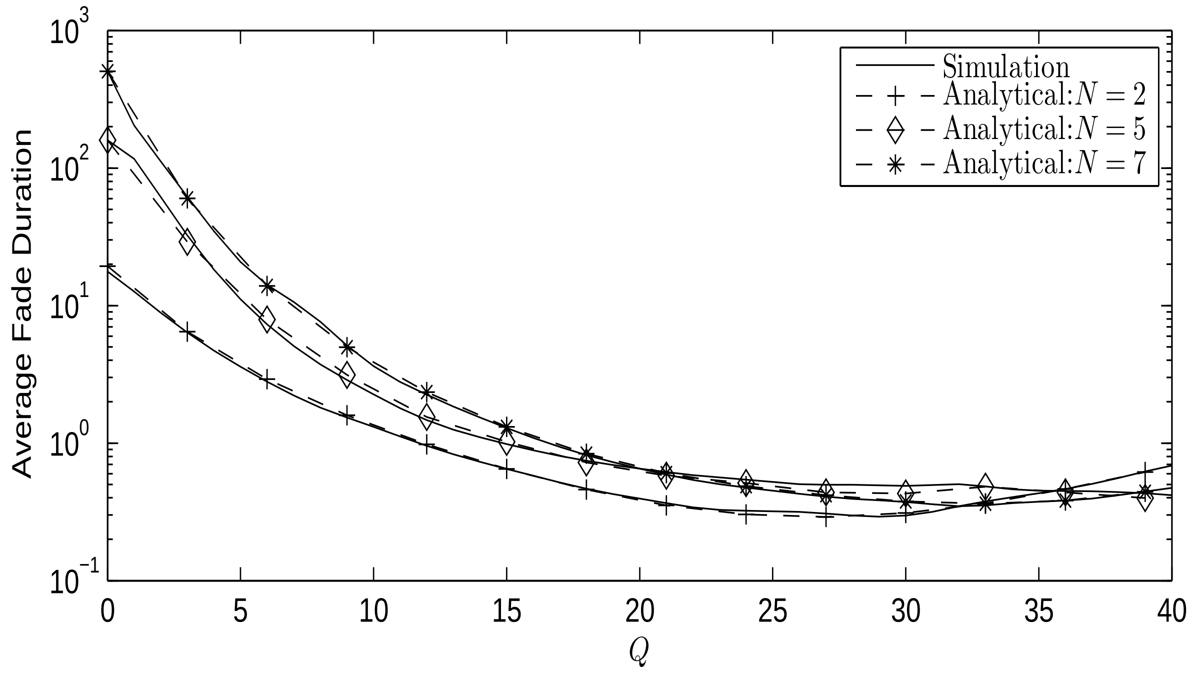

Figure 5.3: Average Fade Duration (AFD) for Fixed Maximum Doppler Shift.

Fig. 5.3 shows the AFD of considered system with $N = 2, 5, 7$. From Fig. 5.3 it is clear that as the interference temperature increases the AFD of the system decreases that is, in low SNR regime, the duration for which the received signal envelope lies below the desired threshold value is high whereas in high SNR regime, this duration is low.
As the number of primary receiver is increases the AFD of system become worsen.

CASE 2: For variable maximum Doppler shift. In this case, we take two values maximum Doppler shift $f_0 = 50$ dB and the number of primary receivers are varied as $N = [2, 5, 7]$. The Figures below shows the numerical and analytical plots of normalized LCR, and AFD.
Fig 5.4 shows that the expected rate at which received signal envelope crosses the threshold level is more when maximum Doppler shift is high.

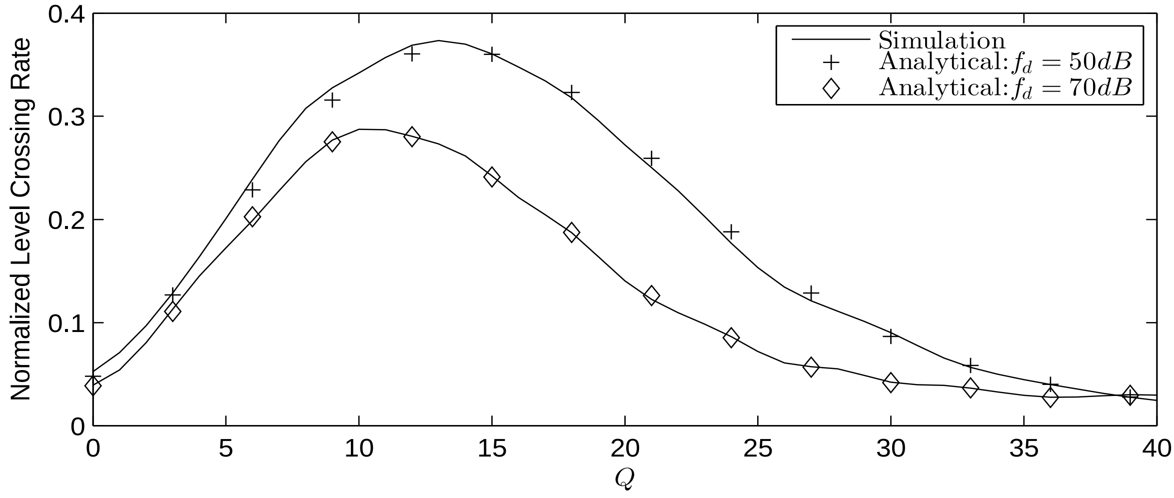

Figure 5.4: Normalized Level Crossing Rate (LCR)for Fixed Number of Primary Receivers.

