B. TECH. PROJECT REPORT

On

Application of Modern Control Theory on DC-DC converters

BY

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DISCIPLINE OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY INDORE

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Application of Modern Control Theory on DC-DC converters

A PROJECT REPORT

Submitted in partial fulfillment of the

requirements for the award of the degrees

of

BACHELOR OF TECHNOLOGY

in

ELECTRICAL ENGINEERING

Submitted by:

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Guided by:

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CANDIDATE'S DECLARATION

We hereby declare that the project entitled **"Application of Modern Control Theory to DC-DC converters"** submitted in partial fulfillment for the award of the degree of Bachelor of Technology in 'Electrical Engineering' completed under the supervision of **Dr. Amod C. Umarikar, Assistant Professor Electrical Engineering,** IIT Indore is an authentic work.

Further, we declare that we have not submitted this work for the award of any other degree elsewhere.

Signature and name of the student(s) with date

CERTIFICATE by BTP Guide

It is certified that the above statement made by the students is correct to the best of my knowledge.

Signature of BTP Guide with dates and designation

Preface

This report on *Application of Modern Control Theory on DC-DC Converters* is prepared under the guidance of Dr. Amod C. Umarikar.

Through this report, we have tried to give a detailed analysis of the various control techniques that can be used for the voltage control of commonly-used DC-DC converters. We have tried to the best of our abilities and knowledge to explain the content in a lucid manner. We have also added the circuit schematics in Simulink and the output responses in both time and frequency domain to make the presentation more illustrative.

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Abstract

This project report documents the research on the different types of control methods and topologies that can be applied to the self-correcting mechanism of the output of common DC-DC converters, and focuses especially on the still uncharted technique of using State Space analysis to provide a control feedback mechanism for these converters. All the simulations on the various circuits were performed on Simulink in MATLAB. Extensive literature survey was performed to learn the various mathematical techniques employed to devise efficient control systems.

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INTRODUCTION

Voltage regulation is the need of every electrical circuit that we see in our day-to-day lives. A step-up converter that can account for a greater range of voltage (12-48 V) is always preferable over a step-up converter that has a low range for voltage (24-40 V). Naturally, one would want to keep their microcontroller board or any other machine that uses step-up converter as its power source to running even when the voltage range varies for a wider range than the average.

DC-DC converters, particularly the Boost-type converters, suffer from various inaccuracies in their output which is affected by the resistances of the inductors and the capacitors involved. The higher the order of the converter, the more there is the need of a control feedback mechanism. Because of the extensive use of converters in most of the modern appliances, it is very important that efficient and efficacious methods are evolved and perfected to control their behavior.

The most widely-used methods of providing compensation or control are the Root Locus technique and the Bode plot technique. A brief encounter with these techniques would prove – at later stages – that the SS method provides more flexibility and more accurate output than the classical counterparts.

An important aspect of this project was to look into the observer design using SS analysis. Many-a-times, the only sensors available for measuring states are those of the output/s, which severely limits our ability to devise a robust system for control. To deal with this problem, the concept of observers is used to estimate the required states within a band of error tolerance.

The converters worked upon in this project are Boost-type: Second-order Boost converter, Quadratic Boost Converter, and SEPIC/Zeta Converter.

Anyone working on the implementation of SS techniques to control DC-DC converters can use this project as a guideline.

The key assumptions while working on this project:

- The LC charging and discharging time constants are far greater than the operating sample time of the MOSFET driver.
- The resistances of inductor and capacitor are too small and therefore neglected in the steady state and dynamic analysis.
- Simulations and calculations are carried out in ideal states on MATLAB Simulink.

The team members were responsible for collectively documenting the progress report; therefore, this report covers all the aspects of the project.

The major accomplishments of this project are:

1. **Theory:** The participants of this project were involved in an extensive literature survey when it came to SS analysis. Apart from that, classical control techniques were also used, and their relevance is also enumerated here. Various mathematical models and stratagems were learnt so as to be employed for finding out the values of the gain blocks while using the modern control theory.
2. **Simulations:** By using the various modules in MATLAB and Simulink, the control mechanisms were successfully simulated for DC-DC converters of second and fourth orders (Second-order Buck and Boost Converters, Quadratic boost converter, and SEPIC and Zeta converters). The desired transient and steady-state characteristics were also realized.

The dependence of Boost converter output on just the duty cycle made the duty cycle a key factor in setting limits for a DC-DC converter. The constraint of Duty cycle makes it difficult to achieve wider range of input. This constraint is typically 20%-80%; in extremes it can be stretched till 5%-95%.

The theoretical analysis was verified in two stages – open loop and closed loop. The former verified the steady state analysis and the simulations in the latter verified the small signal analysis. These simulations matched the exact theoretical results which can further be used in hardware implementation.

PART – A

THE CLASSICAL APPROACH

CHAPTER 1 THE DC-DC CONVERTER

Before the development of power semiconductors and allied technologies, one way to convert the voltage of a DC supply to a higher voltage, for low-power applications, was to convert it to AC by using a vibrator, followed by a step-up transformer and rectifier. For higher power an electric motor was used to drive a generator of the desired voltage (sometimes combined into a single "dynamotor" unit, a motor and generator combined into one unit, with one winding driving the motor and the other generating the output voltage). These were relatively inefficient and expensive procedures used only when there was no alternative, as to power a car radio (which then used thermionic valves/tubes requiring much higher voltages than available from a 6 or 12 V car battery). The introduction of power semiconductors and integrated circuits made it economically viable to use techniques as described below, for example to convert the DC power supply to high-frequency AC, use a transformer—small, light, and cheap due to the high frequency—to change the voltage, and rectify back to DC. Although by 1976 transistor car radio receivers did not require high voltages, some amateur radio operators continued to use vibrator supplies and dynamotors for mobile transceivers requiring high voltages, although transistorized power supplies were available.

While it was possible to derive a lower voltage from a higher with a linear electronic circuit, or even a resistor, these methods dissipated the excess as heat; energy-efficient conversion only became possible with solid-state switch-mode circuits.

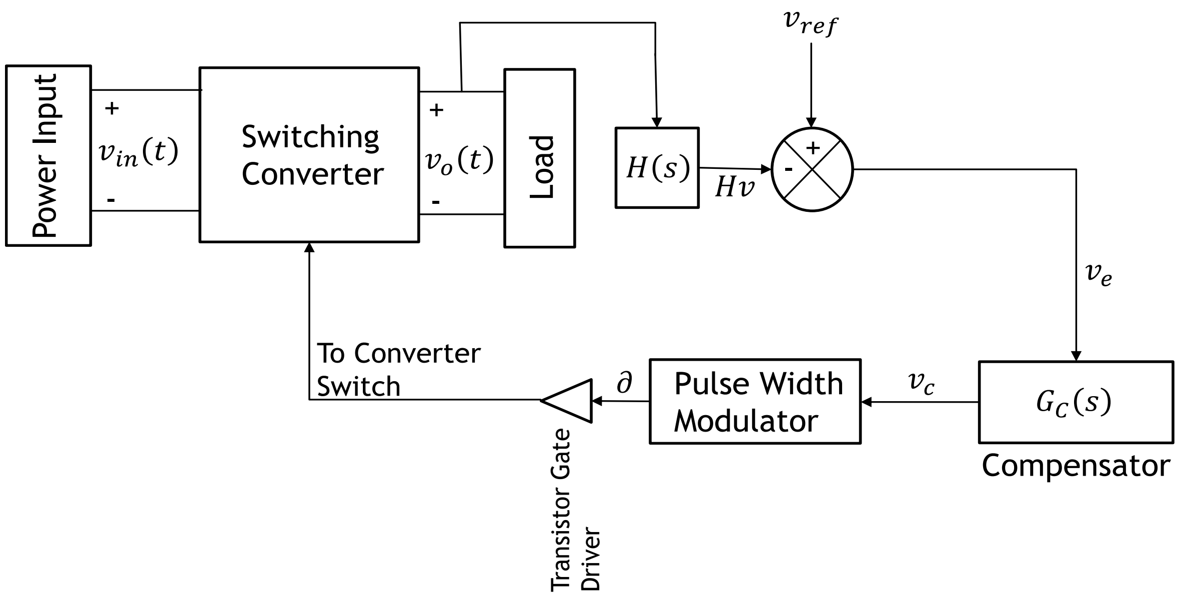
DC to DC converters are used in portable electronic devices such as cellular phones and laptop computers, which are supplied with power from batteries primarily. Such electronic devices often contain several sub-circuits, each with its own voltage level requirement different from that supplied by the battery or an external supply (sometimes higher or lower than the supply voltage). Additionally, the battery voltage declines as its stored energy is drained. Switched DC to DC converters offer a method to increase voltage from a partially lowered battery voltage thereby saving space instead of using multiple batteries to accomplish the same thing.

Most DC to DC converter circuits also regulate the output voltage. Some exceptions include high-efficiency LED power sources, which are a kind of DC to DC converter that regulates the current through the LEDs, and simple charge pumps which double or triple the output voltage.

Transformers used for voltage conversion at mains frequencies of 50–60 Hz must be large and heavy for powers exceeding a few watts. This makes them expensive, and they are subject to energy losses in their windings and due to eddy currents in their cores. DC-to-DC techniques that use transformers or inductors work at much higher frequencies, requiring only much smaller, lighter, and cheaper wound components. Consequently, these techniques are used even where a mains transformer could be used; for example, for domestic electronic appliances it is preferable to rectify mains voltage to DC, use switch-mode techniques to convert it to high-frequency AC at the desired voltage, then, usually, rectify to DC. The entire complex circuit is cheaper and more efficient than a simple mains transformer circuit of the same output.

We performed the steady state analyses on buck and boost converters. A buck converter (step-down converter) is a DC-to-DC power converter which steps down voltage (while stepping up current) from its input (supply) to its output (load). A boost converter, on the other hand, is a DC-to-DC power converter that steps up voltage (while stepping down current) from its input (supply) to its output (load).

CHAPTER 2 CLASSICAL CONTROL APPROACH



2.1 Equivalent block diagram for feedback control

The Bode Plot is a classical approach of dealing with control feedback. The addition of poles and zeros is simple and intuitive. The main focus is on the following two parameters pertaining to a closed-loop system.

- **Gain margin, GM.** The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180^o of phase shift to make the closed-loop system unstable.
- **Phase margin, FM.** The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

$$\frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + B\hat{u}(t) + [(A_1 - A_2)X + (B_1 - B_2)U]\hat{d}(t)$$

$$\hat{y}(t) = C\hat{x}(t) + E\hat{u}(t) + [(C_1 - C_2)X + (E_1 - E_2)U]\hat{d}(t)$$

$$A = DA_1 + D'A_2$$

$$B = DB_1 + D'B_2$$

$$C = CB_1 + D'C_2$$

$$E = EB_1 + D'E_2$$

$$G_{vd}(s) = G_{d0} \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

The salient features of the line-to-output and control-to-output transfer functions of the basic buck, boost, and buck-boost converters are summarized. In each case, the control-to-output transfer function is of the form

Salient features of the small signal CCM transfer functions of some basic DC-DC converters.

Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
Buck	D	$\frac{V}{D}$	$\frac{1}{\sqrt{LC}}$	$R\sqrt{\frac{C}{L}}$	∞
Boost	$\frac{1}{D'}$	$\frac{V}{D'}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{L}$
Buck-Boost	$-\frac{D}{D'}$	$\frac{V}{DD'^2}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{DL}$

Stability

It is well known that adding a feedback loop can cause an otherwise stable system to become unstable. Even though the transfer functions of the original converter, as well as of the loop gain *T(s),* contain no right half-plane poles, it is possible for the closed-loop transfer functions to contain right half-plane poles. The feedback loop then fails to regulate the system at the desired quiescent operating point, and oscillations are usually observed. It is important to avoid this situation. And even when the feedback system is stable, it is possible for the transient response to exhibit undesirable ringing and overshoot. However, a special case of the theorem known as the phase margin test is sufficient for designing most voltage regulators

The Phase Margin Test

The crossover frequency *f*, is defined as the frequency where the magnitude of the loop gain is unity:

II *T(j2πf)* II = 1 => 0 dB

To compute the phase margin ϕ_m, the phase of the loop gain Tis evaluated at the crossover frequency,

and 180° is added. Hence,

$\phi_m = 180^\circ + \angle T(j2\pi f)$

If the phase margin is positive, the feedback system is stable.

2.1 Buck Converter – Classical Approach

To illustrate the design of PI and PD compensators, let us consider the design of a combined PID compensator for the dc-dc buck converter system. The input voltage $V_g(t)$ for this system has nominal value 28 V. It is desired to supply a regulated 14 V to a load. The load is modeled here with a 3Ω resistor.

The quiescent duty cycle is given by the steady-state solution of the converter:

$$D = \frac{V}{V_g} = \frac{15}{28} = 0.536$$

Thus, the quiescent conditions of the system are known.