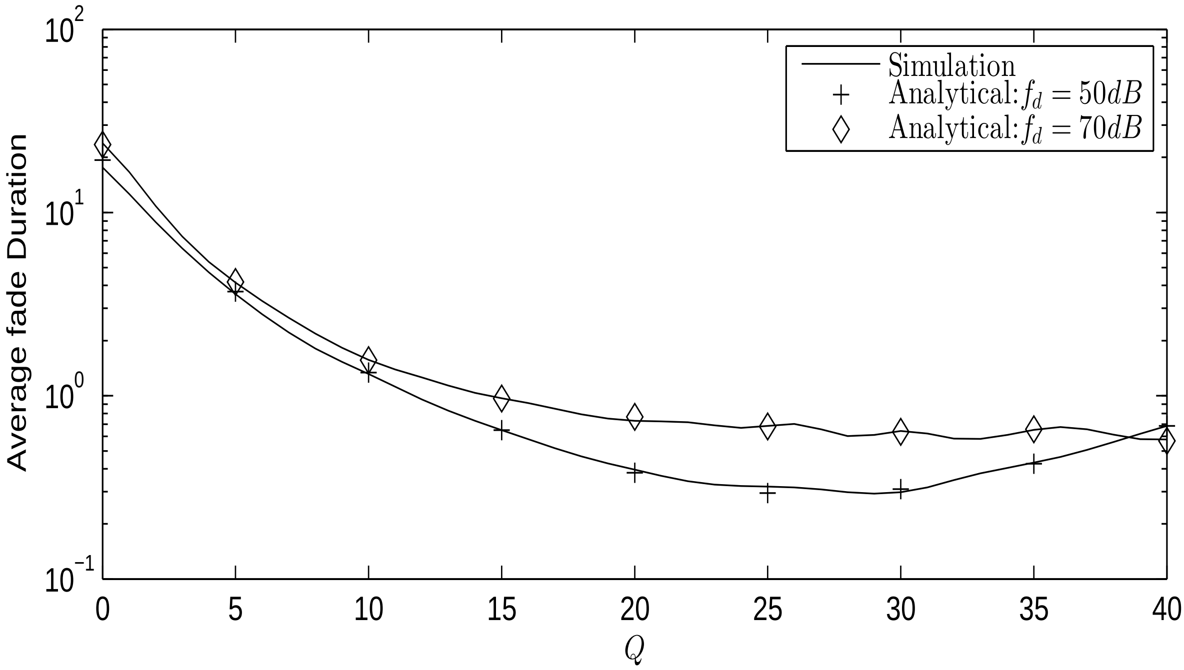

Figure 5.5: Average Fade Duration (AFD) for Fixed Number of Primary Receivers.

Fig 5.5 shows that as the maximum Doppler shift in system increases than the average time for which the system remain in outage condition is more.

The above two Figures shows that as the maximum Doppler shift increases the performance of system gets degraded.

From all plot shown in Figs. 5.1-5.5 we can comment on some important properties of system. As our considered system have stringent energy constraints, the secondary user can be switched off for the least AFD to conserve energy as when value of AFD is least in this case, it is highly unlikely that the transmitted packet can be successfully decoded. The critical velocity of user can be defined over which the system performance gets severely degraded. Because the velocity of user is directly proportional to the maximum Doppler shift.

Chapter 6

Conclusions and Future Work

In this chapter, we conculde our thesis work with possible scope and extension for future work in related field.

6.1 Conclusions

Second order statistics analysis has been investigated for cognitive radio system with multiple primary receivers. We derive outage probability, LCR, and AFD expression of considered system in Rayleigh fading environment. Finally, we verify our analytical results through extensive simulation. From the obtained results, following inferences are reached:

1) From the plots of outage probability, we can easily say that as the interference temperature is increased, the performance of the cognitive radio system gets better.

2) From the LCR and AFD plots, we deduce that in low SNR regime or at low interference temperature, the received signal lies below the threshold level for longer duration and at higher SNR regime or higher interference temperature, the received signal stays above the threshold level for the longer duration.

3) As the maximum Doppler frequency shift increases the system performance gets degraded.

6.2 Future work

We have analyzed the second order statistics of the cognitive radio system with multiple primary receivers in Rayleigh fading. One can extend the work of the thesis by investigating second order statistics of the considered system in more generalized Nakagami fading. Effect of multiple primary transmitters can also be considered by assuming them in proximity of cognitive radio. By incorporating multiple secondary users in considered system, this thesis work can be further extended.

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