$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s\frac{L}{R} + s^2LC}$$

The open-loop control-to-output transfer function is

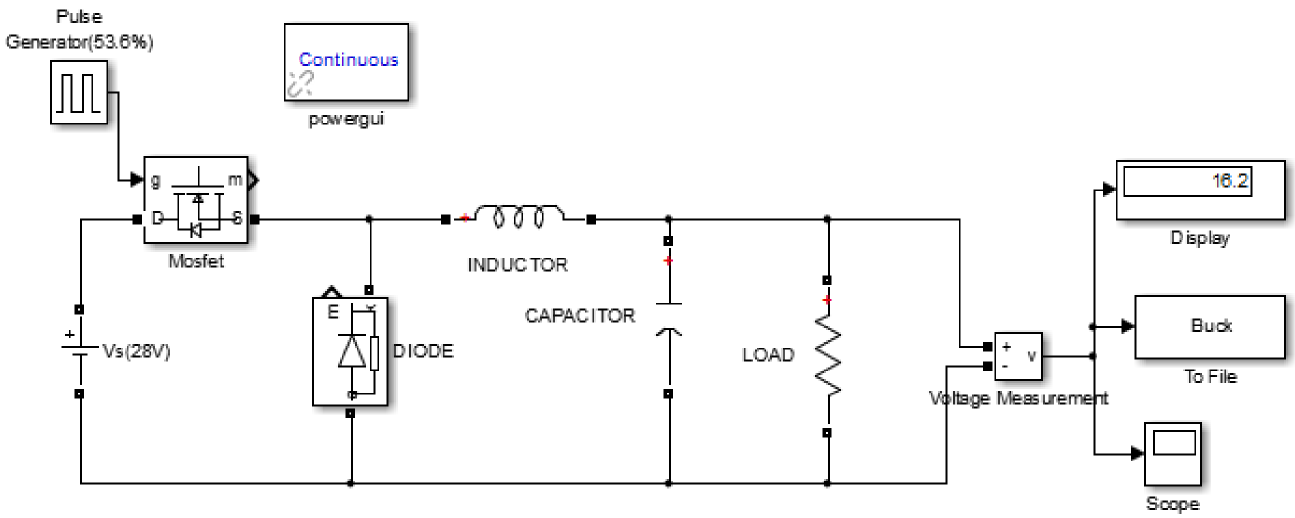
The open loop gain of the system is

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right)G_{vd}(s)$$

Substituting values of G_{vd} in the above equation

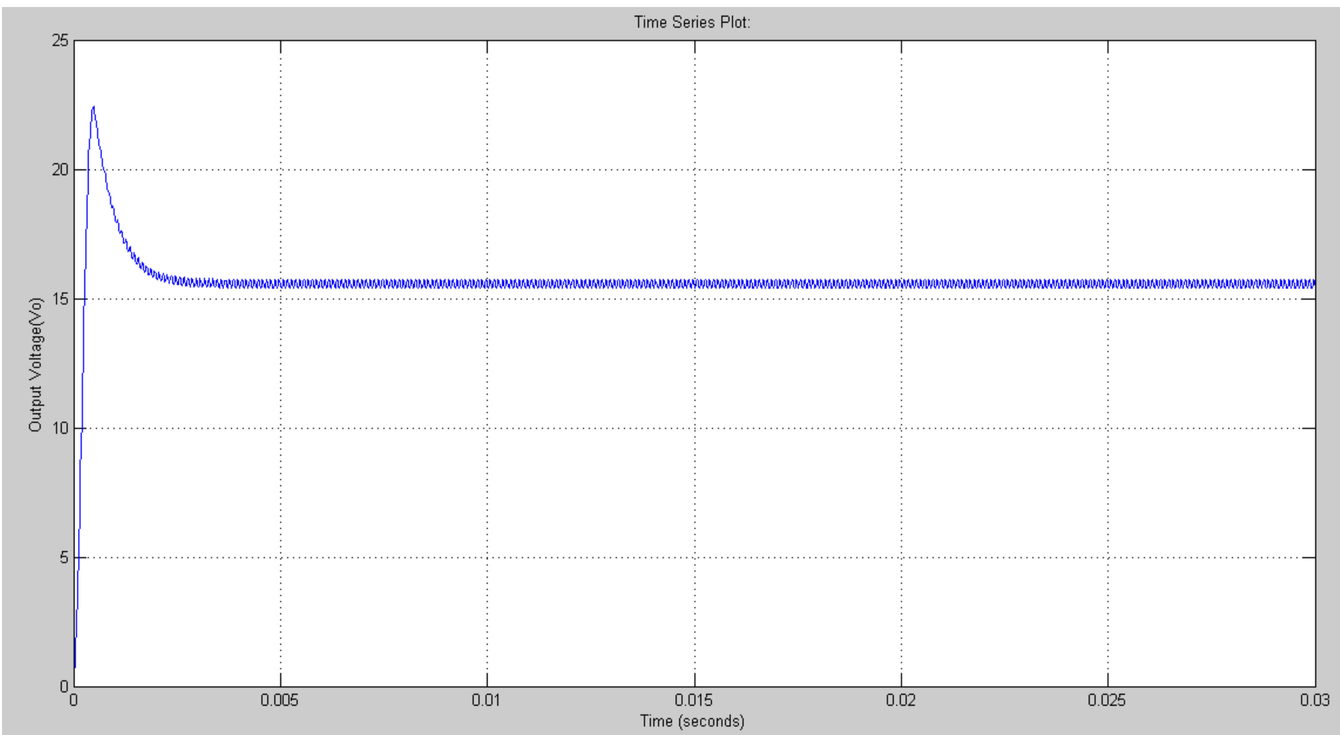
$$T(s) = \frac{G_c(s)}{V_M}\frac{V}{D}\frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Circuit Diagram:



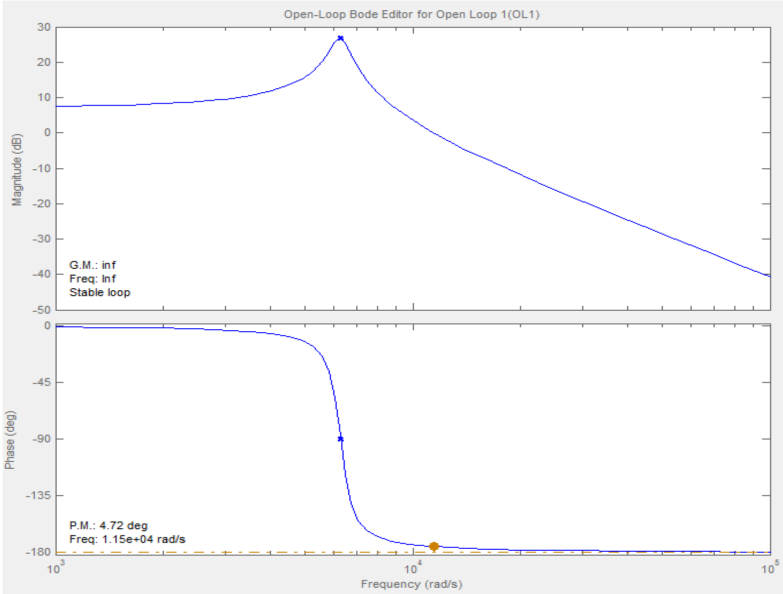
2.2 Circuit diagram of open loop buck converter

Output response:



2.3 Output response of open loop buck converter

The reference voltage was set at 15V. The response settled within 0.004 s with some steady state error.



2.4 Bode plot for open loop transfer function of Buck converter

The uncompensated loop gain has a crossover frequency of approximately 1.8 kHz, with a phase margin of less than five degrees.

Let us design a compensator, to attain a crossover frequency of fc = 5 kHz, or one twentieth of the switching frequency. From Fig. 9.26, the uncompensated loop gain has a magnitude at 5 kHz of approximately $T_{u0}\ (f0\ /fc)^2$ = 0.093 => - 20.6 dB. So, to obtain unity loop gain at 5 kHz, our compensator should have a 5 kHz gain of + 20.6 dB. In addition, the compensator should improve the phase margin, since the phase of the uncompensated loop gain is nearly - 180° at 5 kHz. So a lead (PD) compensator is needed. Let us (somewhat arbitrarily) choose to design for a phase margin of 52°. According to Fig. 9.13, this choice leads to closed-loop poles having a Q-factor of 1. The unit step response, then exhibits a peak overshoot of 16%. Evaluation of f_c= 5 kHz and θ =

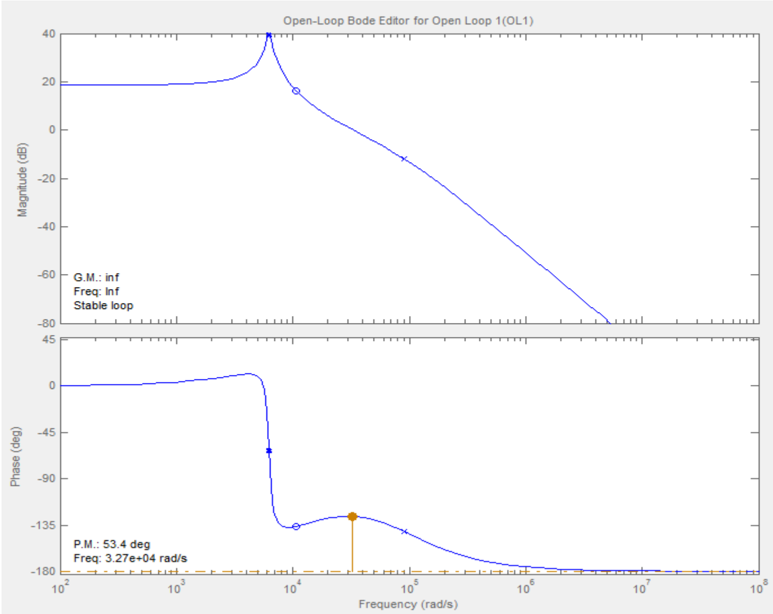
$$f_z = (5\ \text{kHz})\sqrt{\frac{1-\sin(52^\circ)}{1+\sin(52^\circ)}} = 1.7\ \text{kHz}$$

$$f_p = (5\ \text{kHz})\sqrt{\frac{1+\sin(52^\circ)}{1-\sin(52^\circ)}} = 14.5\ \text{kHz}$$

52°, leads to the following compensator pole and zero frequencies:

To obtain a compensator gain of 20.6 dB=> 10.7 at 5kHz, the low-frequency compensator gain must be

$$G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\ \text{dB}$$

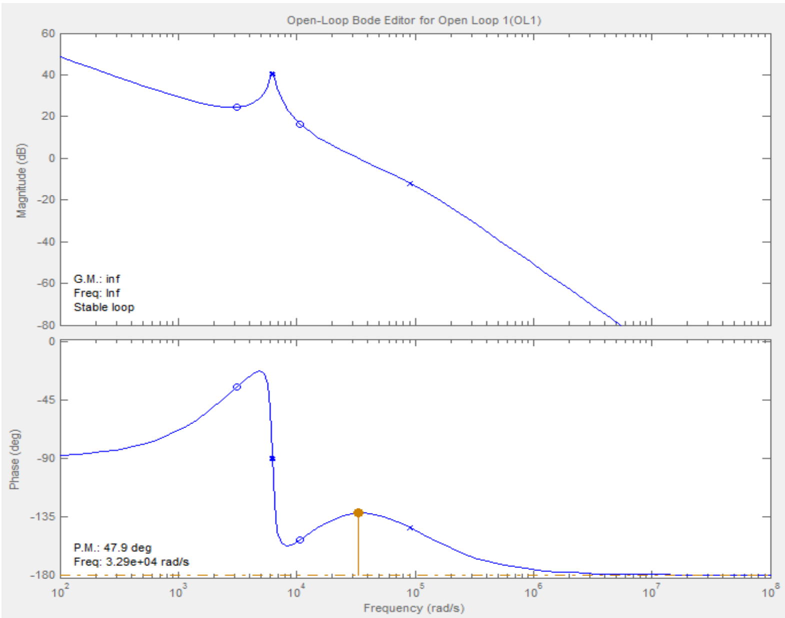


2.5 Bode plot of the closed loop converter transfer function of Buck converter after PD

With this PD controller, the loop gain becomes

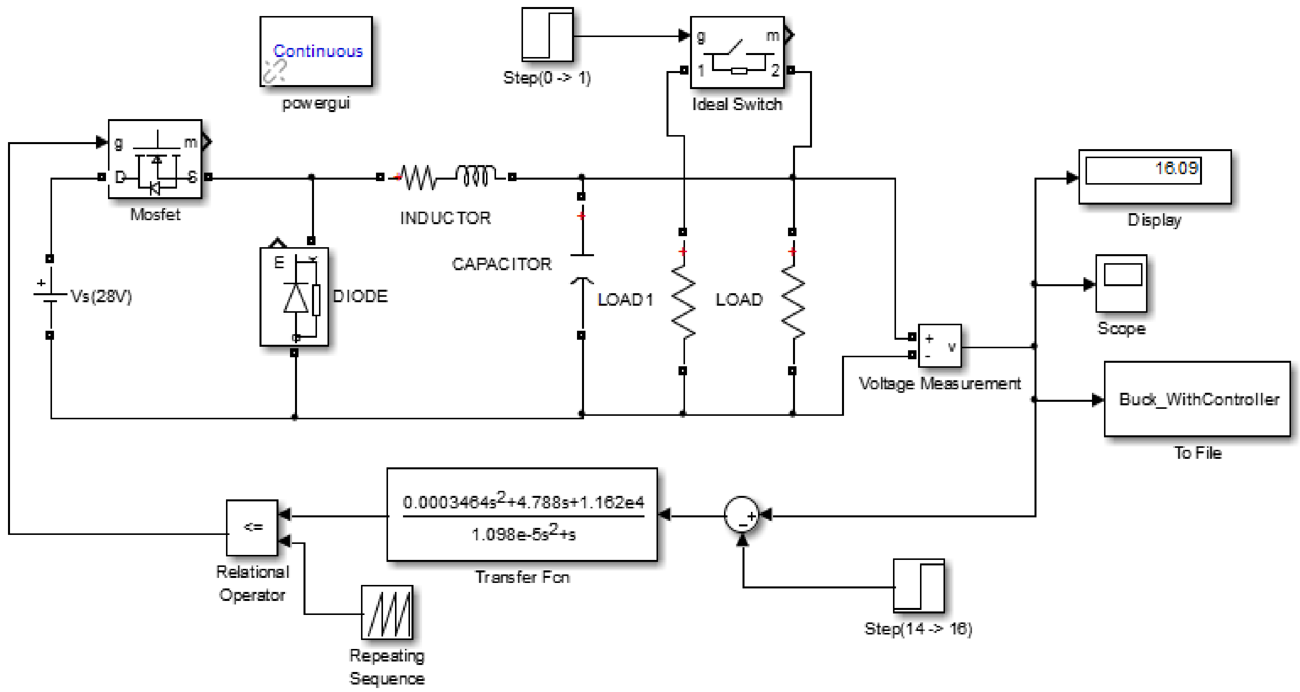
$$T(s) = T_{u0}\, G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

It can be seen that the phase of T (s) is approximately equal to 52° over the frequency range of 1.4 kHz to 17 kHz. Hence variations in component values, which cause the crossover frequency to deviate somewhat from 5 kHz, should have little impact on the phase margin. In addition, it can be seen from Fig. 9.28 that the loop gain has a dc magnitude of Tu0G_{c0} => 18.7 dB



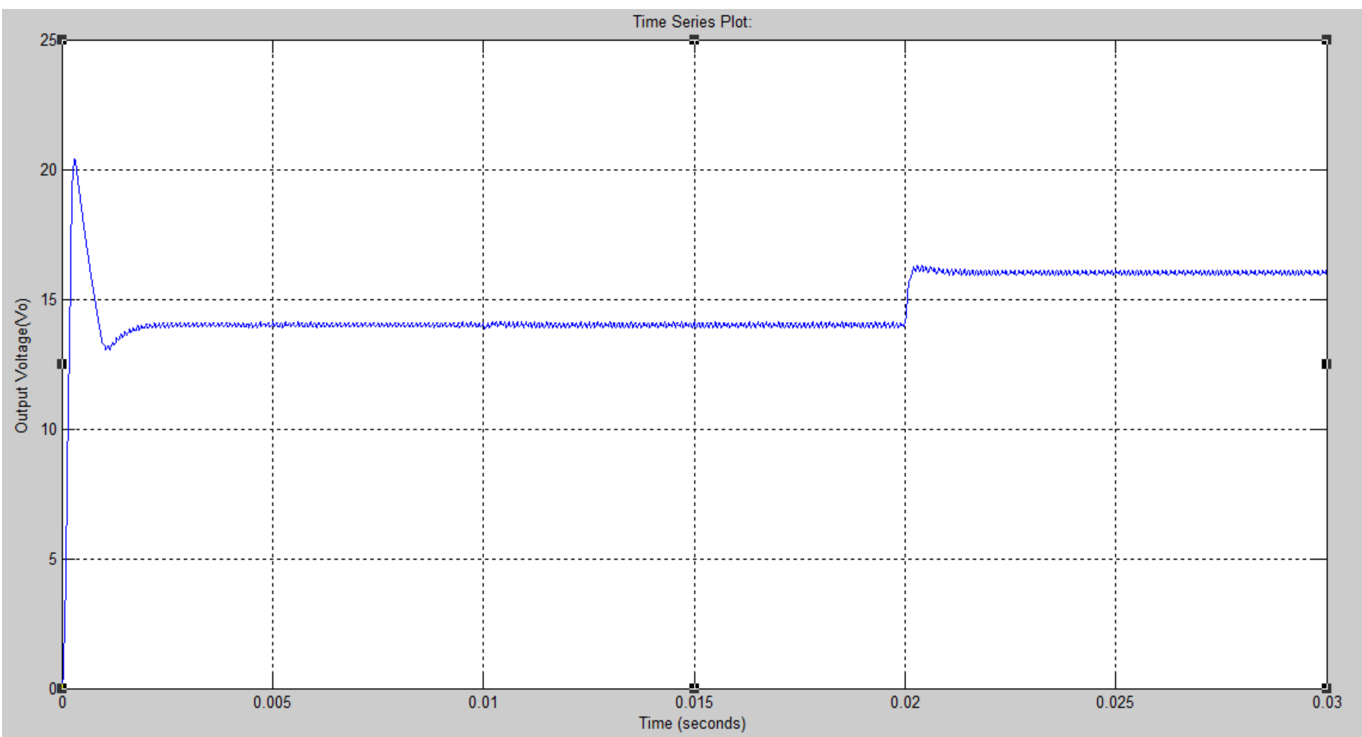
2.6 Bode plot of the closed loop converter transfer function of Buck converter after PID

Circuit Diagram:



2.7 Circuit diagram of closed loop Buck converter after PID

Output Response:

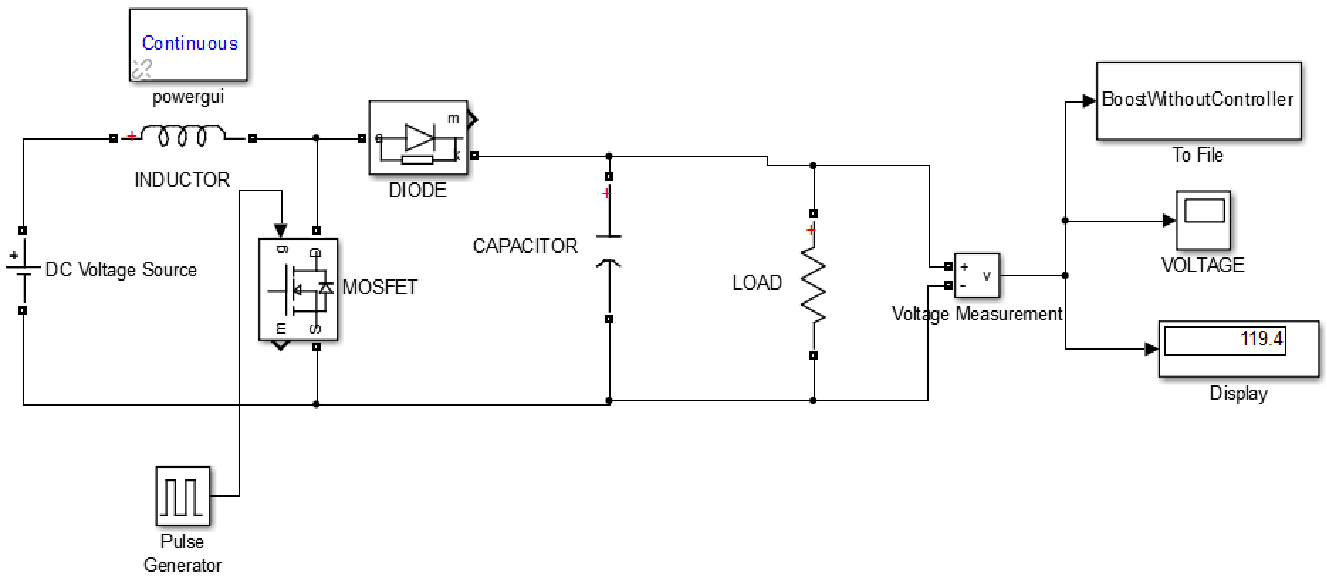


2.8 Output response of Buck converter after PID

The reference voltage was stepped up from 14 V to 16 V at t=0.02 s. To check the effect of load, the load was doubled at t=0.01s. The response settled within 0.004 s with minimum error.

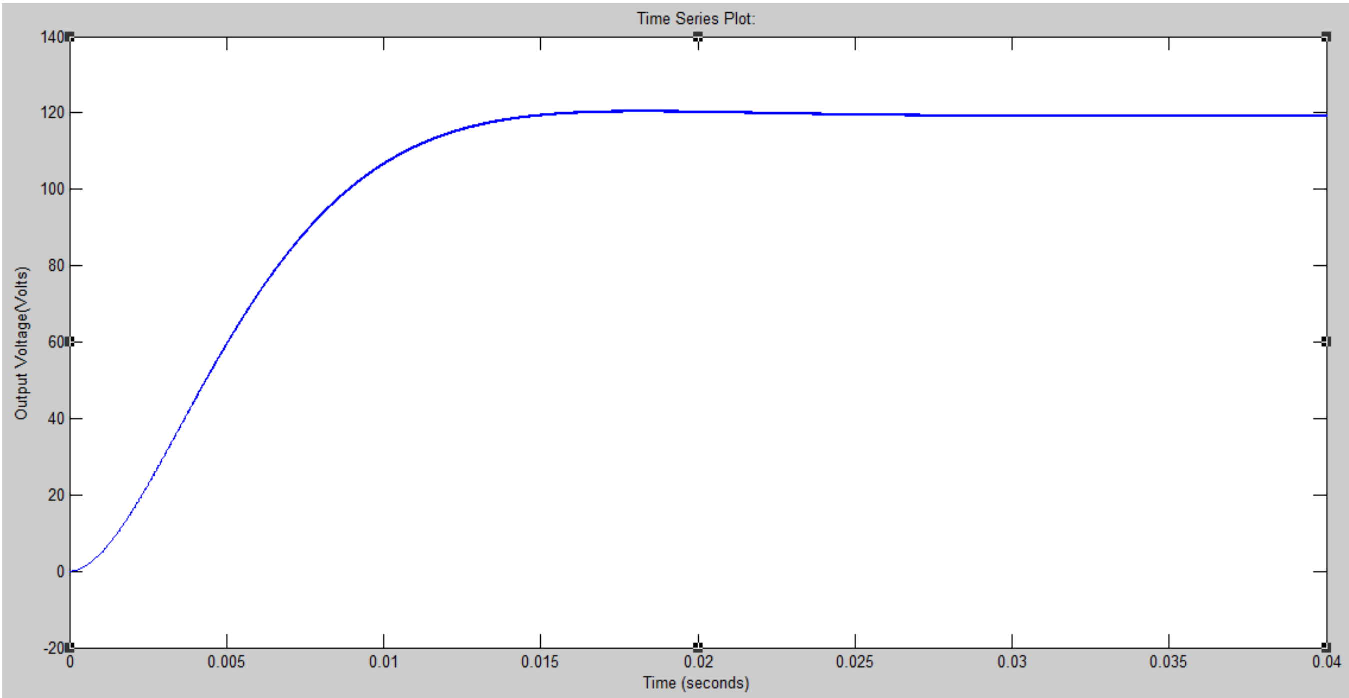
2.2 Boost Converter – Classical Approach

Circuit Diagram:



2.9 Circuit diagram of open loop boost converter

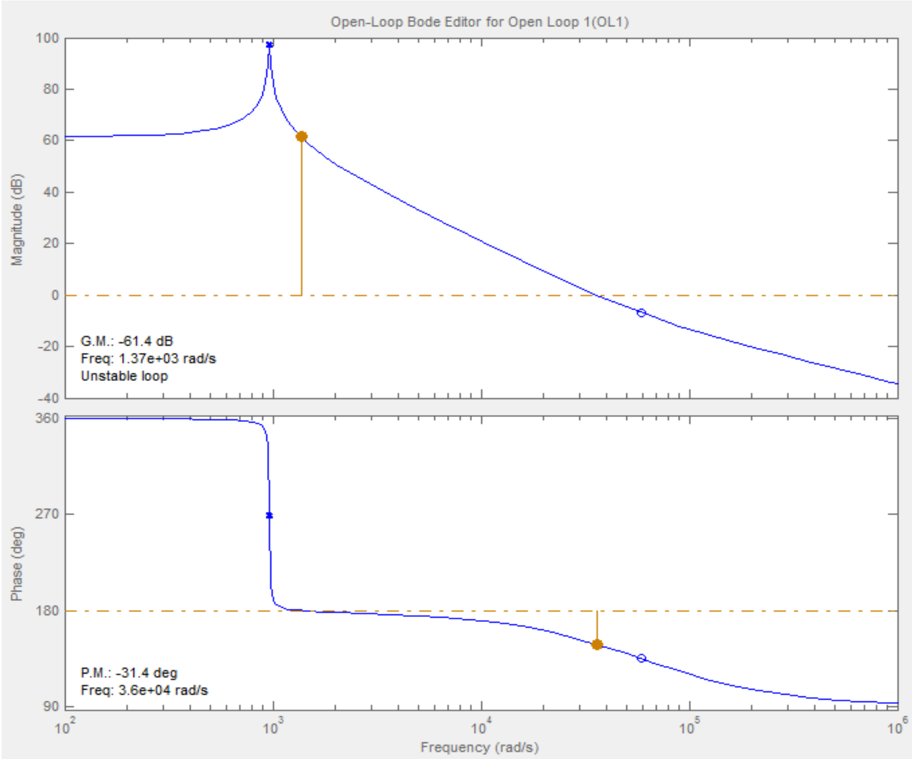
Output Response:



2.10 Output response of open loop boost converter

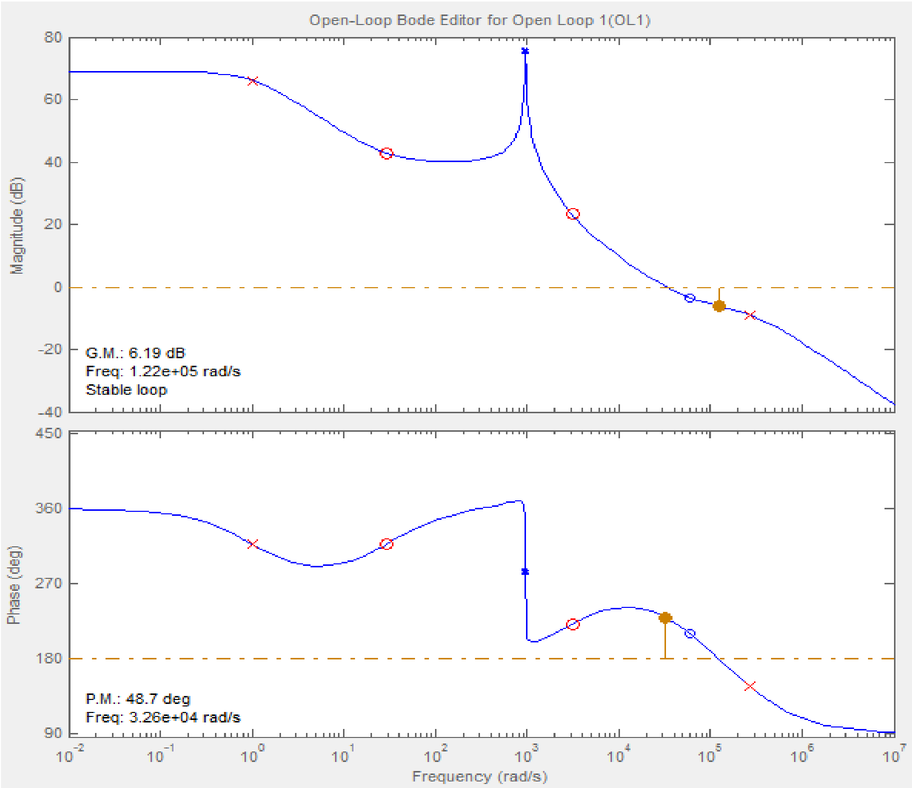
The reference voltage was set at 120V. The response settled within 0.02 s with some steady state error.

Bode plot:



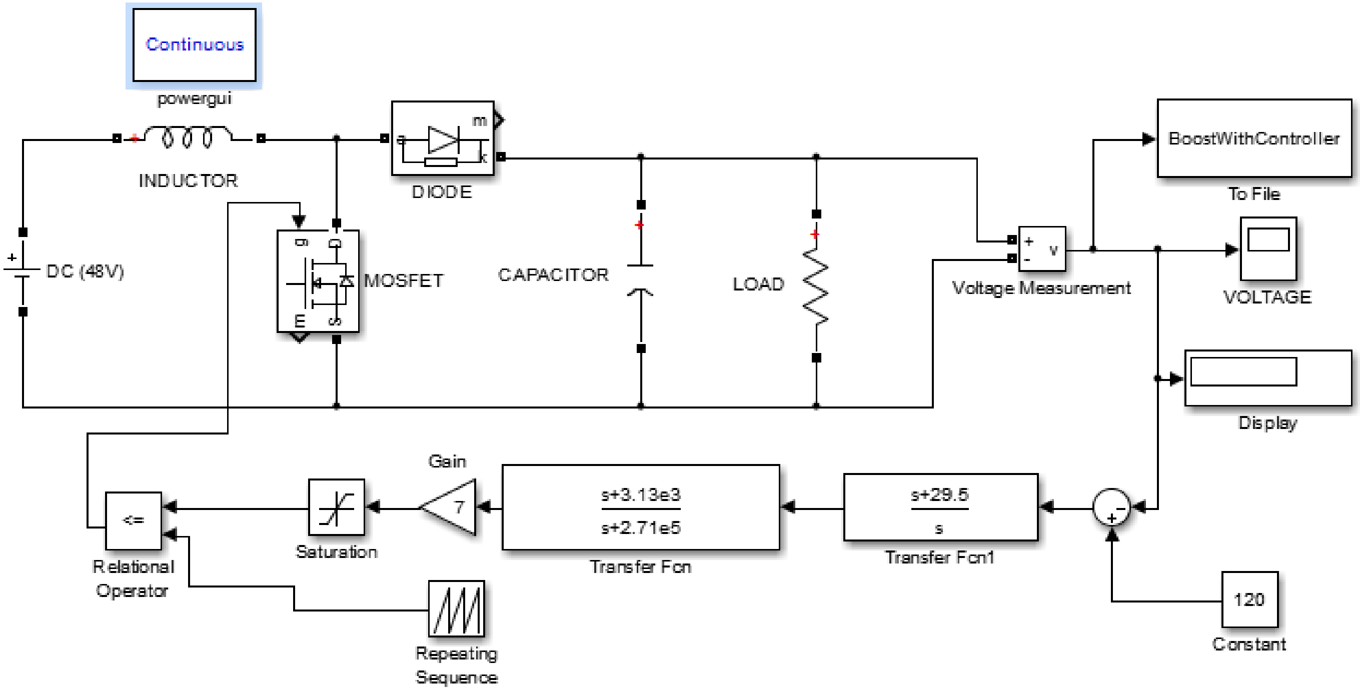
2.11 Bode plot for open loop transfer function of Boost converter

Boost Converter – After PID



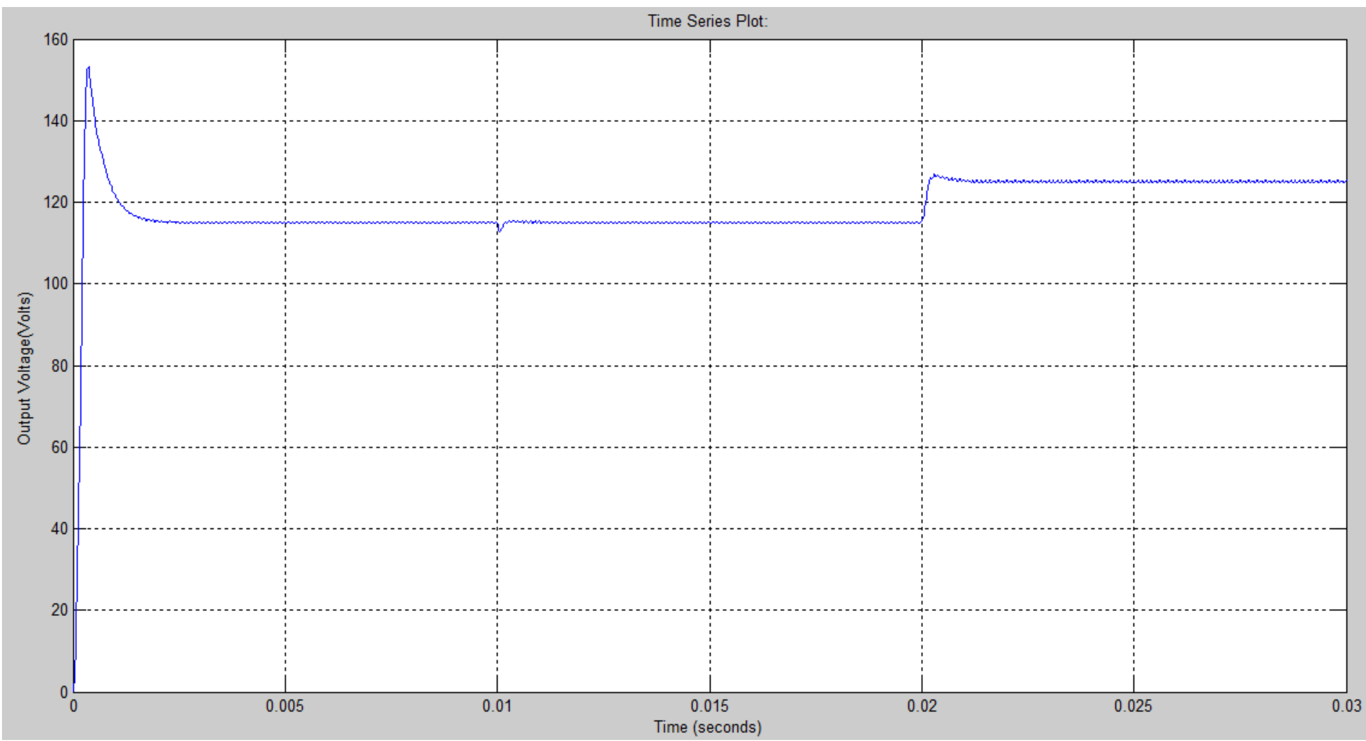
2.12 Bode plot of Boost converter after PID

Circuit Diagram:



2.13 Circuit diagram of closed loop boost converter after PID

Output Response:



2.14 Output response of Boost converter after PID

The reference voltage was stepped up from 115 V to 125 V at t=0.02 s. To check the effect of load, the load was doubled at t=0.01s. The response settled within 0.004 s with minimum error.

PART – B

THE ADVANCED CONTROL METHOD

CHAPTER 3 ADVANCED CONTROL THEORY

In addition to the transform techniques of root locus and frequency response, there is a third major method of designing feedback control systems: the state-space method. This method involves the representation of the system in the form of differential equations in the state variables, which are essentially the minimum-required parameters of the system that we can measure/estimate to implement compensation.

Models in state-variable form enhance our ability to apply the computational efficiency of computer-aided design tools such as MATLAB. The steps of the design method are as follows:

1. Select closed-loop pole (root as referred to in classical theory) locations and develop the control law for the closed-loop system that corresponds to satisfactory dynamic response
2. Design an estimator
3. Combine the control law and the estimator
4. Introduce the reference input

After working through the central design steps, we briefly explore the use of integral control in state-space.

Advantages of state-space design are especially apparent when the system to be controlled has more than one control input or more than one sensed output. However, in this project we shall examine the ideas of state-space design using the simpler single-input-single output (SISO) systems. The design approach used for systems described in state form is “divide and conquer.” First, we design the control as if all of the state were measured and available for use in the control law. This provides the possibility of assigning arbitrary dynamics for the system. Having a satisfactory control law based on full-state feedback, we introduce the concept of an observer and construct estimates

of the state based on the sensed output. We then show that these estimates can be used in place of the actual state variables.

Finally, we introduce the external reference-command inputs, and the structure is complete. Only at this point can we recognize that the resulting compensation has the same essential structure as that developed with the classical transform methods.

CHAPTER 4 ANALYSIS TECHNIQUES

In state space analysis, the given system is defined in terms of its state variables and linear differential equations along with input and output variables. The state is expressed as a vector, and the whole system, being LTI (Linear Time-Invariant), can be represented in terms of matrix equations:

$$\frac{d\boldsymbol{x}}{d\boldsymbol{t}} = \boldsymbol{A}.\boldsymbol{x} + \boldsymbol{B}.\boldsymbol{u} \qquad \textbf{(State equation)}$$

$$\boldsymbol{y} = \boldsymbol{C}.\boldsymbol{x} + \boldsymbol{D}.\boldsymbol{u} \qquad \textbf{(Output equation)}$$

Where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ the input vector, and $\boldsymbol{y}$ the output vector, of orders n, p, and q, respectively. The matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C},$ and $\boldsymbol{D}$ are of the dimensions *(n × n)*, *(n × p)*, *(q × n)*, and *(q × p)*, respectively. The state vector can be treated as a matrix of dimensions *(n × 1)*, with the i^{th} element equal to the i^{th} state variable. In our case, these can be the voltage/s of capacitor/s and/or the currents through the inductor/s.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}.$$

Depending on the number of inductors and capacitors used, the order of the converter can be determined. A simple boost or buck converter employs the use of a single pair of capacitor and inductor, which makes it a second-order converter. Higher order converters like the quadratic boost converter or the SEPIC converter use two pairs of inductor and capacitor, making their order equal to four. The challenge with the higher order converters lies in the hindrance they cause while the desired closed-loop poles are being chosen; second-order approximation cannot be exercised. Although, unlike classical methods – which only provide with pole placement of the dominant poles of the closed-loop – the modern method can be used to place all the closed-loop poles at the respective desired locations.

4.1 Steady-State Analysis

For the steady state analysis, we find the state-space model by using state-space averaging method. The matrices A, B and C have differing elements depending on the duty cycle portion for which the switch is currently in operation. The state variables can be any variables of the system so long as their estimation/ direct measurement can be done. This is done in the same way as done in the case of finding the feedback compensation using the Bode plot: first modeling the converter using the state variables, followed by the application of a small signal (input duty cycle), and then linearizing the perturbed system using approximation. Revisiting the equations derived in the last chapter, we can get the values of the matrices ***A, B, C*** and ***D*** for a given converter.

4.2 Pole Placement

Just like the classical methods, we can place the desired closed-loop poles using the appropriate feedback gain in state-space domain. But for this to be possible, there are certain conditions to be adhered to:

- The system is completely state-controllable
- The state-variables are at least estimable and are available for feedback
- The control input is unconstrained

State-controllability

The state of a deterministic system, which is the set of values of all the system's state variables (those variables characterized by dynamic equations), completely describes the system at any given time. In particular, no information on the past of a system is needed to help in predicting the future, if the states at the present time are known and all current and future values of the control variables (those whose values can be chosen) are known.

Complete state controllability (or simply controllability if no other context is given) describes the ability of an external input (the vector of control variables) to move the internal state of a system from any initial state to any other final state in a finite time interval.

A system with internal state vector ***X*** is called controllable if and only if the system states can be changed by changing the system input. If a given system is controllable, the rank of the controllability should be equal to its order.

The controllability matrix for the system defined by the above equations is; order *(n × n)*:

$$\boldsymbol{P} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A^2B} \quad \boldsymbol{A^3B} \ \ldots \ \boldsymbol{A^{n-1}B}]$$

The system is controllable iff *rank(P) = n*

In general, we can say that any natural system is state-controllable, as is the case with the converters under consideration.

Now, since we want to control the input duty cycle in accordance with the value of the state variables, we can describe the input matrix as:

$$\boldsymbol{u = -K.x}$$

Here, $\boldsymbol{K}$ is a (1 × n) matrix, whose n elements are to be found using mathematical techniques. Substituting the value of $\boldsymbol{u}$ in the state equation, we get:

$$\boldsymbol{\frac{dx}{dt} = (A - B.K)x = A_{CL}x}$$

$$\boldsymbol{A_{CL} = A - B.K}$$

Objective: The closed loop poles should lie at which are their 'desired locations'. Difference from classical approach: Not only the "dominant poles", but "all poles" are forced to lie at specific desired locations.

The gain matrix is designed in such a way that

$$|\boldsymbol{sI} - (\boldsymbol{A} - \boldsymbol{BK})| = (\boldsymbol{s} - \mu_{\boldsymbol{1}})(\boldsymbol{s} - \mu_{\boldsymbol{2}}) \ldots (\boldsymbol{s} - \mu_{\boldsymbol{n}})$$

where $\mu_{\boldsymbol{1}}, \mu_{\boldsymbol{2}} \ldots \mu_{\boldsymbol{n}}$ are the desired closed-loop locations.

There exist three major ways to solve for the feedback gain:

4.2.1 Direct Method

This method is suitable for low-order systems ($n \leq 3$). From the above equation, simply equate powers on both the sides, taking $\boldsymbol{K = [k_1\ k_2\ k_3]}$. The choice of closed-loop poles should be such that they are at least five to ten times farther away from the origin than the most dominant open-loop poles (eigenvalues).

4.2.2 Bass-Gura Method

This method is used for higher-order systems. The system should first be expressed in first companion (controllable canonical) form. In controllable canonical form, the system eigenvalues are written down in the last row of the A matrix; an example is as follows:

We here consider a system defined by

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_{n-1}\dot{u} + b_n u \,,$$

where u is the control input and y is the output. We can write this equation as

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \,.$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \cdots & (b_1 - a_1 b_0) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$

$$\boldsymbol{A_{CL}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ -a_n & -\,a_{n-1} & & \cdots & -a_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ k_1 & k_2 & & \cdots & k_n \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ -(a_n - k_1) & & & \cdots & -(a_1 - k_n) \end{bmatrix}$$

If $\mu_1, \mu_2 \ldots \mu_n$ are the desired closed-loop locations,

$$(s - \mu_1)(s - \mu_2) \ldots (s - \mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_n$$

This characteristic polynomial will lead to the closed-loop system matrix as:

$$\boldsymbol{A_{CL}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ -\alpha_n & -\alpha_{n-1} & & \cdots & -\alpha_1 \end{bmatrix}$$

Comparing the above two values of the closed-loop system matrix:

$$\begin{bmatrix} (& k_1 = \alpha_n - a_n &) \\ \vdots & \ddots & \vdots \\ (& k_n = \alpha_1 - a_1 &) \end{bmatrix}$$

4.2.3 The Ackermann Method

$$\boldsymbol{K} = [\mathbf{0\ 0\ 0} \dots \mathbf{1}]\boldsymbol{P^{-1}\varphi(A)}$$

- $\boldsymbol{P} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A^2B} \quad \boldsymbol{A^3B} \ \dots \quad \boldsymbol{A^{n-1}B}]$
- $\boldsymbol{\varphi(s)}$ is the characteristic equation for the closed-loop poles, which we then evaluate for $\boldsymbol{s} = \mathbf{A}$
- It is explicit that the system is controllable because we are inverting the controllability matrix

This can be implemented easily in MATLAB using acker(A, B, L);

L is a row matrix containing the desired closed-loop pole locations.

CHAPTER 5 OBSERVER DESIGN

In practice, all the states are not always available for direct measurement due to many reasons, which may include:

- Non-availability of sensors
- Use of sensors is expensive (not cost-effective)
- The sensors are not suitable for use in the given system. For example, the system must be sophisticated and the sensors that are available impart high levels of noise, or may consume excess energy.

For this reason, it is more practical to use a state observer that can estimate the value of a particular state based on the measurement of the output over a period of time.

For this to be possible, the system must be observable, which is true iff the rank of the observability matrix is equal to its dimension, ie, *n*.

Consider the system defined by the following equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}.\boldsymbol{x} + \boldsymbol{B}.\boldsymbol{u}$$

$$\boldsymbol{y} = \boldsymbol{C}.\boldsymbol{x}$$

The observer is a subsystem to reconstruct the state vector of the plant. The mathematical model of the observer is basically the same as that of the plant except that we include an additional term that comprises the estimation error to compensate for the inaccuracies in the matrices A and B and the lack of the initial error. The estimation or the observation error is the difference between the measured and the estimated outputs. The initial error is the difference between the original and the estimated values of the initial state.

$$\begin{aligned}\dot{\widetilde{\boldsymbol{x}}} &= \boldsymbol{A}\widetilde{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{K}_{\boldsymbol{e}}(\boldsymbol{y} - \boldsymbol{C}\widetilde{\boldsymbol{x}}) \\ &= (\boldsymbol{A} - \boldsymbol{K}_{\boldsymbol{e}}\boldsymbol{C})\widetilde{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{K}_{\boldsymbol{e}}\boldsymbol{y}\end{aligned}$$

Minimum-order Observer

When the observer plant estimates all-but-one states of the system, it is called a minimum-order observer.

Since the output voltage (Vc) is always to be available for direct measurement, the other state variables are estimated.

Derivation of minimum-order control law

The state equation is divided into two parts as follows: One part corresponds to the measurable part, the other to the unmeasurable one (observed)

$$\begin{bmatrix} \dot{x}_a \\ \dot{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} A_{aa} & \mathbf{A}_{ab} \\ \mathbf{A}_{ba} & \mathbf{A}_{bb} \end{bmatrix} \begin{bmatrix} x_a \\ \mathbf{x}_b \end{bmatrix} + \begin{bmatrix} B_a \\ \mathbf{B}_b \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_a \\ \mathbf{x}_b \end{bmatrix}$$

The upper part relates the measurable to the unmeasurable variables and makes up for the output equation of the modified system

$$\dot{x}_a = A_{aa} x_a + \mathbf{A}_{ab} \mathbf{x}_b + B_a u$$

$$\dot{x}_a - A_{aa} x_a - B_a u = \mathbf{A}_{ab} \mathbf{x}_b$$

This equation represents the dynamics of the unmeasurable portion, and also is the State equation of the modified system

$$\dot{\mathbf{x}}_b = \mathbf{A}_{ba} x_a + \mathbf{A}_{bb} \mathbf{x}_b + \mathbf{B}_b u$$

The Output equation:

$$\dot{x}_a - A_{aa} x_a - B_a u = \mathbf{A}_{ab} \mathbf{x}_b$$

Comparing with the full-state observer equation (right) to get:

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e \mathbf{C}) \tilde{\mathbf{x}} + \mathbf{B} u + \mathbf{K}_e y$$

$$\dot{\tilde{\mathbf{x}}}_b = (\mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab}) \tilde{\mathbf{x}}_b + \mathbf{A}_{ba} x_a + \mathbf{B}_b u + \mathbf{K}_e (\dot{x}_a - A_{aa} x_a - B_a u)$$

In order to eliminate the measured state (which, if noisy, can cause problems with the derivative block), we introduce a new $\dot{\tilde{\mathbf{x}}}_b - \mathbf{K}_e \dot{x}_a$ variable equal to

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}_b - \mathbf{K}_e \dot{x}_a &= (\mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab})\tilde{\mathbf{x}}_b + (\mathbf{A}_{ba} - \mathbf{K}_e A_{aa})y + (\mathbf{B}_b - \mathbf{K}_e B_a)u \\ &= (\mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab})(\tilde{\mathbf{x}}_b - \mathbf{K}_e y) \\ &\quad + [(\mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab})\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}]y \\ &\quad + (\mathbf{B}_b - \mathbf{K}_e B_a)u\end{aligned}$$

$$\mathbf{x}_b - \mathbf{K}_e y = \mathbf{x}_b - \mathbf{K}_e x_a = \boldsymbol{\eta}$$

$$\tilde{\mathbf{x}}_b - \mathbf{K}_e y = \tilde{\mathbf{x}}_b - \mathbf{K}_e x_a = \tilde{\boldsymbol{\eta}}$$

Full-Order State Observer	Minimum-Order State Observer
$\tilde{\mathbf{x}}$	$\tilde{\mathbf{x}}_b$
$\mathbf{A}$	$\mathbf{A}_{bb}$
$\mathbf{B}u$	$\mathbf{A}_{ba}x_a + \mathbf{B}_b u$
y	$\dot{x}_a - A_{aa}x_a - B_a u$
$\mathbf{C}$	$\mathbf{A}_{ab}$
$\mathbf{K}_e$ $(n \times 1 \text{ matrix})$	$\mathbf{K}_e$ $[(n-1) \times 1 \text{ matrix}]$

The final State equation is as depicted, with the modified matrices as shown:

$$\begin{aligned}\dot{\tilde{\boldsymbol{\eta}}} &= (\mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab})\tilde{\boldsymbol{\eta}} + [(\mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab})\mathbf{K}_e \\ &\quad + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}]y + (\mathbf{B}_b - \mathbf{K}_e B_a)u\end{aligned}$$

$$\begin{aligned}\hat{\mathbf{A}} &= \mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab} \\ \hat{\mathbf{B}} &= \hat{\mathbf{A}}\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa} \\ \hat{\mathbf{F}} &= \mathbf{B}_b - \mathbf{K}_e B_a\end{aligned}$$

The modified State equation:

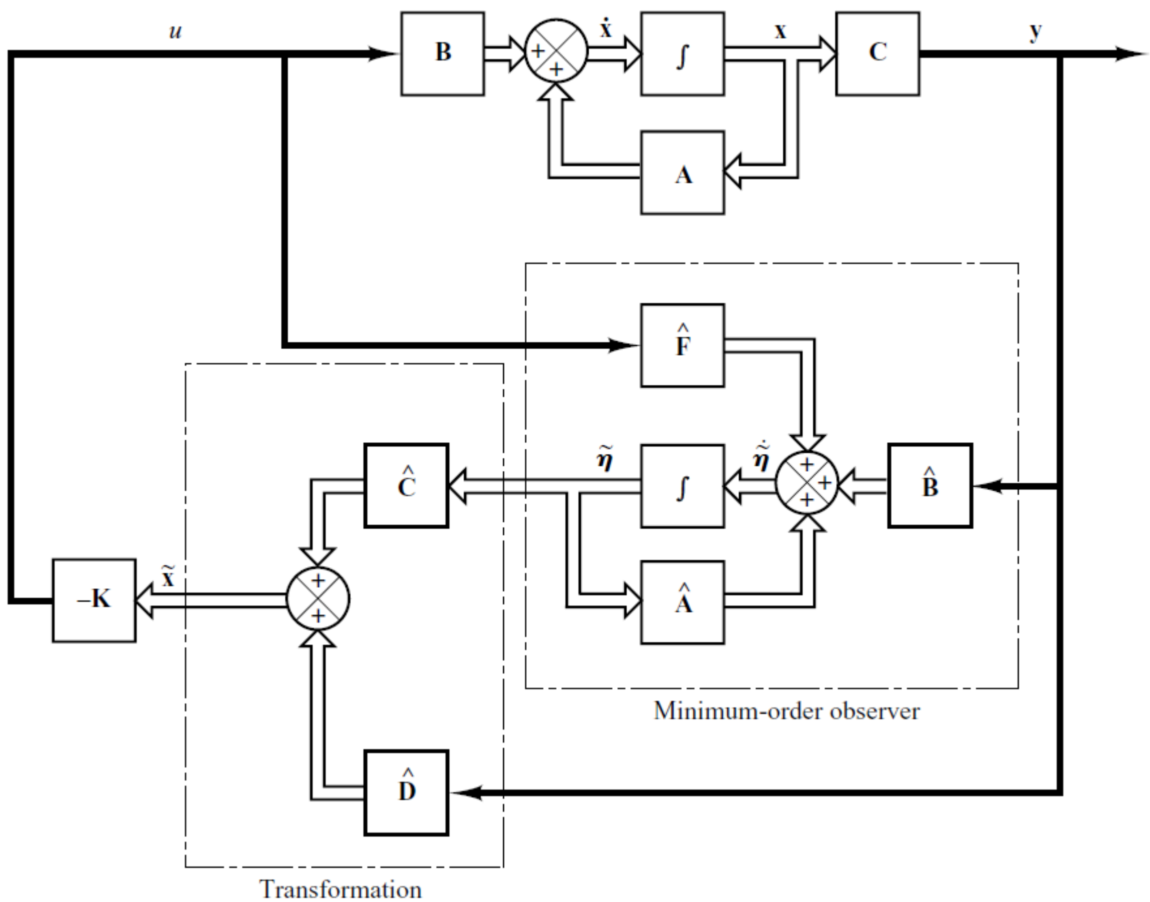
$$\dot{\tilde{\boldsymbol{\eta}}} = \hat{\mathbf{A}}\tilde{\boldsymbol{\eta}} + \hat{\mathbf{B}}y + \hat{\mathbf{F}}u$$

The modified Output equation:

$$y = \begin{bmatrix} 1 & \vdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_a \\ \hdashline \mathbf{x}_b \end{bmatrix}$$

$$\tilde{\mathbf{x}} = \begin{bmatrix} x_a \\ \hdashline \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} y \\ \hdashline \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hdashline \mathbf{I}_{n-1} \end{bmatrix} [\tilde{\mathbf{x}}_b - \mathbf{K}_e y] + \begin{bmatrix} 1 \\ \hdashline \mathbf{K}_e \end{bmatrix} y$$

Block Diagram:



5.1 Block diagram of a minimum-order observer

With the help of this block diagram we will now design minimum-order observer for the DC-DC converters.

CHAPTER 6 APPLICATION OF ADVANCED CONTROL THEORY

6.1 Boost Converter – Pole Placement

$$\boldsymbol{x} = \begin{bmatrix} v_C \\ i_L \end{bmatrix} \qquad \boldsymbol{u} = [d] \qquad \boldsymbol{y} = [v_C]$$

$$\boldsymbol{A} = \begin{bmatrix} -\frac{1}{RC} & \frac{D\prime}{C} \\ -\frac{D\prime}{L} & \frac{R_L}{L} \end{bmatrix} = \begin{bmatrix} -960 & 6000 \\ -12000 & 2000 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} -\frac{i_L}{C} \\ \frac{v_C}{L} \end{bmatrix} = \begin{bmatrix} -40000 \\ 5000000 \end{bmatrix} \qquad \boldsymbol{C} = [1 \;\; 0]$$

$$L = 0.5e\text{-}4\ H; \quad C = 1e\text{-}4\ F; \quad R = 10.42\ \Omega; \quad R_L = 0.1\ \Omega$$

In the above expressions, $D' = 1 - D$, where D is the desired duty cycle given by $1 - \frac{v_i}{v_O}$; $i_L = \frac{v_i}{R(1-D)2}$; R_L is the resistance offered by the inductor.

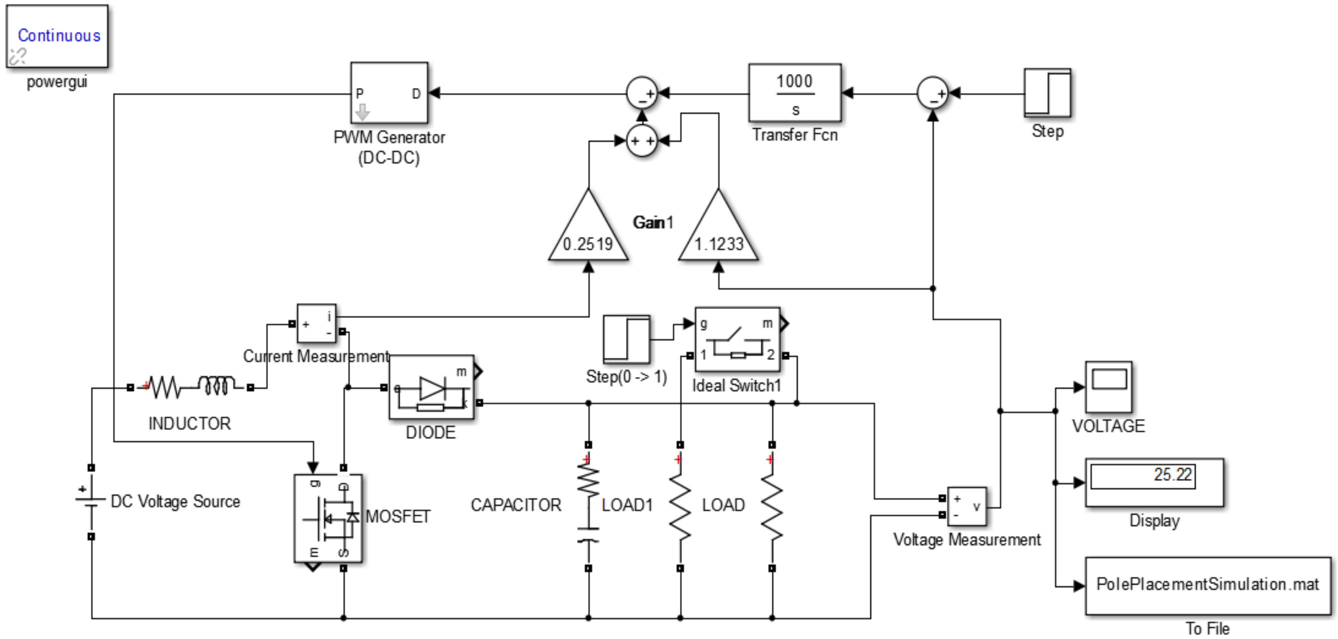
A new behavior, expressed by a new set of poles, must be imposed to the inner plant, with higher natural frequency (bandwidth) and stronger damping.

If these latter are chosen as $\omega n = 6000$ rad and $\zeta n = 0.7$, the imposed closed-loop set of poles turns out to be $-4.2\times10^3 \pm j4.28\times10^3$.

The determinant of the matrix $(sI - A + B.K)$ after taking the Laplace transform on both the sides should give the same equation as the one specified by the desired poles. One can also find the value of $\boldsymbol{K}$ by using the MATLAB function acker(A, B, L); L denotes the location of the desired poles in the form of a matrix.

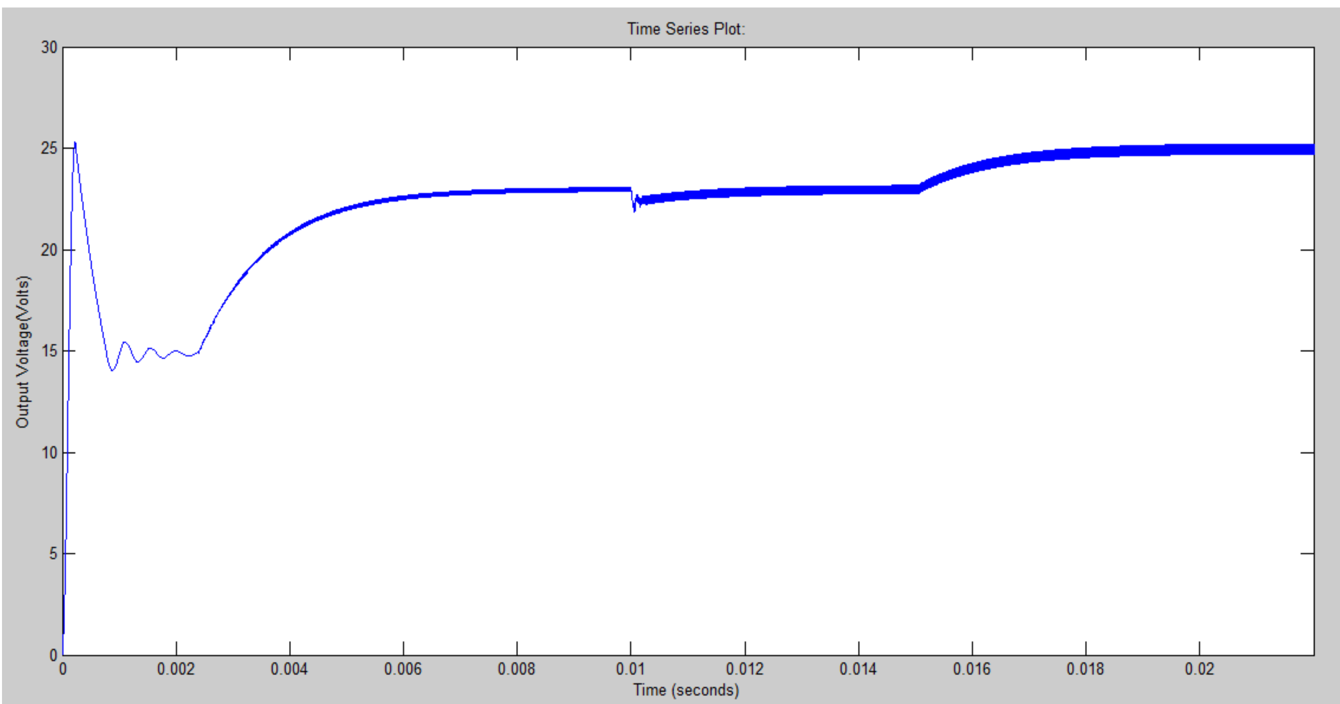
$$\boldsymbol{K} = [1.1233 \;\; 0.2519]$$

Circuit Diagram:



6.1 Circuit diagram of boost converter after pole placement

Output Response:



6.2 Output response of boost converter after pole placement

The reference voltage was stepped up from 23 V to 25 V at t=0.014 s. To check the effect of load, the load was doubled at t=0.01s. The response settled within 0.008 s with minimum error.

6.2 Boost Converter – Minimum-Order Pole Placement

$\boldsymbol{K_e}$ = [0.0833] for the desired pole location of the observer at [0.5e4] for the same $\boldsymbol{K}$ matrix as before. The Ackerman method is applied for $A_{aa}, A_{ab}, A_{ba}, A_{bb}, B_a, B_b, and\ L_o$, where

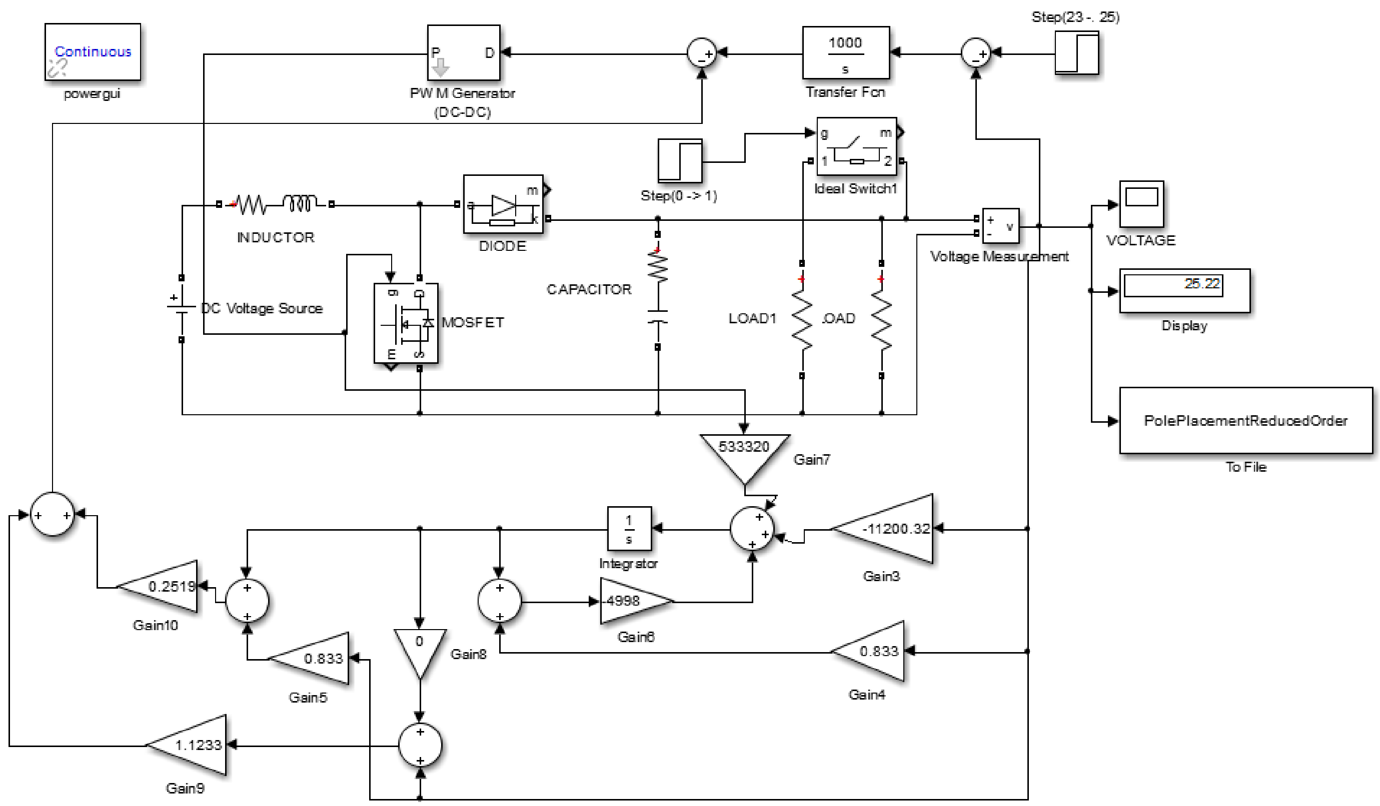
$$\boldsymbol{A} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix}; \qquad \boldsymbol{B} = \begin{bmatrix} B_a \\ B_b \end{bmatrix}$$

$$A_{aa} = -960 \quad A_{ab} = 6000 \quad A_{ba} = -12000 \quad A_{bb} = 2000$$

$$B_a = -40000 \quad B_b = 500000 \quad L_o = 0.5e4$$

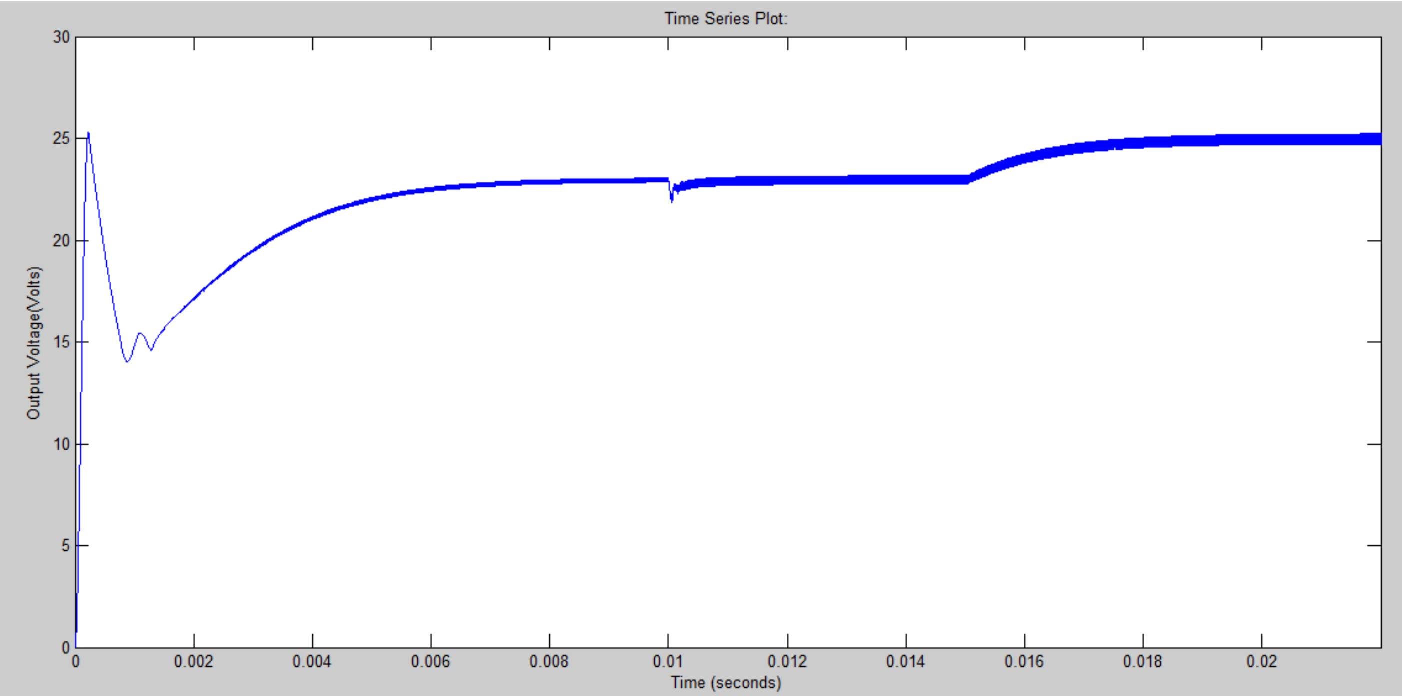
The MATLAB function was used as: acker(Abb', Aab', Lo)' which gave the value of observer gain equal to 0.0833. The following figure depicts the simulation.

Circuit Diagram:



6.3 Circuit diagram of boost converter controlled using minimum-order observer

Output Response:



6.4 Output response of boost converter controlled using minimum-order observer

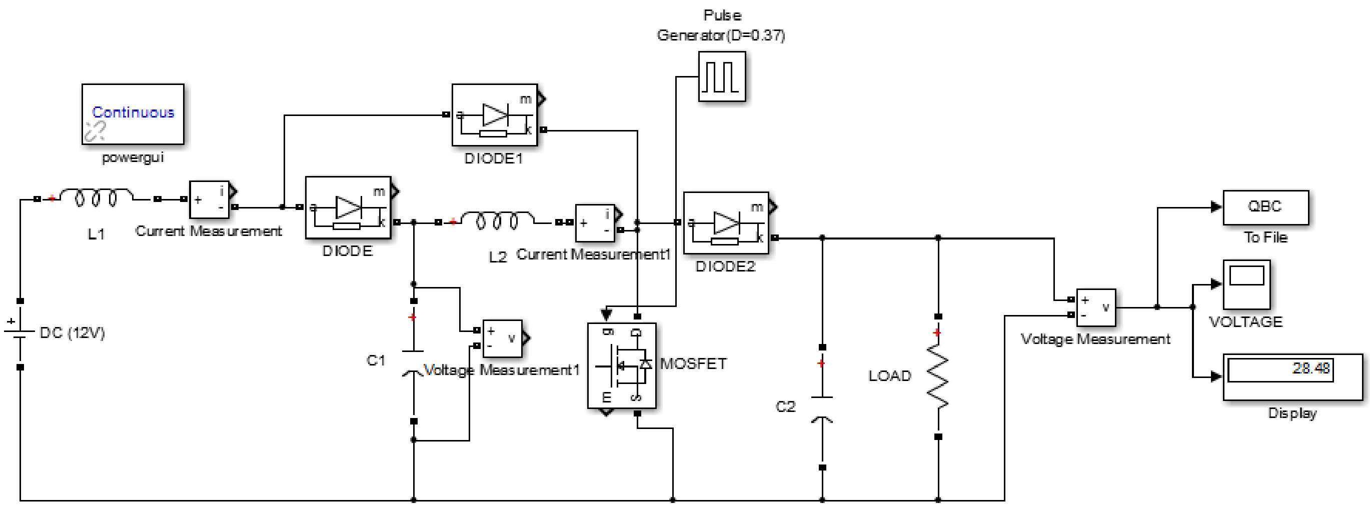
The reference voltage was stepped up from 23 V to 25 V at t=0.014 s. To check the effect of load, the load was doubled at t=0.01s. The response settled within 0.008 s with minimum error.

6.3 Quadratic Boost Converter – Pole Placement

1. **Open Loop converter: -**

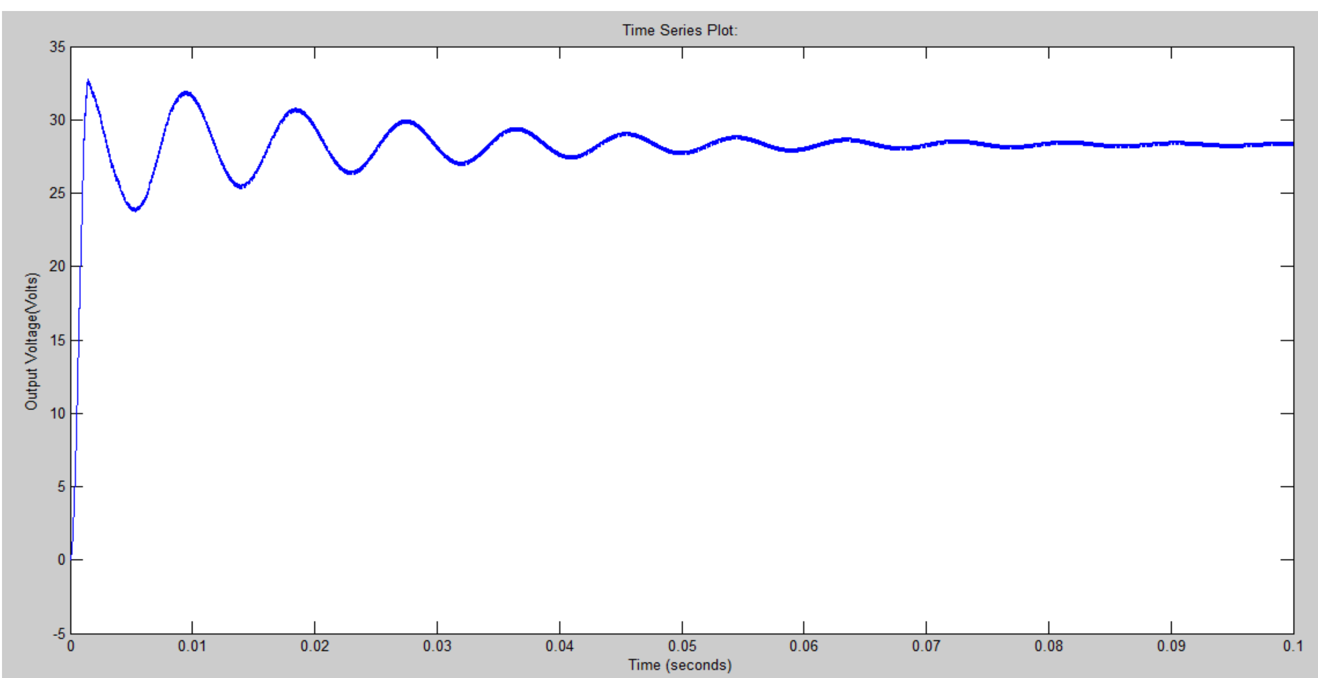
System Parameters: Vin = 12 V, L1 = 0.1 mH, L2 = 0.56 mH, C1 = 22 uF, C2 = 100 uF, Rl1 = 0.047 ohm, Rl2 = 0.087 ohm, Rc1 = 0.063 ohm, Rc2 = 0.2 ohm

Circuit Diagram:



6.5 Circuit diagram of open loop quadratic boost converter

Output Response:

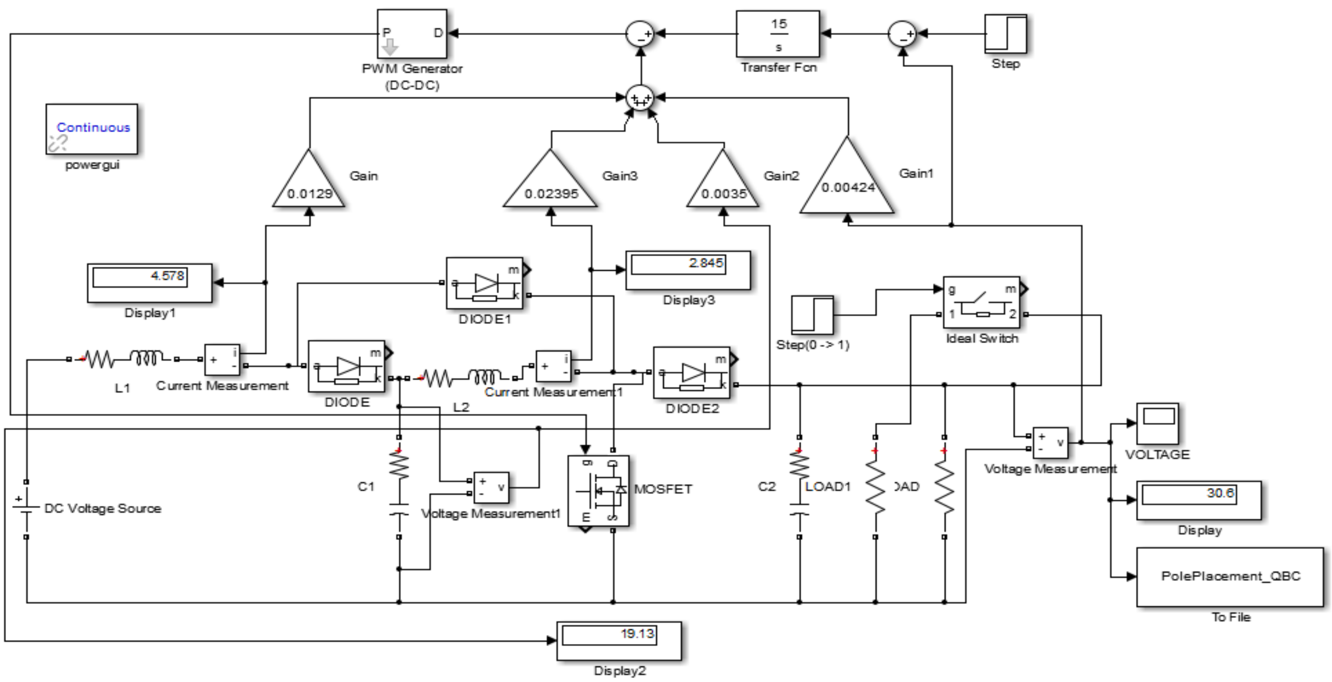


6.6 Output response of open loop quadratic boost converter

The duty cycle was set at D=0.37, so the output voltage should be 30V. The response settled within 0.08s with significant steady state error.

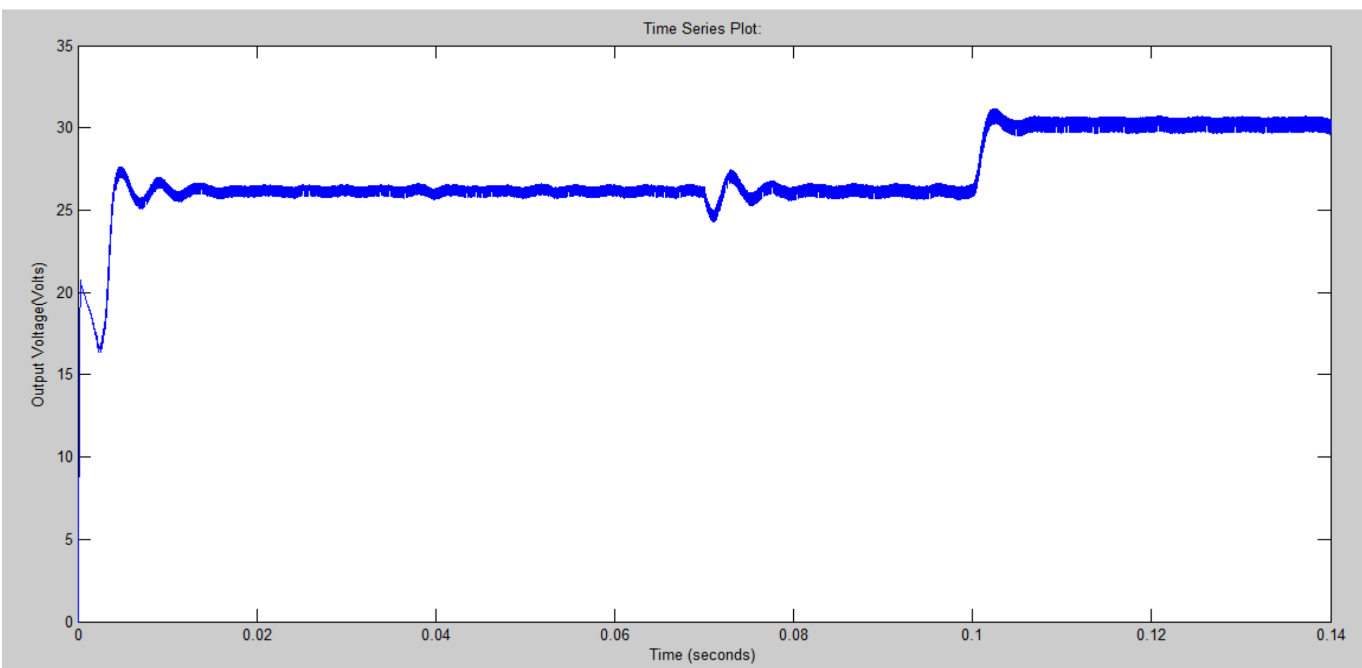
2. After Pole Placement

Circuit Diagram:



6.7 Circuit diagram of closed loop quadratic boost converter after pole placement

Output Response:



6.8 Output response of closed loop quadratic boost converter after pole placement

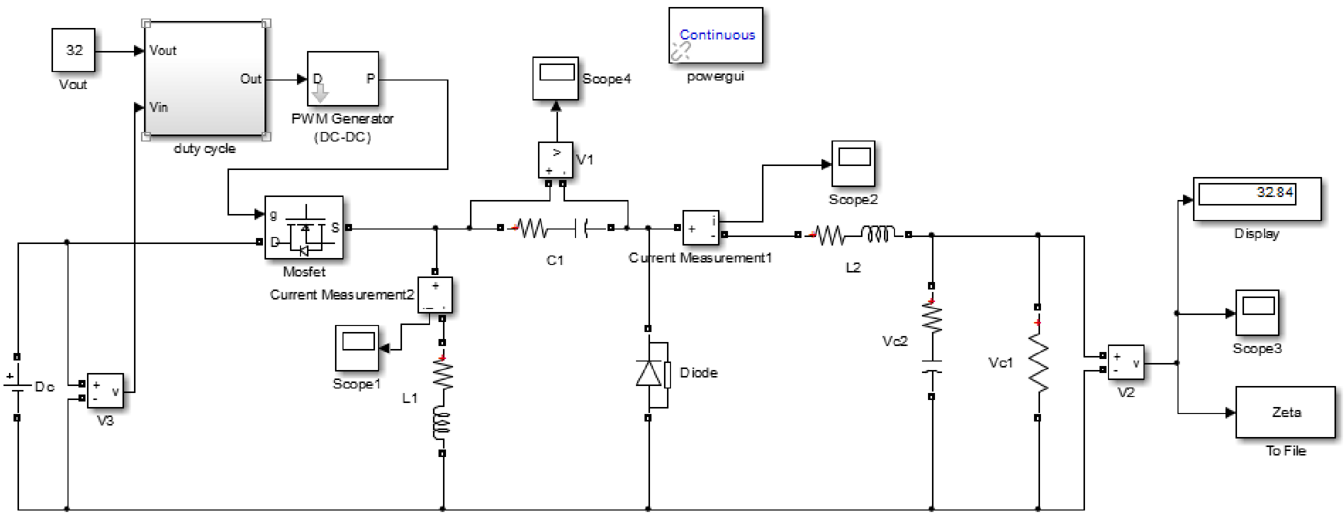
The reference voltage was stepped up from 25 V to 30 V at t=0.1 s. To check the effect of load, the load was doubled at t=0.7s. The response settled within 0.02 s with minimum error

6.4 SEPIC – Pole Placement

1. **Open Loop converter: -**

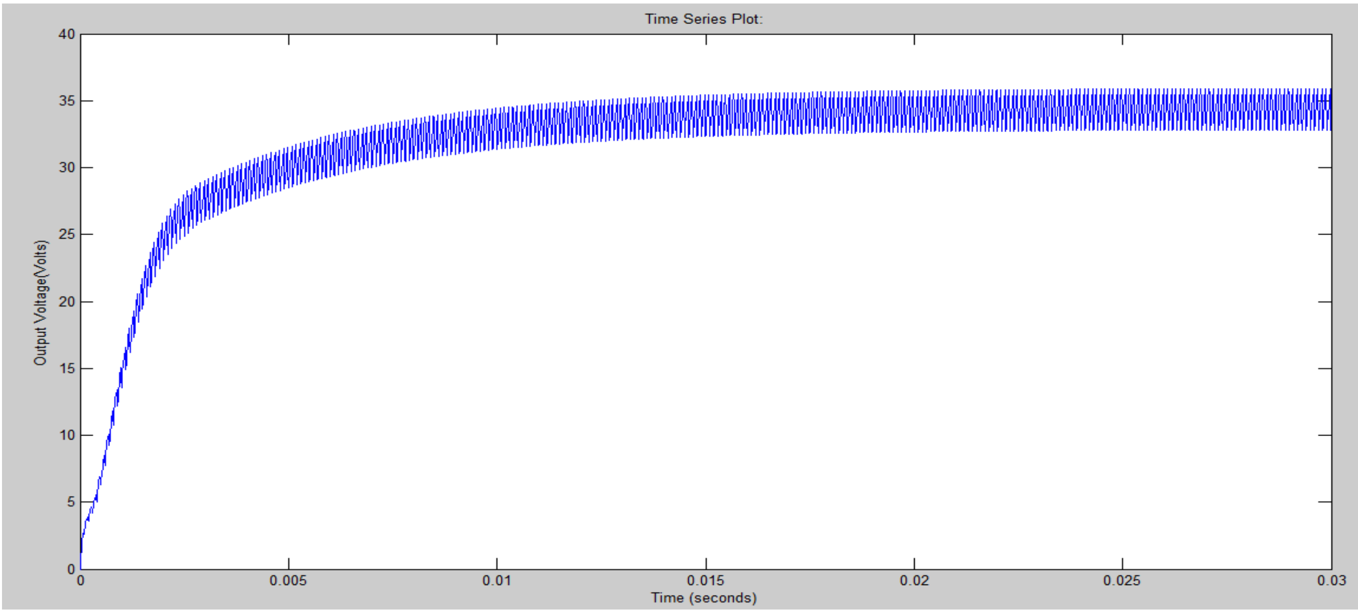
   System Parameters: Vin = 9 V, C1 = 100 uF, C2 = 220 uF, L1 = 100 uH, L2 = 68 uH, Rl1 = 0.034 ohm, Rl2 = 0.028 ohm, Rc1 = 0.8 ohm, Rc2 = 0.35ohm

Circuit Diagram:



6.9 Circuit diagram of open loop SEPIC converter

Output Response:

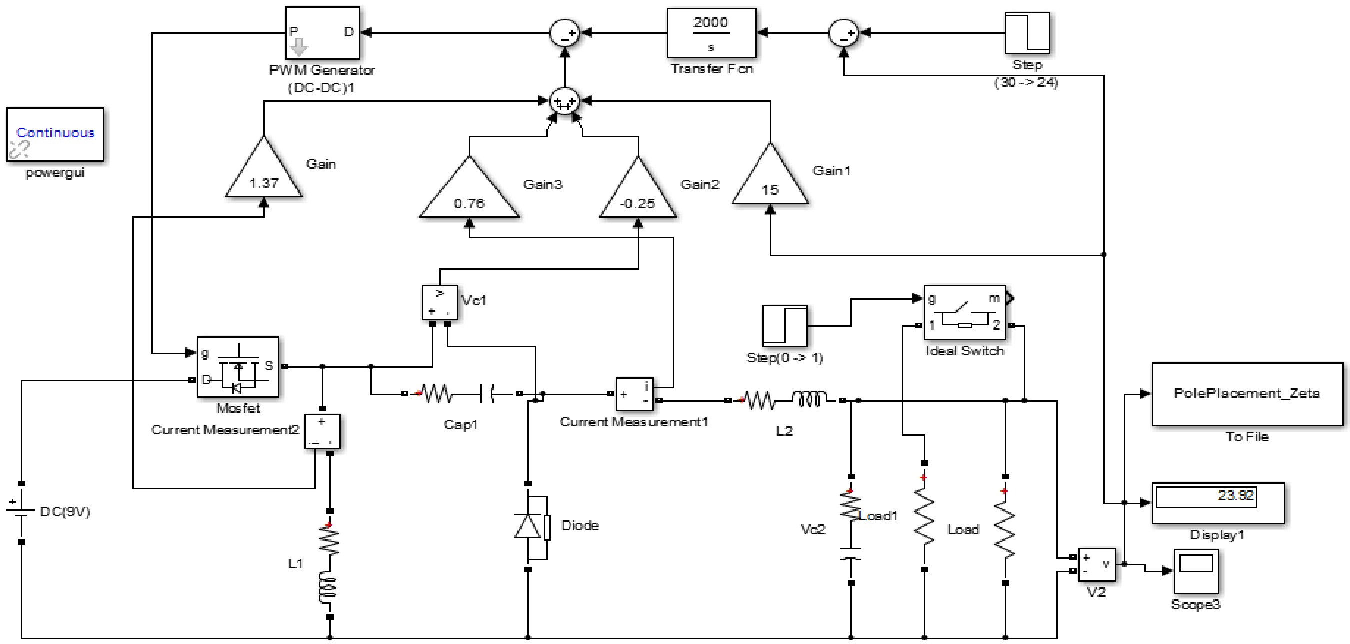


6.10 Output response of open loop SEPIC converter

The reference voltage is set to be 32V, so the steady state output voltage should be 32V. The response shows significant steady state error ripple at output.

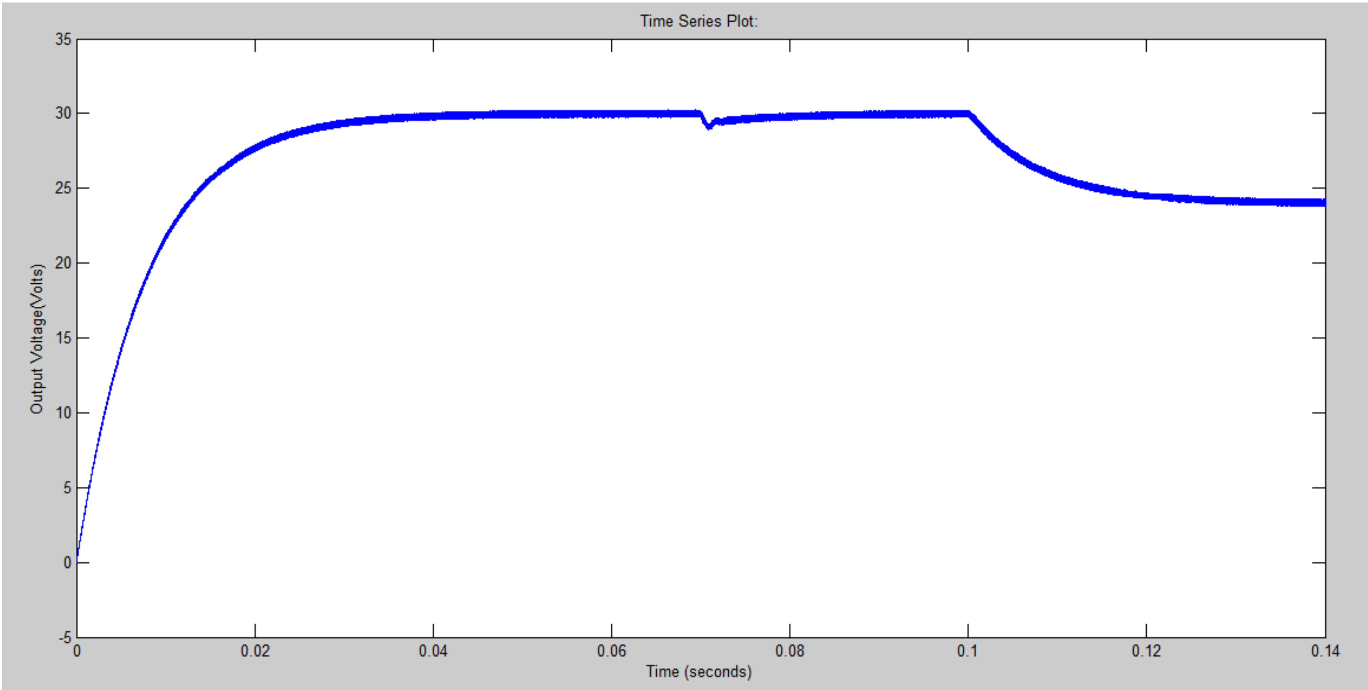
2. After Pole Placement

Circuit Diagram:



6.11 Circuit diagram of closed loop SEPIC converter after pole placement

Output Response:



6.12 Output response of closed loop SEPIC converter after pole placement

The reference voltage was stepped down from 30 V to 25 V at t=0.1 s. To check the effect of load, the load was doubled at t=0.07s. The response settled within 0.04s with minimum error

CHAPTER 7 DISCUSSION

The idea of state-space comes from the state-variable method of describing a dynamic system are organized as a set of first-order differential equations in the vector-valued state of the system, and the solution is visualized as a trajectory of this state vector in space. State-space control design is the technique in which the control engineer designs a dynamic compensation by working directly with the state-variable description of the system. We know that the ordinary differential equations (ODEs) of physical dynamic systems can be manipulated into state-variable form. In the field of Normal form mathematics, where ODEs are studied, the state-variable form is called the normal form for the equations. Studying equations in this form is advantageous for several reasons:

- *To study more general models:* The ODEs do not have to be linear or stationary. Thus, by studying the equations themselves, we can develop methods that are very general. Having them in state-variable form gives us a compact, standard form for study. Furthermore, the techniques of state-space analysis and design easily extend to systems with multiple inputs and/or multiple outputs.
- *To introduce the ideas of geometry into differential equations:* In physics the Phase plane of position versus velocity of a particle or rigid body is called the phase plane, and the trajectory of the motion can be plotted as a curve in this plane. The state is a generalization of that idea to include more than two dimensions. While we cannot plot more than three dimensions, the concepts of distance, of orthogonal and parallel lines, and other concepts from geometry can be useful in visualizing the solution of an ODE as a path in state-space.
- *To connect internal and external descriptions:* The state of a dynamic system often directly describes the distribution of internal energy in the system. The internal energy can always be computed from the state-variables. We can relate the state to the system inputs and outputs and thus connect the internal variables to the external inputs and to the sensed outputs. In contrast, the transfer function relates only the input to the output and does not show the internal behavior. The state form keeps the latter information, which is sometimes important.

The advantages of using modern control theory, hence, can be listed as follows:

1. Convenient tool for MIMO systems
2. Uniform platform for representing time-invariant systems, time-varying systems, linear systems as well as nonlinear systems

3. Can describe the dynamics in almost all systems (mechanical systems, electrical systems, biological systems, economic systems, social systems etc.)
4. It can be performed with non-zero initial conditions.
5. Variables used to represent system can be any variables in the system.
6. Using this analysis, the internal states of the system at any time instant can be predicted.
7. As the method involves matrix algebra, can be conveniently adopted for digital computers.
8. The system can be made cost-effective by using an observer.

CHAPTER 8 CONCLUSION AND SCOPE FOR FUTURE WORK

Having dealt with the applications of the modern control theory in depth, we arrived at the following conclusion. This is in addition to the advantages already listed in last chapter:

- The state-space provides a powerful tool to deal with control feedback compensation
- The major advantage of the modern control theory over its classical counterpart is that the former can be conveniently adopted for digital computers because of the involvement of matrix algebra
- Further research into the topic will help in building more robust, efficient and efficacious control mechanisms for various applications

The future work on this topic should comprise research on applying state-space analysis for even higher order systems using observers that can drastically reduce the costs gone into sensors. The modeling is going to be challenging once it is taken up for implementing on hardware. Even further in to the future, the hardware part can be optimized into becoming smaller and more suitable for use in common gadgets and appliances. By devising the most efficient control systems, it can be hoped to improve the cost-effectiveness of control mechanisms ranging from handheld devices and simulations to automobiles industry-level controllers.

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